Adaptive Modulation and Coding

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Chap. 9 AMC

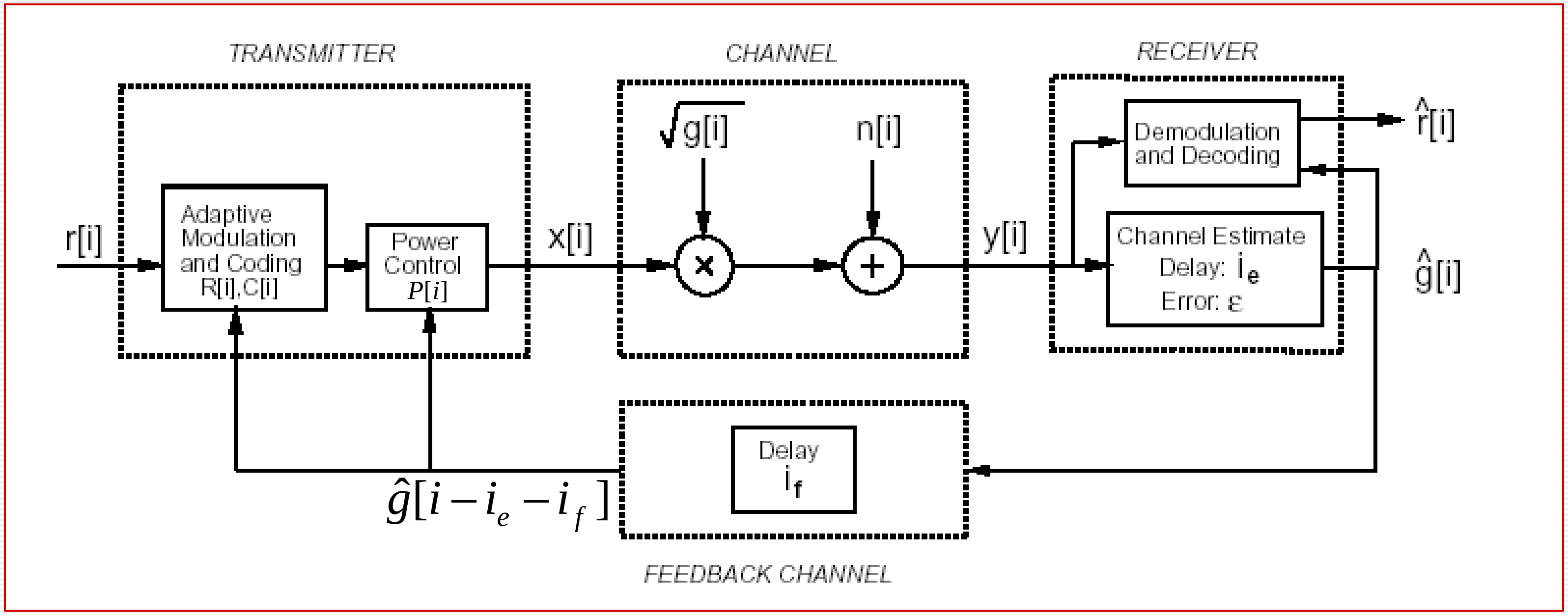
Introduction (1)

- Adaptive modulation and coding enable robust and efficient transmission over time-varying channel.
- AMC requires the channel estimation at the receiver and the feedback of this estimation to the transmitter (rate and/or power adaptation)
  - The transmitter takes advantage of favorable channel condition to send at higher data rate (the increased average throughput) or lower power (the reduced required tx power)
  - The transmitter reduces the data rate or increases tx power as the channel degrades (the lower error probability)

Introduction (2)

- Several practical constraints
  - A feedback channel is required
  - If the channel is changing faster than it can be reliably estimated, adaptive techniques will poorly perform.
  - Adaptive modulation typically varies the data rate but, in fixed-rate applications with tight delay constraint, it should minimize the outage probability.

Adaptive Transmission System

- We estimate the power gain at time *i*: $\hat{g}[i]$
- Suppose that the tx power, data rate, and/or coding parameter are estimated based on the SNR estimate $\hat{\gamma}[i] = \overline{P}\hat{g}[i] / N_0 B$ ($\overline{P}$: avarage power)
- Adaptive transmission system adjusts the data date $R[i]$, the tx power $P[i]$, the coding parameter $C[i]$ according to $\hat{\gamma}[i]$
- 

Variable-Rate Adaptive Technique

- In variable-rate modulation, the data rate $R[\gamma]$ is varied relative to the channel gain γ
  - Fixed symbol rate and multiple modulation schemes or constellation size (easy practically)
  - By fixing the modulation and changing the symbol rate
- In general, the modulation parameters are fixed over a frame
- Most of wireless systems vary their modulation and coding (transmission rate) according to channel gain
- In usual, a discrete set of modulation types or constellation sizes are used: MCS level mapping relative to the channel gain

Variable-Power Adaptive Technique

- Adapting the transmit power for compensating for SNR variation due to fading
  - The goal is to maintain a fixed error probability or a constant received SNR
  - The power adaption inverts the channel fading so that the channel appears as an AWGN channel to the modulator/demodulator
  - Under the truncated channel inversion
    - $$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \sigma/\gamma & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

      where $\sigma = \left(\mathrm{E}_{\gamma_0}[1/\gamma]\right)^{-1} = \left(\int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma) d\gamma\right)^{-1}$

      and is a constant received SNR

Variable-Coding Adaptive Technique

- Adaptive coding can be implemented by multiplexing together codes with different error correction probability
- The channel should remain roughly constant over the block or constraint length of the code
- Adaptive coding is particularly useful when the modulation must remain fixed, over slowly varying channel
- Rate-compatible punctured convolutional code
  - An alternative technique to code multiplexing
  - The basic premise of RCPC code is to have a single encoder and decoder
  - Error correction capability is modified by not transmitting certain coded symbols (puncturing)

Hybrid Adaptation Techniques

- Rate adaptation us often combined with power adaption
- Adaptive modulation and coding is currently used in both cellular system and wireless LANs

Variable-Rate Variable-Power MQAM

- The rate and power of MQAM are varied to maximize spectral efficiency while meeting a given instantaneous P_b target.
- Consider a family of MQAM with a fixed symbol time T_s (= $1/B$ with ideal Nyquist pulse shaping)

Error Probability Bound

- The rate and power of MQAM are varied to maximize spectral efficiency while meeting a given instantaneous P_b target.
- Consider a family of MQAM with a fixed symbol time T_s ($= 1/B$ with ideal Nyquist pulse shaping)
  - In AWGN with the tx power $\overline{P}$, $\dfrac{E_s}{N_0} = \dfrac{\overline{P}T_s}{N_0} = \dfrac{\overline{P}}{N_0 B} = \gamma$
- The BER for an AWGN channel with MQAM modulation, ideal coherent phase detection, SNR γ
  - since it is easy to invert, we use $P_b \le 2e^{-1.5\gamma/(M-1)}$
  - A tight bound (good to within 1 dB for $M \ge 4$ and $0 \le \gamma \le 30$ dB)
    - $P_b \le 0.2e^{-1.5\gamma/(M-1)}$
- In fading channel with nonadaptive transmission (constant tx power and rate), the average BER is obtained by integrating the BER in AWGN over the fading distribution.

Adaptive Rate and Power Scheme (1)

- Consider the tx power $P(\gamma)$ relative to γ , subject to the average power constraint $\bar{P}$ and an instantaneous BER constraint $P_b(\gamma) = P_b$
  - $P_b(\gamma) \le 0.2\exp\left[-\frac{1.5\gamma}{M-1}\frac{P(\gamma)}{\bar{P}}\right]$
  - $M(\gamma) = 1 + \frac{1.5\gamma}{-\ln(5P_b)}\frac{P(\gamma)}{\bar{P}} = 1 + K\gamma\frac{P(\gamma)}{\bar{P}}$
  - Maximizing the following subject to $\int_0^\infty P(\gamma)p(\gamma) = \bar{P}$
    - $E[\log_2 M(\gamma)] = \int_0^\infty \log_2\left(1 + \frac{K\gamma P(\gamma)}{\bar{P}}\right)p(\gamma)d\gamma$
    - Optimal power adaption policy: $K\frac{P(\gamma)}{\bar{P}} = \begin{cases} 1/\gamma_K - 1/\gamma & \gamma \ge \gamma_K \\ 0 & \gamma < \gamma_K \end{cases}$ Water filing
    - γ_K is a cutoff to satisfy $\int_{\gamma_K}^\infty \left(\frac{1}{\gamma_K} - \frac{1}{\gamma}\right)p(\gamma)d\gamma = K$
    - The average spectral efficiency: $\frac{R}{B} = E[\log_2 M(\gamma)] = \int_{\gamma_K}^\infty \log_2\left(\frac{\gamma}{\gamma_K}\right)p(\gamma)d\gamma$

- Adjust the tx power and modulation order
  - power adaption
  - adjustment of modulation order

$$M(\gamma) = \gamma/\gamma_K$$

Adaptive Rate and Power Scheme (2)

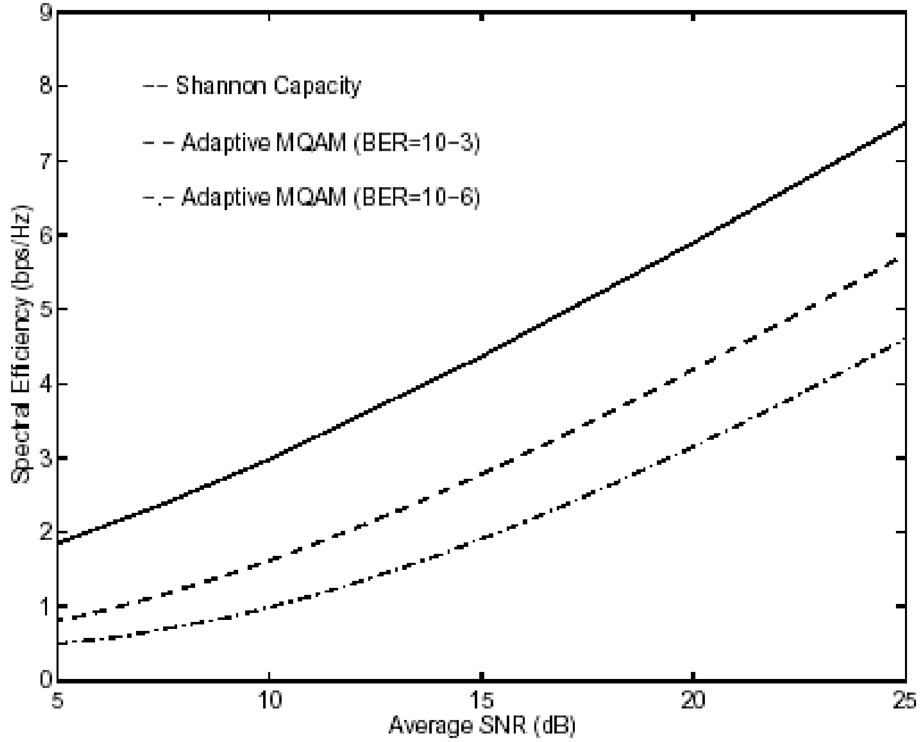


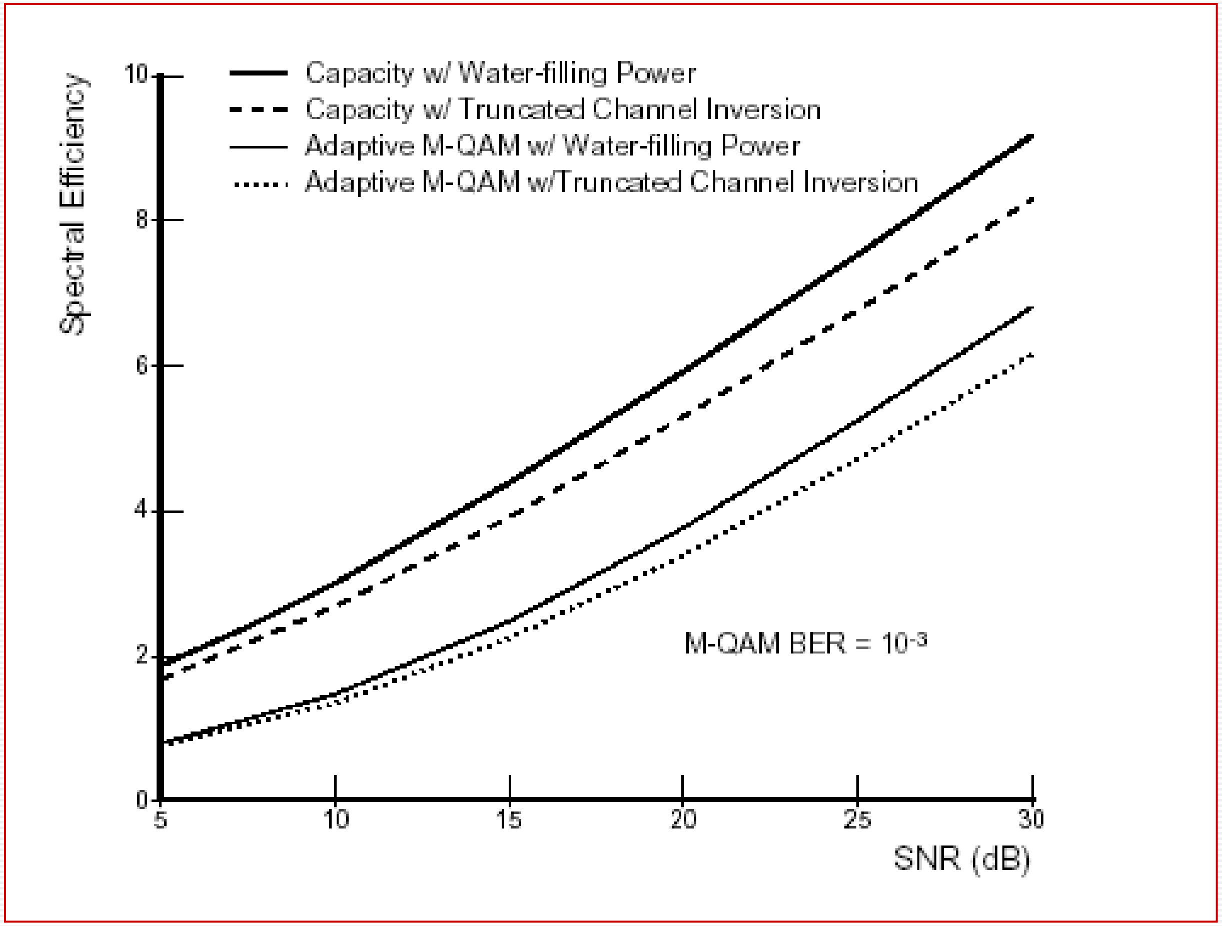
Figure 9.3: Average Spectral Efficiency in Rayleigh Fading.

Channel Inversion with Fixed Rate (1)

- Channel inversion power adaptation to maintain a fixed received SNR
  - $P(\gamma)/\overline{P} = \sigma/\gamma$ with $\sigma = 1/\text{E}[1/\gamma]$ (1)
  - $$\frac{R}{B} = \log_2 M = \int_0^\infty \log_2\left(1 + \frac{1.5\gamma}{-\ln(5P_b)} \frac{P(\gamma)}{\overline{P}}\right) p(\gamma)dr$$
    $$= \log_2\left(1 + \frac{1.5}{-\ln(5P_b)\text{E}[1/\gamma]}\right) \quad (2)$$
- Truncated channel inversion
  - $$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \sigma/\gamma & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases} \quad \text{where } \sigma = \frac{1}{\text{E}_{\gamma_0}[1/\gamma]} = \int_{\gamma_0}^\infty \frac{1}{\gamma} p(\gamma)d\gamma \quad (3)$$
  - $$\frac{R}{B} = \max_{\gamma_0} \log_2\left(1 + \frac{1.5}{-\ln(5P_b)\text{E}_{\gamma_0}[1/\gamma]}\right) p(\gamma > \gamma_0) \quad (4)$$
- Determine the modulation order using (2) or (4)
- Adjust the tx power according to γ using (1) or (3)

Channel Inversion with Fixed Rate (2)

- Rayleigh fading channel



Discrete-Rate Adaption (1)

- Assume a set of square constellations of size M_0=0, M_1=2, and $M_j=2^{2(j-1)}$, j = 2, … , N-1
- The choice of constellation depends on the fade level over the symbol time
- Choosing the M_0 constellation corresponds to no data transmission
- For each value of γ, we must decide which constellation to transmit and what the associated transmit power should be
- A continuous-power discrete-rate adaptation scheme
  - The range of γ : N fading region $R_j = [\gamma_{j-1}, \gamma_j)$
  - Choose the constellation M_j when $\gamma \in R_j$ (spectral efficiency: $\log_2 M_j$)
  - The boundaries can be optimized to maximize spectral efficiency but cannot be found in closed form
    - suboptimal solution: find j such that $M_j \le M(\gamma) < M_{j+1}$, where $M(\gamma) = \gamma / \gamma_K^*$
    - spectral efficiency : $\dfrac{R}{B} = \sum_{j=1}^{N-1} \log_2(M_j) p(M_j \le \gamma / \gamma_K^* < M_{j+1})$

Discrete-Rate Adaption (2)

- We find γ_K^* to maximize
  - $$\frac{R}{B} = \sum_{j=1}^{N-1} \log_2 \left(M_j\right) p(M_j \le \gamma / \gamma_K^* < M_{j+1})$$
  - subject to the power constraint

$$\sum_{j=1}^{N-1} \int_{\gamma_K^* M_j}^{\gamma_K^* M_{j+1}} \frac{P_j(\gamma)}{\overline{P}} p(\gamma) d\gamma = 1,$$

$$M_j = 1 + K\gamma P_j(\gamma) / \overline{P}$$

where $$\frac{P_j(\gamma)}{\overline{P}} = \begin{cases} (M_j - 1)(1/K\gamma) & \text{if } M_j < \gamma/\gamma_K^* \le M_{j+1} \\ 0 & \text{if } M_j = 0 \end{cases}$$

- Discrete rate adaptation
  - Choose M_j such that $M_j \le \gamma/\gamma_K^* < M_{j+1}$, according to γ
- Continuous power adaptation
  - $$P_j(\gamma) = \begin{cases} \overline{P}(M_j - 1)(1/K\gamma) & \text{if } j \ne 0 \\ 0 & \text{if } j = 0 \end{cases}$$

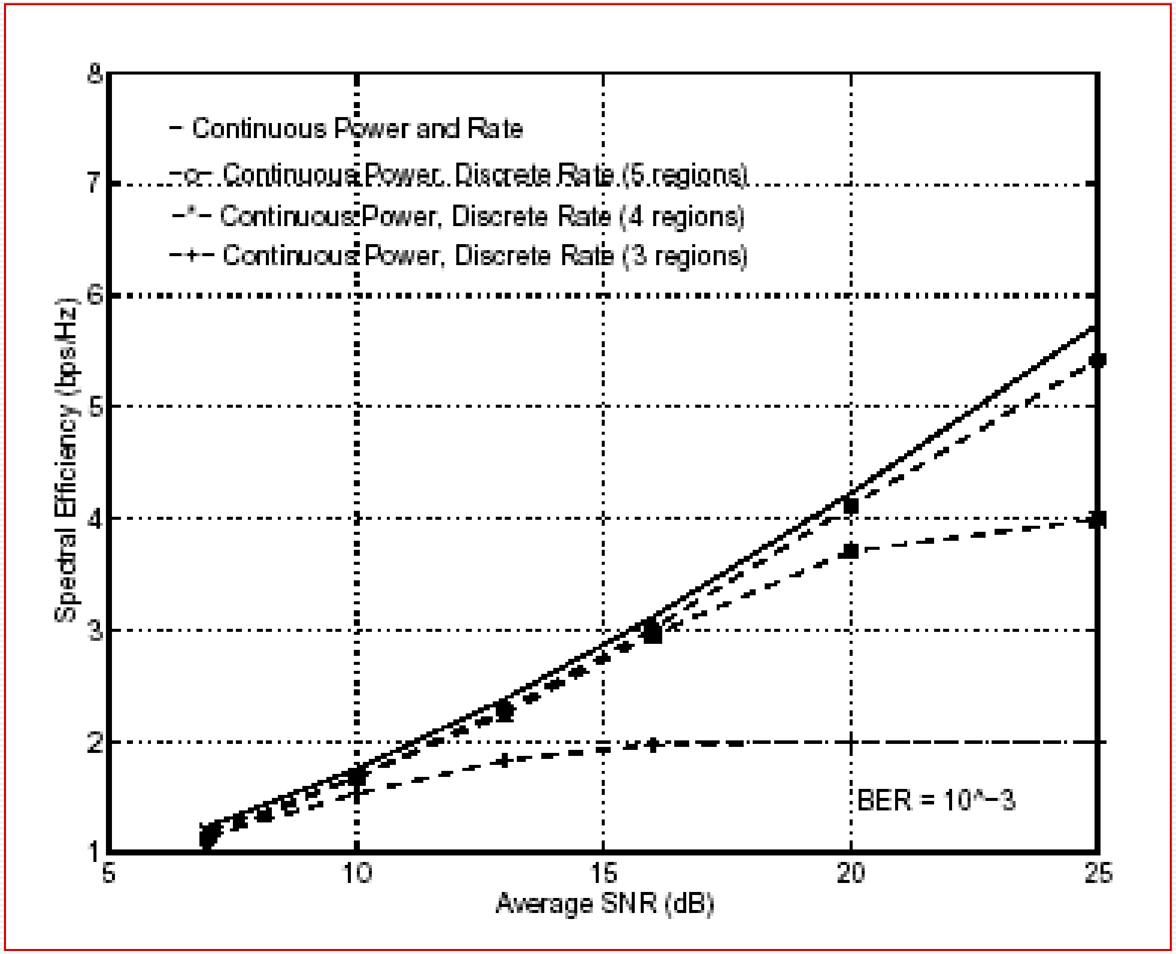
Discrete-Rate Adaption (3)

- Rate and power adaptation for five regions

Region, R_j	γ range	M_j	$P_j(\gamma)/\overline{P}$
0	$0 \le \gamma/\gamma_K^* < 2$	0	0
1	$2 \le \gamma/\gamma_K^* < 4$	2	$1/K\gamma$
2	$4 \le \gamma/\gamma_K^* < 16$	4	$3/K\gamma$
3	$16 \le \gamma/\gamma_K^* < 64$	16	$15/K\gamma$
4	$64 \le \gamma/\gamma_K^* < \infty$	64	$63/K\gamma$

Discrete-Rate Adaption (4)

Rayleigh Fading



Discrete-Rate Adaption (5)

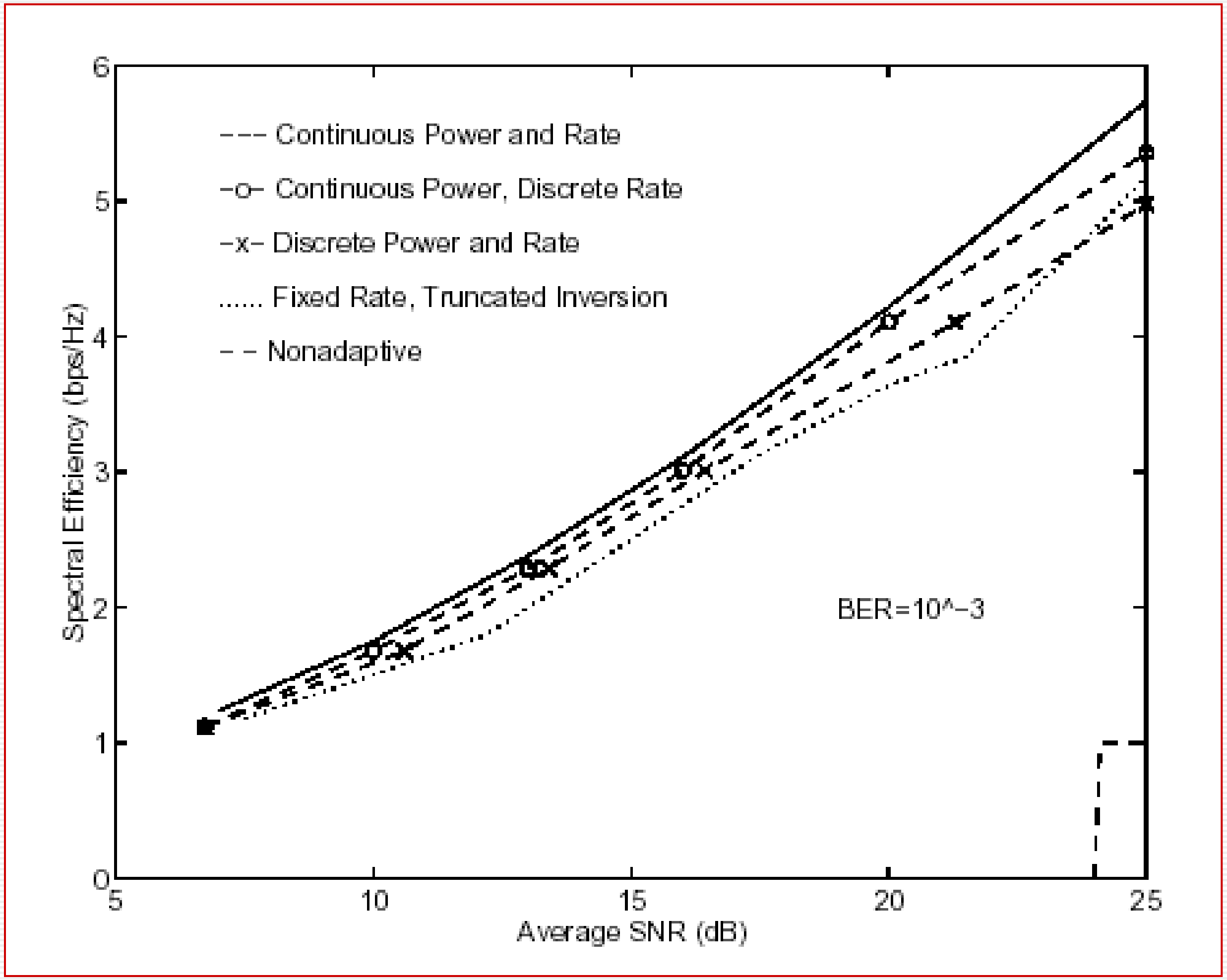
- Total channel inversion
  - $$\frac{R}{B} = \log_2 \left\lfloor 1 + \frac{1.5}{-\ln(5P_b)\mathrm{E}[1/\gamma]} \right\rfloor_{\mathsf{M}},$$
    where $\lfloor x \rfloor_{\mathsf{M}}$ is the largest number in set $\mathsf{M} = \{M_0, \ldots, M_{N-1}\}$ less than or equal to x.
  - $$P(\gamma) = \frac{\overline{P}}{\gamma\,\mathrm{E}[1/\gamma]}$$
- Truncated channel inversion
  - $$\frac{R}{B} = \max_{\gamma_0} \log_2 \left\lfloor 1 + \frac{1.5}{-\ln(5P_b)\mathrm{E}_{\gamma_0}[1/\gamma]} \right\rfloor_{\mathsf{M}} p(\gamma > \gamma_0)$$
  - $$P(\gamma) = \frac{\overline{P}}{\gamma\,\mathrm{E}_{\gamma_0}[1/\gamma]}$$

Discrete-Rate Adaption (6)

- Discrete-rate and discrete-power
  - A constant transmit power for each constellation M_j
  - Since the tx power and constellation size are fixed in each region, the BER varies with γ in each region
  - The boundaries and tx power must be set to achieve a given target average BER

Discrete-Rate Adaption (7)

Rayleigh Fading



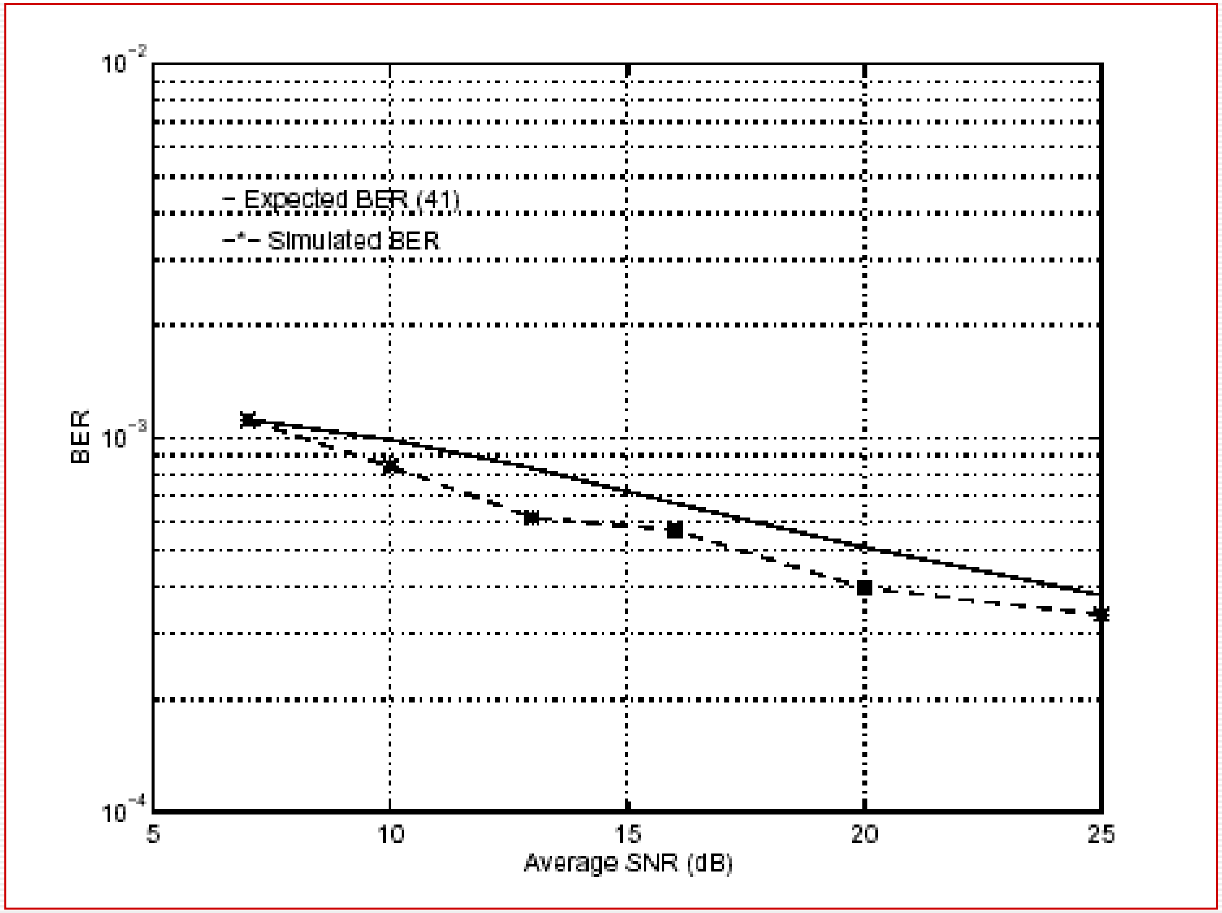
Exact versus Approximate Bit Error Probability (1)

- The adaptive policies in prior sections are based on the BER upper bounds $P_b \le 0.2e^{-1.5\gamma/(M-1)}$ and this leads to a lower BER than the target.
- For MQAM with Gray coding at high SNR
  - BER: $P_b \approx \dfrac{4}{\log_2 M} Q\left(\sqrt{\dfrac{3\gamma}{M-1}}\right)$ ← Received SNR
  - For the *j*th signal constellation: $\dfrac{E_s(j)}{N_0} = \dfrac{\gamma P_j(\gamma)}{\overline{P}} = \dfrac{M_j - 1}{K}$
  - A more accurate analytical expression for the average BER associated with the adaptive policies.

$$\overline{P}_b = \sum_{j=1}^{N-1} \frac{4Q\left(\sqrt{3/K}\right)}{\log_2 M_j} \int_{\gamma_K^* M_j}^{\gamma_K^* M_{j+1}} p(\gamma) d\gamma$$

Exact versus Approximate Bit Error Probability (2)

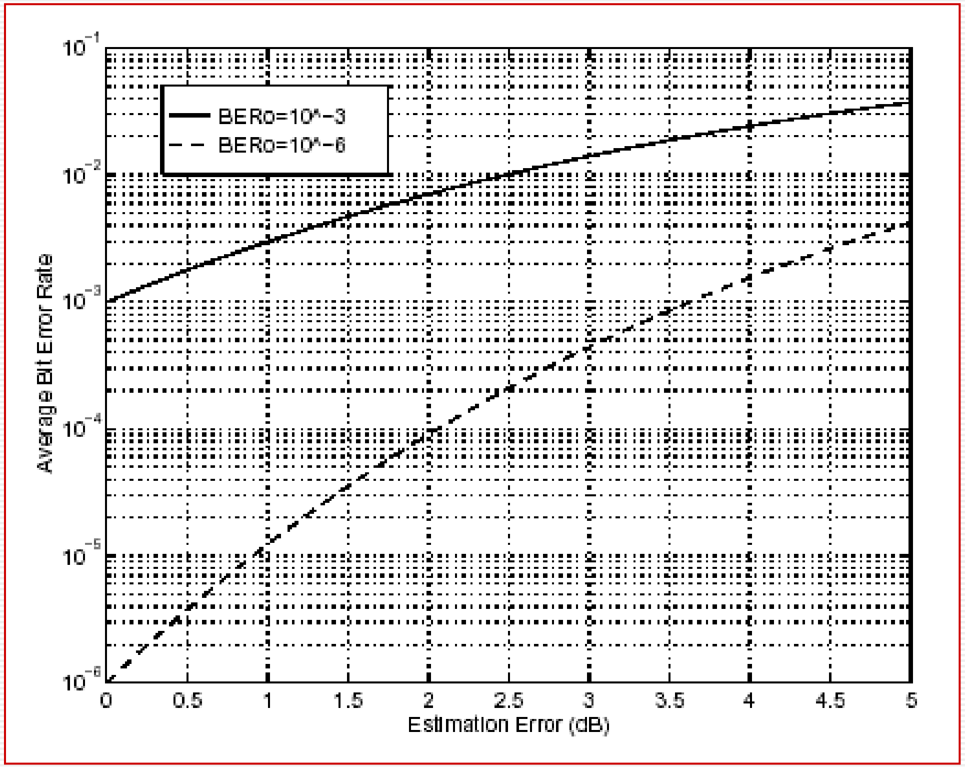
- BER for Rayleigh fading (five regions)



Channel Estimation Error (1)

- Suppose the transmitter adapts its power and rate relative to target BER P_b, based on the channel estimate $\hat{\gamma}$ instead of the true value γ
- Estimation error: $\varepsilon = \hat{\gamma} / \gamma$
- BER
  - $$P_b(\gamma, \hat{\gamma}) \leq \frac{1}{5} \exp\left(\frac{-1.5\gamma}{M(\hat{\gamma}) - 1} \frac{P(\hat{\gamma})}{\overline{P}} \right) = \frac{1}{5} (5P_b)^{1/\varepsilon}$$
- Effect of estimation error on BER
  - $$\overline{P_b} \leq \int_0^\infty \frac{1}{5} (5P_b)^{1/\varepsilon} p(\varepsilon) d\varepsilon$$

Channel Estimation Error (2)



$$\text{constant } \varepsilon : \overline{P_b} = 1\,\text{dB}\,(\text{BER} = 10^{-3}),\ \overline{P_b} = 0.5\,\text{dB}\,(\text{BER} = 10^{-6})$$

Channel Estimation Delay (1)

- Suppose the channel is estimated perfectly and the estimation delay is i_d
  - At time i, the transmitter uses the delay version of channel estimate $\hat{\gamma}[i] = \gamma[i - i_d]$
  - $$P_b(\gamma[i], \gamma[i-i_d]) \le 0.2 \exp\left[\frac{-1.5\gamma[i]}{M(\gamma[i-i_d]) - 1} \frac{P(\gamma[i-i_d])}{\overline{P}} \right]$$
    $$= 0.2\left[5P_{b0}\right]^{\gamma[i]/\gamma[i-i_d]}$$
  - $$P_b(i_d) \le \int_{\gamma_K}^{\infty} \int_{0}^{\infty} \left[0.2\left[5P_{b0}\right]^{\xi} p_{i_d}(\xi \mid \gamma) d\xi\right] p(\gamma) d\gamma$$
    - $\xi[i, i_d] = \gamma[i] / \gamma[i - i_d]$
    - γ_K : the cutoff region of the optimal policy

Channel Estimation Delay (2)