Direct Sequence Spread Spectrum

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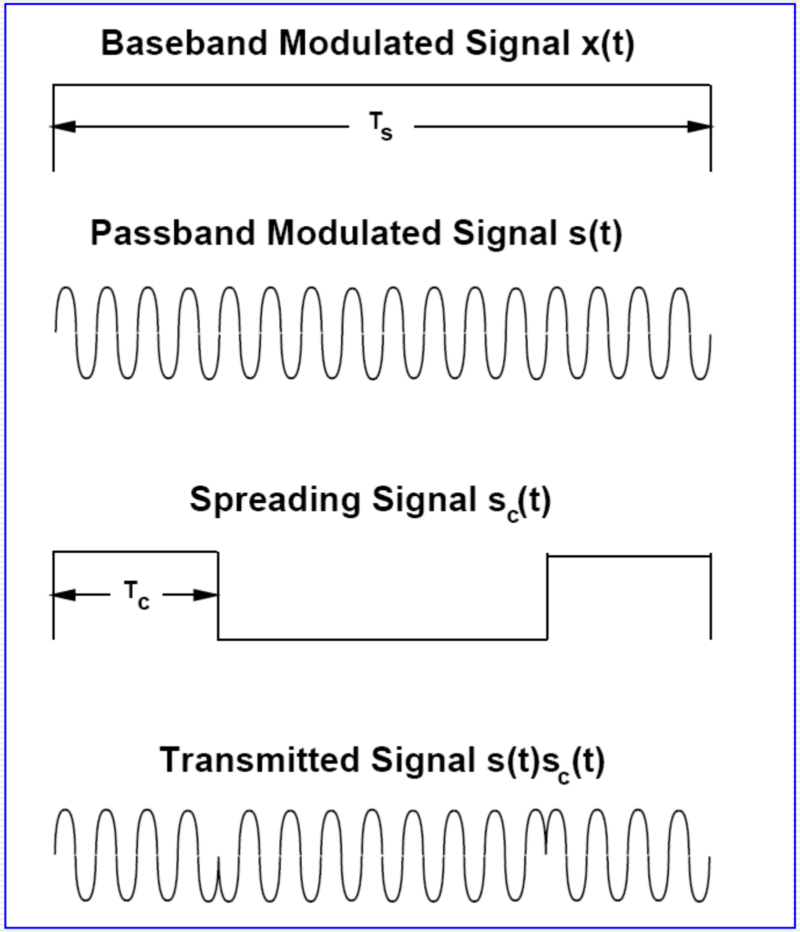
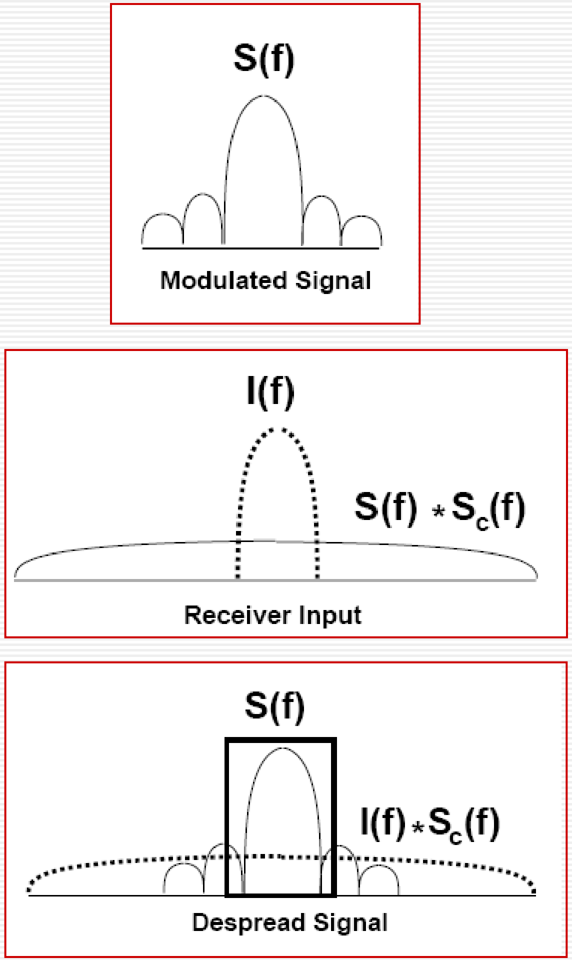
Spread Spectrum Principle (1)

- Spreading signal multiplication



- Narrowband interference rejection

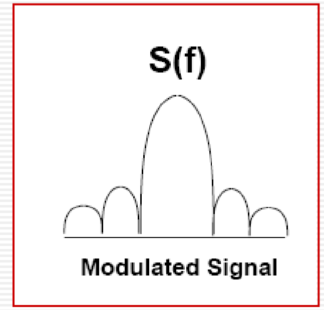


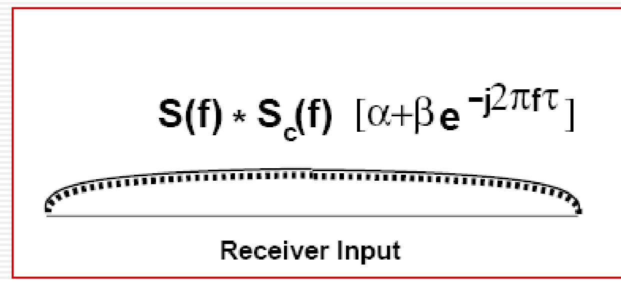
Spread Spectrum Principle (2)

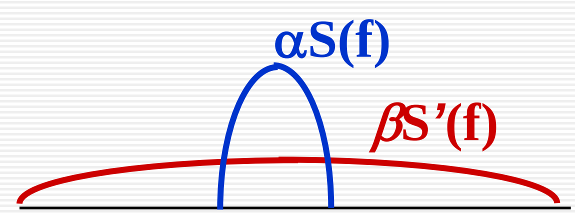
- Suppose the spread signal is transmitted through a two-ray channel with impulse response
  $h(t) = \alpha\delta(t) + \beta\delta(t-\tau)$
  - Frequency response: $H(f) = \alpha + \beta e^{-j2\pi f\tau}$
- The receiver input
  - $[s(t)s_c(t)] * h(t) = \alpha s(t)s_c(t) + \beta s(t-\tau)s_c(t-\tau)$
  - $H(f)[S(f) * S_c(f)]$
    $= \alpha[S(f) * S_c(f)] + \beta e^{-j2\pi f\tau}[S(f) * S_c(f)]$
- Suppose that the receiver despreading process multiplies this signal by a copy of $s_c(t)$ synchronized to the first path
  - $\alpha s(t)s_c^2(t) + \beta s(t-\tau)s_c(t-\tau)s_c(t)$

Autocorrelation of spreading code at delay τ

ISI Rejection







Despread Signal

Spread Spectrum Principle (3)

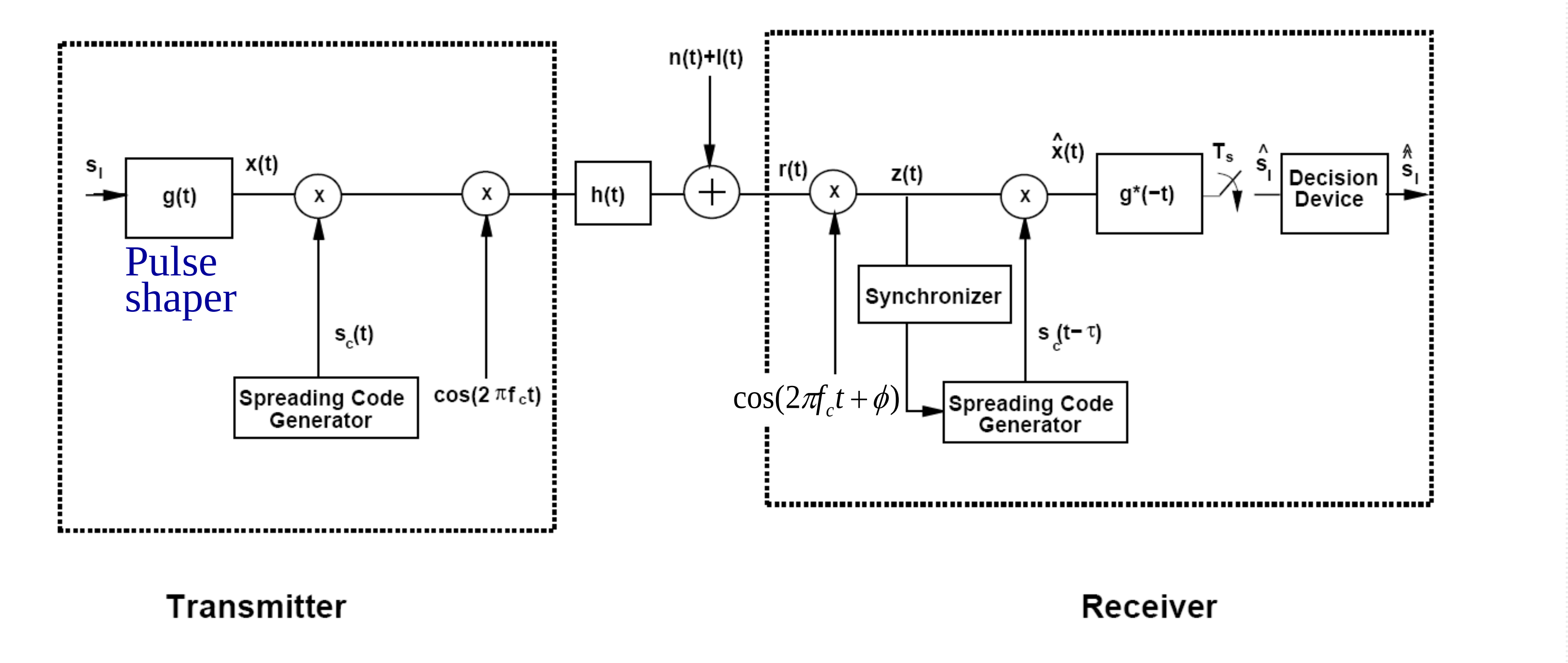
The autocorrelation of spreading code determines the ISI rejection

=> Spreading code with good autocorrelation

In multiuser system, interference between users is determined by the cross-correlation of their spreading codes

=> Spreading code with good cross-correlation

DSSS System Model (1)



$$r(t) = [x(t)s_c(t)\cos(2\pi f_c t)] * h(t) + n(t) + I(t)$$

- Synchronizer: align the delay of the receiver's spreading code generator with ideally the multipath component having the largest amplitude or typically the first component having amplitude above a given threshold. => the multipath diversity is not utilized

DSSS System Model (2)

$$r(t) = [x(t)s_c(t)\cos(2\pi f_c t)] * h(t) + n(t) + I(t)$$

- $$\hat{x}(t) = ([x(t)s_c(t)\cos(2\pi f_c t)] * h(t))\cos(2\pi f_c t + \phi)s_c(t-\tau) + n(t)\cos(2\pi f_c t + \phi)s_c(t-\tau) + I(t)\cos(2\pi f_c t + \phi)s_c(t-\tau)$$
- In the absence of multipath and interference, i.e., $h(t) = \delta(t),\ \phi = 0,\ \tau = 0,\ I(t) = 0$
  - $$\hat{x}(t) = x(t)s_c^2(t)\cos^2(2\pi f_c t) + n(t)s_c(t)\cos(2\pi f_c t) = x(t)\cos^2(2\pi f_c t) + n(t)s_c(t)\cos(2\pi f_c t)$$
  - $n(t)s_c(t)$ has approximately the same statistics as $n(t)$: zero mean AWGN random process with PSD $N_0/2$
  - Matched filter output for $g(t) = \sqrt{2/T_s}$

$$\begin{aligned}\hat{s}_l &= \int_0^{T_s} x(t)g(t)dt \\ &= \sqrt{2/T_s}\int_0^{T_s} x(t)\cos^2(2\pi f_c t)dt + \sqrt{2/T_s}\int_0^{T_s} n(t)s_c(t)\cos(2\pi f_c t)dt \\ &= 2/T_s\int_0^{T_s} s_l\cos^2(2\pi f_c t)dt + \sqrt{2/T_s}\int_0^{T_s} n(t)s_c(t)\cos(2\pi f_c t)dt \\ &\approx s_l + n_l\end{aligned}$$

DSSS System Model (3)

- Interference signal at the carrier frequency, $I(t) = I'(t)\cos(2\pi f_c t)$
  - $\hat{x}(t) = x(t)\cos^2(2\pi f_c t) + n(t)s_c(t)\cos(2\pi f_c t) + I'(t)s_c(t)\cos^2(2\pi f_c t)$
  - $n(t)s_c(t)$ is assumed to be a zero mean AWGN random process with PSD $N_0/2$
  - Matched filter output for $g(t) = \sqrt{2/T_s}$

$$\hat{s}_l = 2/T_s \int_0^{T_s} s_l \cos^2(2\pi f_c t)dt + \sqrt{2/T_s}\int_0^{T_s} n(t)s_c(t)\cos(2\pi f_c t)dt$$

$$+ \sqrt{2/T_s}\int_0^{T_s} I'(t)s_c(t)\cos^2(2\pi f_c t)dt$$

$$\approx s_l + n_l + I_l$$

Spread interference

- Multipath channel: $h(t) = \alpha_0\delta(t) + \alpha_1\delta(t-\tau_1)$ $(\alpha_0 > \alpha_1,\ \tau_1 = kT),\ \phi = 0,\ I(t) = 0$
  -

$$\hat{s}_l = 2/T_s \int_0^{T_s} \alpha_0 s_l \cos^2(2\pi f_c t)dt$$

$$+ 2/T_s \int_0^{T_s} \alpha_1 s_{l-k} s_c(t) s_c(t-\tau_1)\cos(2\pi f_c t)\cos(2\pi f_c(t-\tau_1))dt$$

$$+ \sqrt{2/T_s}\int_0^{T_s} n(t)s_c(t)\cos(2\pi f_c t)dt$$

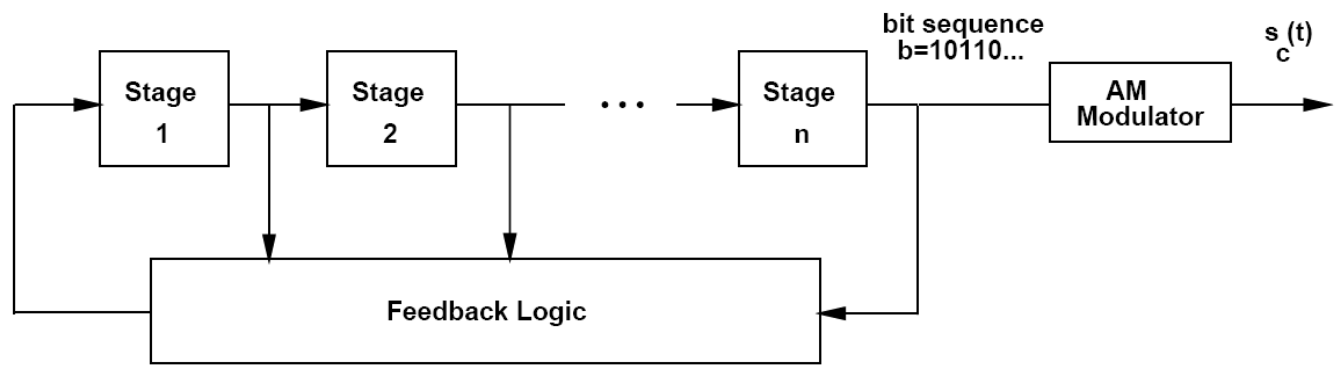
$$\approx \alpha_0 s_l + \alpha_1 s_{l-k}\cos(2\pi f_c \tau_1)\rho_c(\tau_1) + n_l$$

ISI

Autocorrelation of spreading code at delay τ_1

Spreading Code – m sequences

- Pseudo-Noise (PN) sequence
  - Binary sequence: $s_c(t) \in \{0,1\}$
  - Periodic sequence



- m-sequences
  - PN sequence with ideal autocorrelation property
  - Maximal length linear spreading code
  - Has the maximum period $N = 2^n - 1$ that can generated by a shift register of length n

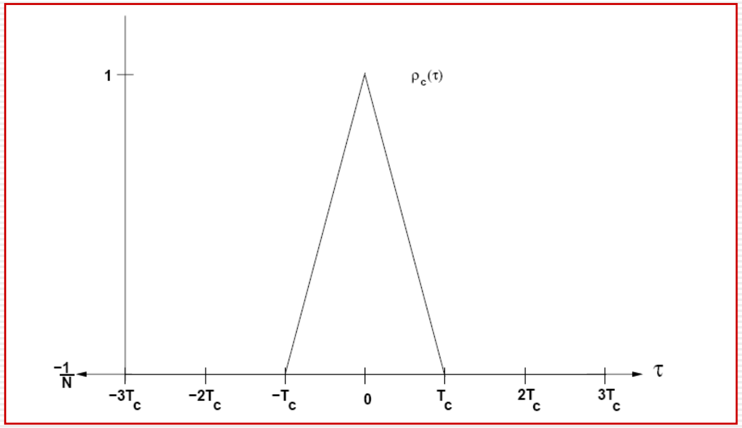
m-sequences (1)

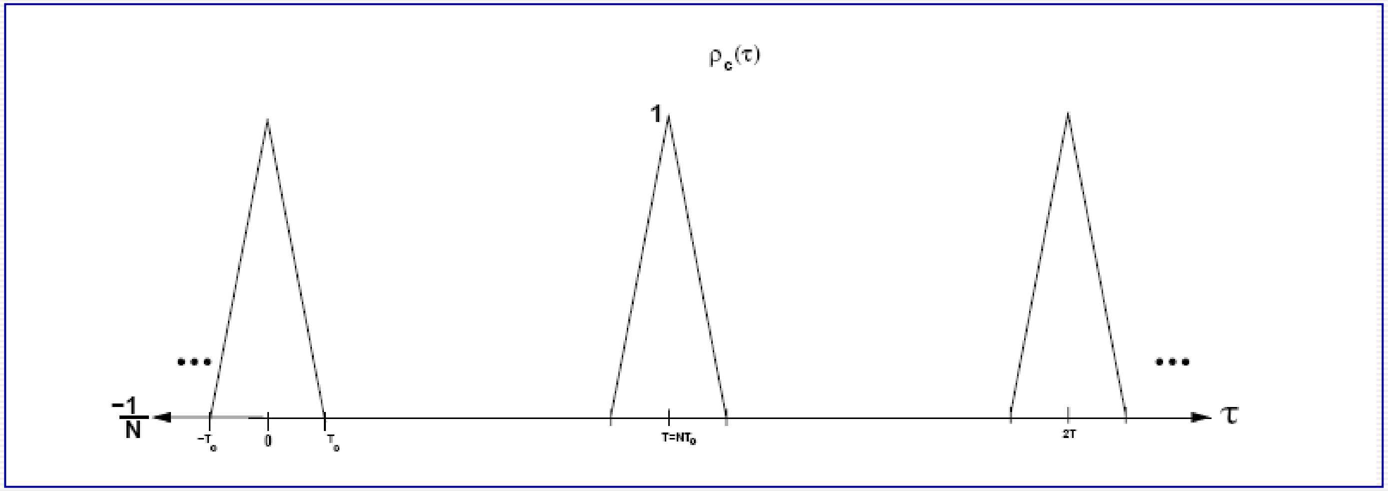
- Balance property
  - Roughly the same number of 0s and 1a over a period: 2^{n-1}-1 0-bits and 2^{n-1}1-bits
  - Has very small DC component
- Run property
  - Run: the consecutive 1s and 0s
  - Run length: the number of consecutive 1s and 0s (the smaller is the better)
  - The number of runs with length *r* in an *n*-length sequence: a fraction $1/2^r$ of all runs
- Shift and add property
  - Any time shift of an *m*-sequence is itself an *m*-sequence.
  - The modulo-2 addition of an *m*-sequence and a time shift of itself results in a different *m*-sequence corresponding to a different time-shift of the original sequence
- Example: m-sequence with period 15: 011110101100100

m-sequences (2)

- Autocorrelation $\rho_c(\tau)$ taken over a full period $T = NT_c$

$$\rho_c(\tau) = \begin{cases} 1 - |\tau|(1+1/N)/T_c & |\tau| \le T_c \\ -1/N & |\tau| > T_c \end{cases}$$



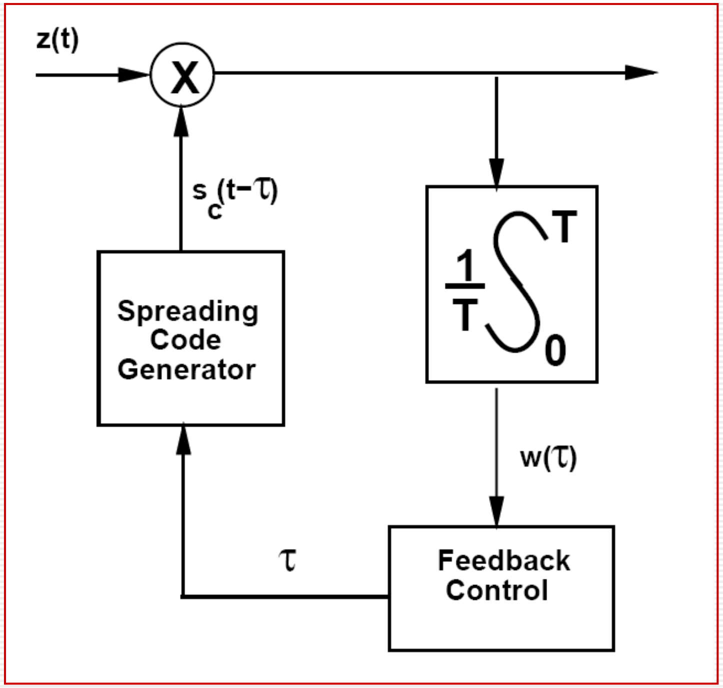


Synchronization (1)

- The synchronizer must align the timing of the spreading code generator in the receiver with the spreading code associated with one of the multipath components.
  - Separate from the carrier phase recovery: we assume that the carrier in the demodulator is coherent in phase with the received carrier
- A feedback control loop
  - Adjust the delay τ until w(τ) reaches peak value.
  - Consider a channel with impulse response $h(t) = \delta(t - \tau_0)$

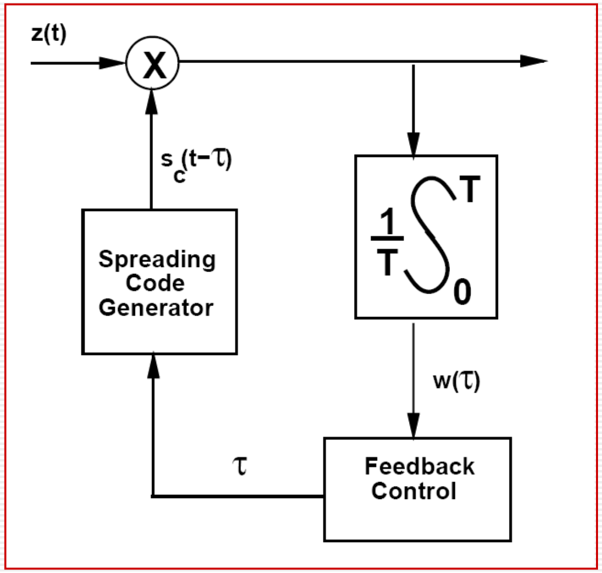
$$z(t) = x(t - \tau_0) s_c(t - \tau_0) \cos(2\pi f_c (t - \tau_0)) \cos(2\pi f_c + \phi)$$

  - Assuming perfect phase recovery
    - $\cos(2\pi f_c + \phi) = \cos(2\pi f_c (t - \tau_0))$
    - $z(t) = x(t - \tau_0) s_c(t - \tau_0) \cos^2(2\pi f_c + \phi)$
  - A BPSK with rectangular pulse shaping
    - $x(t) = \pm\sqrt{2E_b / T_b} \quad (s_I = \pm\sqrt{E_b})$



Synchronization (2)

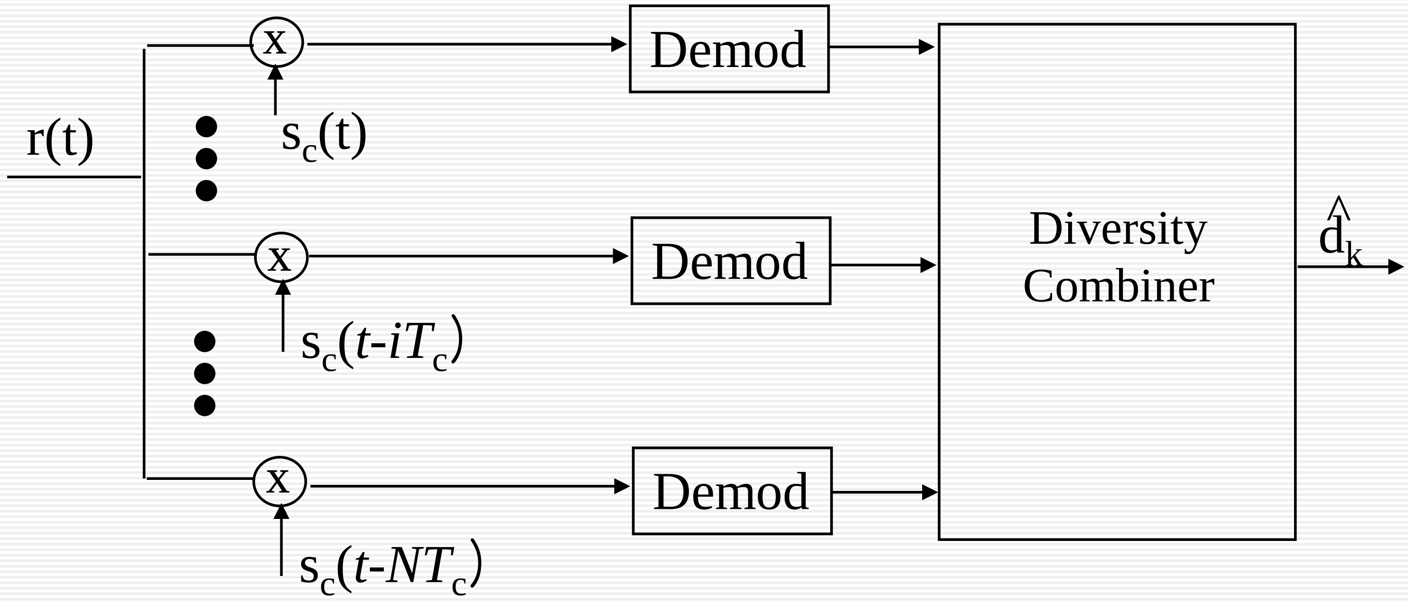
- Assume the spreading code has the period $T = NT_c = T_b$
  - $$w(\tau) = \frac{1}{T}\sqrt{\frac{2}{T_b}}\int_0^T s_I s_c(t-\tau_0) s_c(t-\tau)\cos^2(2\pi f_c + \phi)dt$$
    $$\approx \frac{s_I}{\sqrt{2T_b}}\int_0^T s_c(t-\tau_0)s_c(t-\tau)dt = s_I\sqrt{\frac{T_b}{2}}\rho_c(\tau-\tau_0)$$
  - Since $\rho_c(\tau-\tau_0)$ is maximized at $\tau = \tau_0$ the feedback control loop will adjust τ such that $|w(\tau)|$ increases.



- Acquisition (Coarse synchronization)
  - When $|\tau-\tau_0| > T_c$, $\rho_c(\tau-\tau_0) = -1/N$.
  - The synchronizer is operating outside the triangular region of the autocorrelation
  - The feedback control computes the autocorrelation $\rho_c(\tau-\tau_0)$ and compares it with threshold
  - If the autocorrelation is larger, acquisition is done; Otherwise, the PN code is time shifted by $2T_c$ and computes autocorrelation again
- Code Tracking (Fine tuning)
  - Align the receiver PN code phase to PN code phase of received signal as small as possible

Rake Receiver

- Multibranch receiver
  - Branches synchronized to different MP components



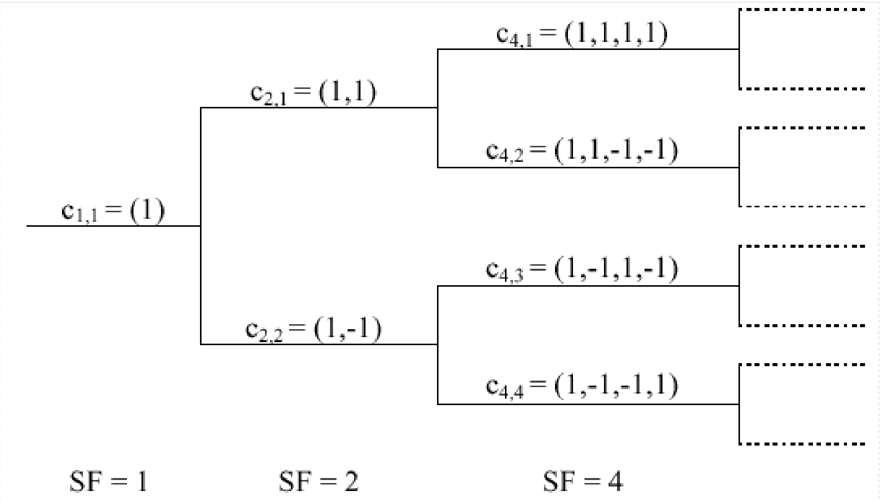
- These components can be coherently combined
  - Use SC, MRC, or EGC

Multiuser DSSS System (1)

- Spreading Codes
  - Interference between users is dictated by the cross-correlation of the spreading code.
  - Cross-correlation: $\rho_{ij}(\tau) = \frac{1}{T_s}\int_0^{T_s} s_{ci}(t)s_{cj}(t-\tau)dt = \frac{1}{N}\sum_{n=1}^{N} s_{ci}(nT_c)s_{cj}(nT_c-\tau)$
  - Orthogonal code set
    - $\rho_{ij}(\tau) = 0$ for all τ (asynchronous users)
    - $\rho_{ij}(0) = 0$ (synchronous users)
  - It is not possible to obtain orthogonal code set for asynchronous users
  - For synchronous users, there is only a finite number of spreading codes that are orthogonal within any given bandwidth
  - Ex) Gold, Kasami, Walsh-Hadamard code

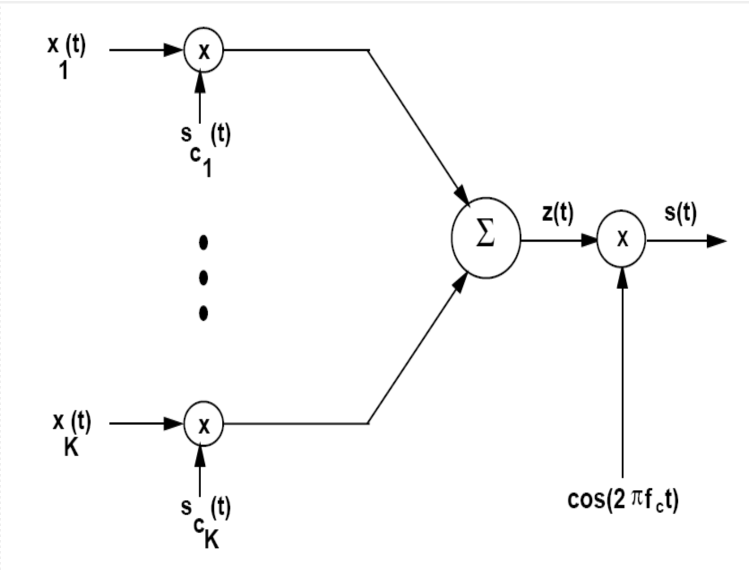
Orthogonal Code

- cross-correlation between othogonal codes is zero
- Walsh code (Hadamard matrix)
- $$H_0 = [0], \quad H_n = \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & \overline{H_{n-1}} \end{bmatrix}$$
- tree-structured code

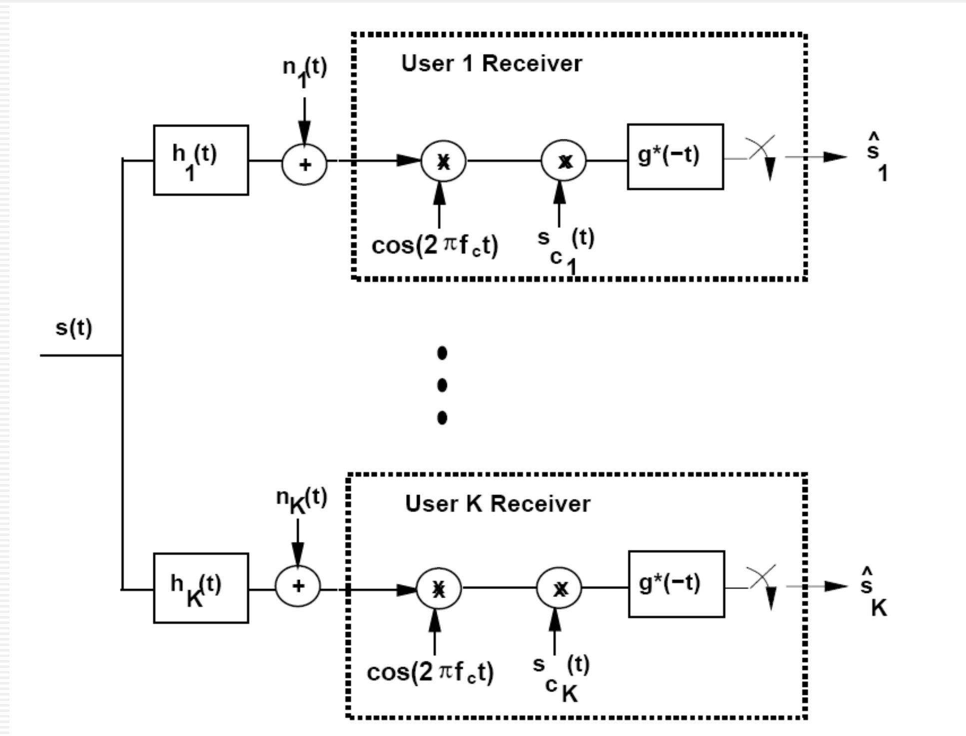


Multiuser DSSS System (2)

- Downlink Channels



Transmitter



Receiver

Multiuser DSSS System (3)

- Downlink Channels (over the lth symbol time)

$$\hat{s}_k = \sqrt{2/T_s}\int_0^{T_s}[s(t)*h_k(t)+n_k(t)]s_{ck}(t)\cos(2\pi f_c t)dt$$

$$= \sqrt{2/T_s}\int_0^{T_s}[z(t)*h_k^{LP}(t)]s_{ck}(t)\cos^2(2\pi f_c t)dt + \sqrt{2/T_s}\int_0^{T_s} n_k(t)s_{ck}(t)\cos(2\pi f_c t)dt$$

$$= 2/T_s\int_0^{T_s}\left[\sum_{j=1}^{K} s_{jl}s_{cj}(t)*h_k^{LP}(t)\right]s_{ck}(t)\cos^2(2\pi f_c t)dt + \sqrt{2/T_s}\int_0^{T_s} n_k(t)s_{ck}(t)\cos(2\pi f_c t)dt$$

$$= 2/T_s\int_0^{T_s}[s_{kl}s_{ck}(t)*h_k^{LP}(t)]s_{ck}(t)\cos^2(2\pi f_c t)dt$$

$$+ 2/T_s\int_0^{T_s}\left[\sum_{\substack{j=1\\ j\neq k}}^{K} s_{jl}s_{cj}(t)*h_k^{LP}(t)\right]s_{ck}(t)\cos^2(2\pi f_c t)dt +$$

$$+ \sqrt{2/T_s}\int_0^{T_s} n_k(t)s_{ck}(t)\cos(2\pi f_c t)dt$$

$$\approx \alpha_k s_{kl} + \boxed{\alpha_k \sum_{\substack{j=1\\ j\neq k}}^{K} s_{jl}\rho_{jk}(0)} + n_k \qquad \text{for } h_k(t) = \alpha_k\delta(t)$$

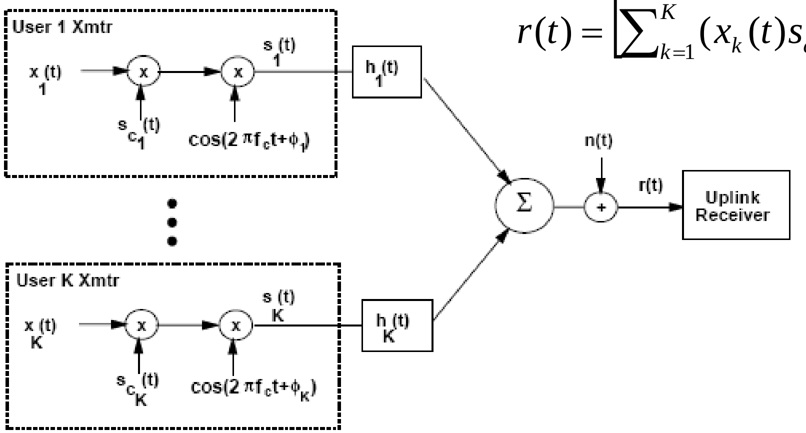
Multiuser interference

Interference Limited System

Multiuser DSSS System (4)

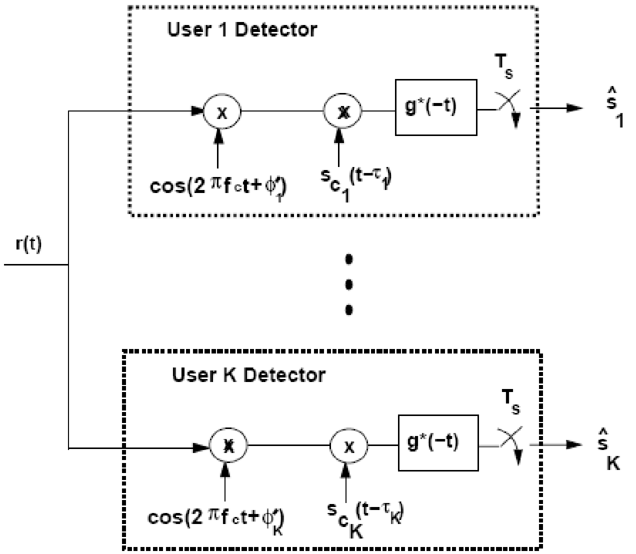
- Uplink Channels

Transmitter



$$r(t) = \left[\sum_{k=1}^{K} (x_k(t) s_{ck}(t) \cos(2\pi f_c t + \phi_k)) * h_k(t)\right] + n(t)$$

Receiver



Assumption

The LOS path of the kth user's channel introduces a delay of τ_k.

The phase offset resulting from this delay and the transmitter phase offset is matched by the phase offset ϕ'_k obtained by the carrier recovery loop in the kth receiver branch

Multiuser DSSS System (5)

- Uplink Channels

$$\hat{s}_k = \sqrt{2/T_s}\int_0^{T_s}\left[\sum_{j=1}^{K}(x_j(t)s_{cj}(t)) * h_k^{LP}(t)\right]s_{ck}(t-\tau_k)\cos(2\pi f_c t+\phi_k')\cos(2\pi f_c t+\phi_j')dt + n_k$$

$$= 2/T_s\int_0^{T_s}[s_{kl}s_{ck}(t) * h_k^{LP}(t)]s_{ck}(t-\tau_k)\cos^2(2\pi f_c t+\phi_k')dt$$

Multiuser interference

$$+ 2/T_s\int_0^{T_s}\left[\sum_{\substack{j=1\\ j\neq k}}^{K}s_{jl}s_{cj}(t) * h_j^{LP}(t)\right]s_{ck}(t-\tau_k)\cos(2\pi f_c t+\phi_k')\cos(2\pi f_c t+\phi_j')dt + n_k$$

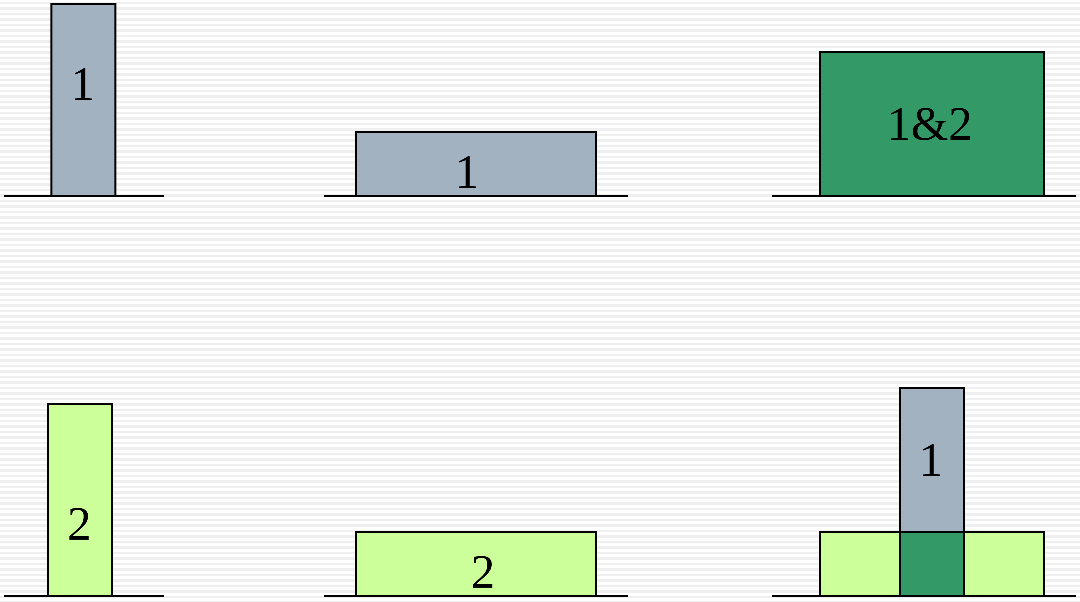
$$I_{kl} = 2/T_s\int_0^{T_s}\left[\sum_{\substack{j=1\\ j\neq k}}^{K}s_{jl}s_{cj}(t) * \alpha_j\delta(t-\tau_j)\right]s_{ck}(t-\tau_k)\cos(2\pi f_c t+\phi_k')\cos(2\pi f_c t+\phi_j')dt$$

$$= 1/T_s\int_0^{T_s}\left[\sum_{\substack{j=1\\ j\neq k}}^{K}\alpha_j s_{jl}s_{cj}(t-\tau_j)\right]s_{ck}(t-\tau_k)[\cos(\Delta\phi_{kj})+\cos(4\pi f_c t+\phi_k'+\phi_j')]dt$$

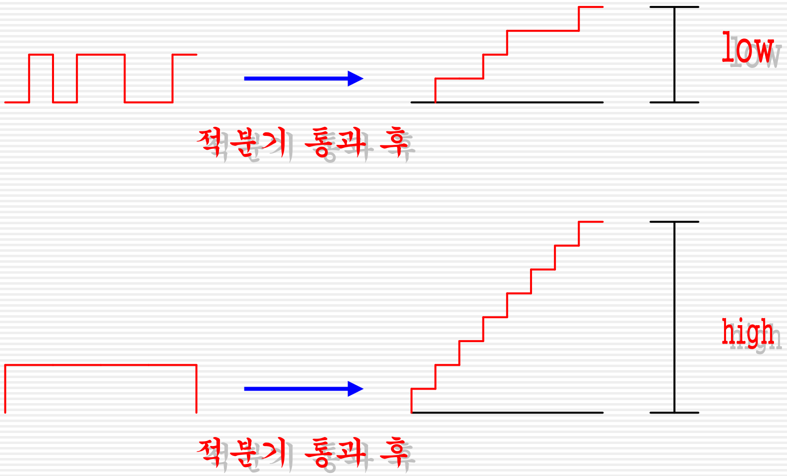
$$\approx \sum_{\substack{j=1\\ j\neq k}}^{K}\alpha_j\cos(\Delta\phi_{kj})s_{jl}\frac{1}{T_s}\int_0^{T_s}s_{cj}(t-\tau_j)s_{ck}(t-\tau_k)dt = \sum_{\substack{j=1\\ j\neq k}}^{K}\alpha_j\cos(\Delta\phi_{kj})s_{jl}\rho_{jk}(\tau_j-\tau_k)$$

Property of Direct Sequence SS (1)

- Multiple access capability



Soft Capacity



Property of Direct Sequence SS (2)

- Mutipath interference
  - If the code sequence has an ideal autocorrelation function, the correlation function is zero outside the interval $[-T_c, T_c]$, where T_c is chip duration.

  => the desired signal and a version delayed for more than $2T_c$ are received, despreading will treat the delayed version as an interfering signal.

  =>In conjunction with RAKE receiver: multipath diversity
- Privacy
  - Transmitted signal can only be despread and the data can be recovered if the code is known to the receiver
  - code: secret key