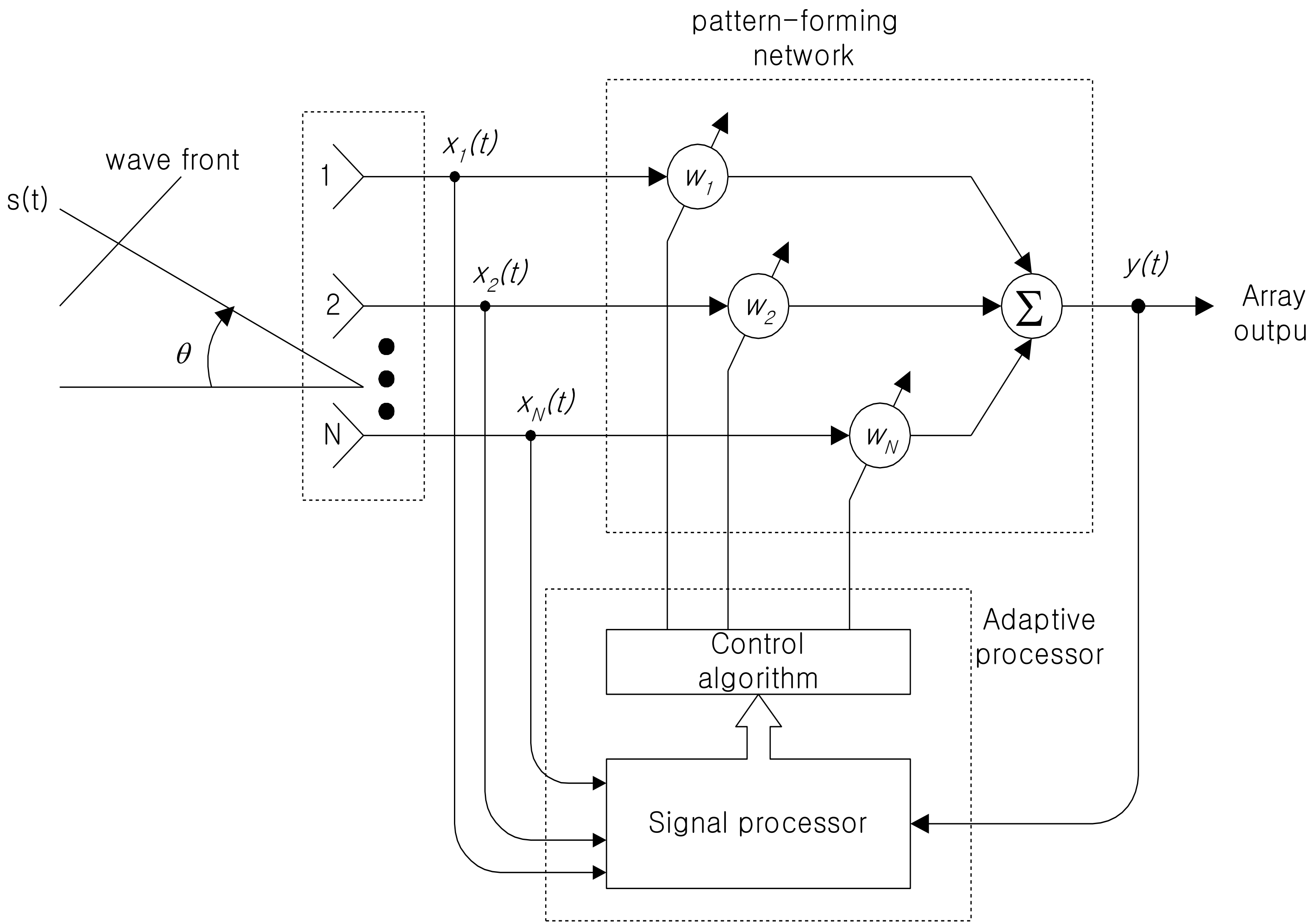
Smart Antennas (Adaptive array)

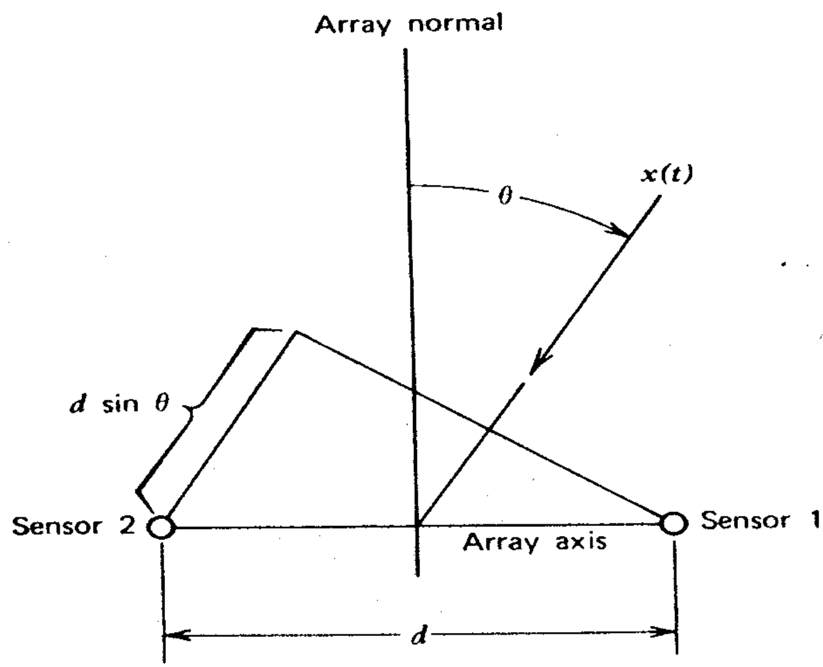
- Omnidirectional
- Directional
- Adaptive array

Ref: R. A. Monzingo and T. W. Miller, ***Introduction to Adaptive Arrays*.** Wiley, 1980



Functional diagram of an N-dimensional adaptive array

Adaptive array 개념



Element 2 experiences a time delay w.r.t element 1

$$\tau = \frac{d \sin\theta}{v}$$

Array output signal

$$y(t) = x(t) + x(t-\tau)$$

Phase shift $\psi = 2\pi \frac{d \sin\theta}{\lambda_0}$ (wave length = $\frac{v}{f_0}$)

$$y(t) = x(t) + x(t)e^{-j\psi}$$

$$= \sum_{i=1}^{2} x(t) e^{-j(i-1)\psi} = x(t)\sum_{i=1}^{2} e^{-j(i-1)\psi} = x(t)\left(1 + e^{-j\psi}\right)$$

$$\text{Gain} = y(t)/x(t) = \sum_{i=1}^{2} e^{-j(i-1)\Psi} = 1 + e^{-j\Psi} = 1 + e^{-j\left(2\pi \frac{d\sin\theta}{\lambda_0}\right)}$$

Directional pattern $A(\theta) = \sum_{i=1}^{2} e^{-j(i-1)\left(2\pi \frac{d\sin\theta}{\lambda_0}\right)}$

Normalized $G(\theta)$ (decibels) $= 10\log\left\{\frac{|A(\theta)|^2}{N^2}\right\}$

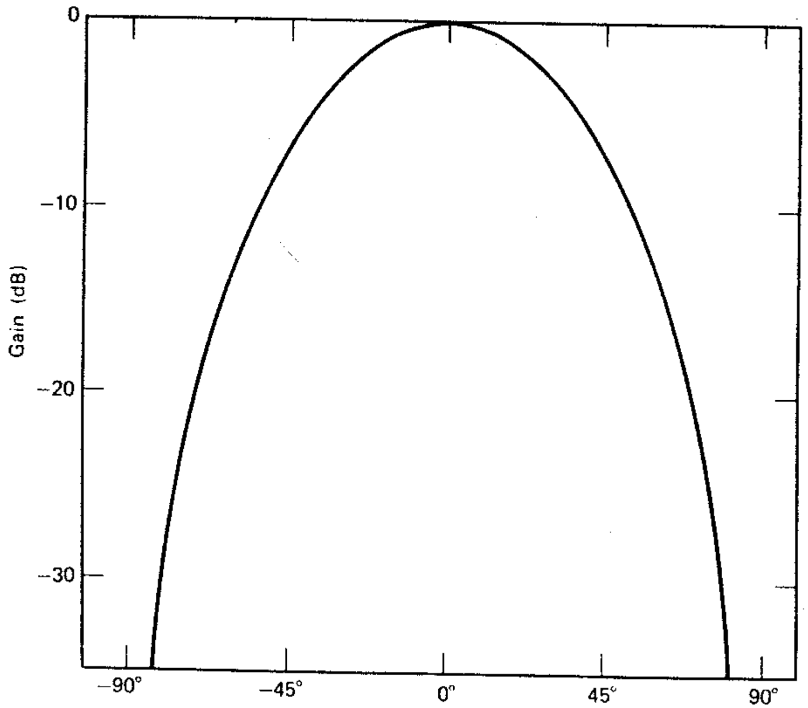
$$\cos(\omega_c t) \qquad \cos(\omega_c t - \psi)$$

$$\cos(\omega_c t) \qquad \cos\psi \cos(\omega_c t) + (-\sin\psi)(-\sin(\omega_c t))$$

complex 1
envelope

$$\cos\psi - j\sin\psi = e^{-j\psi}$$

For $d = 0.5\lambda_0$



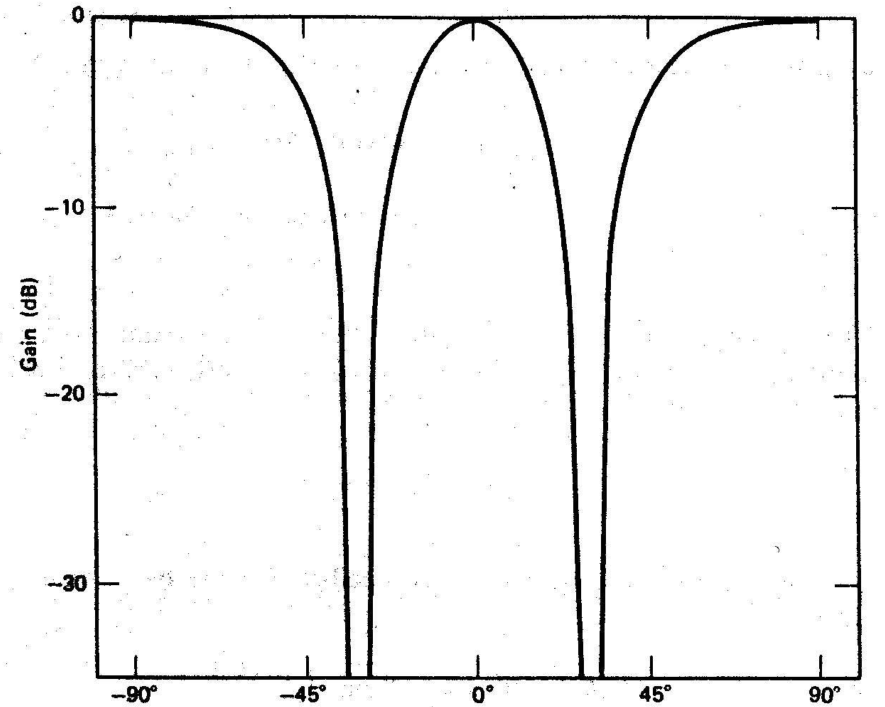
$$A(\theta) = 1 + e^{-j\pi \sin\theta}$$

when $\sin\theta = +1 \ or \ -1$

$$\theta = 90^\circ \ or \ -90^\circ$$

$$A(\theta) = 0$$

For $d = \lambda_0$



$$A(\theta) = 1 + e^{-j2\pi \sin\theta}$$

Find which satisfies

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ \quad A(\theta) = 0$$

$$d\sin\theta = \frac{\lambda_0}{2}$$ (path delay=$\frac{\lambda_0}{2}$)

Linear Array

$$y(t) = \sum_{i=1}^{N} x(t) e^{-j(i-1)\psi}$$

$$\left(H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h(k) e^{-j\omega k} \right)$$

$$A(\theta) = \sum_{i=1}^{N} e^{-j(i-1)\psi}$$

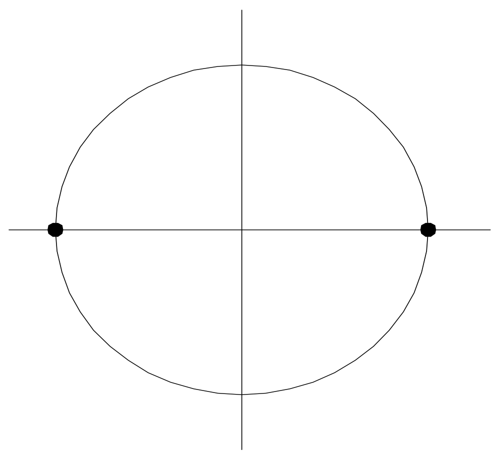
$$G(\theta) = 10 \log_{10} \left\{ \frac{|A(\theta)|^2}{N^2} \right\}$$

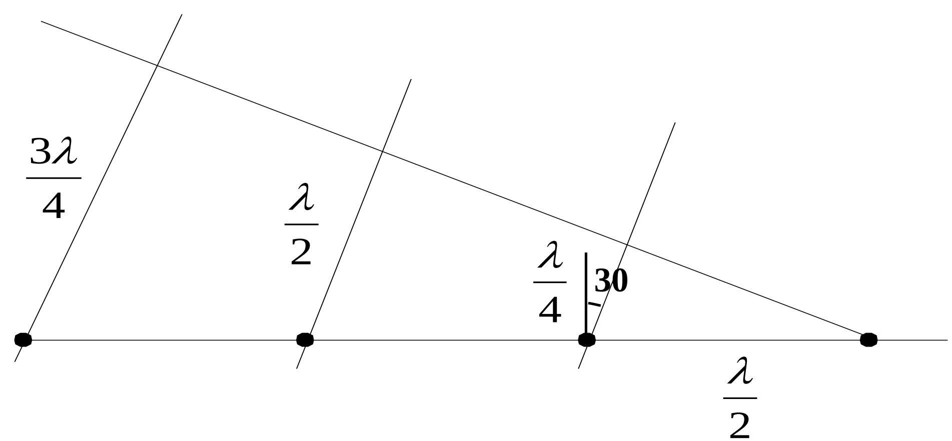
Linear array with 4 elements, $d = \frac{\lambda}{2}$

For $N = 4$, $A(\theta) = \sum_{i=1}^{4} e^{-j(i-1)\psi} = \sum_{i=0}^{3} e^{-ji\psi}$

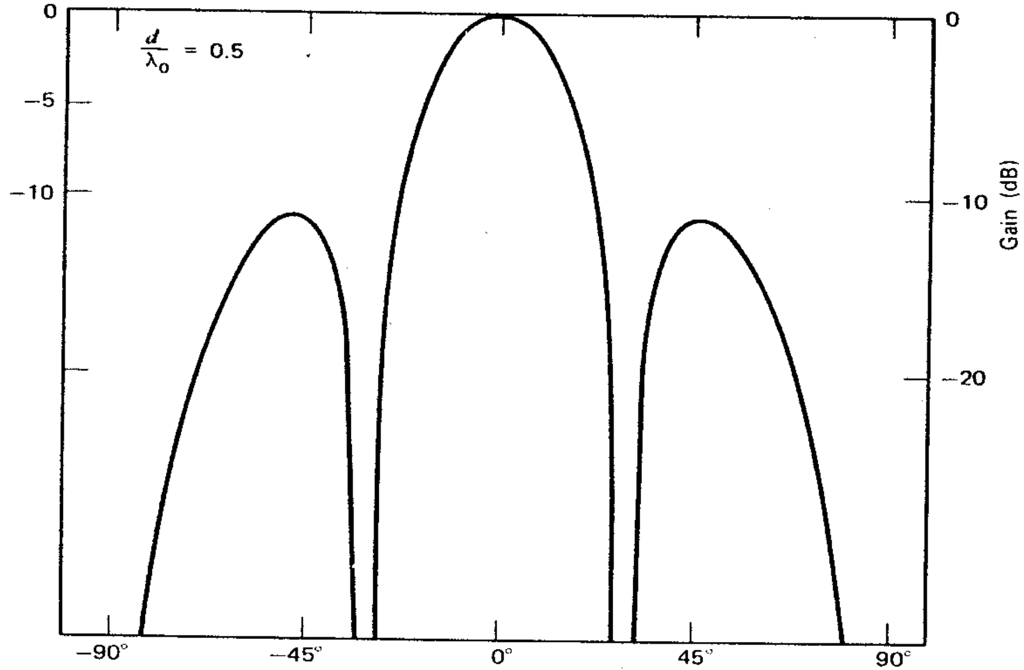
For $\psi = \pi, A(\theta) = e^{-j0} + e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} = 0 \quad (\theta = 90^\circ)$

For $\psi = \frac{\pi}{2} A(\theta) = e^{-j0} + e^{-j\frac{\pi}{2}} + e^{-j\pi} + e^{-j\frac{3\pi}{2}} = 0 \quad (\theta = 30^\circ)$





$* \Psi = 2\pi d \sin\theta / \lambda_o$



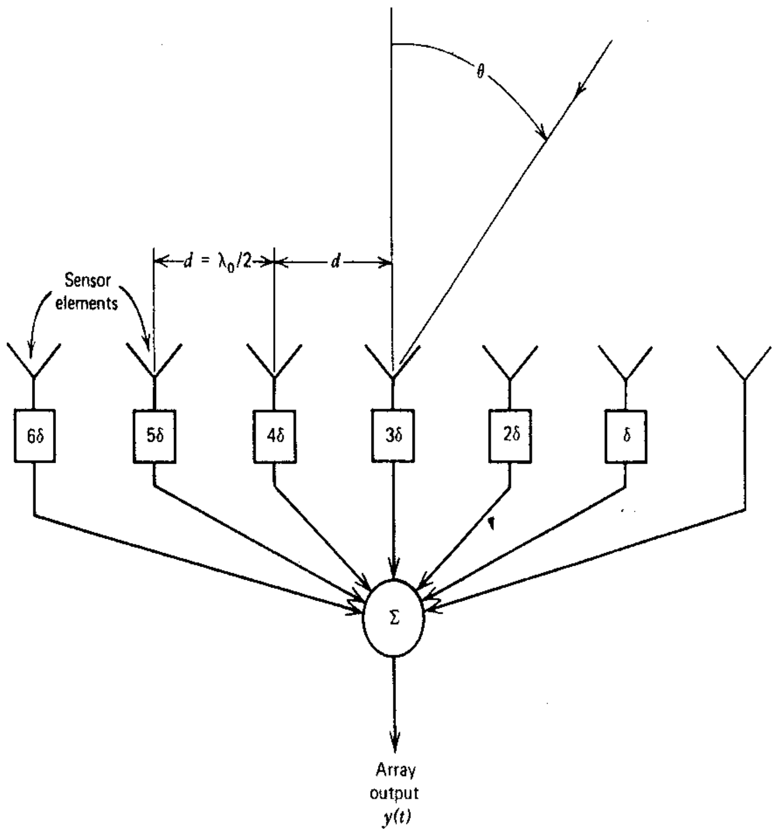
As the number of elements ↑

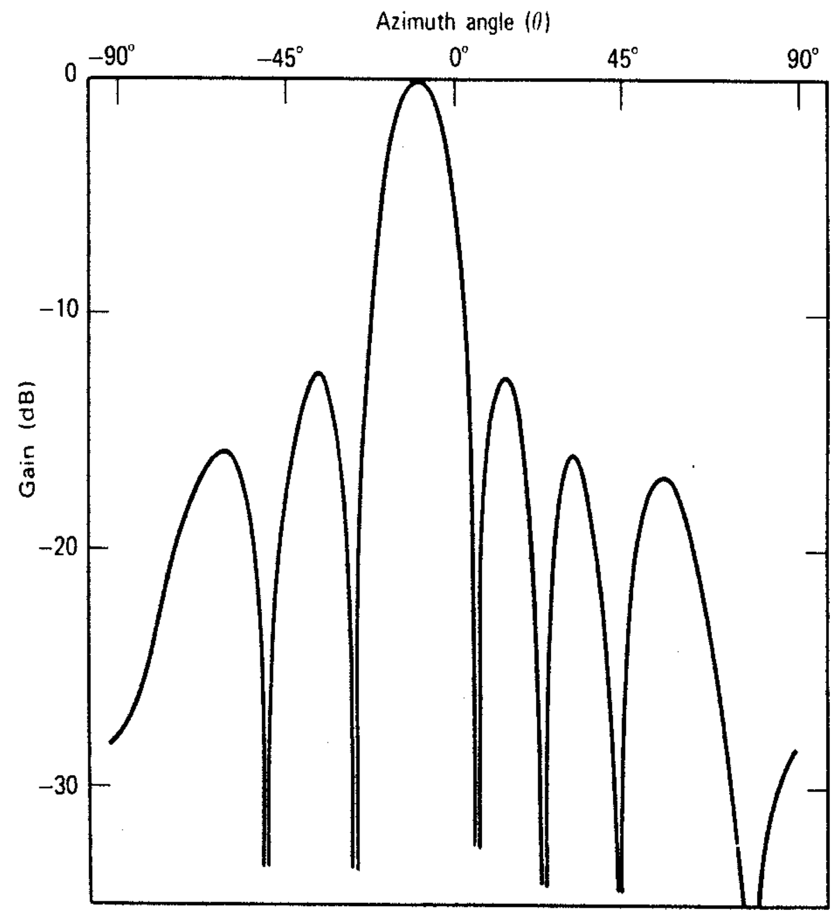
the main lobe beamwidth ↓

the number of sidelobes ↑

the number of nulls ↑

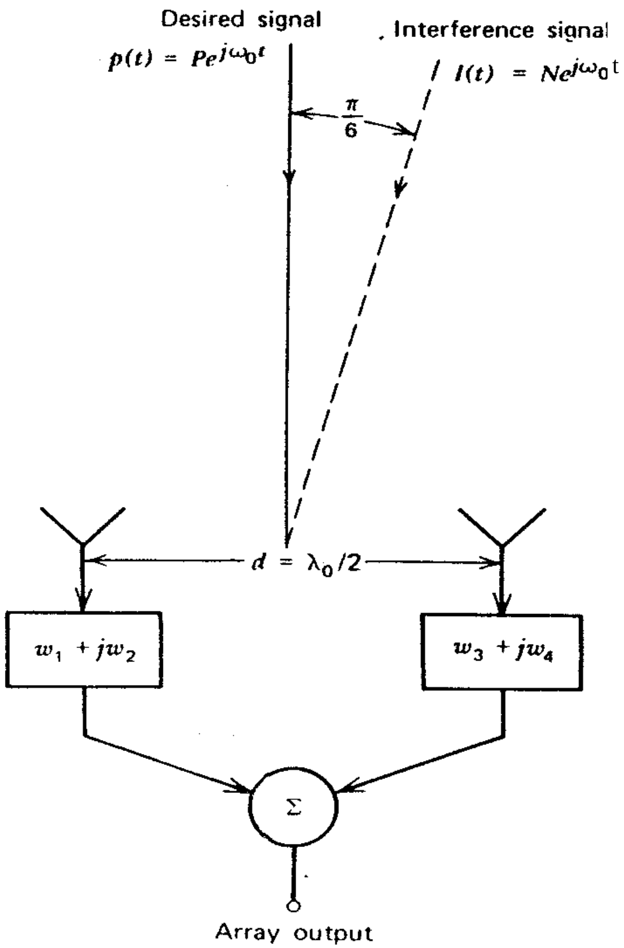
Linear Array with Phase Shifters





* Planar Arrays

Signal Reception by Weight Adjustment



- For output signal: $p(t) = Pe^{j\omega_0 t}$

$$w_1 + w_3 = 1$$
$$w_2 + w_4 = 0$$

- The array output due to the incident noise:

$$Ne^{j\left(\omega_0 t-\frac{\pi}{4}\right)}\left[w_1+jw_2\right]+Ne^{j\left(\omega_0 t+\frac{\pi}{4}\right)}\left[w_3+jw_4\right]$$

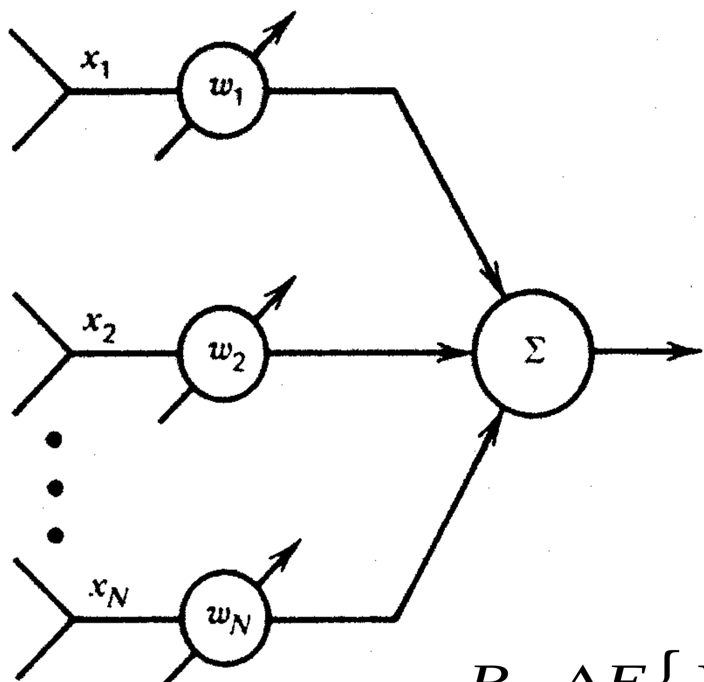
$$w_1+w_2+w_3-w_4=0$$ (phase diff: $\frac{\pi}{2}$)

$$-w_1+w_2+w_3+w_4=0$$

$$\Rightarrow w_1=\frac{1}{2},\quad w_2=-\frac{1}{2},\quad w_3=\frac{1}{2},\quad w_4=\frac{1}{2}$$

- The directions of signals are known

Determination of Optimal Weights (MSE)



$$y(t) = \underline{w}^T \underline{x}(t)$$

$$R_{xx} \underline{\underline{\Delta}} E\left\{ \underline{X}(t) \underline{X}^T(t) \right\} = R_{ss} + R_{nn}$$

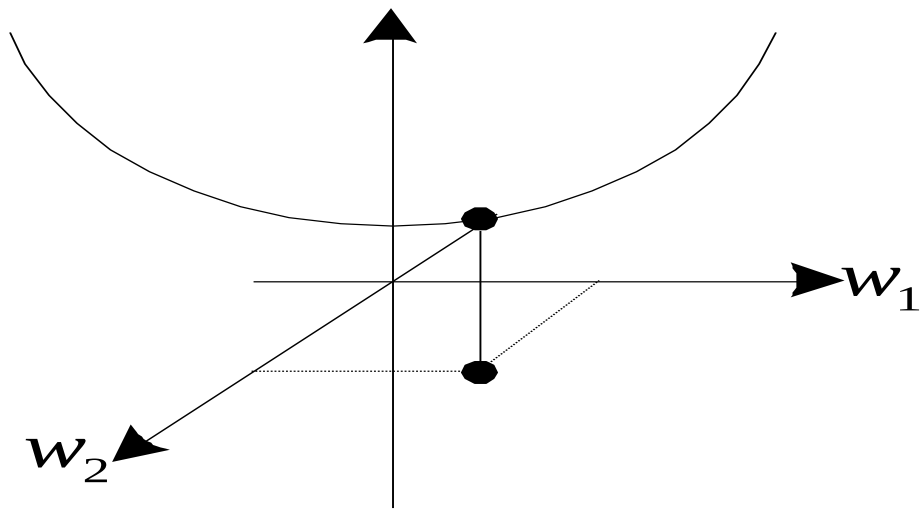
$$\varepsilon(t) = d(t) - \underline{w}^T \underline{x}(t)$$

$$\varepsilon^2(t) = d^2(t) - 2d(t)\underline{w}^T \underline{x}(t) + \underline{w}^T \underline{x}(t)\underline{x}^T(t)\underline{w}$$

$$J = E\left\{\varepsilon^2(t)\right\} = \overline{d^2(t)} - 2\underline{w}^T \underline{\gamma}_{xd} + \underline{w}^T R_{xx}\,\underline{w}$$

$$\text{where } \underline{r}_{xd} = \left[\overline{\underline{X}_1(t)d(t)},\ \overline{\underline{X}_2(t)d(t)},\ldots\overline{\underline{X}_N(t)d(t)}\right]$$

$$\nabla_w(J) = \frac{\partial J}{\partial \underline{w}} = -2\underline{\gamma}_{xd} + 2\underline{R}_{xx}\,\underline{w}$$



$$\underline{w}_{opt} = \underline{R}_{xx}^{-1}\underline{\gamma}_{xd}$$

Wiener-Hopf-solution

* Different Criteria

Adaptive Beam Forming

unknown 환경, time varying channel

Steepest Descent

$$\underline{w}(n+1) = \underline{w}(n) + \frac{1}{2}\mu\left[-\nabla_w J(n)\right]$$
$$= \underline{w}(n) + \mu\left[\underline{r}_{xd} - R_{xx}\underline{w}(n)\right]$$

LMS

$$\hat{\underline{R}}_{xx}(n) = \underline{X}(n)\underline{X}^T(n)$$

$$\hat{\underline{r}}_{xd}(n) = \underline{X}(n)d(n)$$

$$\hat{\nabla}J_w(n) = -2\underline{X}(n)d(n) + 2\underline{X}(n)\underline{X}^T(n) \quad \underline{W}(n)$$

$$= -2\underline{X}(n)\left(d^*(n) - \hat{d}(n)\right) = -2\underline{X}(n)e(n)$$

$$\underline{w}(n+1) = \underline{w}(n) + \mu\underline{X}(n) \times e(n)$$

step size μ: convergence, stability

simple, slow(convergence depends on the eigenvalues of R)

RLS

$$R(n) = \delta_0 R(n-1) + \underline{X}(n)\underline{X}^H(n)$$

* Use of a reference signal
* Blind Estimation

CMA (Constant Modulus algorithm)

$$J(n) = \frac{1}{2} E\left\{ \left(\left| y(n) \right|^2 - y_0^2 \right)^2 \right\}$$

The existence of an interference causes fluctuation in the amplitude of the array output.

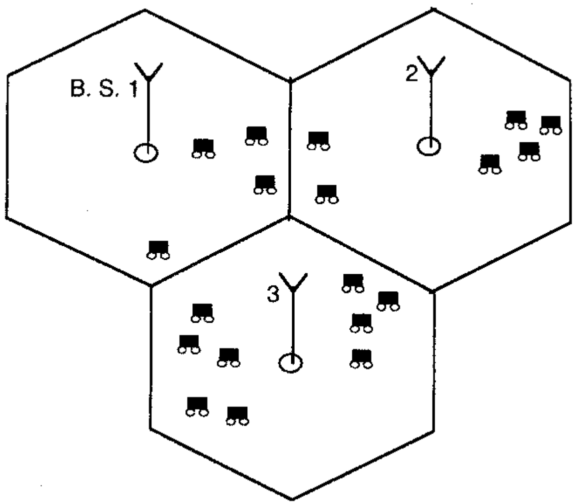
DOA Estimation Methods

- Spectral Estimation Methods
- Linear Prediction Method
- Music...

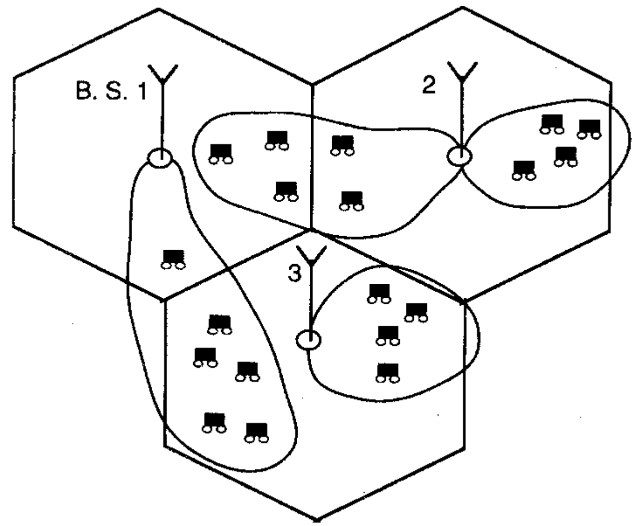
Cost and Complexity

- System requires quick updates for fast moving mobiles.
- Response time is limited by the time required by DOA estimation & beam-forming algorithms to converge.

Dynamic Cell Formation

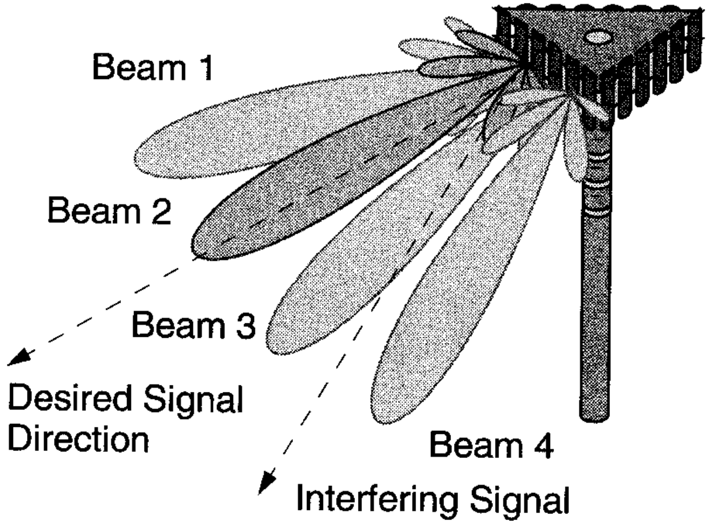


(a)

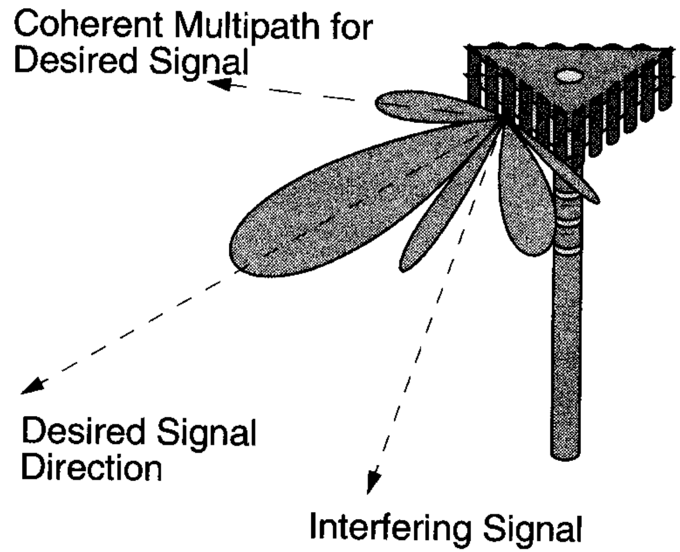


(b)

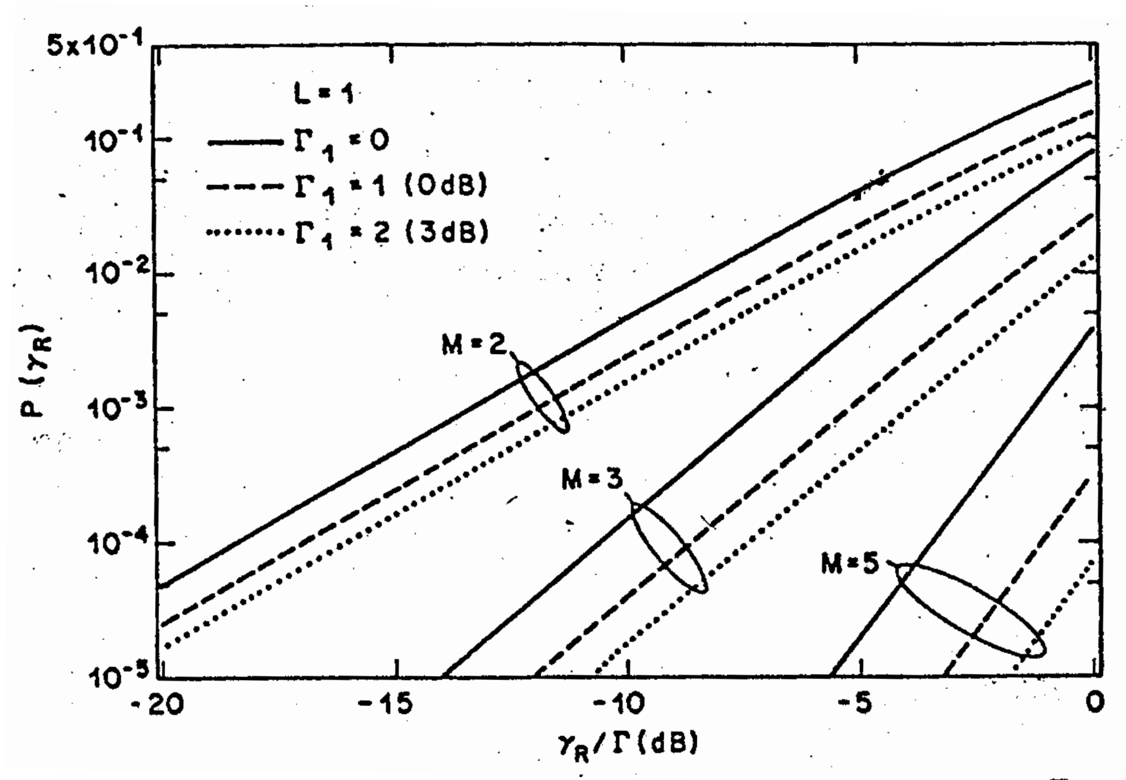
Switched Beam



(a) Switched Beam Systems can select one of several beams to enhance receive signals. Beam 2 is selected here for the desired signal.



(b) An adaptive antenna can adjust its antenna pattern to enhance the desired signal, null or reduce interference, and collect correlated multipath power.



The cumulative distribution of γ_R versus γ_R/Γ for optimum combining when the desired and interfering signals are subject to fading. Results are shown for one interferer with several values of M and Γ_1. The distribution function for γ_R with fixed average received SINR is shown to decrease as the power of the interferer becomes a larger proportion of the total noise plus interference power. The decrease is even larger as M increases.

Space diversity vs Adaptive array

- 수신신호: Desired signal + Interference signal + Noise
- 일반적인 space diversity
  - mitigating multipath fading of the desired signal
  - interference signals at receiving antennas are assumed to be independent
  - maximal ratio combining: max SINR
- Adaptive array diversity
  - Combats multipath fading of desired signal
  - Reduces the power of interfering signals at the receiver
    Interfering signals: correlated
    Correlated interfering signals are combined
  - Achieves higher output SINR than MRC