# OFDM

# (Orthogonal Frequency Division Multiplexing)

# Contents

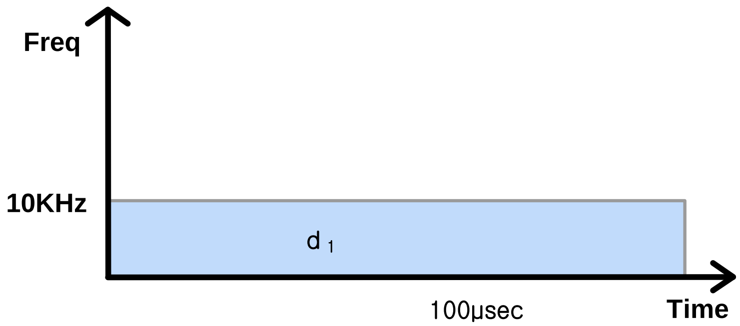
- Introduction
- Generation of subcarriers using the IFFT
- Guard time and cyclic extension
- Windowing
- Choice of OFDM parameters
- OFDM signal processing
- Implementation complexity of OFDM versus signal-carrier modulation

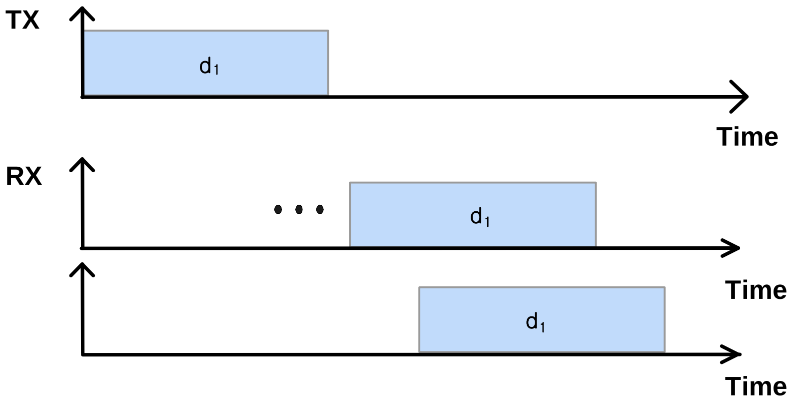
- Ref: OFDM for Wireless Multimedia Communications

  Richard Van Nee & Ramjee Prasad, Artech House

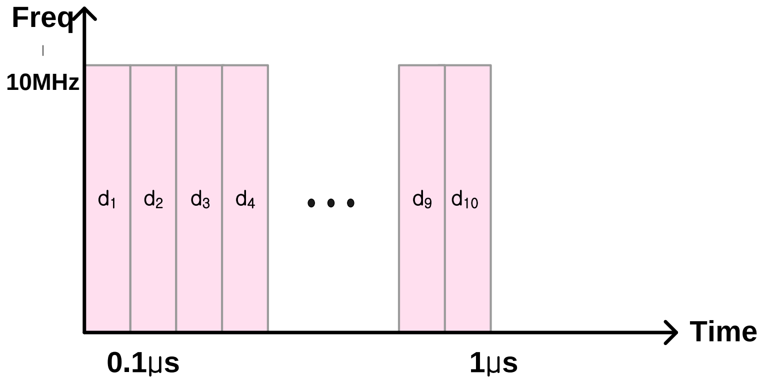
# Why OFDM?

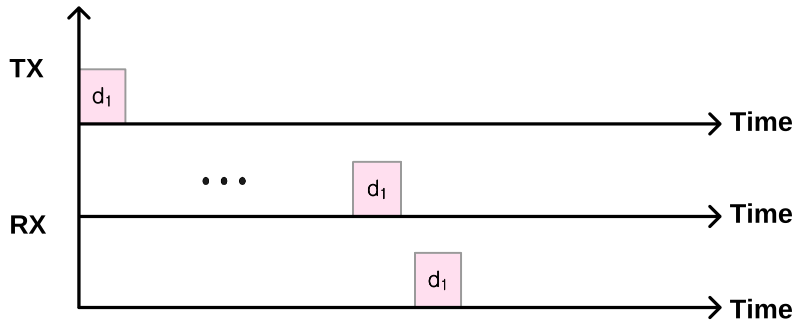
- 전송률 증대 필요 (음성, 데이터,…)
- 10 Kbps -> 100 usec/bit -> 10 KHz

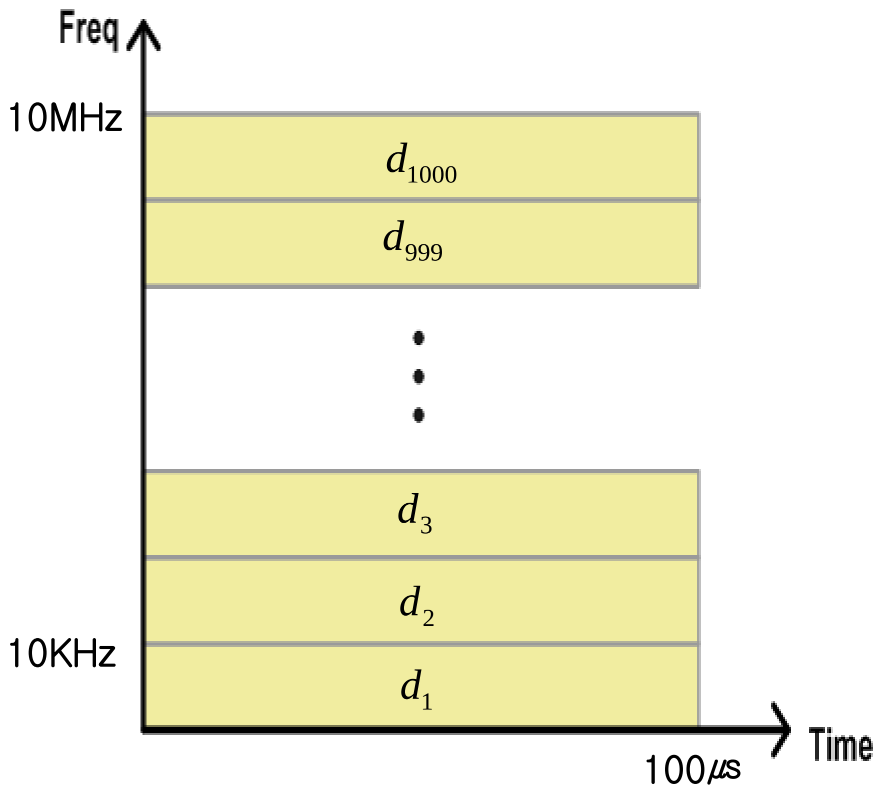


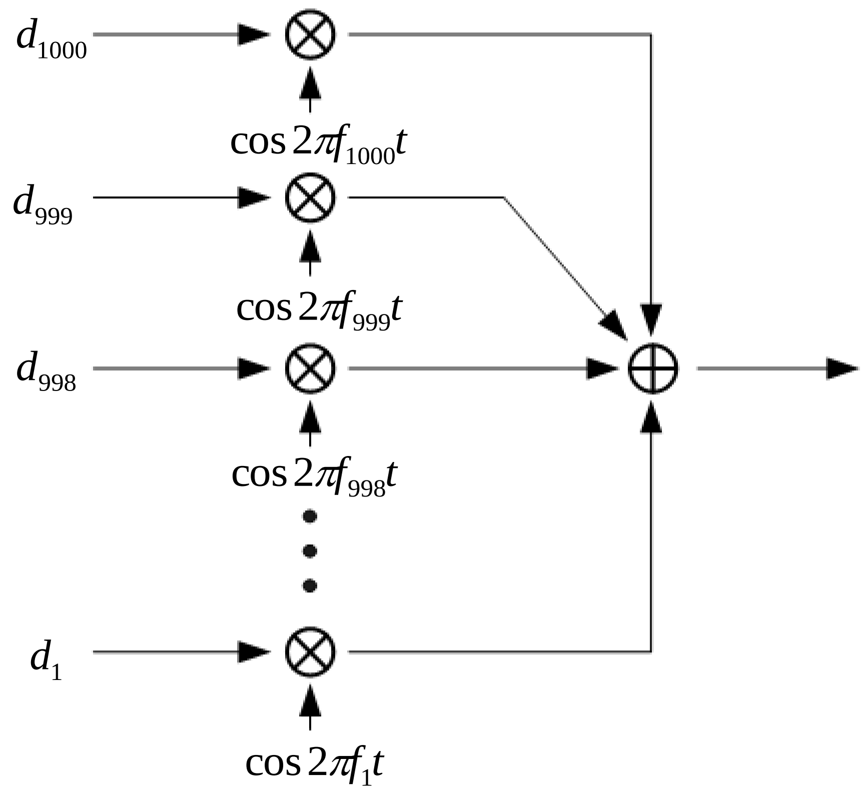




- 10 Mbps -> 0.1 usec/bit -> 10 MHz

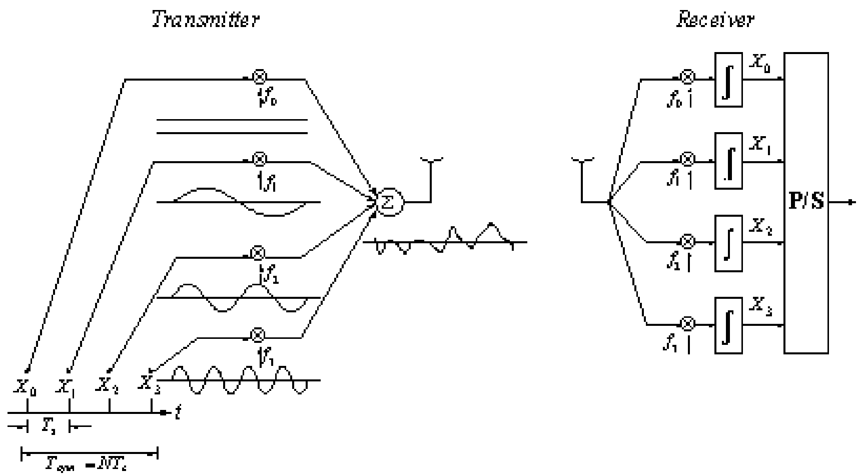


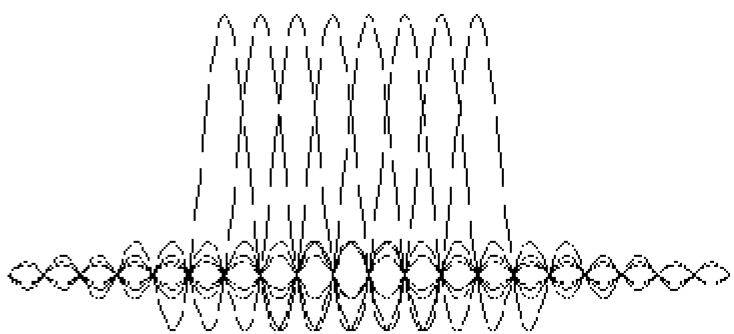

Freq
10MHz
10KHz
$d_{1000}$
$d_{999}$
$d_3$
$d_2$
$d_1$
100㎲
Time


$d_{1000}$
$\cos 2\pi f_{1000} t$
$d_{999}$
$\cos 2\pi f_{999} t$
$d_{998}$
$\cos 2\pi f_{998} t$
$d_1$
$\cos 2\pi f_1 t$

# Intruduction




<그림 1. OFDM 전송방식의 개념>



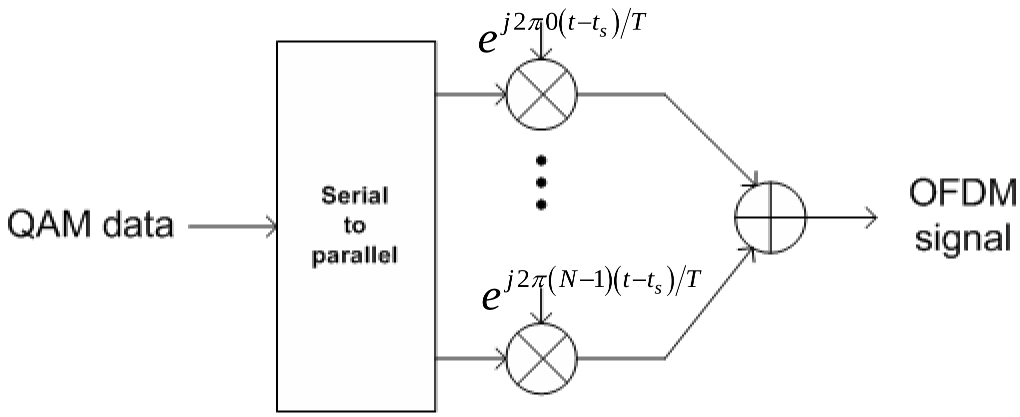
<그림 2. OFDM 방식의 전송 스펙트럼>

- What is OFDM?
  - High-rate data stream
    ➔ Number of lower rate streams
- Parameters under consideration
  - Number of subcarriers
  - Guard time
  - Symbol duration
  - Subcarrier spacing
  - Modulation type
  - Error correction coding
- Sprint Nextel MIRS
  Channel spacing > 1/T

- Orthogonality: subcarrier spacing = 1/T
- Baseband Mathematical Expression
  - Transmitted signal

$$s(t) = \sum_{i=0}^{N-1} d_i \exp\left( j2\pi \frac{i}{T}(t - t_s) \right), \qquad t_s \le t \le t_s + T$$

$$s(t) = 0, \qquad t < t_s \quad and \quad t > t_s + T$$




<그림 3. OFDM modulator>

- Frequencies: 0, 1/T, 2/T,…(N-1)/T

- Orthogonality
  - Each subcarrier has exactly an integer number of cycles
  - Adjacent subcarriers ➔ exactly 1 cycle difference

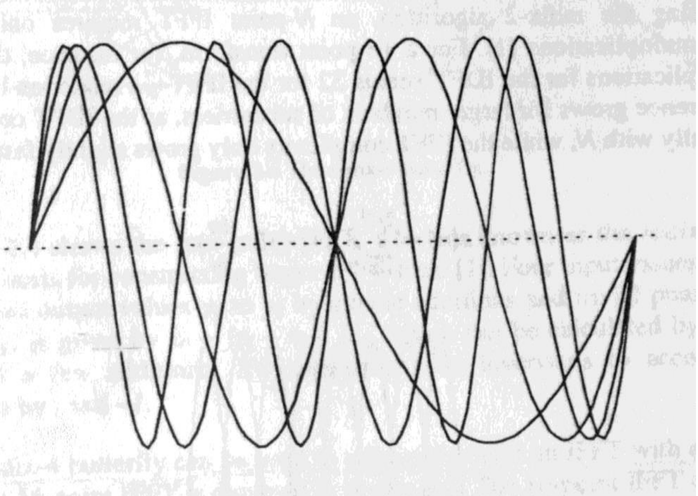


Figure 2.2 Example of four subcarriers within one OFDM symbol.

$$\int_0^T e^{j2\pi\frac{k}{T}t}dt$$

$$=\int_0^T\left(\cos\frac{2\pi kt}{T}+j\sin\frac{2\pi kt}{T}\right)dt$$

$$=0$$

$$\int_0^T e^{j\left(\frac{2\pi kt}{T}+\theta\right)}dt$$

$$=\int_0^T\left(\cos\left(\frac{2\pi kt}{T}+\theta\right)+j\sin\left(\frac{2\pi kt}{T}+\theta\right)\right)dt$$

$$=0$$

- Integration
  - integration over T ➔ desired output

$$\int_{t_s}^{t_s+T}\exp\left(-j2\pi\frac{l}{T}(t-t_s)\right)\sum_{i=0}^{N-1}d_i\exp\left(j2\pi\frac{i}{T}(t-t_s)\right)dt$$

$$=\sum_{i=0}^{N-1}d_i\int_{t_s}^{t_s+T}\exp\left(-j2\pi\frac{l-i}{T}(t-t_s)\right)dt=d_lT$$

- Spectrum of One OFDM Signal
  - ICI (inter carrier interference) is avoided by having the maximum of one subcarrier spectrum occur at zero crossings of all the others.

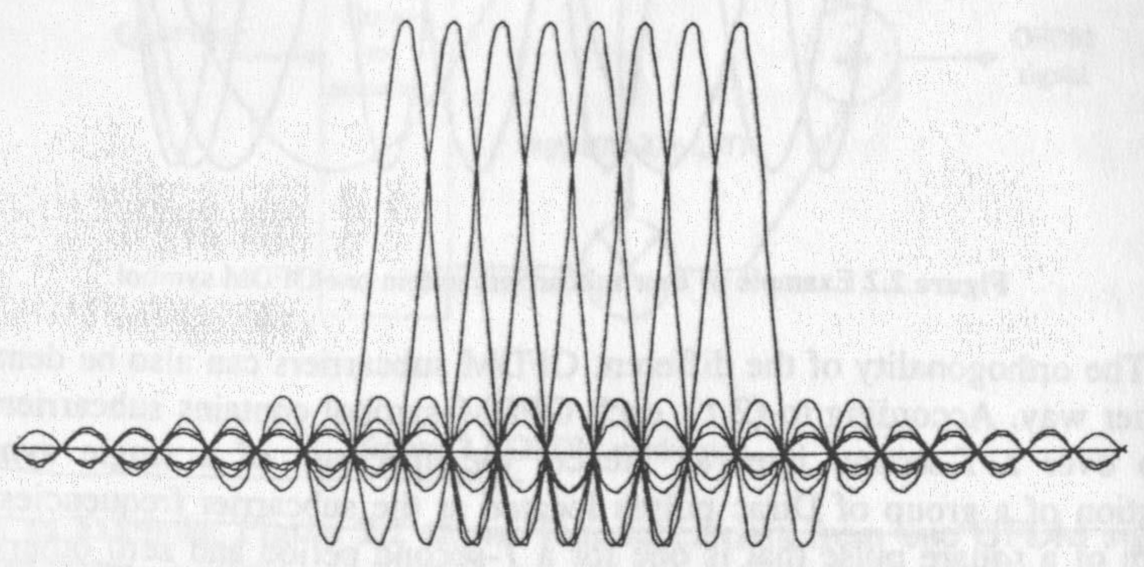


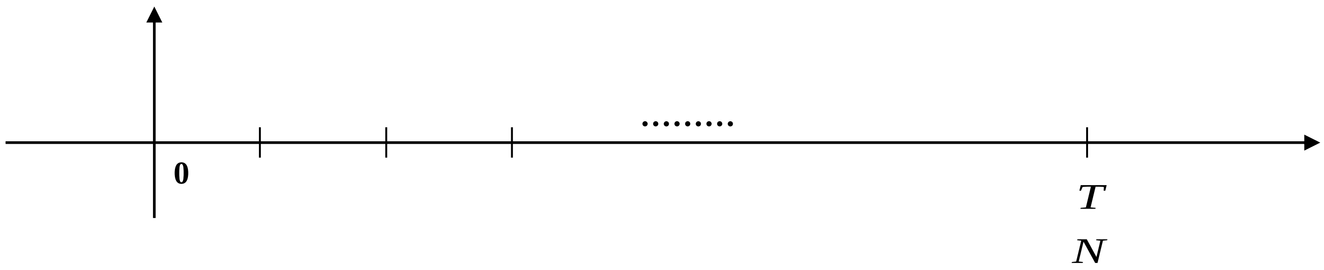
Figure 2.3 Spectra of individual subcarriers.

- IDFT implementation….

$$S(t)=\sum_{i=0}^{N} d_i e^{+\frac{j2\pi i}{T}(t)}, \qquad 0 \le t \le T$$

$$S(t)=0, \qquad t<0, \qquad t>T$$

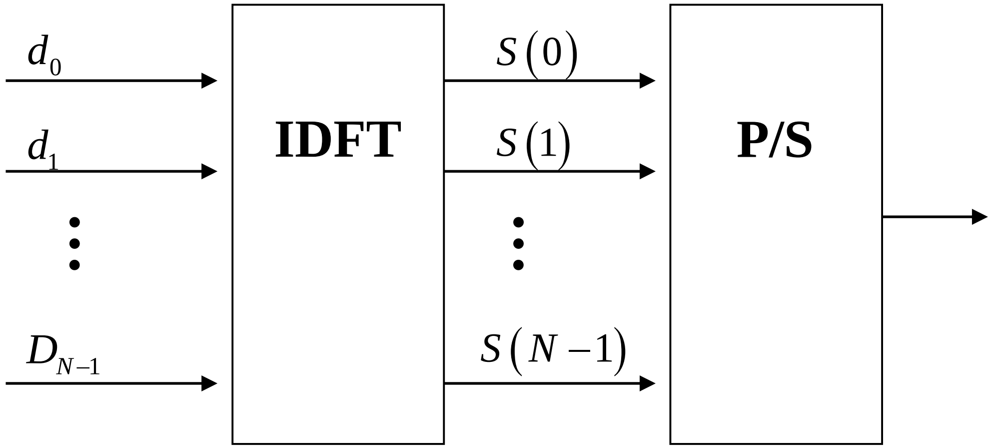
$$S(n)=\sum_{i=0}^{N-1} d_i e^{+\frac{j2\pi in}{N}}$$



$$S(0) = d_0 + d_1 + d_2$$

$$S(1) = d_0 + d_1 e^{\frac{j2\pi}{N}} + d_2 e^{\frac{j2\pi 2}{N}} + \cdots$$

$$S(2) = d_0 + d_1 e^{\frac{j2\pi}{N}\cdot 2} + d_2 e^{\frac{j2\pi 2\cdot 2}{N}} + \cdots$$



- **IFFT**
  - OFDM signal generation = inverse Fourier Transform
  - The number of multiplications in the IFFT can be reduced even further by using a radix-4 algorithm. This technique makes use of the fact that in a four-point IFFT, there are only multiplications by {1,-1,j,-j}.

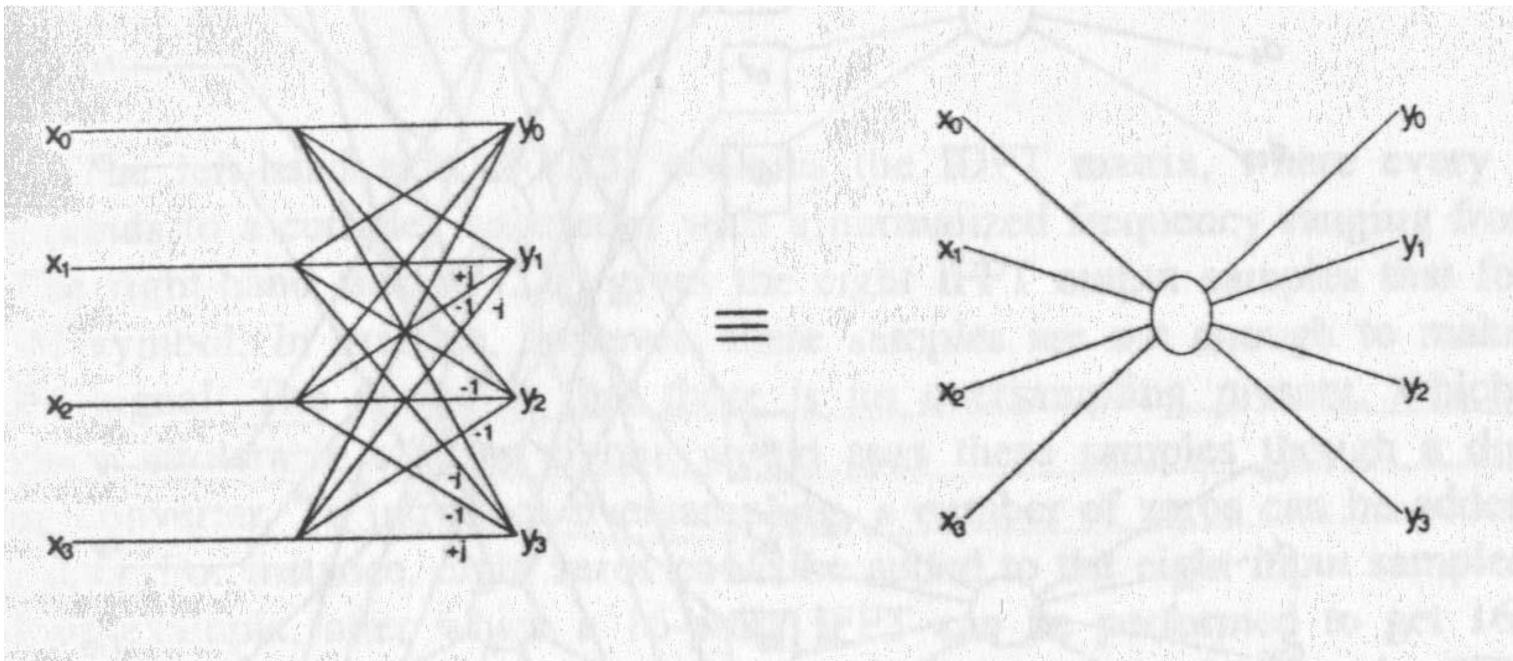
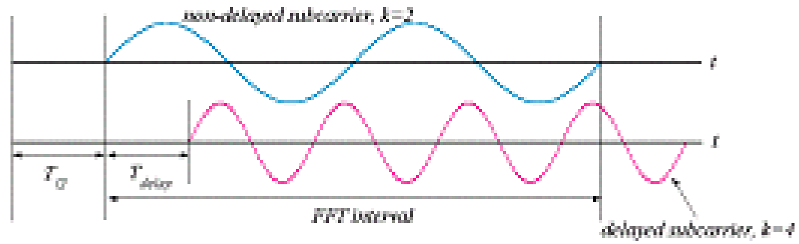


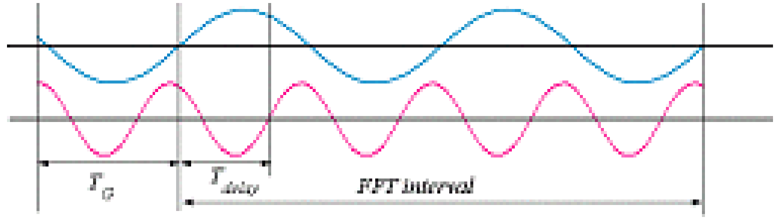

Figure 2.4 The radix-4 butterfly.

# Guard time & Cyclic extension

- Guard Time
  - The guard time is chosen larger than the expected delay spread, so that the delayed symbol cannot interfere with the next symbol.
  - If the guard time consist of no signal…



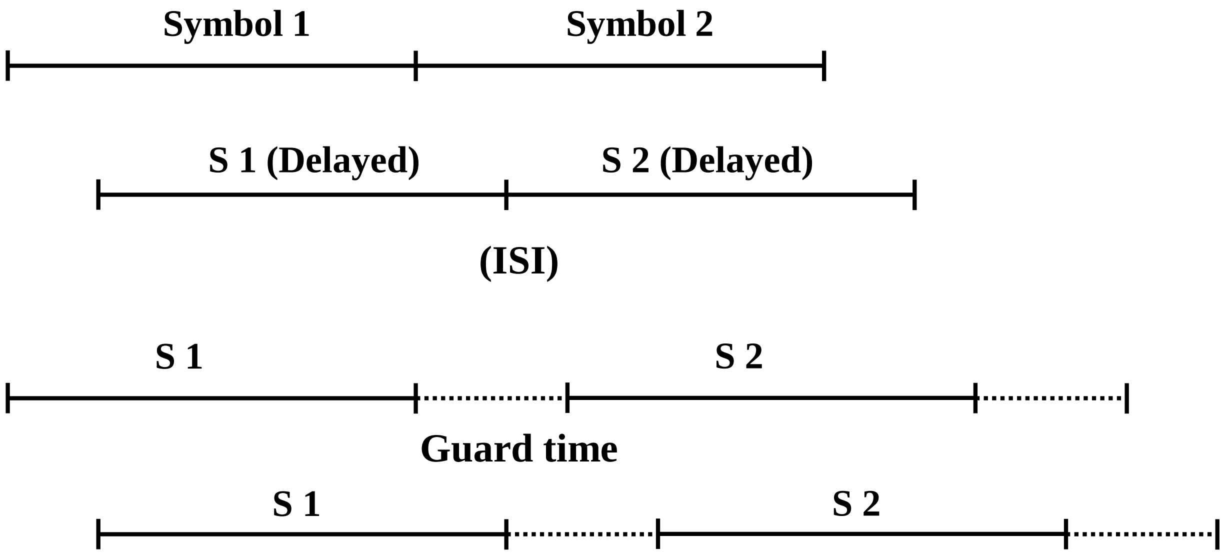

(a) ‘0’ 신호 삽입

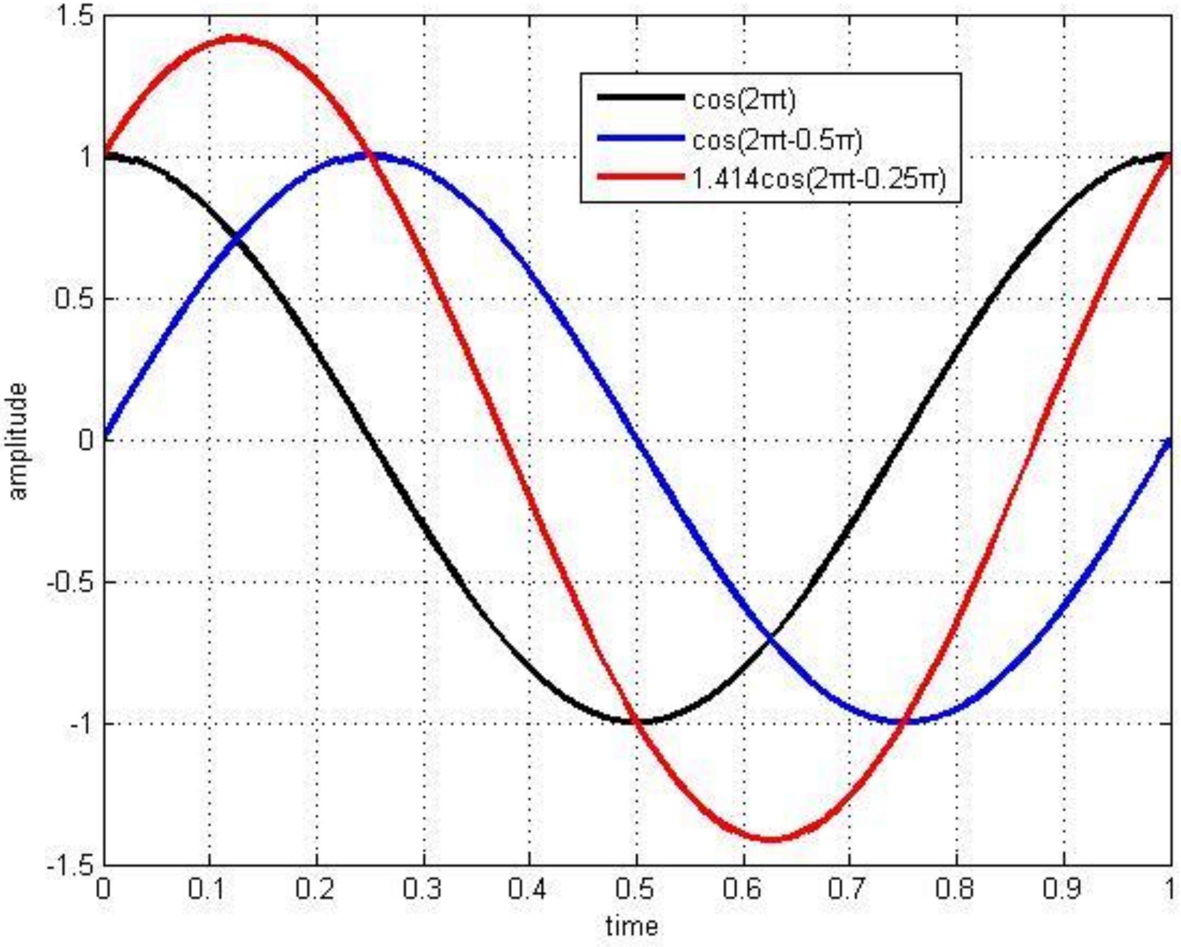



(b) Cyclic prefix의 삽입

<그림 4. 보호구간의 삽입을 위한 두 가지 방식>

- Single Carrier (ISI)






- Multiple Carrier (Inter carrier interference) Cyclic Extension

- Cyclic Extension
  - OFDM signal should be extended in guard time
  - Subcarriers should have integer number of cycle difference in FFT interval
  - Orthogonality lost when multipath delay > guard time

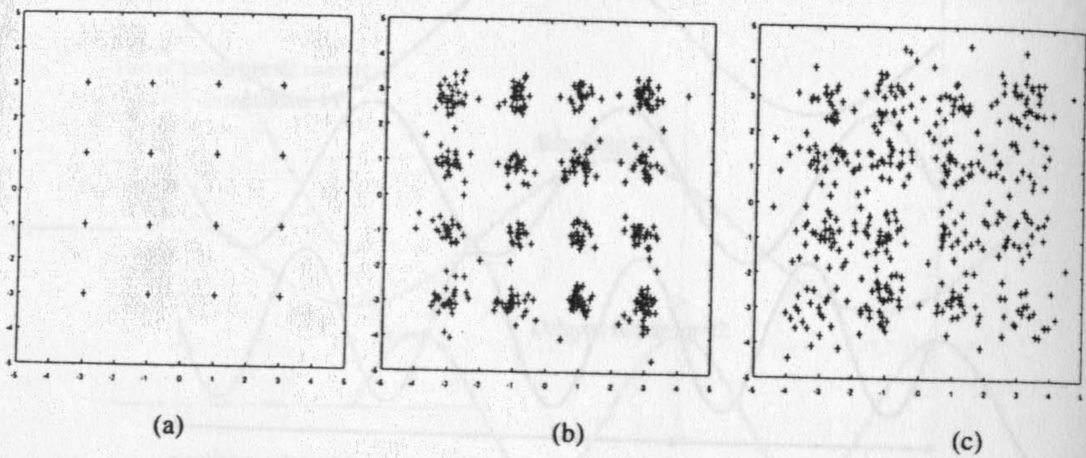
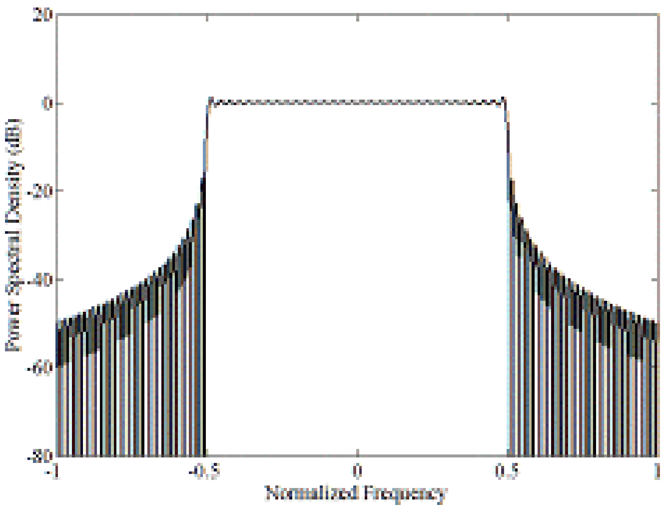



**Figure 2.9** 16-QAM constellation for a 48-subcarrier OFDM link with a two-ray multipath channel, the second ray being 6 dB lower than the first one. (a) delay < guard time; (b) delay exceeds guard time by 3% of the FFT interval; (c) delay exceeds guard time by 10% of the FFT interval.

# Windowing

- Windowing: results in lower sidelobes
- Raised Cosine Window

$$w(t)=\begin{cases}0.5+0.5\cos\left(\pi+t\pi/(\beta T_s)\right) & 0\le t\le \beta T_s\\ 1.0 & \beta T_s\le t\le T_s\\ 0.5+0.5\cos\left((t-T_s)\pi/(\beta T_s)\right) & T_s\le t\le (1+\beta)T_s\end{cases}$$




<OFDM 방식에서의 전송 스펙트럼>

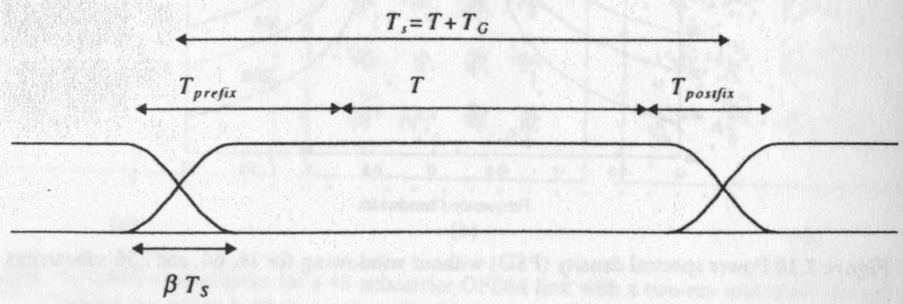



**Figure 2.11** OFDM cyclic extension and windowing. $T_s$ is the symbol time, $T$ the FFT interval, $T_G$ the guard time, $T_{prefix}$ the preguard interval, $T_{postfix}$ the postguard interval, and $\beta$ is the rolloff factor.

# OFDM system design parameters

– Guard time >> Delay spread

– Symbol duration >> Guard time

– Number of subcarrier

– Subcarrier spacing

# OFDM system design

- System requirements
  - Bit rate : 20Mbps
  - Tolerable delay spread : 200ns
  - Bandwidth : < 15MHz

- guard time = 800ns
- OFDM symbol duration = 6 * guard time = 4.8us
- FFT time = 4.8 - 0.8 = 4us ($T_s = T + T_G$)
- subcarrier spacing = 1 / 4us = 250kHz
- symbol rate = 1 / $T_{symbol\ duration}$=1/4.8us

# OFDM system design (cont'd)

- No. of bits/OFDM symbol = Bit rate/OFDM symbol rate

$$= \frac{20Mbps}{(1/4.8us)} = 96bits/\text{OFDM symbol}$$

(1) 16-QAM

4bits/subcarrier, 1/2rate coding -> 2 information bit/subcarrier

number of subcarriers = 48

requiered Bandwidth = 48 * 250 kHz = 12MHz < 15MHz

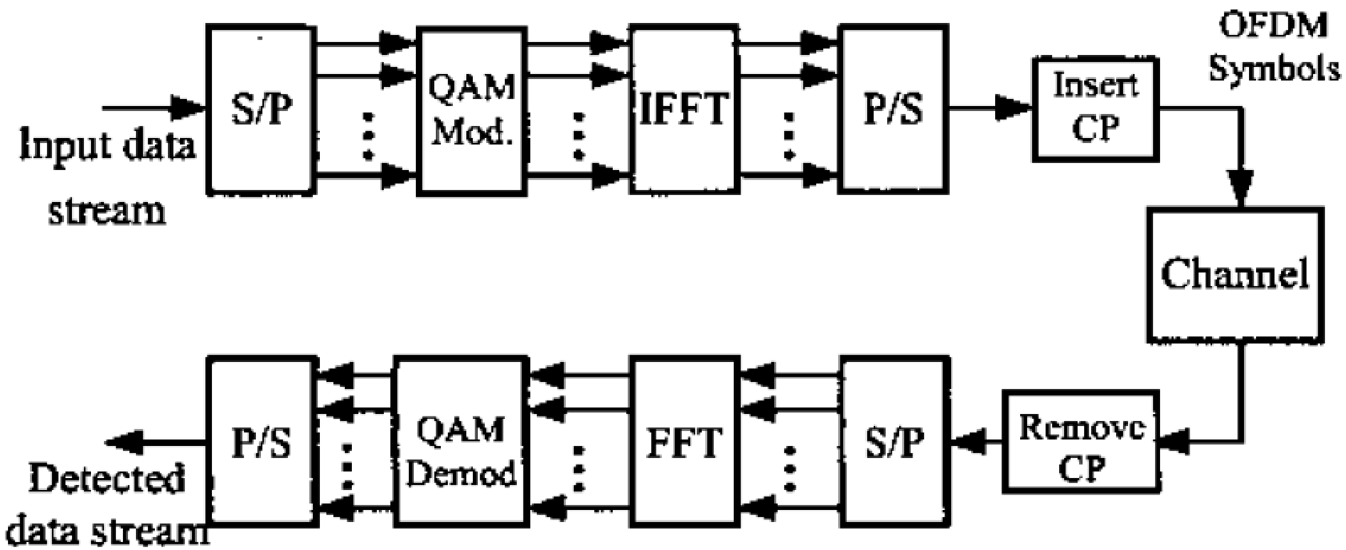
(2) QPSK

2bits/symbol, 3/4rate coding -> 1.5information bits/subcarrier

number of subcarrier = 96/1.5 = 64

required Bandwidth = 64 * 250kHz = 16MHz > 15MHz

# OFDM signal processing



# Implementation complexity of OFDM vs. Single-carrier modulation

- Complexity
  - Single-carrier systems need equalizer when delay spread over 10% but OFDM systems don’t
  - FFT does not need full multiplication but rather phase rotation
  - The complexity in OFDM grows slightly faster than linear

- Robustness
  - Sigle-carrier system performance degrade abruptly when delay spread exceeds the value which equalizer is desiged
  - OFDM systems are robust against delay spread

# OFDM 특징

- 구현 용이 (FFT)
- 채널 지연 문제에 강인

- 사용자에게 다양한 용량 할당 용이
- 주파수별 전력할당과 복조방법 변화
- 사용자에게 주파수 할당 방법


- **J. Jang and K. B. Lee, "Transmit Power Adaptation for Multiuser OFDM Systems," IEEE Journal on Selected Areas in Communications, vol. 21, no. 2, pp. 171-178, Feb. 2003.**