Multicarrier Modulation

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Contents

- Concept of multicarrier modulation
  - Data transmission over multiple carriers
  - Multicarrier modulation with overlapping subchannels
- Mitigation of subcarrier fading

Chap. 12

- Discrete implementation
- Orthogonal frequency division multiplexing
- Challenges in Multicarrier Systems (Chap. 12)
- Multi-user

Data Transmission Using Multiple Carriers (1)

- The simplest form of multicarrier modulation
  - divides the data stream into multiple substreams to be transmitted over different orthogonal subchannels centered at different frequencies
- Consider a system with data rate R and bandwidth B
  - If coherence bandwidth (B_c) is assumed to be $B_c < B$, the signal experiences frequency-selective fading => The effect of ISI cannot be negligible
  - The wideband system is broken into N subsystems in parallel
    - Each with subchannel bandwidth $B_N = B / N$ and data rate $R_N = R / N$
    - For N sufficiently large, $B_N << B_c$,
      - relatively flat fading on each subchannel
      - The symbol time on each subchannel is much greater than the delay spread of the subchannel => experiences little ISI

Data Transmission Using Multiple Carriers (2)

- Multicarrier transmitter
  - The ith substream is modulated via QAM or PSK with the subcarrier frequency f_i
  - The modulated signals associated with all the subcarriers are summed together

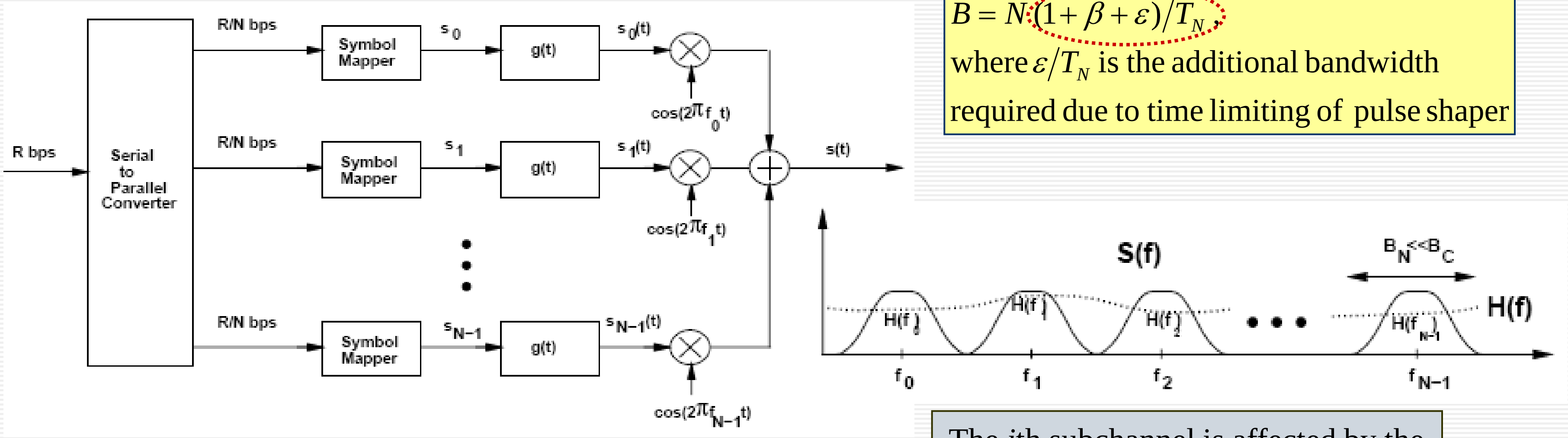
$$s(t) = \sum_{i=0}^{N-1} s_i g(t) \cos(2\pi f_i t + \phi_i)$$

where s_i is the complex symbol, ϕ_i is phase offset of the ith carrier, and $g(t)$ is raised cosine pulses with the rolloff factor β.

  - This system does not change the data rate or signal bandwidth relative to the original system but eliminates ISI for $B_N << B_c$

B_N

$B = N\underbrace{(1+\beta+\varepsilon)/T_N}_{B_N}$,
where ε/T_N is the additional bandwidth required due to time limiting of pulse shaper



The ith subchannel is affected by the flat fading corresponding to a channel gain $H(f_i)$

Multicarrier Modulation with Overlapping Subchannels

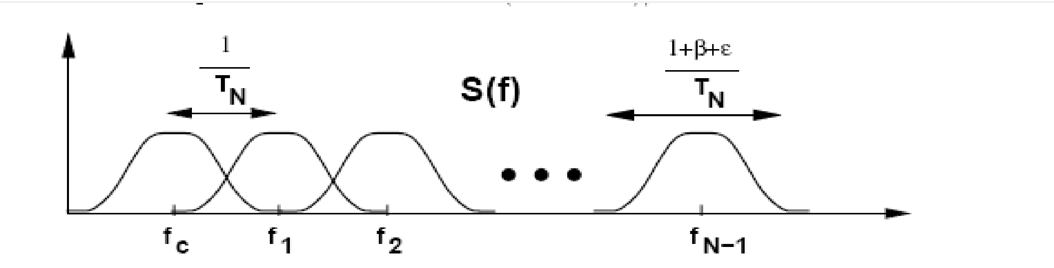
- The spectral efficiency of multicarrier modulation by overlapping the subchannel
  - The subcarrier must still be orthogonal so that they can be separated out by the demodulator in the receiver
- The subcarrier set: $\{\cos(2\pi(f_0 + i/T_N)t + \phi_i), i = 0, 1, 2, \cdots\}$
  - Orthonomal basis set:

$$\int_0^{T_N} \cos(2\pi(f_0 + i/T_N)t + \phi_1)\cos(2\pi(f_0 + j/T_N)t + \phi_2)dt = 0 \quad \text{if } i \neq j$$

  - Even if the subchannels overlap, the modulated signal transmitted in each subchannel can be separated out in the receiver
  - The total system bandwidth

$$B = \frac{N + \beta + \varepsilon}{T_N} \approx \frac{N}{T_N} \quad \text{for a large } N$$

  - Overlapping subcarriers



Auxiliary Material

$$\frac{1}{T_N}\int_0^{T_N} \cos\left(2\pi(f_0+i/T_N)t+\phi_i\right)\cos\left(2\pi(f_0+j/T_N)t+\phi_j\right)dt$$

$$=\frac{1}{2T_N}\int_0^{T_N}\cos\left(2\pi\frac{(i-j)t}{T_N}+\phi_i-\phi_j\right)dt$$

$$+\frac{1}{2T_N}\int_0^{T_N}\cos\left(2\pi\left(2f_0+\frac{i+j}{T_N}\right)t+\phi_i+\phi_j\right)dt$$

$$\approx\frac{1}{2T_N}\int_0^{T_N}\cos\left(2\pi\frac{(i-j)t}{T_N}+\phi_i-\phi_j\right)dt$$

$$=0.5\,\delta(i-j)$$

$$\hat{s}_i=\int_0^{T_N}\left(\sum_{j=0}^{N-1}s_j g(t)\cos\left(2\pi f_j t+\phi_j\right)\right)g(t)\cos\left(2\pi f_i t+\phi_i\right)dt$$

$$=\sum_{j=0}^{N-1}s_j\int_0^{T_N}g^2(t)\cos\left(2\pi(f_0+j/T_N)t+\phi_j\right)\cos\left(2\pi(f_0+i/T_N)t+\phi_i\right)dt$$

$$=\sum_{j=0}^{N-1}s_j\,\delta(j-i)=s_i$$

Mitigation of Subcarrier Fading (1)

- Coding with interleaving over time and frequency
  - encode data bits into codeword, interleave coded bits over both time and frequency, and then transmit the coded bits over different subcarriers
  - The coded bits within a given codeword experience independent fading
- Precoding
  - The transmitter has to have knowledge of the subchannel flat fading gain through channel estimation
  - If the desired received signal power in the *i*th subchannel is P_i and the flat fading gain in the *i*th subchannel is α_i , the transmitted power in the *i*th subchannel under precoding is P_i / α_i^2
  - The received signal power: $\alpha_i^2 \, P_i / \alpha_i^2 = P_i$
  - quite common on wireline multicarrier systems like HDSL
  - Two main problem with precoding in a wireless setting
    - power inefficient in fading channel because precoding is basically channel inversion.
    - The need for accurate estimates at the transmitter (it is very difficult to obtain)

Mitigation of Subcarrier Fading (2)

- Adaptive Loading
  - Based on the adaptive modulation techniques
    - It is to vary the data rate and power of each subchannel relative to its gain
    - The channel estimation at transmitter is required
    - It is commonly used on slowly changing channel like digital subscriber line, where the channel estimation at transmitter can be obtained fairly easily.
  - Consider the capacity of multicarrier system with *N* independent subchannel of bandwidth B_N and gain $\{\alpha_i, i = 0, \cdots N-1\}$ with total power constraint *P*
    - Total capacity: $C = \max_{P_i : \sum P_i = P} \sum_{i=0}^{N-1} B_N \log_2\left(1 + \frac{\alpha_i^2 P_i}{N_0 B_N}\right)$

Multicarrier Modulation (1)

- Multicarrier modulation, which is invented in the 1950s, can be used widely by the development of the discrete Fourier transform and inverse DFT
- Multicarrier Modulation

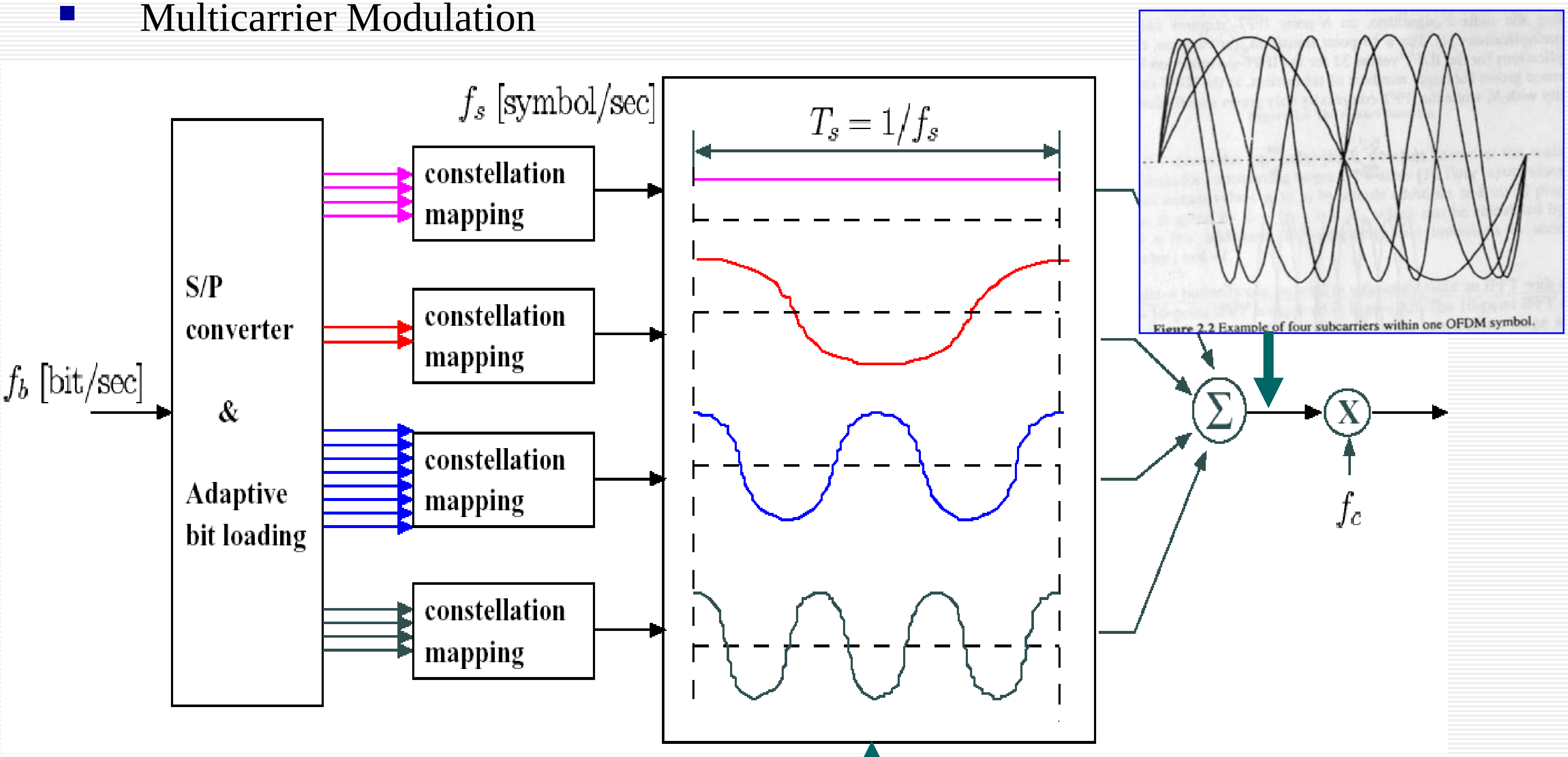
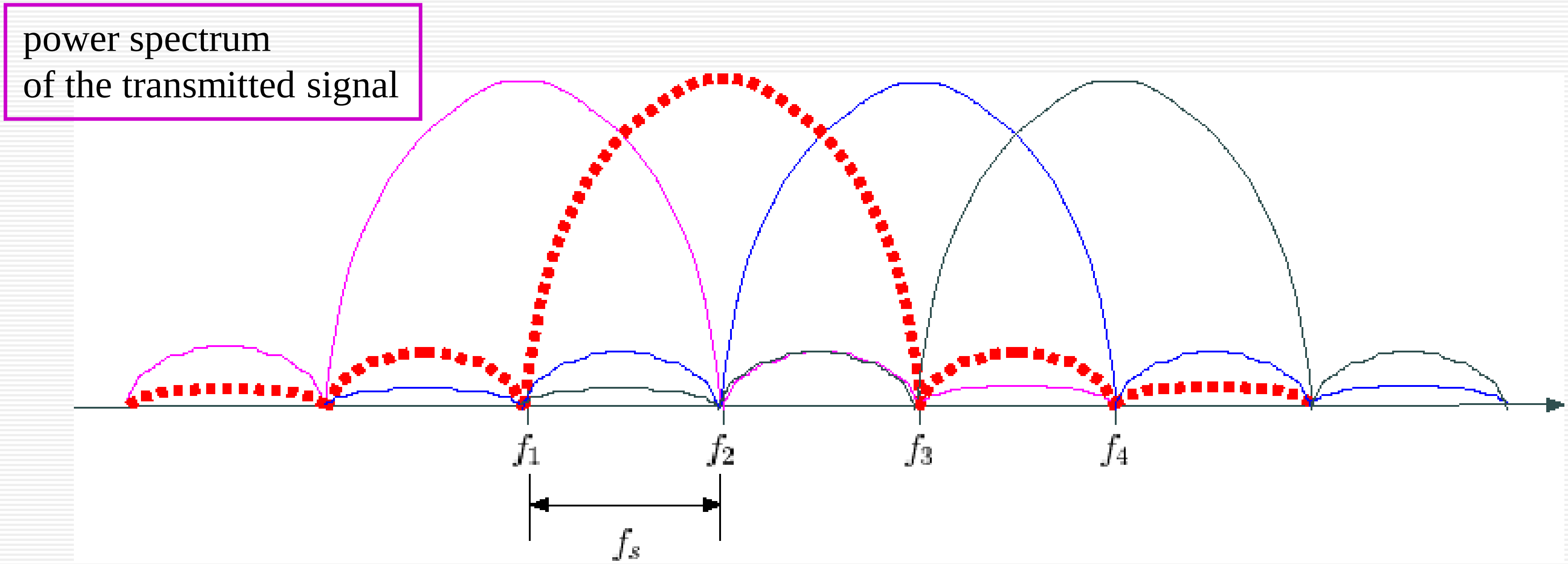


Figure 2.2 Example of four subcarriers within one OFDM symbol.

each waveform is orthogonal with each other for a symbol interval (T_s)

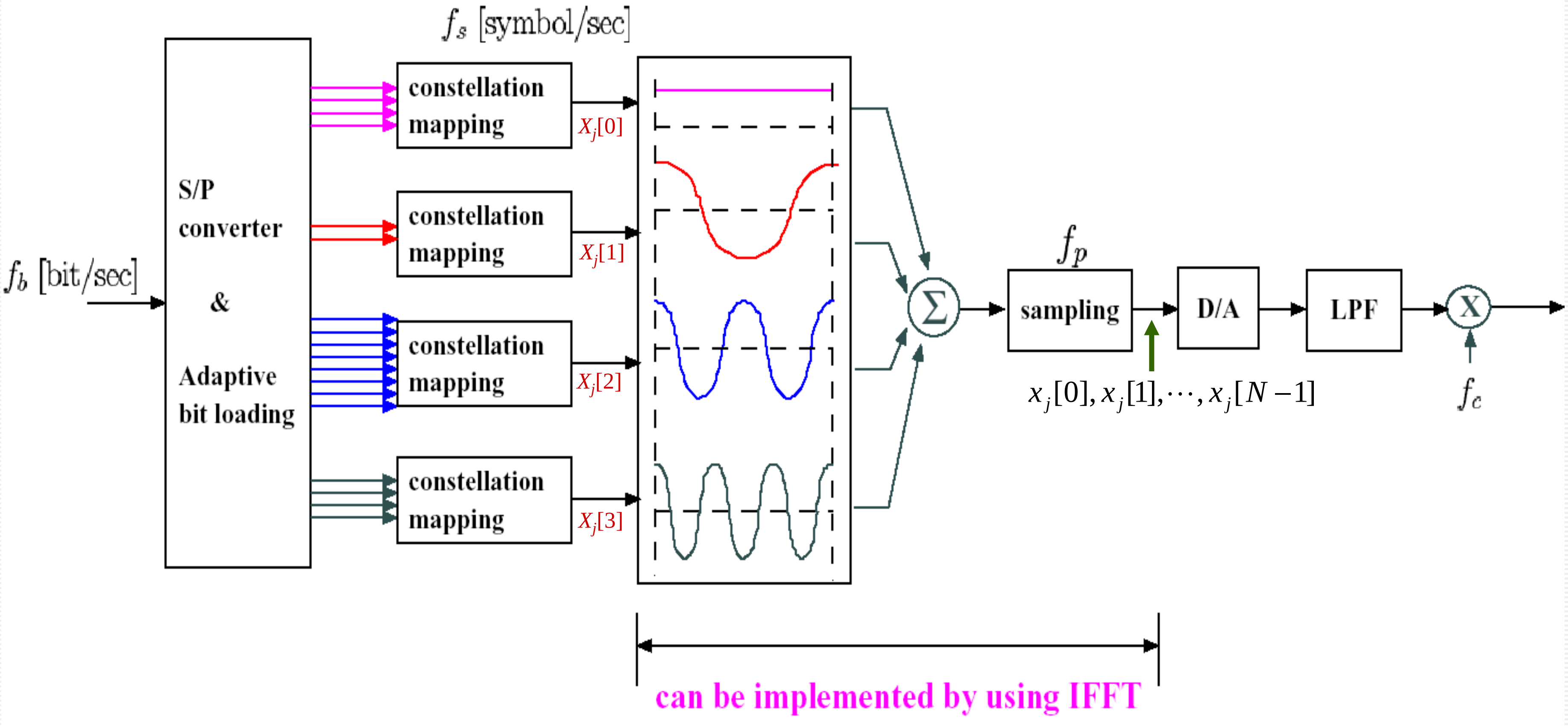
Multicarrier Modulation (2)


power spectrum
of the transmitted signal

- ⇨ Signal spectrum on each subcarrier has sinc function for a rectangular pulse shape
- ⇨ Signal on each subcarrier can be detected by using baseband processing (not bandpass filtering)
- ⇨ Implementation via FFT/IFFT

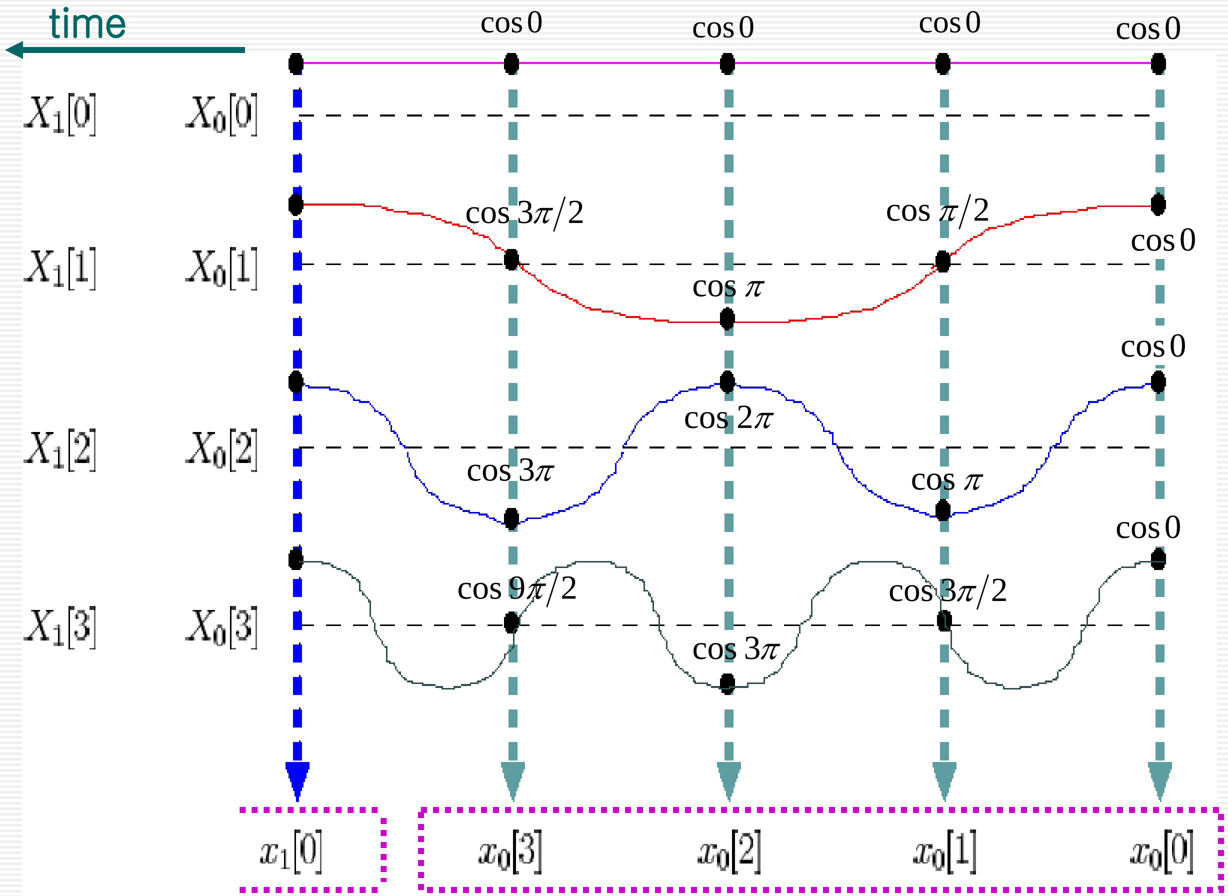
Discrete Implementation of Multicarrier Modulation (1)

- Implementation



Discrete Implementation of Multicarrier Modulation (3)

- $N=4$



1-th OFDM symbol

0-th OFDM symbol

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] e^{j\frac{2\pi i}{N} n}$$

Auxiliary Material

$$x[0] = \frac{1}{\sqrt{4}}\left\{X[0] + X[1] + X[2] + X[3]\right\}$$

$$x[1] = \frac{1}{\sqrt{4}}\left\{X[0] + X[1]e^{j2\pi/4} + X[2]e^{j4\pi/4} + X[3]e^{j6\pi/4}\right\}$$

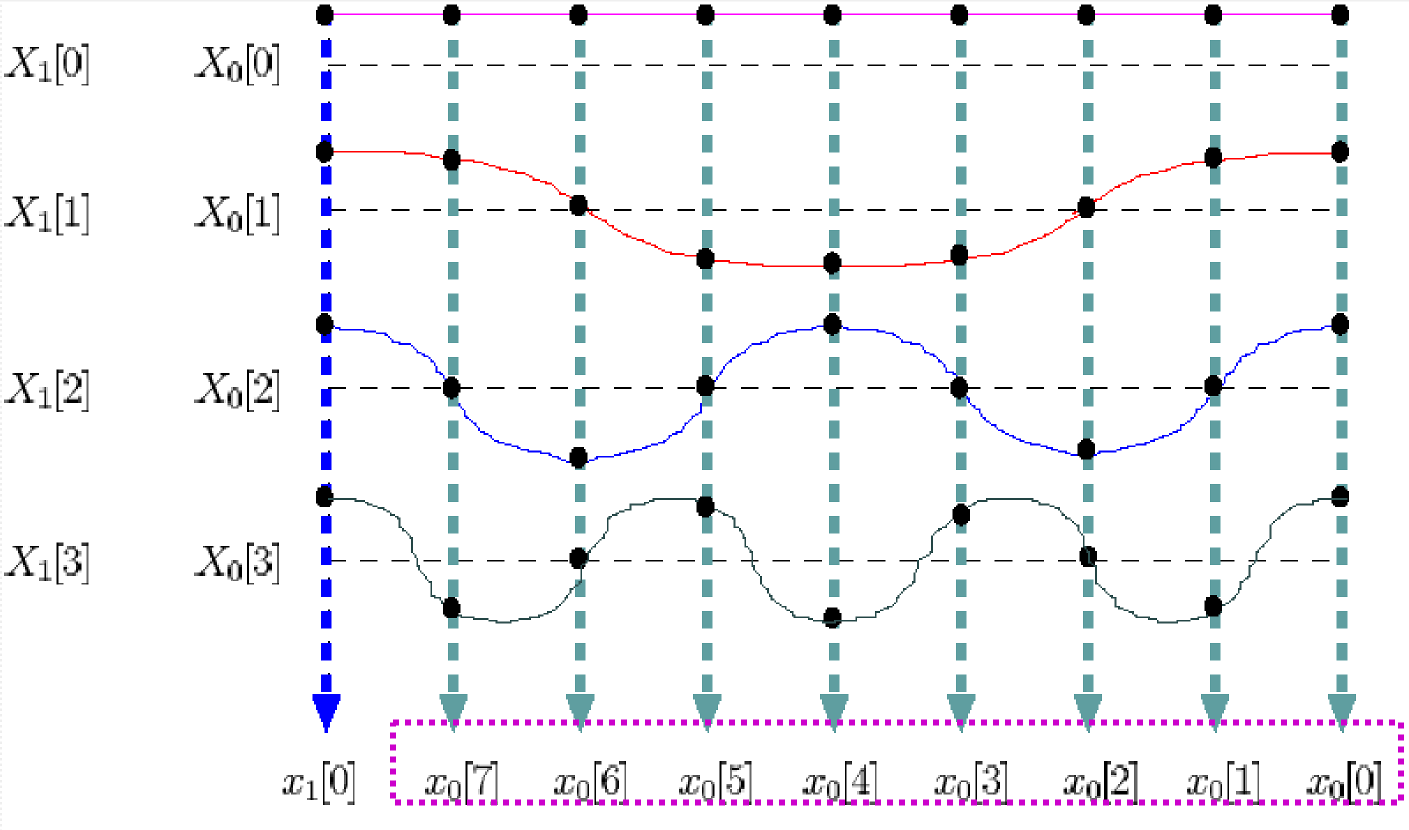
$$x[2] = \frac{1}{\sqrt{4}}\left\{X[0] + X[1]e^{j4\pi/4} + X[2]e^{j8\pi/4} + X[3]e^{j12\pi/4}\right\}$$

$$x[3] = \frac{1}{\sqrt{4}}\left\{X[0] + X[1]e^{j6\pi/4} + X[2]e^{j12\pi/4} + X[3]e^{j18\pi/4}\right\}$$

$$x[n] = \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1} X[i]e^{j\frac{2\pi i}{N}n}$$

Discrete Implementation of Multicarrier Modulation (4)

- N=8



⇨ N값이 증가하면 D/A의 입력 frequency가 증가한다.

⇨ N값은 sub-carrier의 개수 (M)와는 무관하며, 단지 N >= M 이면 된다.

Discrete Implementation of Multicarrier Modulation (5)

◆ N-point DFT

$$x[n], \quad n = 0, \cdots, N-1 \quad \underset{\text{IDFT}}{\overset{\text{DFT}}{\rightleftarrows}} \quad X[k], \quad k = 0, \cdots, M-1$$

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] \times \boxed{e^{j(2\pi i/N)n}} \qquad X[i] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] \times \boxed{e^{-j(2\pi i/N)n}}$$

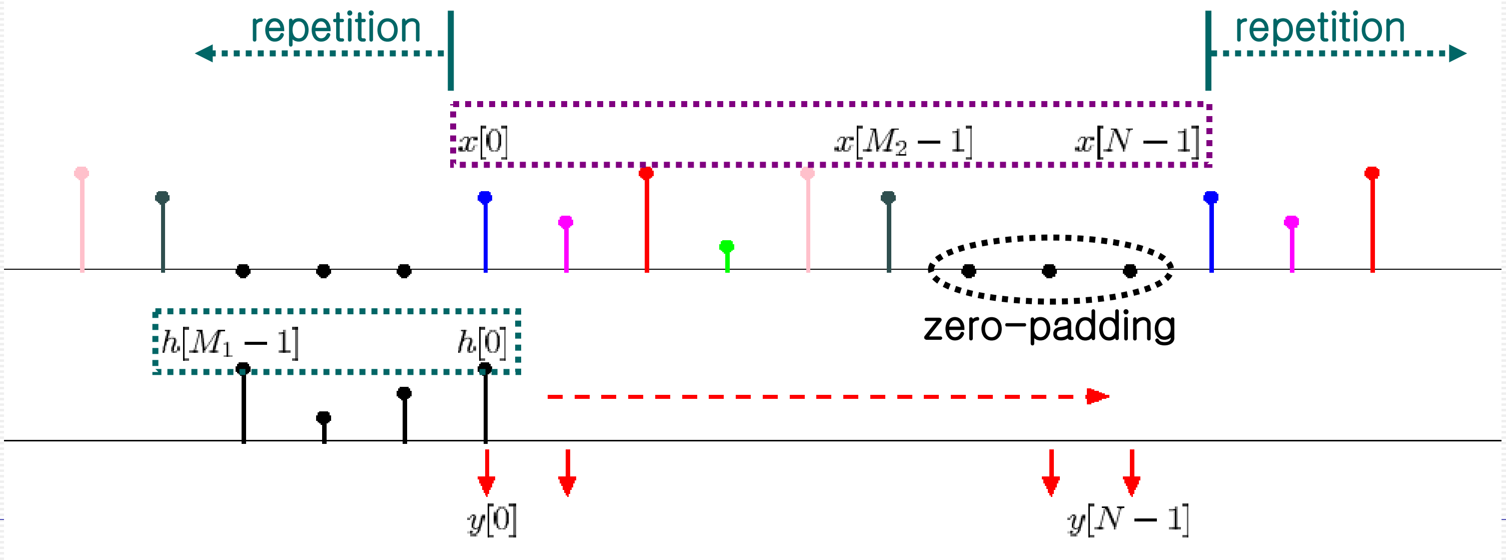
⇨ "N = power of two" : FFT/IFFT (fast processing)

Discrete Implementation of Multicarrier Modulation (6)

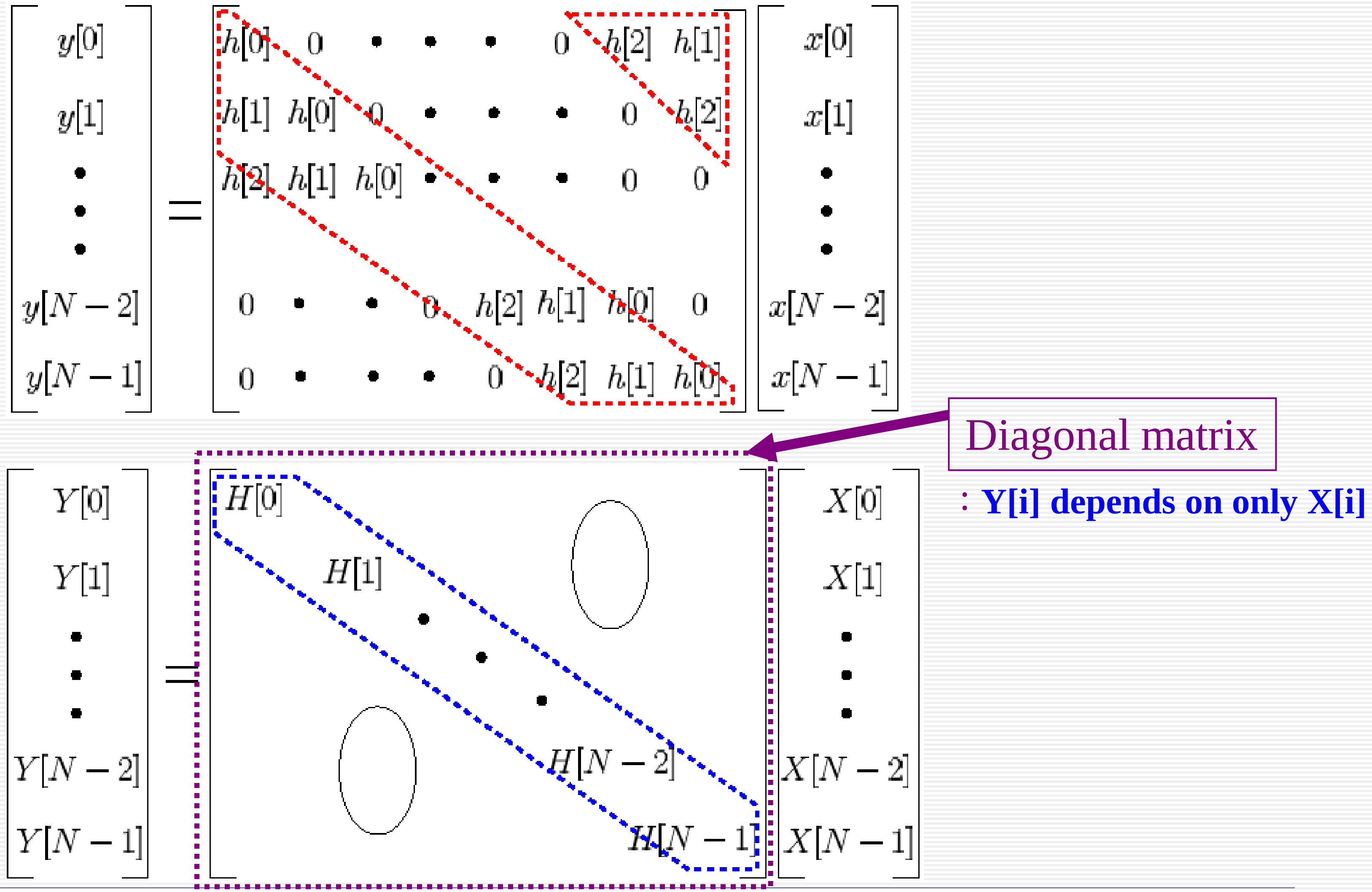
◆ Circular convolution

⇨
- h[n] with duration M1,   x[n] with duration M2   (N >= max(M1,M2))
- DFT of N-point circular convolution of h[n] and x[n] is equal to the multiplication of DFT(h[n]) and DFT(x[n])

$$y[n] = h[n] \circledN x[n] \quad (n = 0, \cdots, N-1) \quad \underset{\text{IDFT}}{\overset{\text{DFT}}{\rightleftarrows}} \quad Y[k] = H[k]X[k] \quad (k = 0, \cdots, N-1)$$



Discrete Implementation of Multicarrier Modulation (7)



$$\begin{bmatrix} y[0] \\ y[1] \\ \bullet \\ \bullet \\ \bullet \\ y[N-2] \\ y[N-1] \end{bmatrix} = \begin{bmatrix} h[0] & 0 & \bullet & \bullet & \bullet & 0 & h[2] & h[1] \\ h[1] & h[0] & 0 & \bullet & \bullet & \bullet & 0 & h[2] \\ h[2] & h[1] & h[0] & \bullet & \bullet & \bullet & 0 & 0 \\ & & & & & & & \\ 0 & \bullet & \bullet & 0 & h[2] & h[1] & h[0] & 0 \\ 0 & \bullet & \bullet & \bullet & 0 & h[2] & h[1] & h[0] \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \bullet \\ \bullet \\ \bullet \\ x[N-2] \\ x[N-1] \end{bmatrix}$$

Diagonal matrix

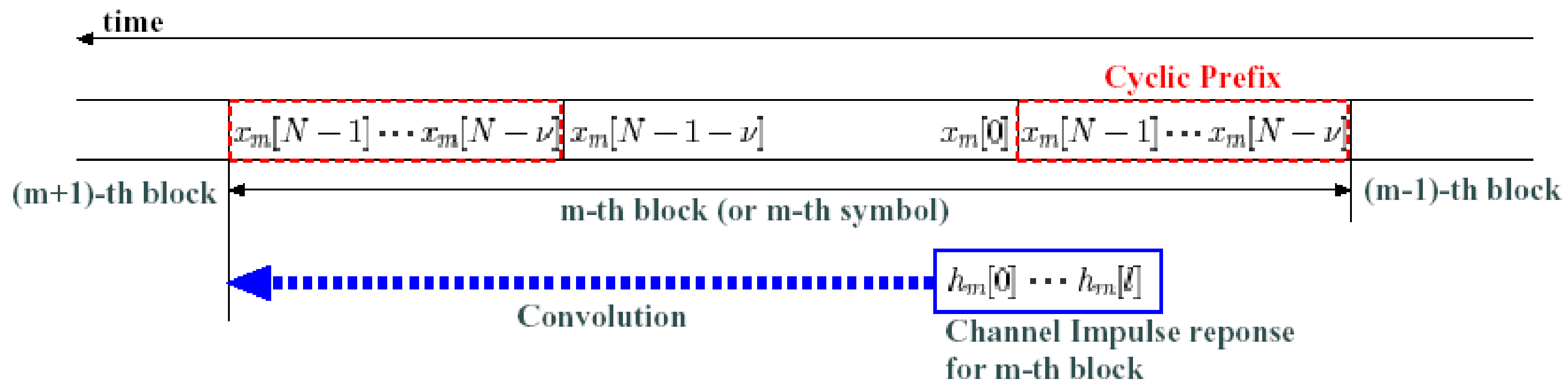
$$\begin{bmatrix} Y[0] \\ Y[1] \\ \bullet \\ \bullet \\ \bullet \\ Y[N-2] \\ Y[N-1] \end{bmatrix} = \begin{bmatrix} H[0] & & & & & 0 \\ & H[1] & & & & \\ & & \bullet & & & \\ & & & \bullet & & \\ & 0 & & & \bullet & \\ & & & & H[N-2] & \\ & & & & & H[N-1] \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ \bullet \\ \bullet \\ \bullet \\ X[N-2] \\ X[N-1] \end{bmatrix}$$

: **Y[i] depends on only X[i]**

Cyclic Prefix (CP) (1)

- 전송된 신호와 채널 impulse response의 convolution으로 주어지는 수신신호가 전송된 신호와 채널 impulse response의 N-point circular convolution된 신호처럼 보이도록 해 주기 위하여 CP를 추가한다.

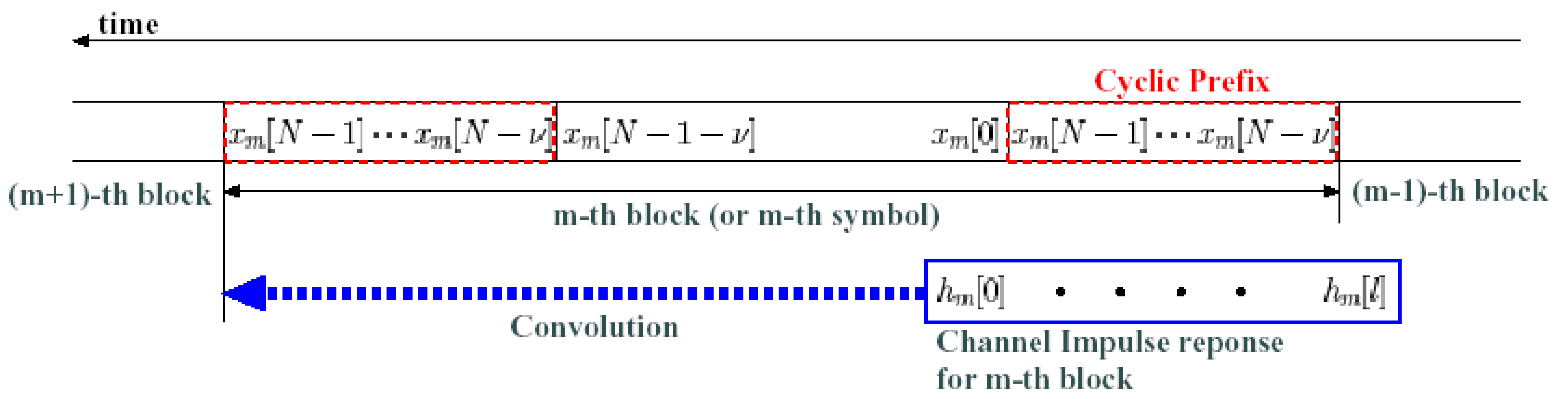
⇨ CP의 길이가 channel impulse response의 길이보다 긴 경우



$$y_m[n] = h_m[n] \textcircled{N} x_m[n] + \eta_m[n] \xrightarrow[\text{DFT of the received signal}]{} Y_m = \underbrace{H_m}_{\text{Diagonal matrix}} X_m + \Upsilon_m$$

Cyclic Prefix (CP) (2)

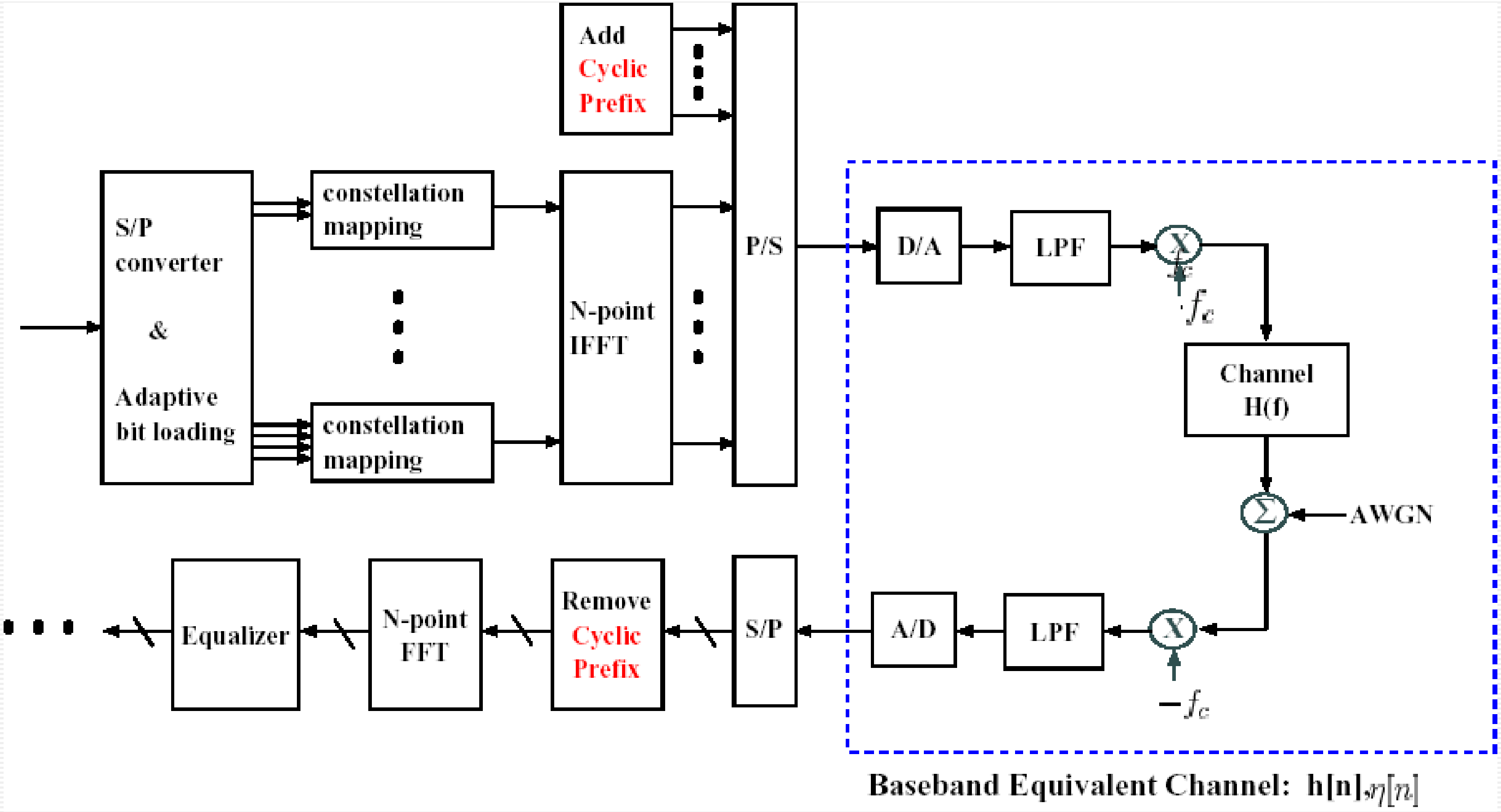
- CP의 길이가 channel impulse response의 길이보다 짧은 경우



$$Y_m = H_m X_m + \Gamma_m + \underline{H_{\mathrm{ICI}} X_{m-1}} + \underline{H_{\mathrm{ISI}} X_{m-1}}$$

- CP의 길이를 channel impulse response의 길이보다 길게 잡아주면, received signal에 대해 N-point DFT를 하고 난 후에는 inter-carrier interference (ICI) 및 inter-symbol interference (ISI)가 생기지 않는다.

Transmitter and Receiver (OFDM)



Challenges in Multicarrier Systems (1)

- Peak-to-Average Power Ratio

  : PAR grows with the number of subcarriers

  - Average power

$$E\left[\frac{1}{\sqrt{N}}\left|x_0+x_1+\cdots+x_{N-1}\right|^2\right]=\frac{1}{N}E\left[\left|x_0+x_1+\cdots+x_{N-1}\right|^2\right]=\frac{E[|x_0|^2]+E[|x_1|^2]+\cdots+E[|x_{N-1}|^2]}{N}=1$$

  - Maximum power: when all the x_i add coherently

$$\max\left[\frac{1}{\sqrt{N}}\left|x_0+x_1+\cdots+x_{N-1}\right|^2\right]=\left|\frac{N}{\sqrt{N}}\right|^2=N$$

  - The maximum PAR is *N* for *N* subcarriers
  - The observed PAR is typically less than N because full coherent addition of all N symbols is highly improbable.

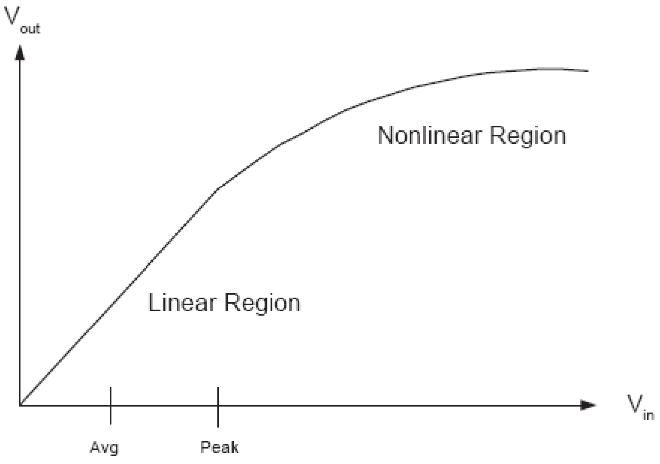


Figure 12.11: A typical power amplifier response.

Challenges in Multicarrier Systems (2)

- Frequency Offset
  - Orthogonality is assured by the subcarrier separation $\Delta f = 1/T_N$
  - In practice, the frequency separation of the subcarriers is imperfect
    - Due to mismatched oscillators, Doppler shifts, or timing sync error
  - Intercarrier interference (ICI)
    - The received samples of the FFT will contain interference from adjacent subcarriers due to the degradation in the orthogonality of the subcarriers
- Timing Offset
  - The effect of timing error is less than that from the frequency offset due to guard time (cyclic prefix), that is, as long as a full N-sample OFDM symbol is used at the receiver without interference from previous or subsequent symbols

Multiuser OFDM

◆ OFDM-TDMA: 각각의 사용자에게 predetermined time slot이 할당.
time slot안에서는 모든 sub-carrier 사용.

◆ OFDM-FDMA: 각각의 사용자에게 predetermined band of sub-carriers 할당.
특정 sub-carrier band를 사용하므로 frequency-selective fading 환경에서 사용자간 성능차이가 많이 생길 수 있다.

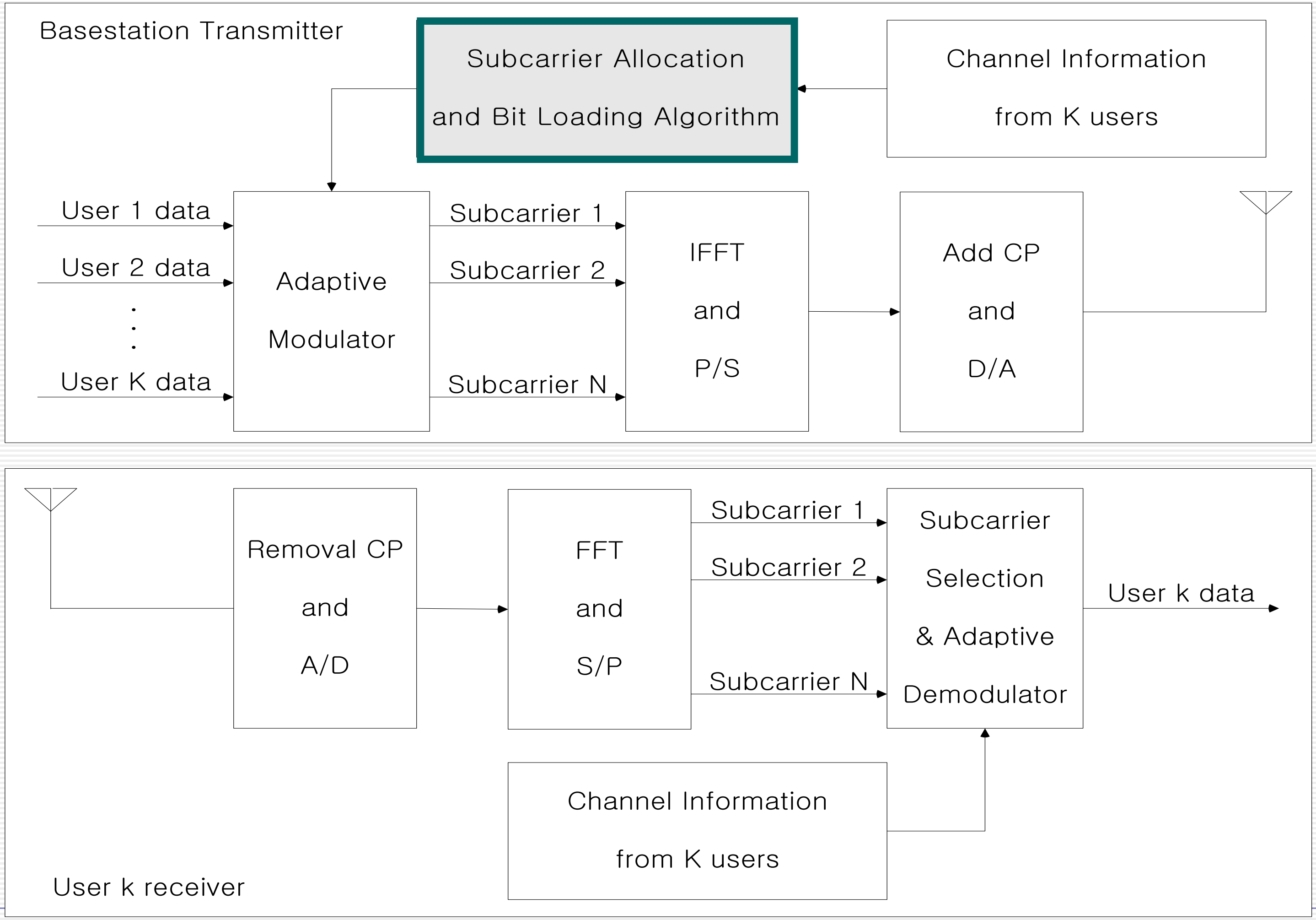
◆ OFDM Interleaved-FDMA: band of sub-carriers 가 아니라 서로서로 떨어져 있는 sub-carrier들을 각각의 사용자에게 할당.

OFDMA (Orthogonal Frequency Division Multiple Access)

◆ OFDM-CDMA

OFDM-FDMA (or OFDMA)

- OFDMA with Adaptive modulation



Case Study I – IEEE 802.11a

- IEEE 802.11a, which occupies a bandwidth of 20 MHz in the 5GHz band, is based on OFDM
  - $N = 64$ subcarriers are generated, although only 48 are actually used for data transmission
  - The cyclic prefix length is 1/4 of OFDM symbol time
  - Possible coding rates are 1/2, 2/3, 3/4
  - The modulation types can be used are BPSK, QPSK, 16QAM, 64QAM
- The subcarrier bandwidth $B_N = 20\text{MHz}/64 = 312.5\,\text{kHz}$
- Symbol time per subcarrier is $T_N = 1/B_N \times 5/4 = 4us$
- Maximum data rate is $R_{Max} = 48 \times \frac{3}{4} \times 6 \times \frac{1}{4 \times 10^{-6}} = 54\,\text{Mbps}$

Case Study II – IEEE 802.16a

- IEEE 802.16a, which occupies a bandwidth of 10 MHz, is based on OFDMA
  - N = 1024 subcarriers are generated, although only 768 are actually used for data transmission
  - The cyclic prefix length is 1/8 of OFDM symbol time
  - Possible coding rates are 1/2, 2/3, 3/4
  - The modulation types can be used are QPSK, 16QAM, 64QAM
- The subcarrier bandwidth $B_N = 10\text{MHz}/768 = 13.02\,\text{kHz}$
- Symbol time per subcarrier is $T_N = 1/B_N \times 9/8 = 115.2\,us$
- Maximum data rate is $R_{Max} = 768 \times \frac{3}{4} \times 6 \times \frac{1}{115.2 \times 10^{-6}} = 30\ \text{Mbps}$