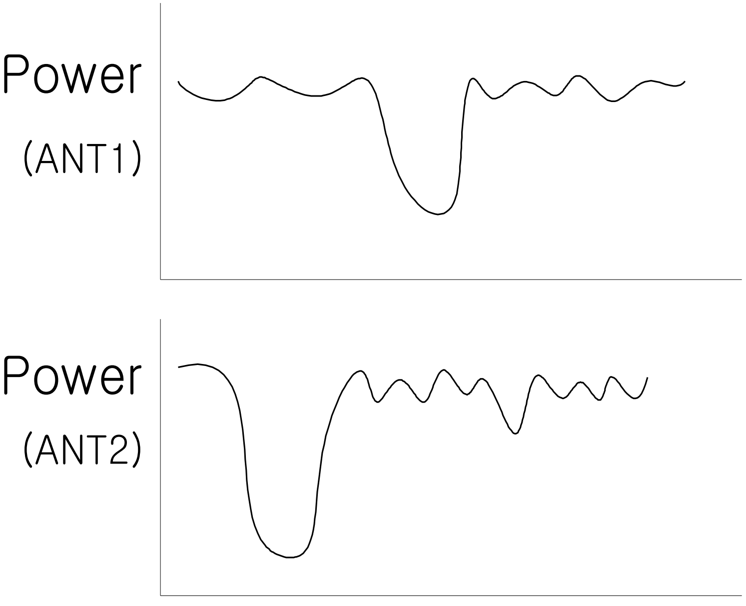
Diversity Techniques for Fading Channel

Contents

- Diversity & Combining techniques
- Space diversity
- Time diversity
- Interleaving

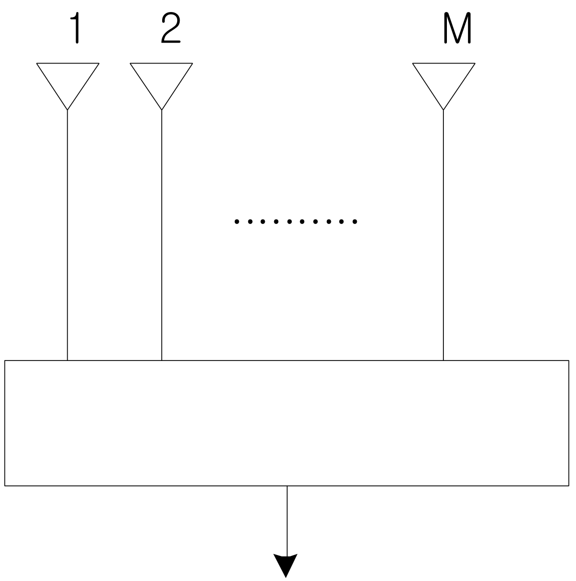
Diversity

- Spatial
- Frequency
- Time (Temporal)
- Polarization

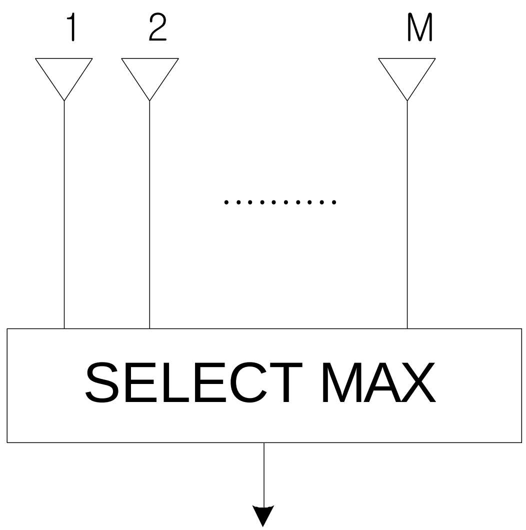


Combining techniques

- Selection
- EGC (Equal Gain Combining)
- MRC
  (Maximal Ratio Combining)



• Selection Diversity



$$P_b(BER) = \int P_b(R_s)\cdot p_s(R_s)dR_s$$

where $P_b(R_s)$: Prob. of error at SNR=R_s

$p_s(R_s)$: pdf of SNR for selection diversity

$$p(R) = \frac{1}{R_0}\exp\left(-\frac{R}{R_0}\right)$$

$$\text{where } R_0 = \frac{\text{mean signal power/branch}}{\text{mean noise power/branch}}$$

Find the PDF of R_s

$$F_X(x) = P[X < x]$$

$$F_X(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$f(x) = \frac{dF_X(x)}{dx}$$

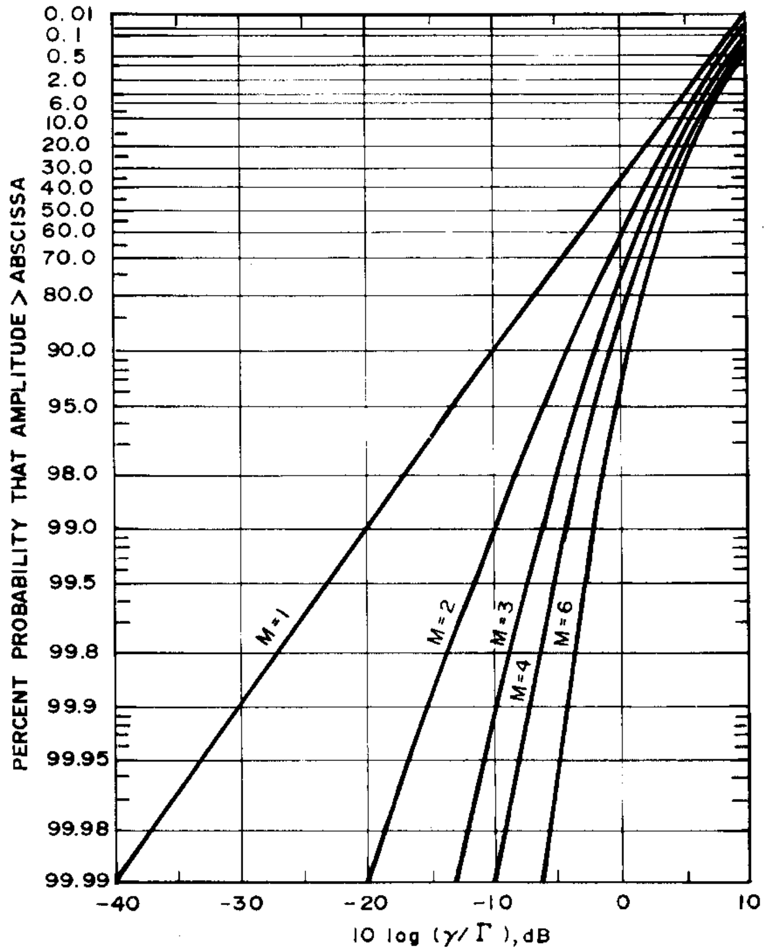
$$P[R < R_s] = \int_0^{R_s} \frac{1}{R_0} \exp\left(-\frac{R}{R_0}\right) dR = 1 - \exp\left(-\frac{R_s}{R_0}\right)$$

$$P[R_1, R_2, \ldots, R_M \le R_s] = \left\{1 - \exp\left(-\frac{R_s}{R_0}\right)\right\}^M$$

$$p[R_s] = \frac{dP[R_1, R_2, \ldots R_M \le R_s]}{dR_s} = \frac{M}{R_0} \exp\left(-\frac{R_s}{R_0}\right) \cdot \left(1 - \exp\left(-\frac{R_s}{R_0}\right)\right)^{M-1}$$

$$E[R_s] = \int R_s \cdot p[R_s]\,dR_s = R_0 \sum_{k=1}^{M} \frac{1}{k};$$

R_0: Mean SNR at the output of the selector

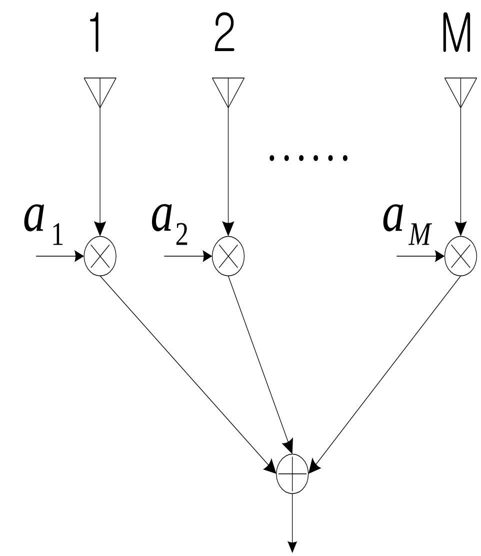


Prob. distribution of SNR R_s for
M branch selection diversity

• MRC (Maximal Ratio Combining)

– *kth* antenna : envelope r_k

gain α_k



– Combined signal envelope:

$$r_R = \sum_{k=1}^{M} a_k r_k$$

* cophasing

– Combined noise power $= N_{tot} = N \sum a_k{}^2$

N: Noise power per branch

– Combined signal SNR = R_R

$$= \frac{(r_R)^2}{2} \cdot \frac{1}{N_{t_0 t}}$$

$$= \frac{1}{2} \frac{\left(\sum a_k r_k\right)^2}{N \sum a_k{}^2} \le \frac{1}{2} \frac{\sum a_k{}^2 \sum r_k{}^2}{N \sum a_k{}^2}$$

At $a_k = \frac{r_k}{N}$, SNR is maximized

$$R_R = \frac{1}{2} \cdot \frac{\left(\sum \left(\frac{r_k}{N}\right) r_k\right)^2}{N \sum \left(\frac{r_k}{N}\right)^2} = \frac{1}{2} \sum \frac{r_k{}^2}{N} = \sum_{i=1}^{M} R_i$$

– Find $p(R_R)$, given $p(R_i)$

$$p(R_i) = \frac{1}{R_0}\exp\left(-\frac{R_i}{R_0}\right)$$

$$\Phi_{R_i}(\omega) = E\left[e^{j\omega R_i}\right] = \int_0^\infty p(R_i)e^{j\omega R_i}dR_i$$

$$= \int_0^\infty \frac{1}{R_0}\exp\left(-\frac{R_i}{R_0}\right)e^{j\omega R_i}dR_i$$

$$= \frac{1}{1-j\omega R_0}$$

$$\therefore \Phi_{R_R}(\omega) = \left(\frac{1}{1-j\omega R_0}\right)^M$$

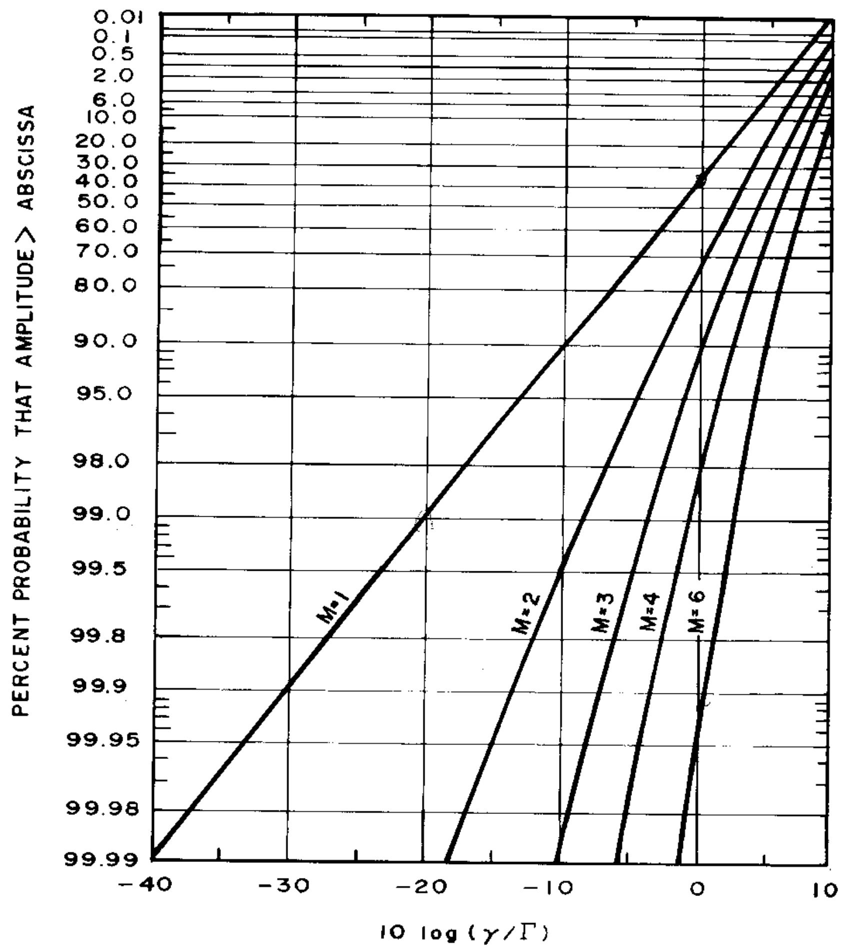
$$p(R_R) = \frac{R_R^{M-1}}{R_0{}^M(M-1)!}\exp\left(-\frac{R_R}{R_0}\right)$$

- For BPSK

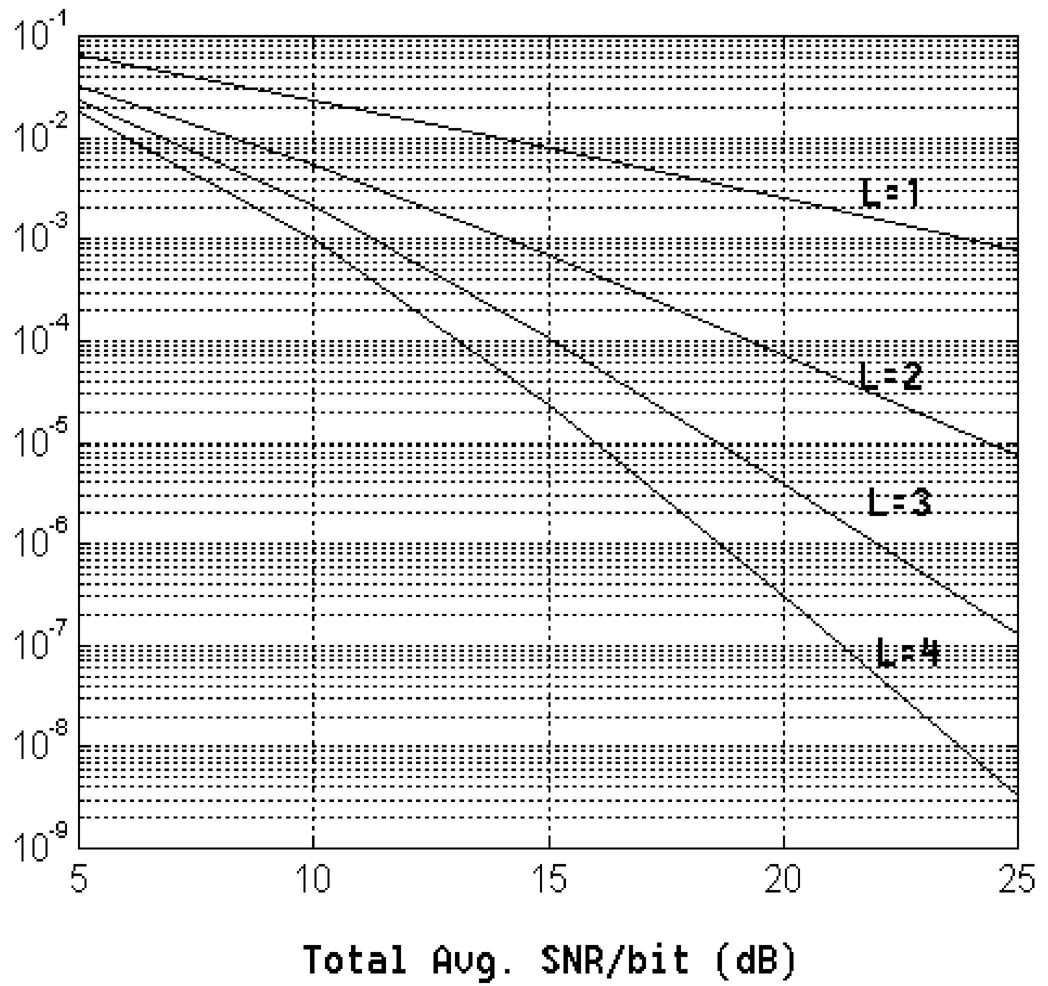
$$P_{b(\,MRC\ system)} = \int_0^\infty P_b(R)\, p_{R_R}(R)\, dR$$

$$= \frac{1}{2}\left[1 - \mu \sum_{k=0}^{M-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4} \right)^k \right]$$

$$\text{where } \mu = \sqrt{\frac{R_0}{1+R_0}}$$



Prob. distribution of SNR R_s for *M* branch MRC



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
5
10
15
20
25
L=1
L=2
L=3
L=4
Total Avg. SNR/bit (dB)

- EGC (Equal Gain Combining)

  - Combined signal env. $r_E = \sum_{k=1}^{M} r_k$
  - Combined noise power =N·M

    (N : Noise power per branch)

  - SNR $= R_E = \frac{r_E^{\ 2}}{2NM} = \frac{\left(\sum r_k\right)^2}{2NM}$

$$\overline{R_E} = \frac{1}{2NM}\overline{\left(\sum_k r_k\right)^2} = \frac{1}{2NM}\sum_j\sum_k \overline{(r_j r_k)}$$

$$= \frac{1}{2NM}\left(\sum_{k=1}^{M}\overline{r_k^{\,2}} + \sum_{j=1}^{M}\sum_{\substack{k=1\\k\neq j}}^{M}\overline{(r_j r_k)}\right)$$

$$= \frac{1}{2N}\overline{r_k^{\,2}} + \frac{M(M-1)\left(\overline{r_k}\right)^2}{2MN}$$

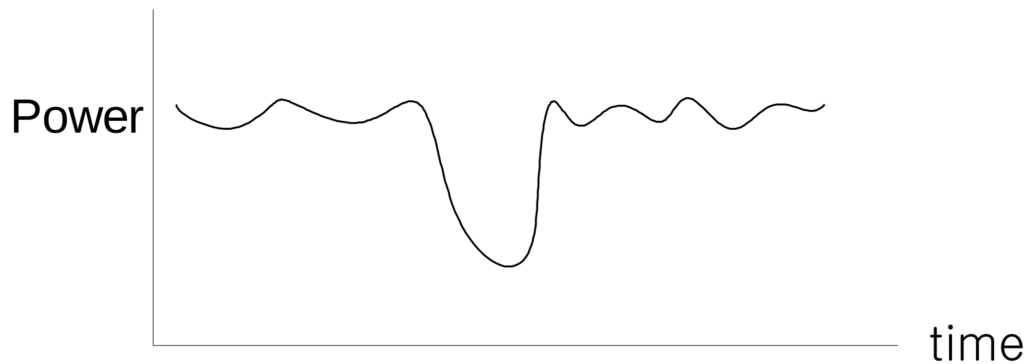
$$\because \overline{r_j r_k} = \overline{r_j}\cdot\overline{r_k} \qquad (\overline{r_j} \text{ and } \overline{r_k} \text{ is independent})$$

$$= R_0 + (M-1)\left(\frac{\pi}{4}\right)R_0$$

$$\because \frac{\left(\overline{r_k}\right)^2}{2N} = \frac{\left(\sqrt{\frac{\pi}{2}}\sqrt{mean\ signal\ \ power\ \ per\ branch}\right)^2}{2N}$$

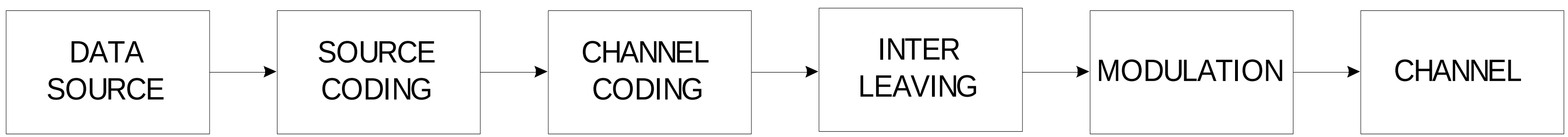
$$= R_0\left(1 + (M-1)\frac{\pi}{4}\right)$$

Time diversity



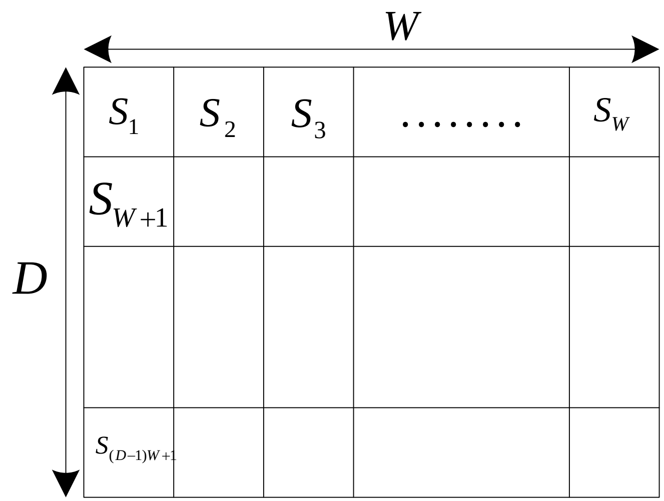
Data : 111222333 • • • $\Longleftrightarrow$ 12 • • • 1 • • • 1 • • •

Interleaving



- Process of rearranging the ordering of a sequence of symbols in some one-to-one deterministic manner.

• Block interleaving



Tx data sequence : $S_1, S_{W+1}, S_{2W+1} \cdots$

Tx delay : $W(D-1)+1 \approx WD$

Rx delay : $D(W-1)+1 \approx WD$

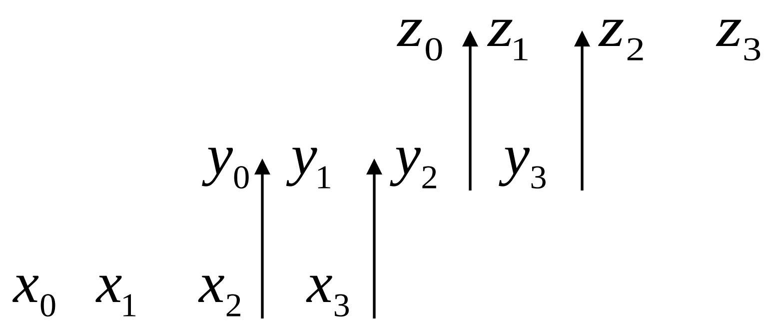
Total delay $\approx 2WD$

Example:

S_1	S_2
S_3	S_4
S_5	S_6
S_7	S_8
S_9	S_{10}

$$S_1 S_3 \ldots . S_9 S_2 S_4 \ldots S_{10}$$

- Diagonal Interleaving



Tx data sequence:

$$x_2 y_0 x_3 y_1 y_2 z_0$$