Fusion Reactor Technology I

(459.760, 3 Credits)

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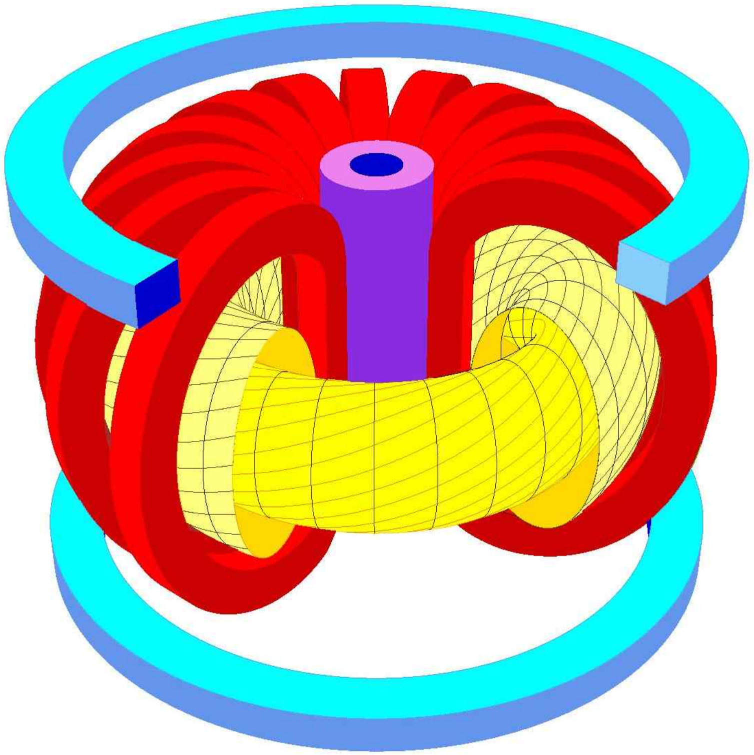
Contents

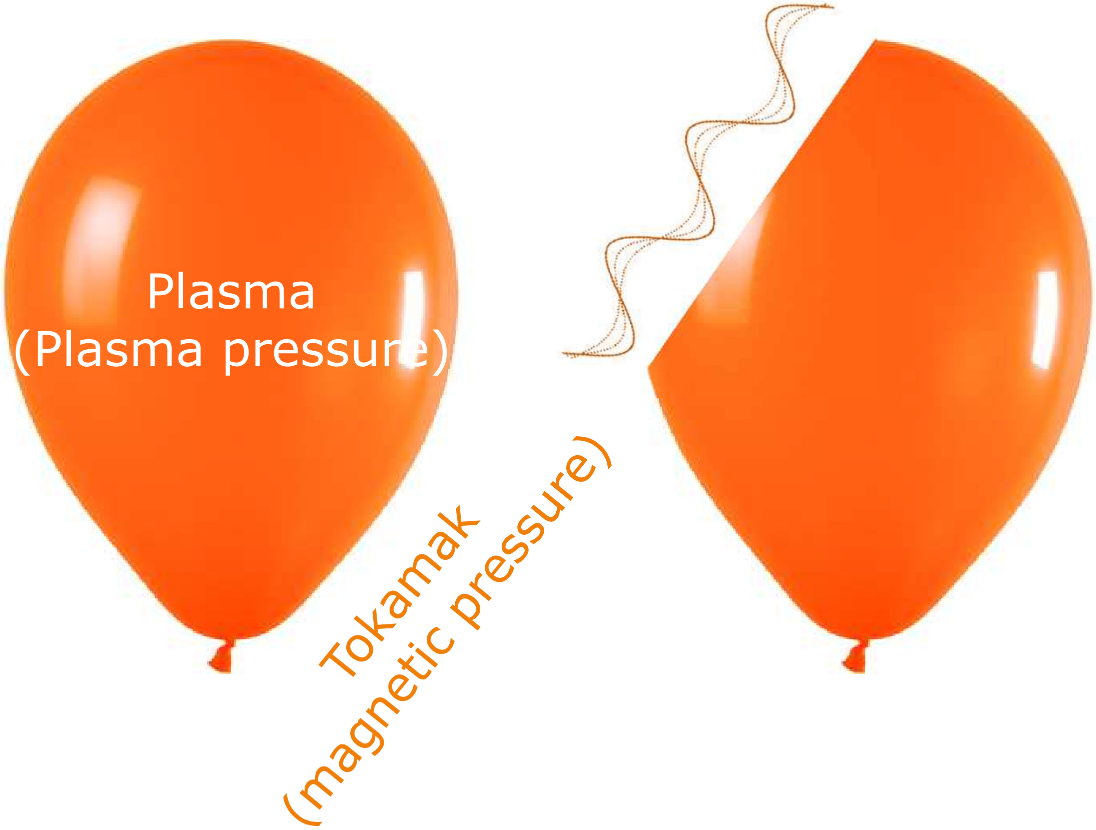
Objectives of the Tokamak Operation

$$n\tau_E T \geq 3\times 10^{21} m^{-3} keVs = 5\, bar\cdot s$$

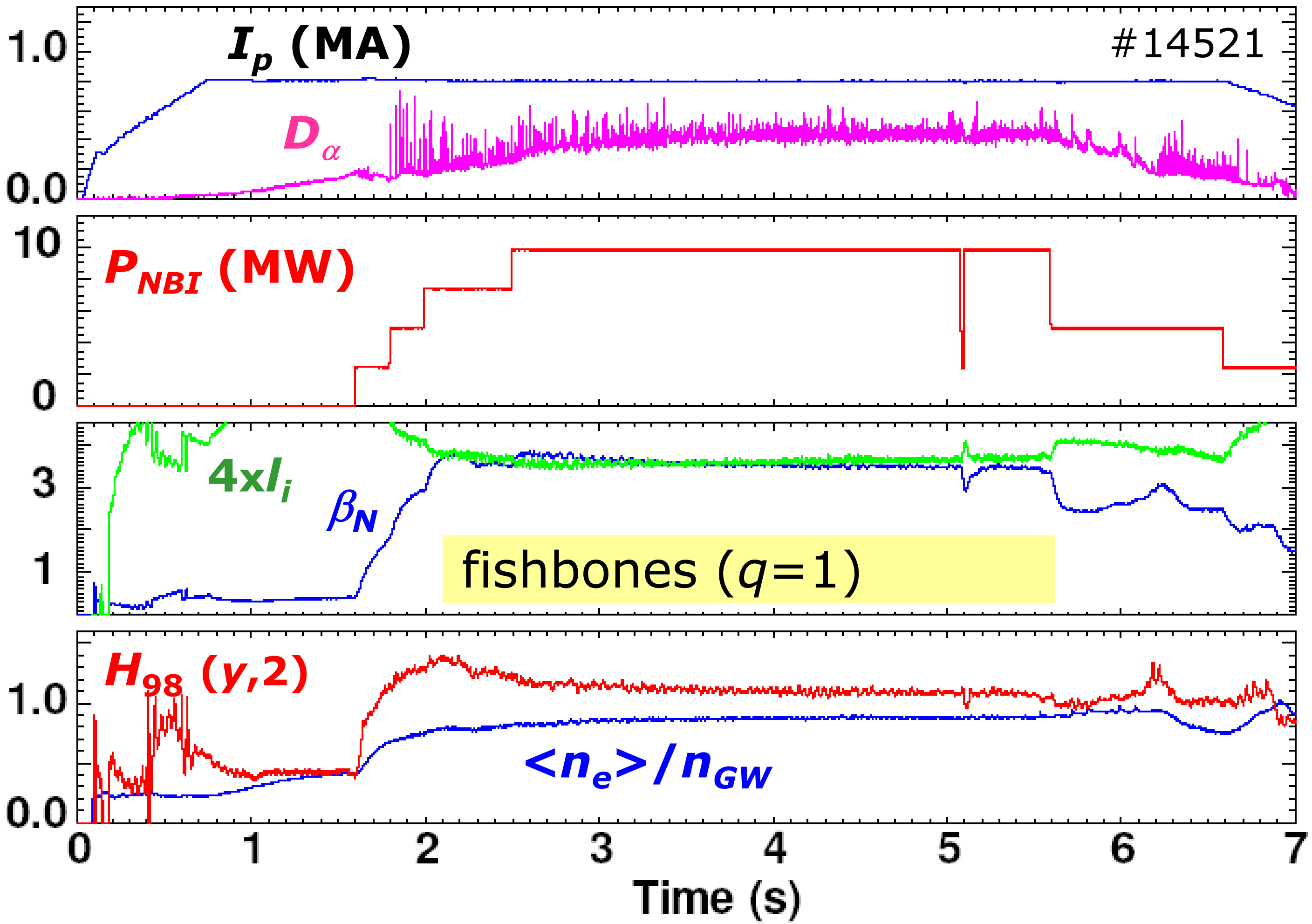


- High $\langle n_e\rangle/n_{GW}$
- High β_N
- High $H_{98}(y,2)$
- Pulse length

Plasma Equilibrium, Stability and Transport

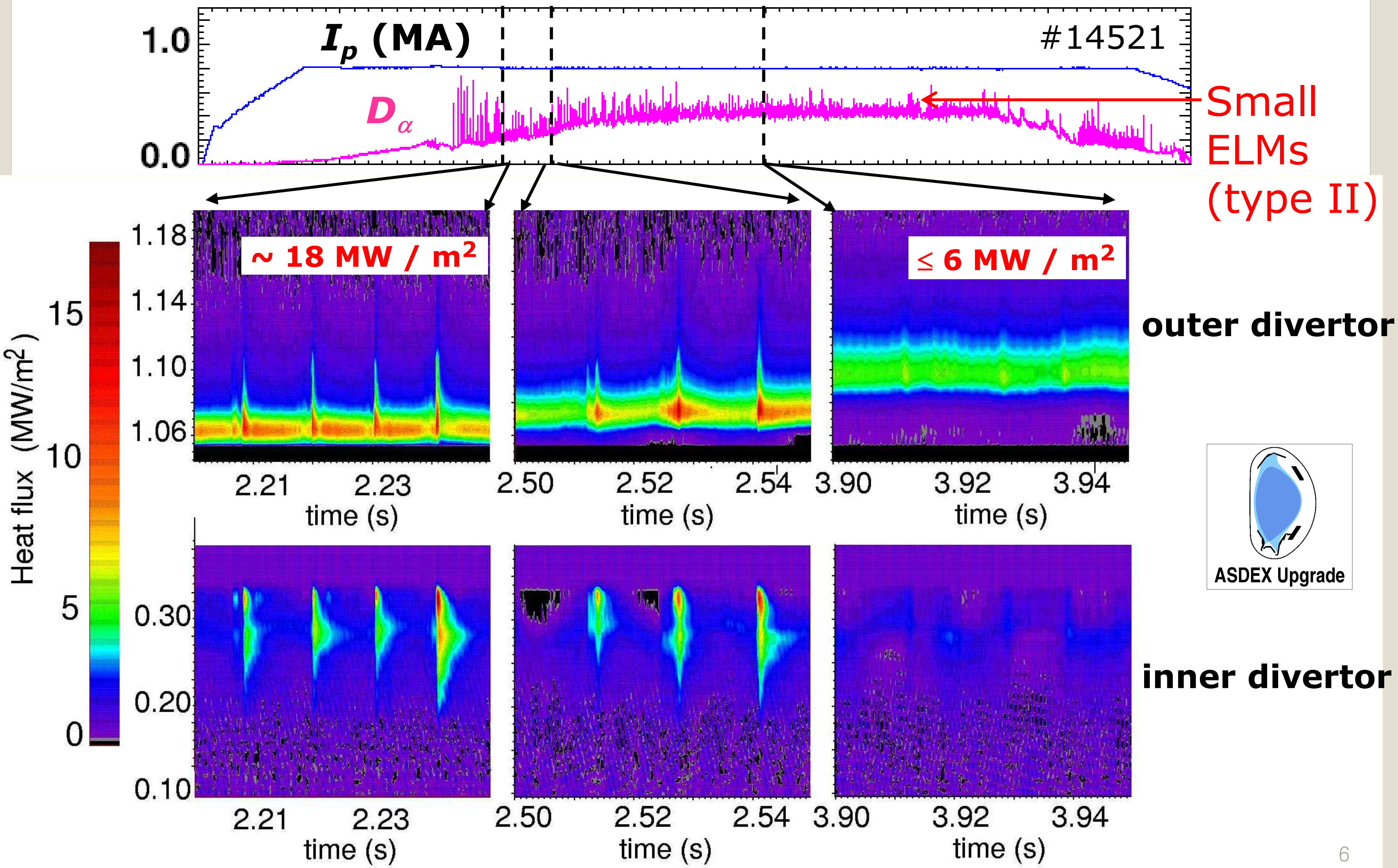


Objectives of the Tokamak Operation



- No sawteeth, good confinement, and $\beta_N \sim 3.5$, $T_i \sim T_e$, $\langle n_e \rangle / n_{GW} \sim 0.88$, averaged over 3.6 seconds ($\sim 50\ \tau_E$).

Objectives of the Tokamak Operation



Plasma as a Complex System

- **High-temperature plasma, confined by a magnetic field, is an exceptionally unusual physical object**
  → complex physical system

- Presence of macroscopic instabilities
- Local entropy production due to local plasma transport
- Rare Coulomb collisions (anomalous transport)
- Non-linear phenomena (noise source) in the edge plasma propagating inside the plasma core leading to transport enhancement
- Heating resulting in additional noise generation

  cf) OH heating: drift current velocity of electrons ~ 10^5 m/s

  $\ll$ sound velocity (j ~ 1 MA/m^2, n_e ~ 10^{20} m^{-3})

$$V_{Alfven} = \sqrt{\frac{B_0^2}{\mu_0 \rho_0}}, \quad V_{adiabatic\ sound} = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

Plasma as a Complex System

- A rational approach to study complex systems consists of a large number of experiments aimed at understanding empirical laws supported by development of a theoretical description and computer models.
- All this is actively used in modern tokamak studies.
- As experience with other complex systems shows, the general method of scaling and dimensional approach represents a powerful tool for their description.

Dimensional Analysis of Tokamaks

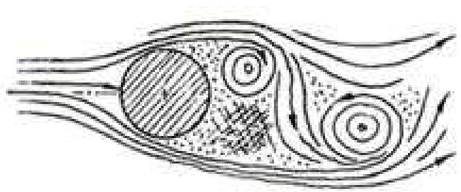
- **Dimensional approach**

- All the laws of physics are based on mechanics.
- Mechanics uses conventionally chosen units for mass, length, and time.
- The objective laws of nature cannot depend on those units. These laws are invariant with respect to variations of measurement units chosen by man.
- This invariance is seen more precisely when non-dimensional combinations of dimensional values are used.
- The non-dimensional parameters define the internal physics of a complex system: indicators of the fundamental state of the system
- Dimensional parameters look like some projection of a given system on the external world.

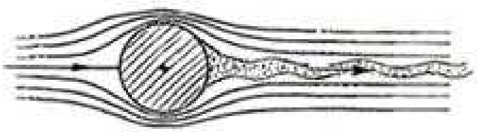
Dimensional Analysis of Tokamaks

- **Dimensional approach - Example**
  - Reynolds number

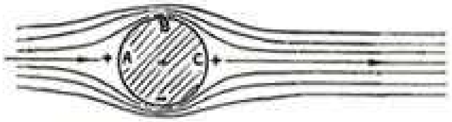
$$\mathrm{Re} = \frac{\rho v_s^2 / L}{\mu v_s / L^2} = \frac{\rho v_s L}{\mu} = \frac{v_s L}{\nu} = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$



(C) CYLINDER BETWEEN $R_d = 10^4$ and 10^5; VORTEX STREET WITH $C_{D_*} = 1.2$.



(E) CYLINDER ABOVE CRITICAL REYNOLDS NUMBER WITH $C_{D_*} = 0.3$.



(A) FLOW PATTERN OF CIRCULAR CYLINDER IN NON-VISCOUS FLOW; NO DRAG.

ρ: density of the fluid
v_s: mean velocity of the object relative to the fluid
L: travelled length of the fluid
μ: dynamic viscosity
ν: kinetic viscosity

Variation in flow pattern and drag coefficients for cylinders with increase in Reynolds number (Hoerner 1965)

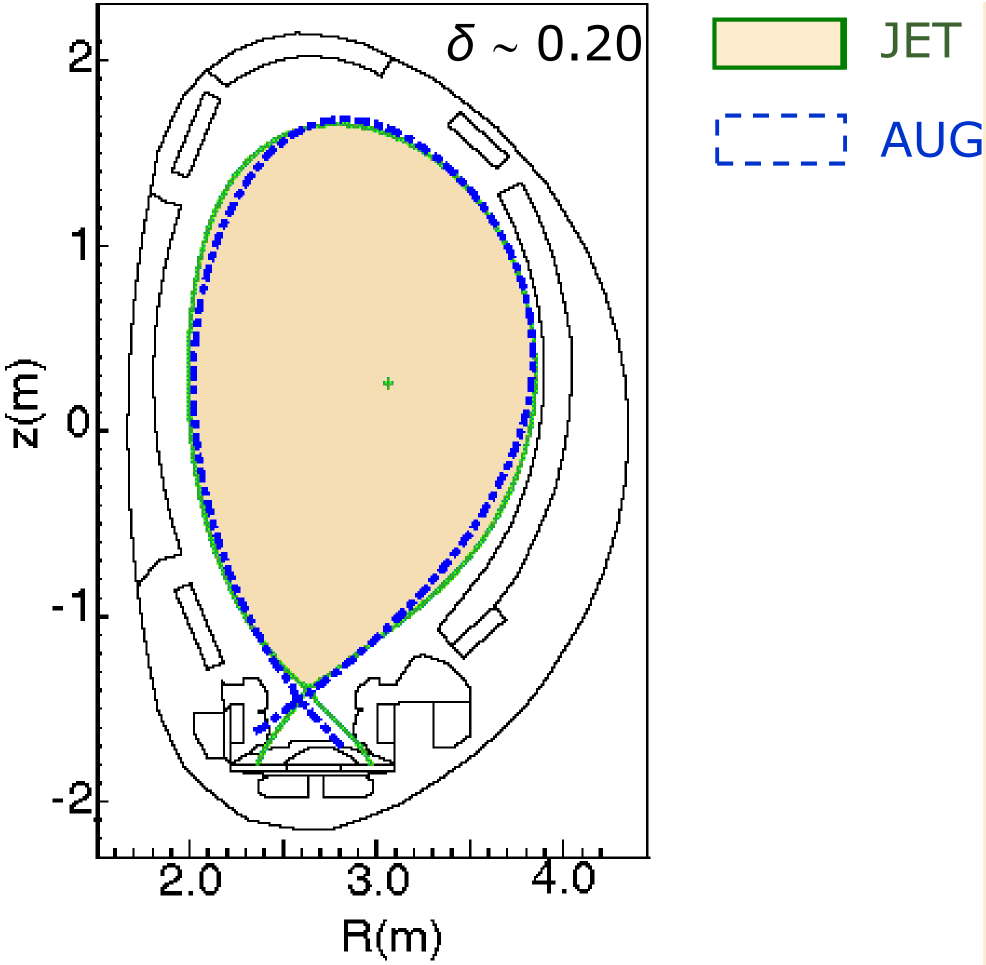
Dimensional Analysis of Tokamaks

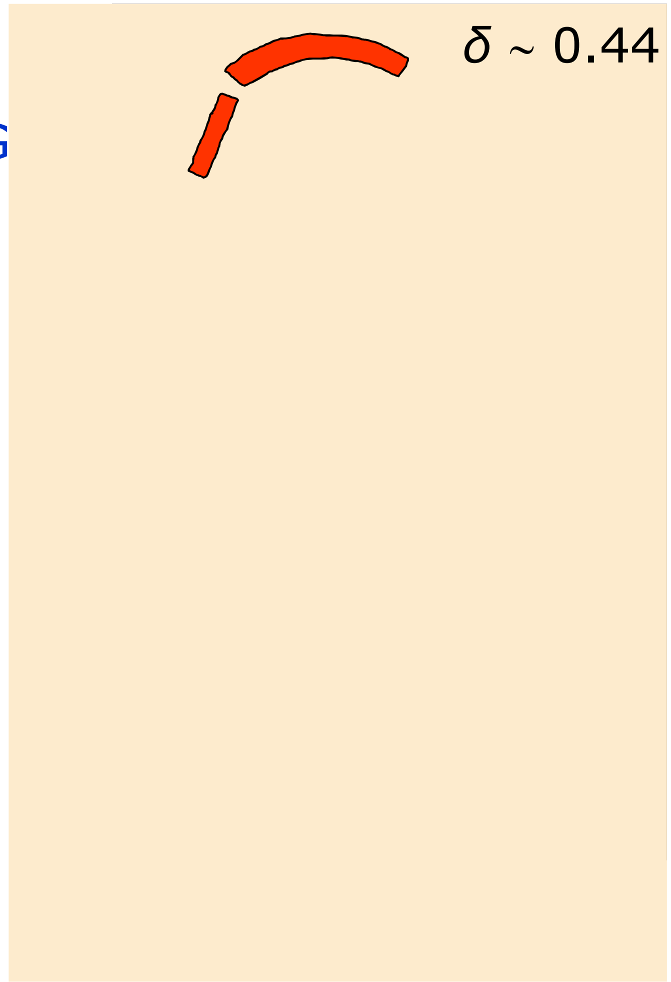
- **Dimensional approach**

- Being immersed in the external physical world, each complex system can possess a non-unique set of dimensional parameters.
- For a given set of dimensionless parameters the family of systems can exist with different sets of dimensional parameters.
  $\rightarrow$ Self-similarity
- Therefore, all the objective laws of physics may be presented as relations between non-dimensional parameters.
- Dimensional analysis should always be based on reasonable physical parameters which are specific for each particular case. Such an approach can allow us to pick out the most relevant parameters and to drop the unimportant ones.

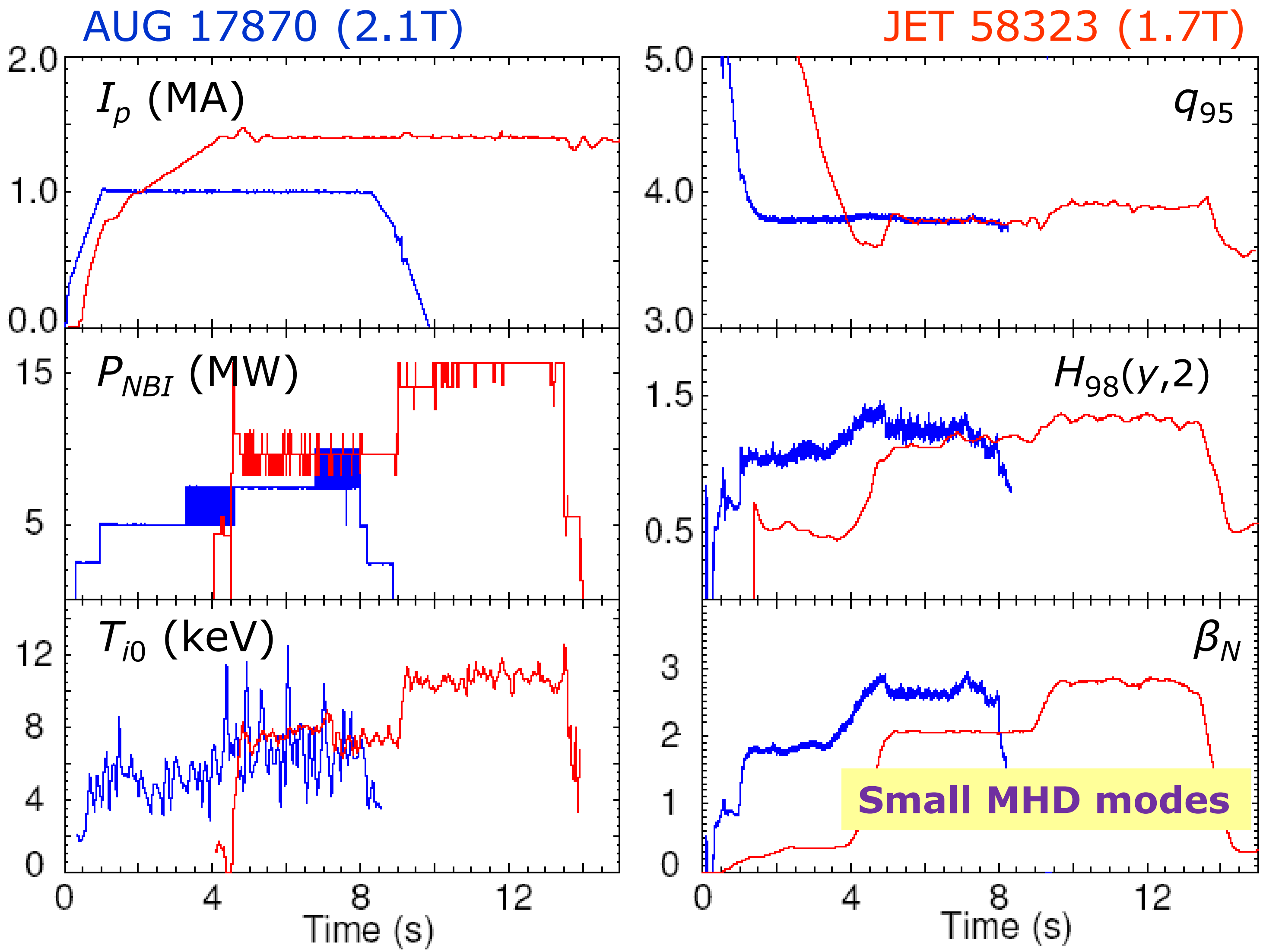
Identity (Similarity) Experiments

- **Plasma shapes used in JET compared to ASDEX Upgrade**





Identity (Similarity) Experiments



Dimensionless Parameters

- All the dimensional parameters

$$a,\ R,\ B_T,\ B_p,\ m_e,\ m_i,\ e,\ n,\ T$$

- Frequently used non-dimensional parameters for tokamak plasmas

$$A = R / a$$

$$q_a = aB_T / RB_p$$

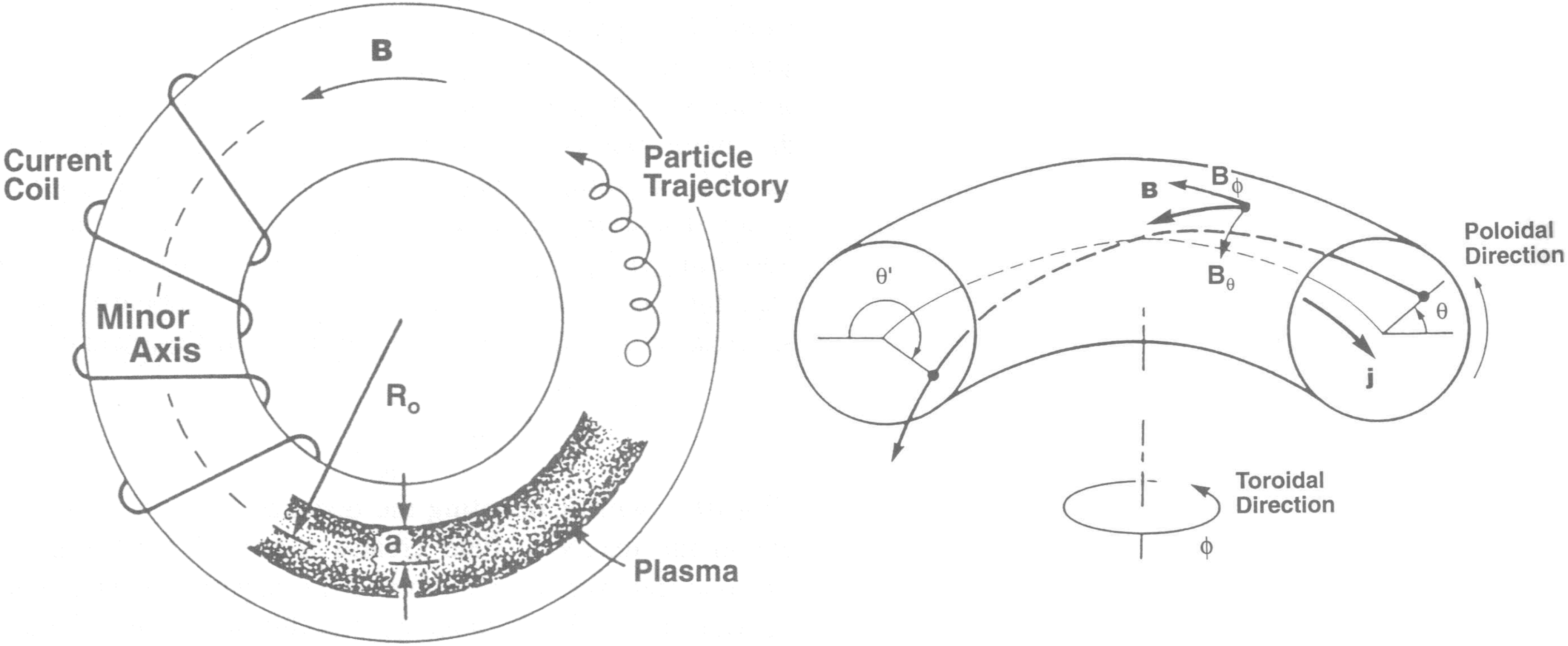
$$\beta = p2\mu_0 / B_T^2 = 4\mu_0 nT / B_T^2$$

$$\rho^* = \frac{\rho_i}{a} = \left(\frac{2T_i}{m_i}\right)^{1/2} \frac{m_i}{eBa}$$

$$\nu^* = \nu_{ii}\left(\frac{m_i}{T_i}\right)^{1/2}\left(\frac{R}{a}\right)^{3/2} qR$$

Basic Tokamak Variables

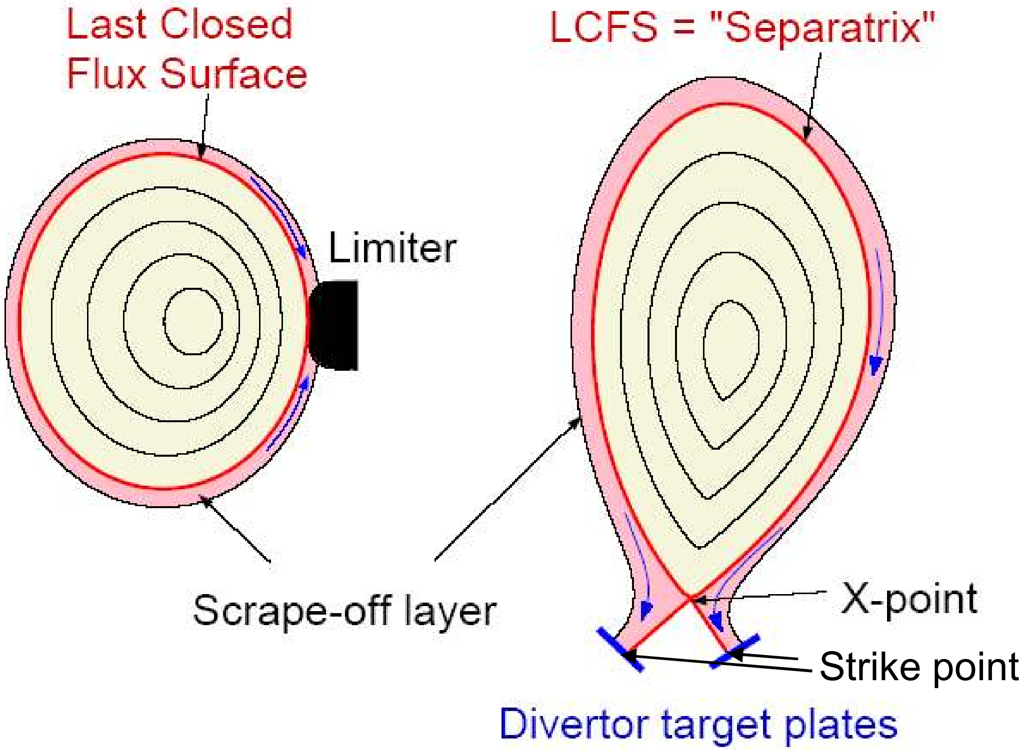
- **Cylindrical and local coordinates for a tokamak**



- Aspect ratio: R_0/a ~ 3-5
  ex) KSTAR: 3.6, ITER: 3.1
- Inverse aspect ratio: $\varepsilon = a/R_0$

Basic Tokamak Variables

- **Plasma configuration**



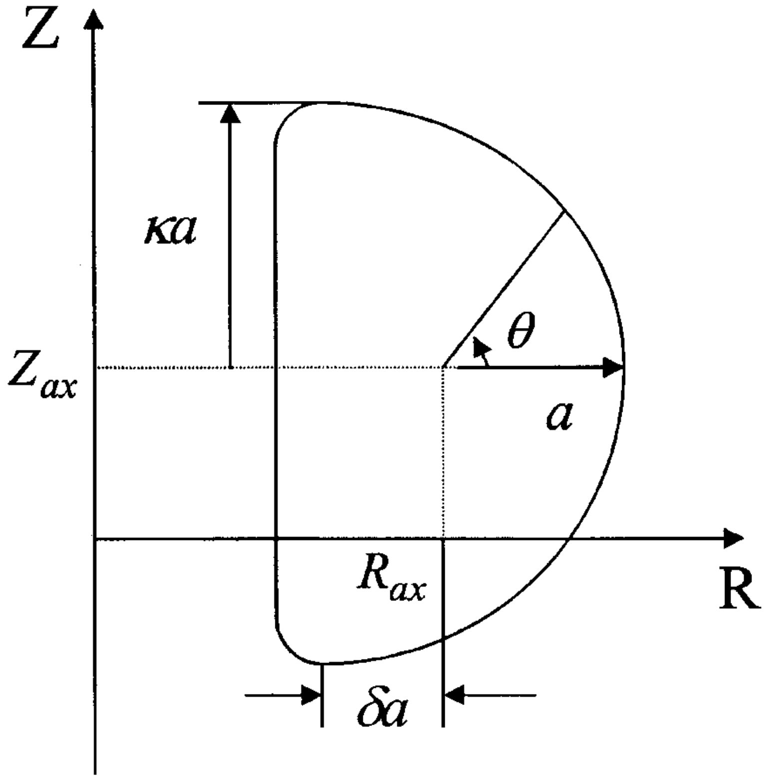
If no limiter and divertor?

Plasma diffusing into the whole vessel along the magnetic field → if touching the wall, impurities coming out

- Advantage of the divertor configuration
- First contact with material surface at a distance from plasma boundary
- Reducing the influx of ionized impurities into the interior of the plasma by diverting them into an outer „SOL"

Basic Tokamak Variables

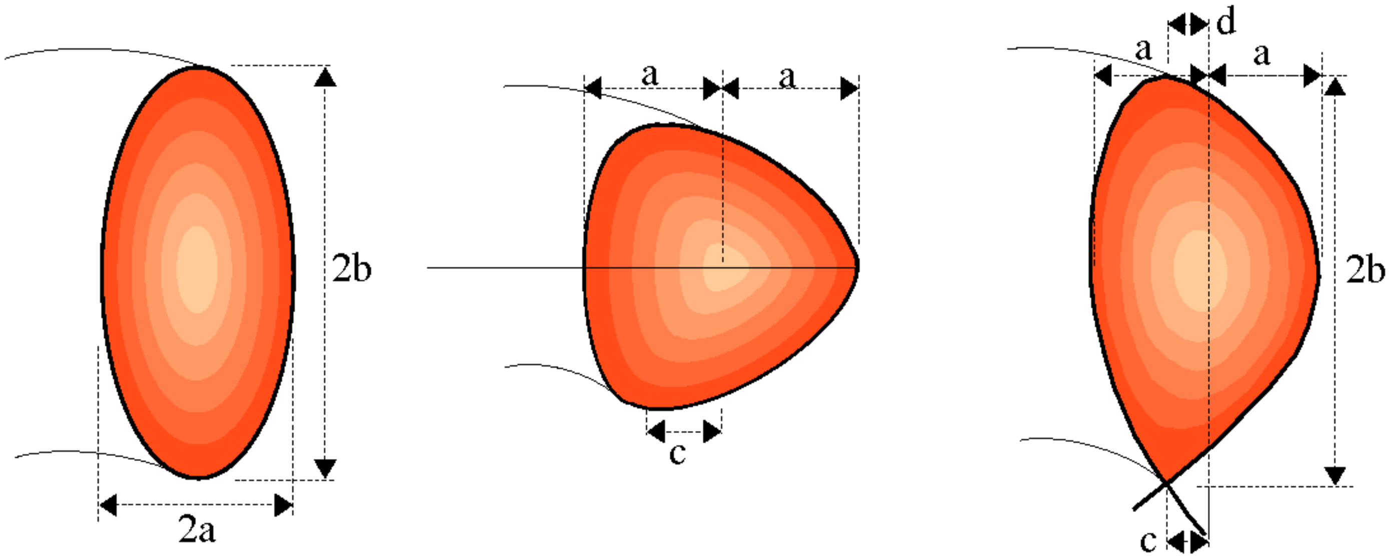
- **Plasma equilibrium parameters**



- Elongation: κ
- Triangularity: δ
- Squareness: ζ

Basic Tokamak Variables

- **Plasma equilibrium parameters**

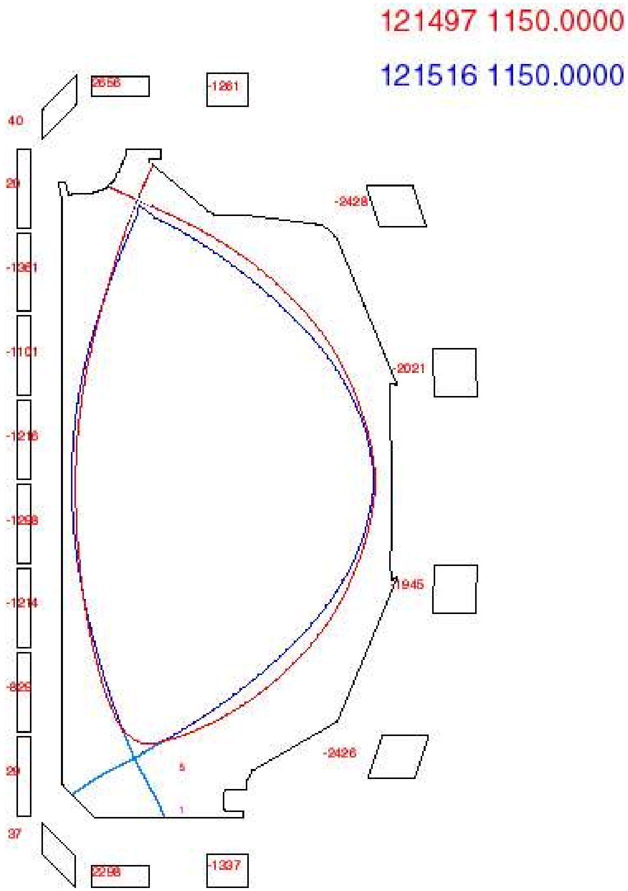


$$\kappa = \frac{b}{a} \qquad \delta = \frac{c}{a} \qquad \delta = \frac{c+d}{2a}$$

$$R = R_0 + a\cos(\theta - \delta\sin\theta)$$
$$Z = \kappa a \sin\theta$$

Basic Tokamak Variables

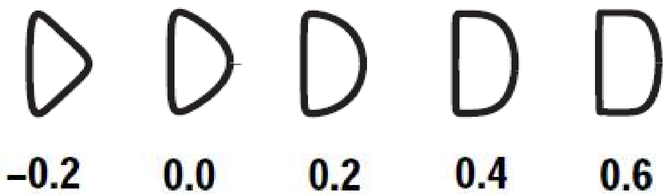
- **Plasma equilibrium parameters**



- Outer and inner squareness: $\zeta_{o,i}$

$$R = R_0 + a\cos\left(\theta + \sin^{-1}\delta\sin\theta\right)$$
$$Z = \kappa a\sin\left(\theta + \zeta_{o,i}\sin 2\theta\right)$$

Homework: derive!



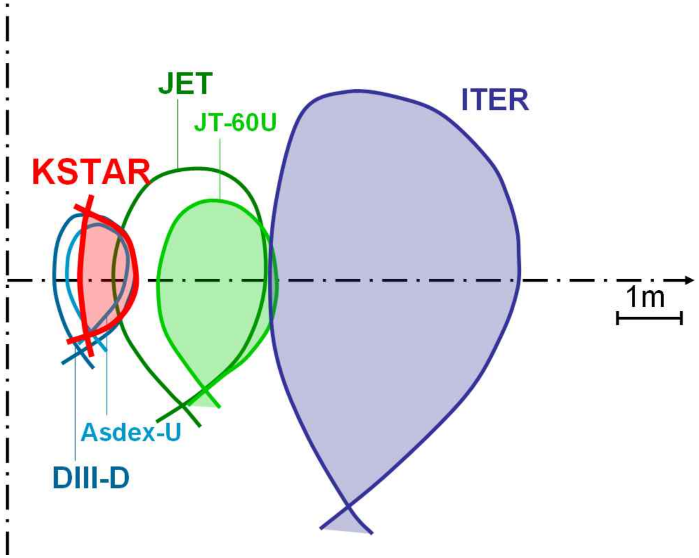
NSTX

Basic Tokamak Variables

- **Plasma equilibrium parameters**

Parameters	KSTAR	ITER
Major Radius, R_0	1.8 m	6.2 m
Minor Radius, a	0.5 m	2.0 m
Plasma Current, I_P	2.0 MA	15 MA
Elongation, κ_x	2.0	1.85
Triangularity, δ_x	0.8	0.5
Toroidal Field, B_0	3.5 T	5.3 T
Pulse Length	300 s	500 s
Fuel	H, D	D, T

- Plasma shape



Basic Tokamak Variables

- **Plasma Equilibrium**

$$\boxed{\begin{aligned}\nabla p &= \vec{J}\times\vec{B}\\ \nabla\times\vec{B} &= \mu_0\vec{J}\\ \nabla\cdot\vec{B} &= 0\end{aligned}}$$

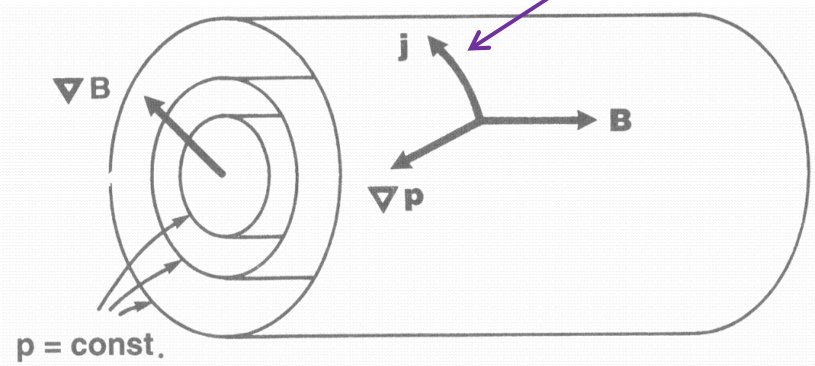
→ Force balance — kinetic pressure balanced by **J**x**B** (Lorentz) force

→ Ampere's law

→ Closed magnetic field lines

$$\vec{B}\cdot\nabla p = 0 \qquad \vec{J}\cdot\nabla p = 0$$

induced by the pressure gradient: causing a decrease in **B** → diamagnetism



Diamagnetic current

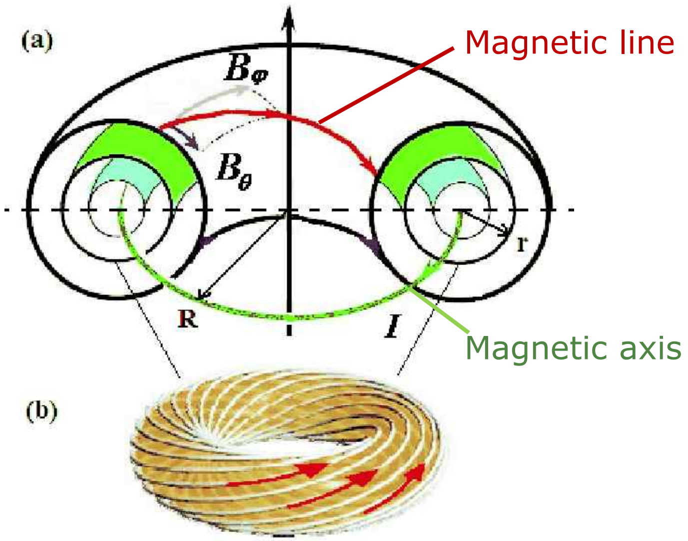
$$\vec{v}_{D,\nabla p} = -\frac{\nabla p\times\vec{B}}{nqB^2}$$

$$\vec{J} = n_i q_i \vec{v}_{D,i} + n_e q_e \vec{v}_{D,e} = \frac{\vec{B}\times\nabla p}{B^2}$$

- If B_Z is applied, plasma equilibrium can be built by itself due to induction of diamagnetic current.

Basic Tokamak Variables

- **Magnetic Flux Surfaces**
  - In fusion configurations with confined plasmas the magnetic lines lie on a set of nested toroidal surfaces called flux surfaces.
  - Pressure is constant along a magnetic field line.
  - Magnetic lines lie in surfaces of constant pressure.
  - Flux surfaces are surfaces of constant pressure.
  - The current lines lie on surfaces of constant pressure.
  - The current flows between flux surfaces and not across them.
  - The angle between **J** and **B** is arbitrary.

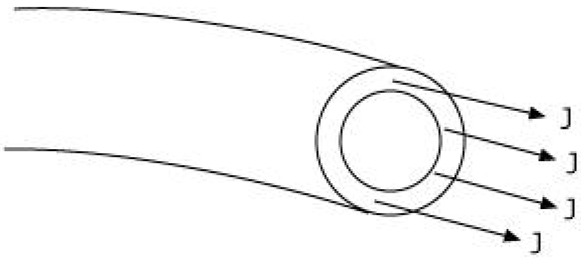


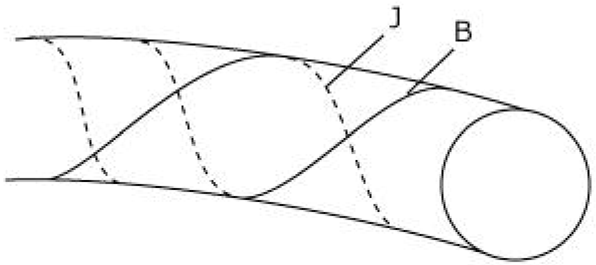
$$\vec{J} \times \vec{B} = \nabla p$$

$$\vec{B} \cdot \nabla p = 0$$

$$\vec{J} \cdot \nabla p = 0$$

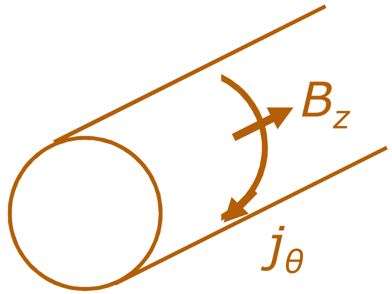
If $J = J_{||}$,
Jx**B** = ∇p = 0





Basic Tokamak Variables

- **Magnetic Flux Surfaces**
  - Consider particle motion in a cylindrically symmetric configuration, i.e. $\partial/\partial\theta = 0$



$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$$ vector potential

$$\vec{B} = \nabla \times A_\theta \vec{e}_\theta = -\frac{\partial A_\theta}{\partial z}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_z$$

Equation of particle motion

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$= q\left\{\vec{E}_A + \left[\vec{v} \times \left(-\frac{\partial A_\theta}{\partial z}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_z\right)\right]\right\}$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

$$\nabla \times \vec{E}_A = -\frac{\partial \vec{B}}{\partial t} = -\nabla \times \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E}_A = -\frac{\partial \vec{A}}{\partial t} = -\dot{A}_\theta \vec{e}_\theta$$

Equation of particle motion

$$m\frac{d\vec{v}}{dt} = q\left\{\vec{E}_A + \left[\vec{v} \times \left(-\frac{\partial A_\theta}{\partial z}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_z\right)\right]\right\}$$

$$= q\left\{-\dot{A}_\theta \vec{e}_\theta + \left[\frac{v_\theta}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_r - \left(\frac{v_r}{r}\frac{\partial}{\partial r}(rA_\theta) + v_z\frac{\partial A_\theta}{\partial z}\right)\vec{e}_\theta + v_\theta\frac{\partial A_\theta}{\partial z}\vec{e}_z\right]\right\}$$

$$v_\theta = \dot{l}_\theta = \dot{r}\theta + r\dot{\theta}, \quad v_r = \dot{r}, \quad v_z = \dot{z}$$

$$\dot{v}_\theta = \ddot{r}\theta + 2\dot{r}\dot{\theta} + r\ddot{\theta}$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

Equation of particle motion

$$m\frac{d\vec{v}}{dt}=q\left\{-\dot{A}_\theta\vec{e}_\theta+\left[\frac{v_\theta}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_r-\left(\frac{v_r}{r}\frac{\partial}{\partial r}(rA_\theta)+v_z\frac{\partial A_\theta}{\partial z}\right)\vec{e}_\theta+v_\theta\frac{\partial A_\theta}{\partial z}\vec{e}_z\right]\right\}$$

θ-component

$$\begin{aligned}m\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)&=-\frac{q}{r}\left(\dot{r}A_\theta+r\dot{A}_\theta+r\dot{r}\frac{\partial A_\theta}{\partial r}+r\dot{z}\frac{\partial A_\theta}{\partial z}\right)\\&=-\frac{q}{r}\left(\dot{r}A_\theta+r\frac{\partial A_\theta}{\partial t}+r\frac{\partial r}{\partial t}\frac{\partial A_\theta}{\partial r}+r\frac{\partial z}{\partial t}\frac{\partial A_\theta}{\partial z}\right)\\&=-\frac{q}{r}\left(\dot{r}A_\theta+r\frac{dA_\theta}{dt}\right)\\&=-\frac{q}{r}\frac{d}{dt}\left[r(t)A_\theta(r,z,t)\right]\end{aligned}$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

Multiply by *r*

$$m\left(r^2\ddot{\theta}+2r\dot{r}\dot{\theta}\right)+r\frac{q}{r}\frac{d}{dt}\left(rA_\theta\right)=\frac{d}{dt}\left(mr^2\dot{\theta}+qrA_\theta\right)=\frac{d}{dt}(l)=0$$

→ Canonical momentum *l* due to the rotational motion about the z-axis conserved

$$rA_\theta\left(\frac{mr\dot{\theta}}{qA_\theta}+1\right)=\frac{l}{q}$$

$$rA_\theta\left(\frac{mr\dot{\theta}}{qA_\theta}+1\right)=rA_\theta\left(\frac{mr\dot{\theta}}{\frac{q}{2}rB_z(0)}+1\right)\longleftarrow$$

$$B_z=(\nabla\times A)\cdot\vec{e}_z=\frac{1}{r}\frac{\partial}{\partial r}\left(rA_\theta\right)$$

$$A_\theta=\frac{1}{r}\int_0^r r'B_z(r')dr'\approx\frac{1}{2}rB_z(0)$$

$$=rA_\theta\left(\frac{2mv_\perp}{r|q|B_z(0)}+1\right)=rA_\theta\left(\frac{2r_L}{r}+1\right)=\frac{l}{q}=const.$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

$$rA_\theta\left(\frac{2r_L}{r}+l\right)=\frac{l}{q}=const.$$

- $r_L/r << 1 \rightarrow$ The trajectories of the particles must lie on surfaces defined by $rA_\theta = const.$

$\rightarrow$ Flux surface label:
The particle's guiding centers move on them in the absence of other forces (as a consequence of angular momentum conservation)

- Magnetic field lines lie within these surfaces which can be readily demonstrated by proving that the surface's normal is orthogonal to the field.

$$\vec{B}\cdot\nabla(rA_\theta)=B_r\frac{\partial(rA_\theta)}{\partial r}+B_z\frac{\partial(rA_\theta)}{\partial z}=0 \longleftarrow$$

$$B_r=\vec{e}_r\cdot\left(\nabla\times\vec{A}\right)=-\frac{\partial A_\theta}{\partial z}$$

$$B_z=\vec{e}_z\cdot\left(\nabla\times\vec{A}\right)=\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)$$

Basic Tokamak Variables

- **Plasma Equilibrium**

$$\nabla p = \vec{J} \times \vec{B}$$ → Force balance

kinetic pressure balanced by **JxB** (Lorentz) force

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$ → Ampere's law

$$\nabla \cdot \vec{B} = 0$$ → Closed magnetic field lines

$$\nabla p = (\nabla \times B) \times B / \mu_0$$

$$= [(B \cdot \nabla) B - \nabla (B^2 / 2)] / \mu_0$$

$$\nabla (p + B^2 / 2\mu_0) = (B \cdot \nabla) B / \mu_0$$

Assuming the field lines are straight and parallel

$$\frac{E^*_{mag}}{V} = \frac{BH}{2} = \frac{B^2}{2\mu_0}$$

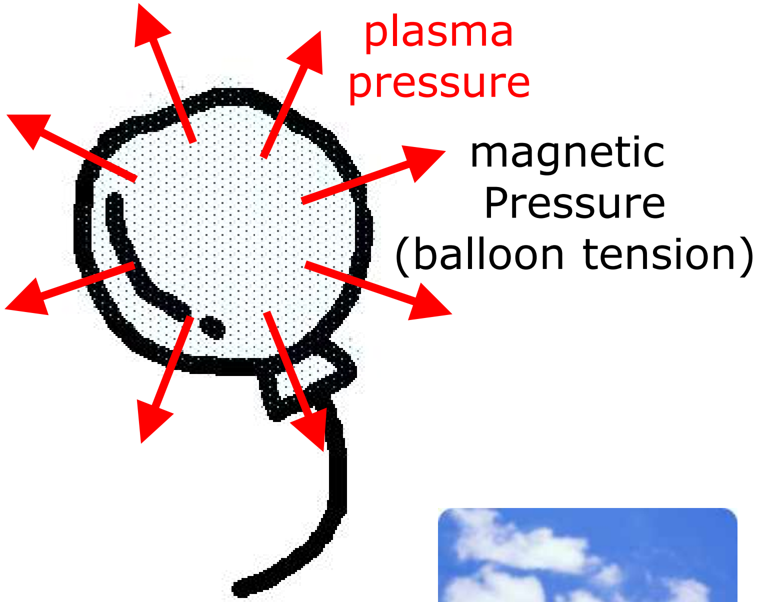
$$p + \frac{B^2}{2\mu_0} = \text{constant}$$

Total sum of kinetic pressure and magnetic field energy density will be a constant

Basic Tokamak Variables

- **Concept of Beta**

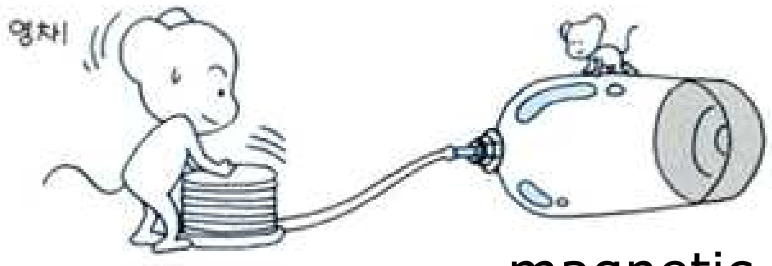
$$\beta = \frac{p}{B^2/2\mu_0} = \frac{(n_i + n_e)kT}{B^2/2\mu_0}$$



- The ratio of the plasma pressure to the magnetic field pressure
- A measure of the degree to which the magnetic field is holding a non-uniform plasma in equilibrium.
- In most magnetic configurations, fusion plasma confinement requires an imposed magnetic pressure significantly exceeding the particle kinetic pressure.

Basic Tokamak Variables

- **Concept of Beta**



plasma pressure

magnetic pressure

$$\beta = 2\mu_0 p / B^2$$

Instability (bad curvature region) when with high *p*

www.waterrocket.com/goachaik-28.htm
the43sunsets.tistory.com/tag/코카콜라

- β is related with fusion reactor economics and technology.
- Maximum allowable value is set by MHD equilibrium requirements and instabilities driven by the pressure gradient.

Basic Tokamak Variables

- **Concept of Beta**

  - Assuming that the magnetic surfaces have concentric, circular CXs and that conditions are independent of φ.

$$\langle p \rangle = \int p dS \Big/ \int dS = \frac{2\pi}{\pi a^2} \int_0^a p(r) r dr$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$$ Ampère's law

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_0 j_\varphi \,, \quad B_\theta = \frac{\mu_0}{r}\int_0^r j_\varphi(r') r' dr'$$

$$I_p = 2\pi \int_0^a j_\varphi r dr = 2\pi a B_{\theta a} / \mu_0 \qquad \beta_t = \frac{2\mu_0 \langle p \rangle}{B_\varphi^2} \,, \quad \beta_p = \frac{2\mu_0 \langle p \rangle}{B_{\theta a}^2} = \frac{8\pi^2 a^2 \langle p \rangle}{\mu_0 I_p^2}$$

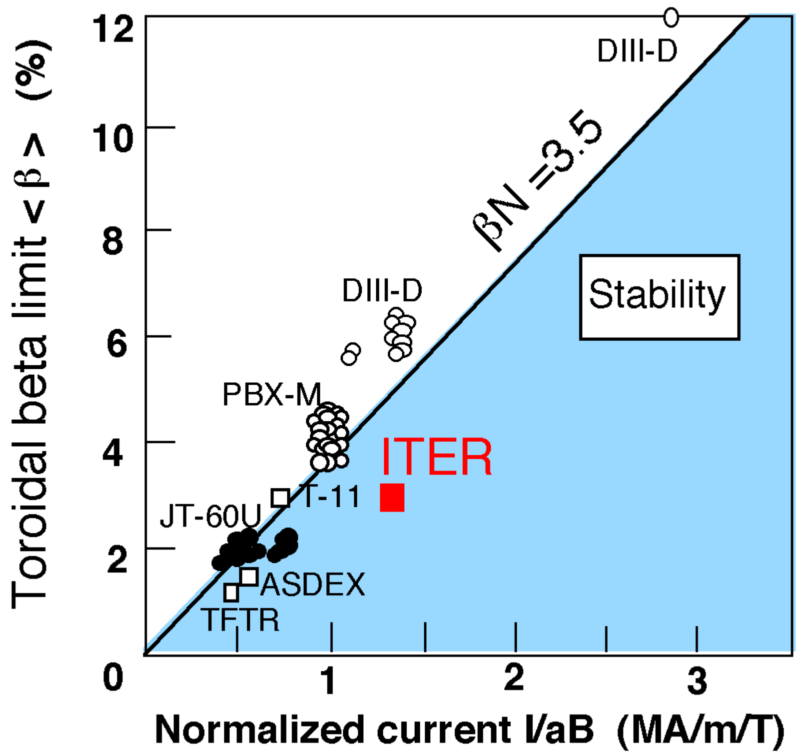
$$\overline{\beta} \equiv \frac{2\mu_0 \langle p \rangle}{B_\varphi^2 + B_{\theta a}^2} \qquad \frac{1}{\overline{\beta}} = \frac{1}{\beta_t} + \frac{1}{\beta_p}$$

- The power output for a given magnetic field and plasma assembly is proportional to the square of beta.
- In a reactor it should exceed 0.1: economic constraint

Basic Tokamak Variables

- **Normalized beta – stability limit**

$$\beta_N = \beta_t \frac{aB_t}{I_p}$$



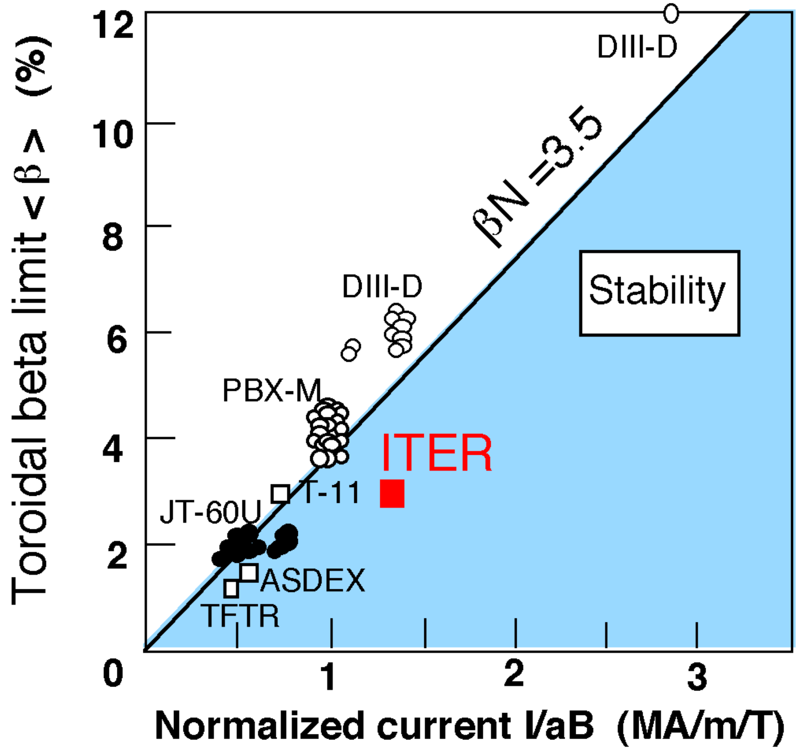
- Fundamental elements affecting the β_N-limit

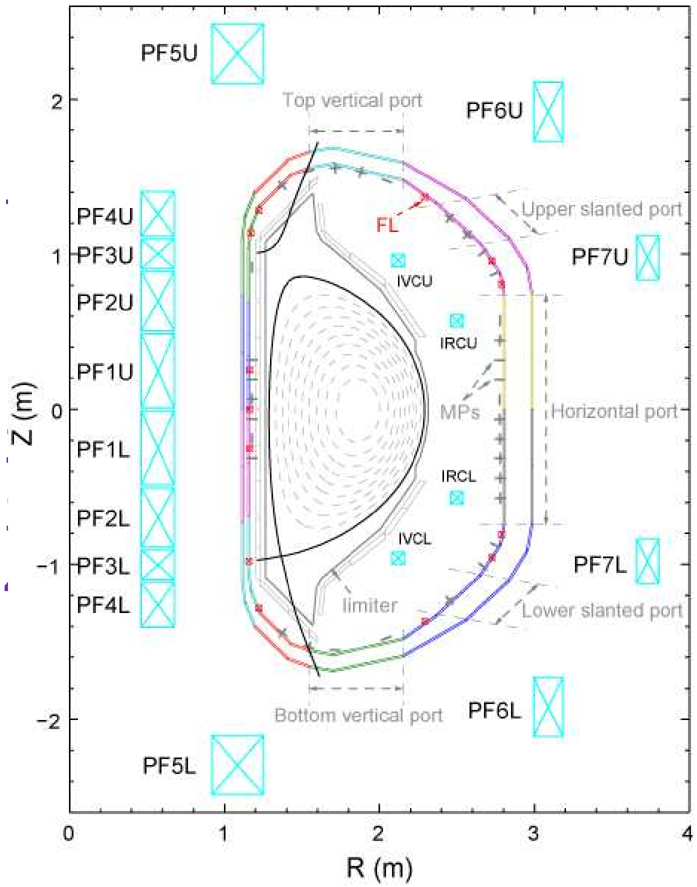
1. Current profile
2. Pressure profile
3. Plasma shape
4. Stabilising wall

Basic Tokamak Variables

- **Normalized beta – stability limit**

$$\beta_N = \beta_t \frac{aB_t}{I_p}$$





Basic Tokamak Variables

- **How to achieve high beta?**

- Providing high heating power and reducing loss by transport (sealing, insulation)
- If without reducing transport loss, transient high beta achievable with high heating power

Basic Tokamak Variables

- **Plasma internal inductance**

$$l_i = \frac{L_i / 2\pi R_0}{\mu_0 / 4\pi} = \frac{2}{\mu_0^2 I_p^2 R_0} \int B_\theta^2(r) d^3V \qquad L_i = \frac{1}{\mu_0 I_p^2} \int B_\theta^2(r) d^3V$$

- For flat current density profile, circular cx

$$J = J_0 \quad (r \le a)$$

$$J = 0 \quad (a < r \le b)$$

$$B_\theta = \frac{\mu_0}{2\pi r} \int J 2\pi r dr \qquad B_{\theta a} = \frac{\mu_0 I_p}{2\pi a}$$

$$B_\theta = \frac{\mu_0 I_p r}{2\pi a^2} \quad (r \le a)$$

$$L_i = \frac{1}{\mu_0 I_p^2} \int B_\theta^2(r) 2\pi R_0 2\pi r dr$$

$$B_\theta = \frac{\mu_0 I_p}{2\pi r} \quad (a < r \le b)$$

$$l_i = \frac{1}{2} - 2 \ln \frac{a}{b}$$

Basic Tokamak Variables

- **Plasma internal inductance**
  - For Bennett current density profile, circular cx

$$J = \frac{I_p a^2}{\pi \left(r^2 + a^2\right)^2} \quad (r \le a)$$

$$J = 0 \quad (a < r \le b)$$

$$B_\theta = \frac{\mu_0 I_p}{2\pi} \left(\frac{r}{r^2 + a^2} \right) \quad (r \le a)$$

$$B_\theta = \frac{\mu_0 I_p}{4\pi r} \quad (a < r \le b)$$

$$B_{\theta a} = \frac{\mu_0 I_p}{4\pi a}$$

$$l_i = \frac{1}{2} \left(\ln \frac{4b}{a} - 1 \right)$$

Basic Tokamak Variables

- **Plasma internal inductance**
  - For more general current density profile, circular cx

$$J = J(0)\left(1-\frac{r^2}{a^2}\right)^{\nu} \quad (r \le a) \qquad J(0)=\frac{I_p(\nu+1)}{\pi a^2}$$

$$J = 0 \quad (a < r \le b)$$

$$B_\theta = \frac{\mu_0 J(0) a^2}{2(\nu+1)r}\left(1-\left(1-\frac{r^2}{a^2}\right)^{\nu+1}\right) \quad (r \le a)$$

$$B_\theta = \frac{\mu_0 J(0) a^2}{2(\nu+1)r} \quad (a < r \le b)$$

$$l_i = ?$$