
Introduction to Spread Spectrum

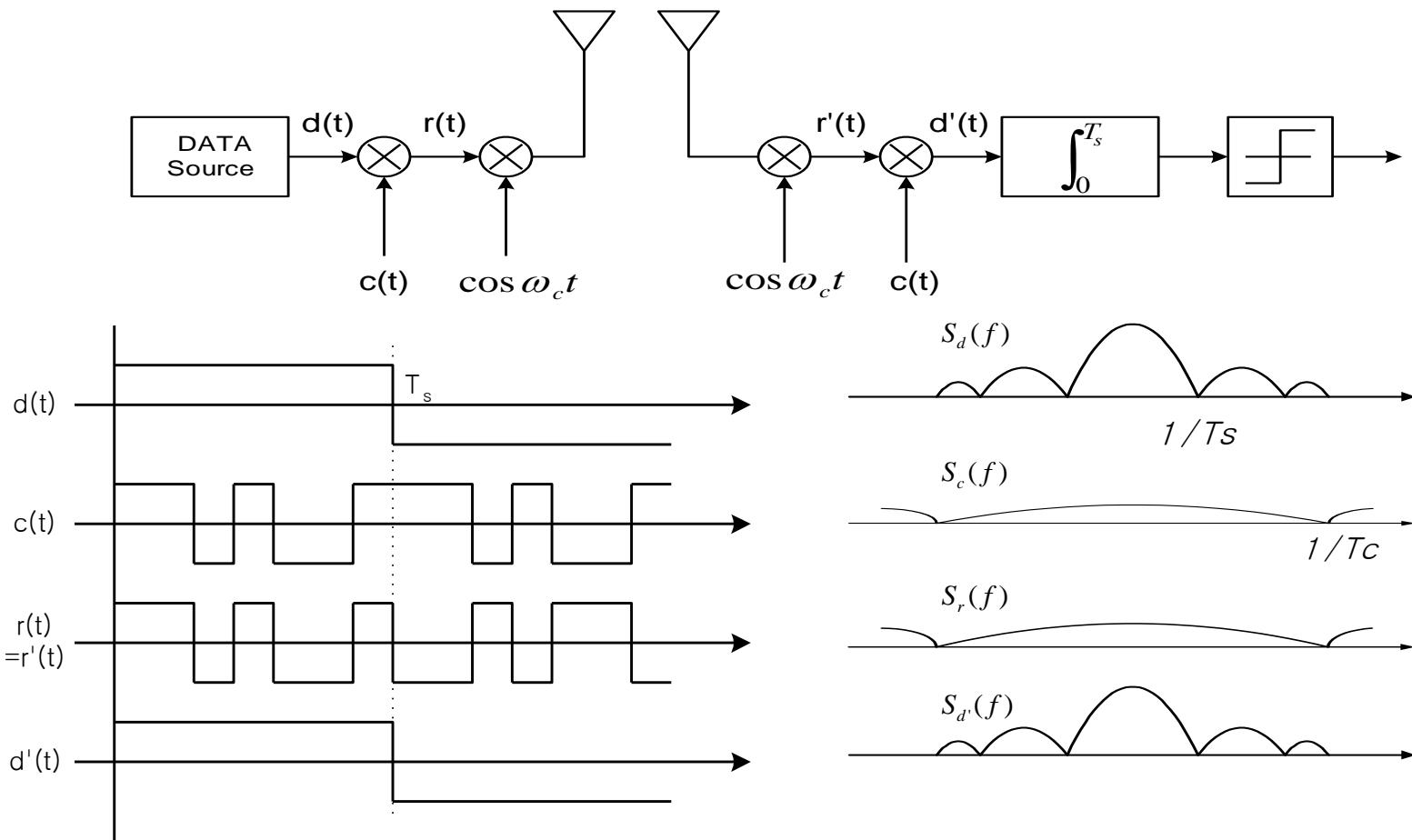
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- Introduction
- DS/SS
- Frequency hopping

Introduction

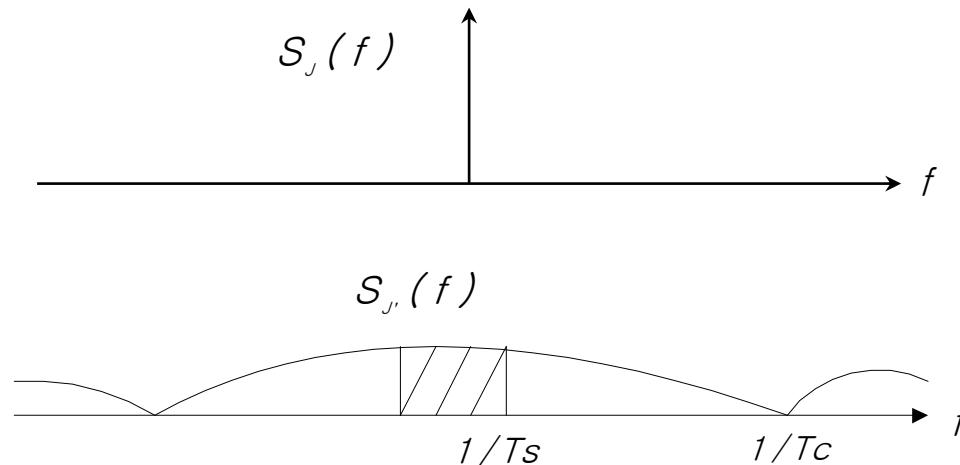
- Initial application: military
- Benefits
 - Anti-jamming
 - Robust to multipath fading
 - Multiple User Access (CDMA)
 - High resolution ranging (DS)

DS/SS



Anti-Jamming effect

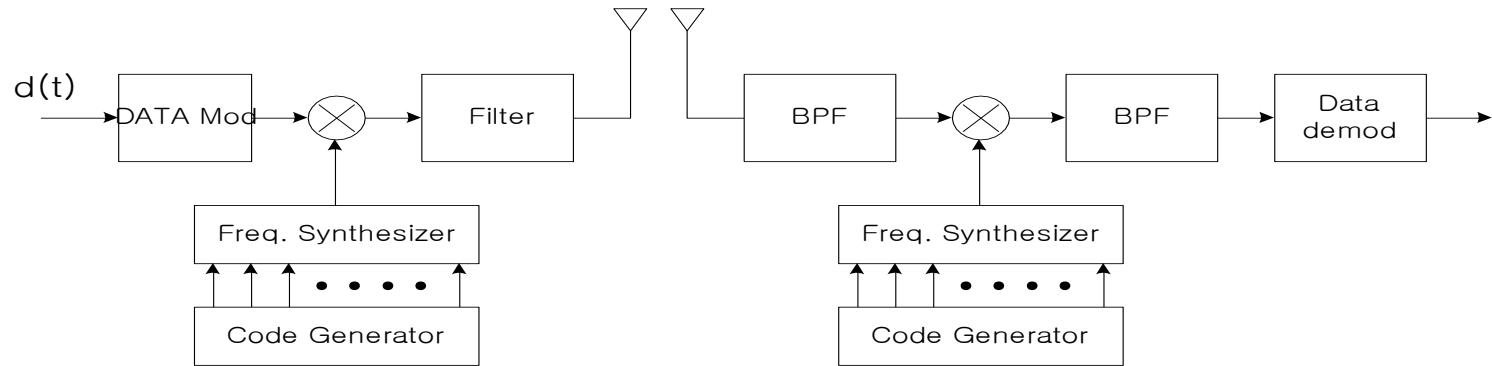
- Interfering Signal: tone



- Tone Jamming signal is spread out by despreading operation.
- Small part of a jamming signal goes through the rx filter.
- Jamming effect reduction factor : $\frac{T_s}{T_c}$

Frequency hopping

- The carrier frequency is pseudo randomly hopped over various frequencies.



Slow hopping : $T_{\text{symbol}} < T_{\text{hopping}}$

Fast hopping : $T_{\text{symbol}} > T_{\text{hopping}}$

* Noncoherent scheme is common.

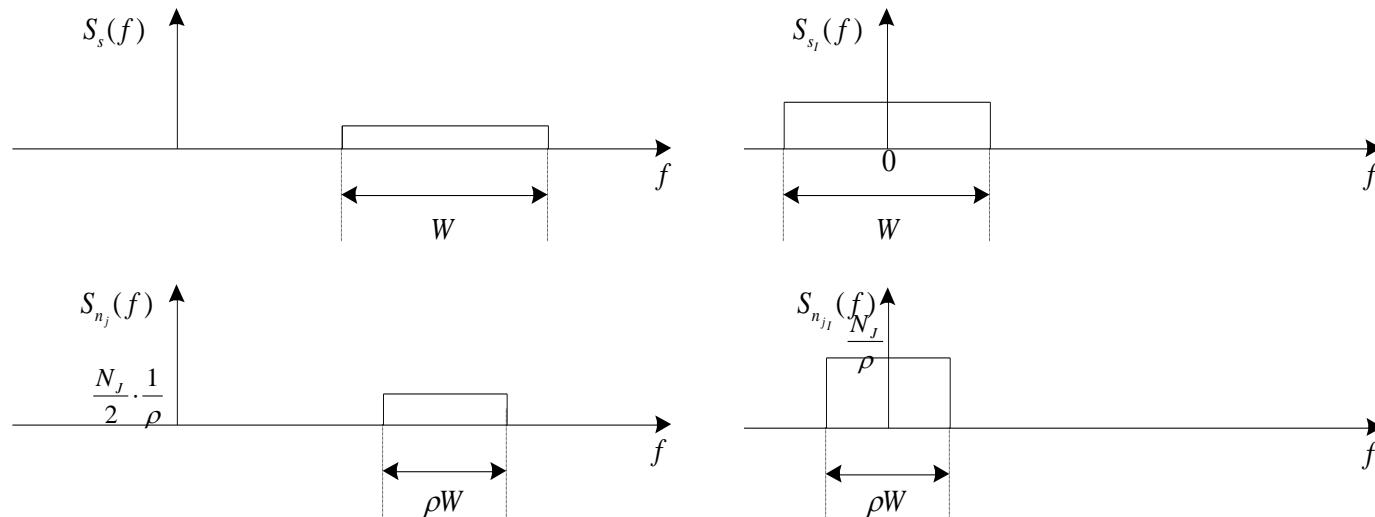
Performance of SS (Jamming)

Contents

- Performance in partial band jamming in DS/CDMA systems
- Pulse noise jammer in DS/CDMA systems
- Partial band jammer in FH systems
- Reference : “Introduction to spread spectrum communications.” by peterson...

Performance in partial band jamming

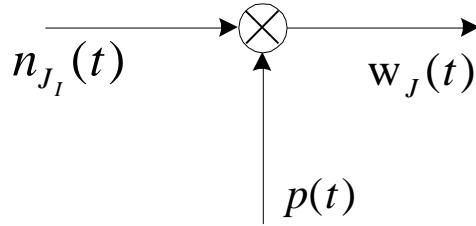
$$r(t) = s(t) + n(t) + n_j(t); \quad s(t) \in [BW = W], \quad PBJ \in [BW = \rho W]$$



Total Jamming Power = J

$$\frac{N_J}{2} = \frac{J}{2W}$$

$$\text{PBJ PSD} = \frac{N_J}{2} \cdot \frac{1}{\rho}$$



$$\begin{aligned}
 W_J(t) &= n_{J_I}(t) \cdot p(t) \\
 \Rightarrow R_{W_J}(\tau) &= R_{N_{J_I}}(\tau) \cdot R_p(\tau) \\
 S_{W_J}(f) &= S_{n_{J_I}}(f) \otimes S_p(f) \\
 &= \int_{-\infty}^{\infty} T_c \sin c^2(\tau T_c) S_{N_{J_I}}(f - \tau) d\tau \\
 &= \int_{f - \frac{\rho W}{2}}^{f + \frac{\rho W}{2}} T_c \sin c^2(\tau T_c) \frac{N_J}{\rho} d\tau \\
 &= \frac{N_J}{\rho} T_c \int_{f - \frac{\rho W}{2}}^{f + \frac{\rho W}{2}} \sin c^2(\tau T_c) d\tau
 \end{aligned}$$

Case 1 :

$$\rho = 1$$

$$S_{W_J}(0) = N_J T_c \int_{-\frac{W}{2}}^{\frac{W}{2}} \sin c^2(\tau T_c) d\tau$$

$$\text{If } \frac{W}{2} = \frac{1}{T_c}, \quad S_{W_J}(0) = 0.9 N_J$$

Case 2 :

$$\text{small } \rho, \quad \rho W \ll \frac{1}{T_c} \Rightarrow \sin c^2(\tau T_c) \cong 1$$

$$S_{W_J}(0) = \frac{N_J}{\rho} T_c \rho W = N_J T_c \mathbf{W}$$

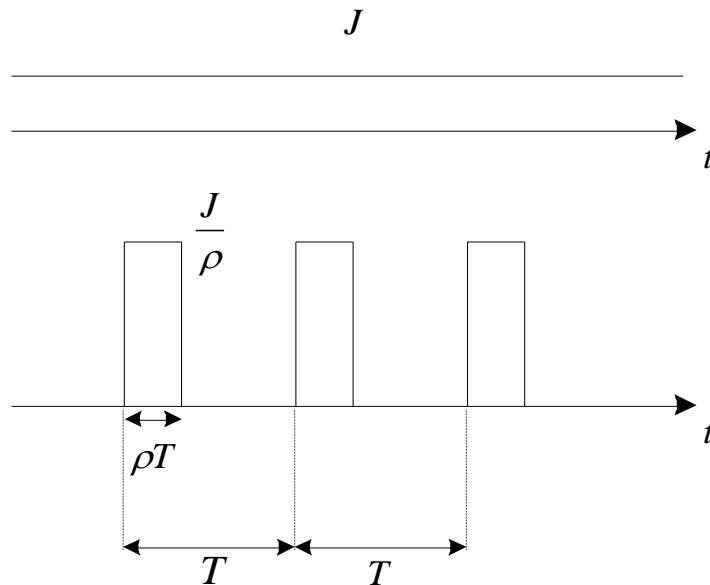
$$= \frac{J}{\mathbf{W}} T_c \cdot \mathbf{W} = JT_c$$

$$\text{If } \frac{W}{2} = \frac{1}{T_c},$$

$$S_{W_J}(0) = JT_c = \frac{J}{\frac{1}{T_c}} = \frac{J}{\frac{W}{2}} = 2N_J$$

Pulse noise jammer

- Jamming power J is fixed



- Prob. of Error = $\rho f\left(\frac{E_b}{N_0 + N_J / \rho}\right) + (1 - \rho) f\left(\frac{E_b}{N_0}\right)$
 $f\left(\frac{E_b}{N_0}\right)$: BER equation

- Assume $f\left(\frac{E_b}{N_0}\right) \cong 0,$

$$N_J = J / W \quad E_b = P / R$$

$$\begin{aligned} P_e &\approx \rho f\left(\frac{\rho E_b}{N_J}\right) = \rho f\left(\rho \frac{P}{R} \cdot \frac{W}{J}\right) \\ &= \rho f\left(\rho \frac{P}{J} \cdot \frac{W}{R}\right) \end{aligned}$$

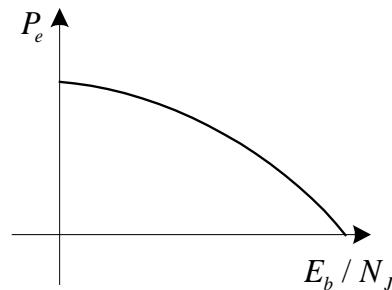
- For BPSK, $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

In pulse noise jamming

$$P_e \approx \rho Q\left(\sqrt{\rho \frac{2E_b}{N_J}}\right) = \rho Q\left(\sqrt{2\rho \frac{P}{R} \cdot \frac{W}{J}}\right)$$

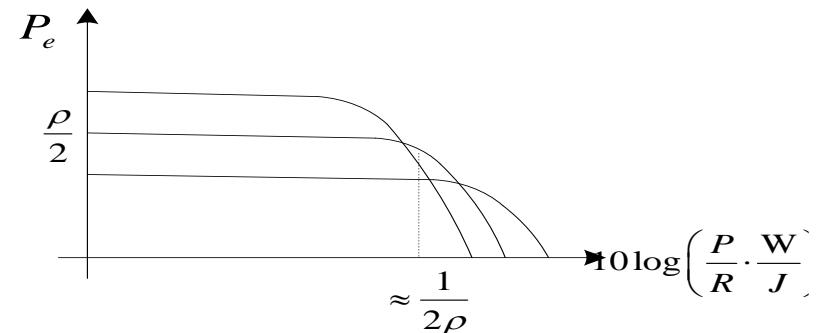
Case 1: $\rho = 1$

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_J}}\right)$$



Case 2: $\rho \frac{2E_b}{N_J}$ is small

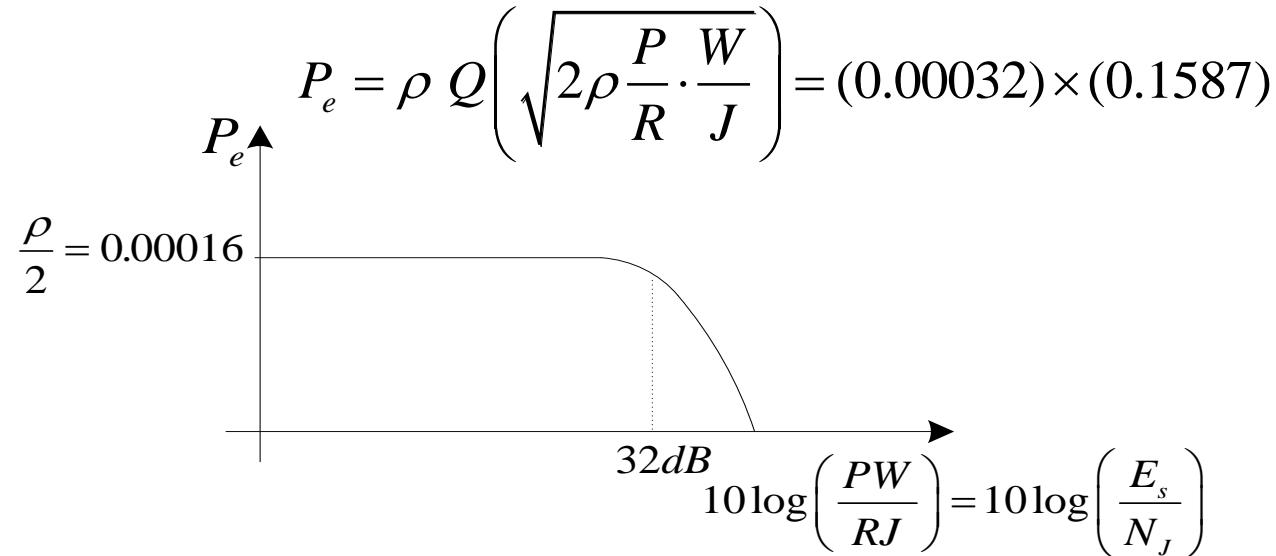
$$P_e = \frac{\rho}{2}$$



- example

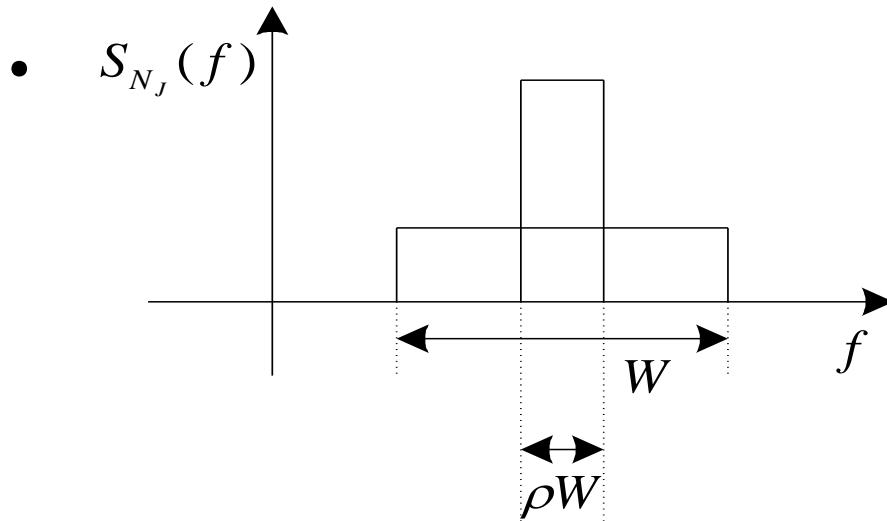
$$\rho = 0.00032$$

$$2\rho \frac{PW}{RJ} = 1 \rightarrow \frac{PW}{RJ} = \frac{1}{2\rho} = 1562.5 \Rightarrow 10\log(1562.5) = 32dB$$



- Optimal ρ
- Pulse noise jammer in DS-CDMA systems and systems without spreading

Partial band jammer in FH system



- $$P_e = \rho f \left(\frac{E_b}{N_0 + \frac{N_J}{\rho}} \right) + (1 - \rho) f \left(\frac{E_b}{N_0} \right) \approx \rho f \left(\frac{\rho E_b}{N_J} \right)$$