458.308 Process Control & Design

Lecture 6: PID Controller Tuning

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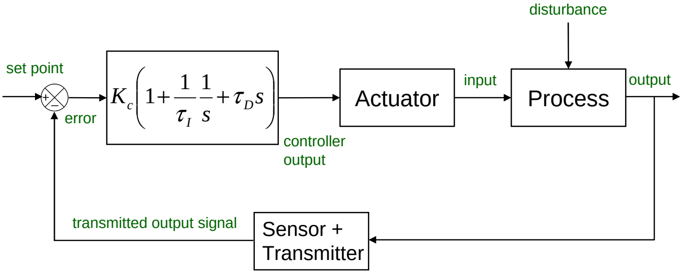
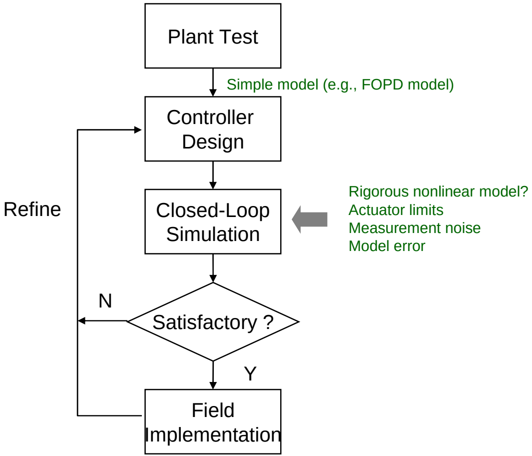
Main Problem



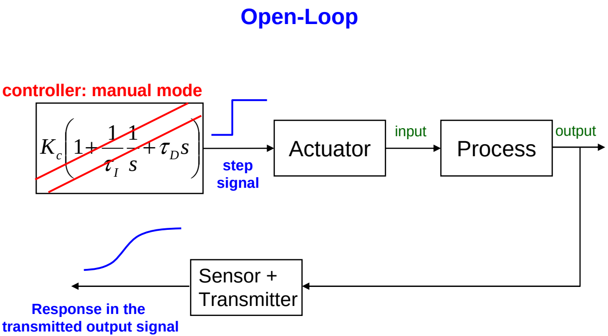
Given a process, choose the parameters K_c, τ_I (and possibly τ_D) so that

- the closed-loop system is stable.
- the output follows the set point fast (rise time, settling time) and smoothly with little or no offset and no excessive overshoot or oscillation (decay ratio).
- the output is insensitive to (recovers quickly from) disturbances.

General Procedure



Principle of Step Response Based Method



- Based on the response curve, fit a simple model (usually First-Order-Plus-Delay)
- Use the model to decide on the PI(D) parameter values
  - Ziegler-Nichols Method
  - Cohen-Coon Method
  - Minimum IAE/ISE/ITAE Method
  - IMC Method

Ziegler-Nichols Method

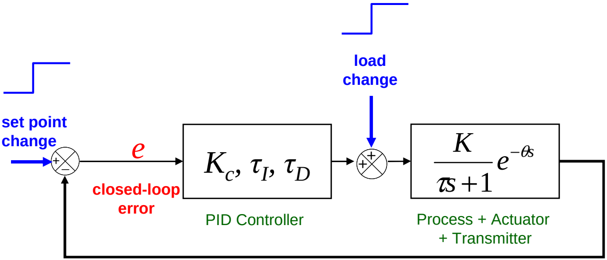
- Based on obtaining a closed-loop response of 1/4 decay ratio ($\zeta = 0.2$) and 50% overshoot for a set point change.
- Cohen-Coon method is a slightly refined version of this.

Controller	K_c	τ_I	τ_D
P	$\frac{1}{K}\left(\frac{\tau}{\theta}\right)$	--	--
PI	$\frac{0.9}{K}\left(\frac{\tau}{\theta}\right)$	3.33θ	--
PID	$\frac{0.9}{K}\left(\frac{\tau}{\theta}\right)$	2.0θ	0.5θ

Note: Better to use for $0.1 < \theta/\tau < 1.0$ (almost negligible delay)

Integral Error Criteria Based Method

Choose PID parameters to minimize time integral of closed-loop error!



- The parameters that minimize the integral error depend on the plant's FOPD parameters
- The parameters that minimize the integral error for a set point change and those for a load change are different

Types of Integral Error

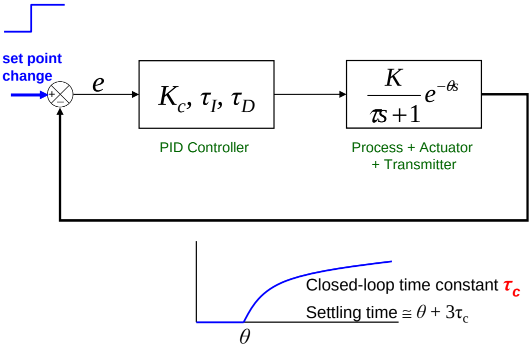
1. Integral Absolute Error (IA): Sum of the area under the error curve
$$\text{IAE} = \int_0^\infty |e(t)|\, dt$$
2. Integral Square Error (ISE): Penalizes high peaks more
$$\text{ISE} = \int_0^\infty e^2(t)\, dt$$
3. Integral of Time-Weighted Absolute Error (ITAE): Penalizes slow settling more
$$\text{ITAE} = \int_0^\infty t\,|e(t)|\, dt$$

ITAE Tuning Rules

Controller Type	Type of Response	K_c	τ_I	τ_D
P	Disturb.	$\frac{0.49}{K}\left(\frac{\tau}{\theta}\right)^{1.084}$	--	--
PI	Set point	$\frac{0.586}{K}\left(\frac{\tau}{\theta}\right)^{0.916}$	$\frac{\tau}{1.03-0.165\left(\frac{\theta}{\tau}\right)}$	--
PI	Disturb.	$\frac{0.859}{K}\left(\frac{\tau}{\theta}\right)^{0.977}$	$\frac{\tau}{0.674}\left(\frac{\theta}{\tau}\right)^{0.680}$	--
PID	Set point	$\frac{0.965}{K}\left(\frac{\tau}{\theta}\right)^{0.855}$	$\frac{\tau}{0.796-0.147\left(\frac{\theta}{\tau}\right)}$	$0.308\tau\left(\frac{\theta}{\tau}\right)^{0.929}$
PID	Disturb.	$\frac{1.357}{K}\left(\frac{\tau}{\theta}\right)^{0.947}$	$\frac{\tau}{0.842}\left(\frac{\theta}{\tau}\right)^{0.738}$	$0.381\tau\left(\frac{\theta}{\tau}\right)^{0.995}$

Similar tables exist for IAE and ISE criteria.

Internal Model Control Tuning



- τ_c: User-chosen parameter -- smaller τ_c, faster response, more aggressive tuning
- Choose PI(D) parameter so that the closed-loop response to a set point change is approximately FOPD with time constant of τ_c.

IMC Tuning Rule

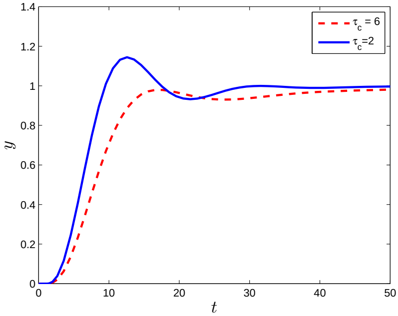
Controller Type	K_c	τ_I	τ_D	Recommended Choice of τ_c
PI	$\frac{\tau}{\tau_c K}$	τ	--	$\frac{\tau_c}{\theta} > 1.7$
``Improved" PI	$\frac{2\tau+\theta}{2\tau_c K}$	$\tau + \frac{\theta}{2}$	--	$\frac{\tau_c}{\theta} > 1.7$
PID	$\frac{2\tau+\theta}{2K(\tau_c+\theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	$\frac{\tau_c}{\theta} > 0.25$

Example

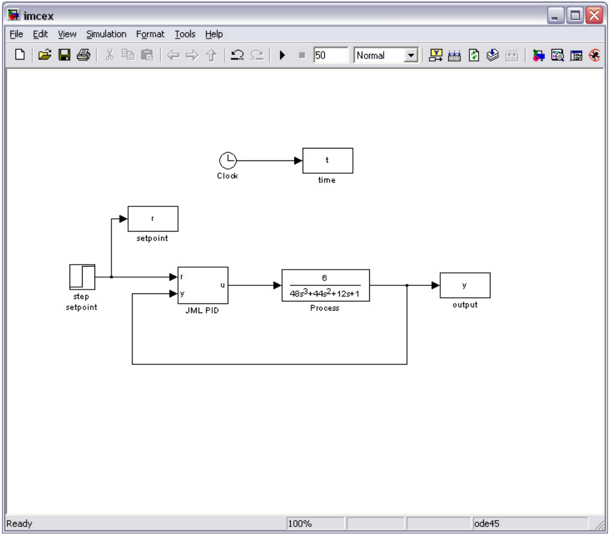
Plant/model mismatch

$$G(s) = \frac{6}{(2s+1)(4s+1)(6s+1)} \quad \Rightarrow \quad \tilde{G}(s) = \frac{6}{15s+1} e^{-3s}$$

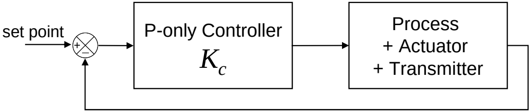
IMC Tuning: $\tau_c = 6,\ 2$



Simulink Implementation



Continuous Cycling



Ziegler-Nichols Tuning Rule

Obtain the ultimate gain† K_{cu} and ultimate period† P_u from the continuous cycling experiment and use the following rule -- designed to give 1/4 decay ratio underdamped closed-loop response to a set point change.

†more relevant information for closed-loop control than step response curve

Controller	K_c	τ_I	τ_D
P	$0.5K_{cu}$	--	--
PI	$0.45K_{cu}$	$P_u/1.2$	--
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$

- Problems
  - 1/4 decay ratio response was judged too oscillatory and not robust enough for process control.
  - Continuous cycling experiment is difficult to conduct.

Modified Ziegler Nichols Method

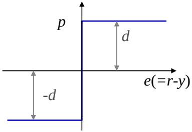
Controller	K_c	τ_I	τ_D
Original	$0.6K_{cu}$	$P_u/2$	$P_u/8$
Some overshoot	$0.33K_{cu}$	$P_u/2$	$P_u/3$
No overshoot	$0.2K_{cu}$	$P_u/2$	$P_u/3$

- Less aggressive than original Z-N
- More gain margin: GM = 1/0.33 = 3, GM = 1/0.2 = 5
- These tunings are more robust than the original Z-N tuning.
- Perhaps better suited for process control.

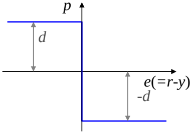
Relay Tuning Method

A different, more practical way to obtain K_{cu} and P_u from a process.

- Relay: on-off controller
- Positive gain process: reverse-acting relay

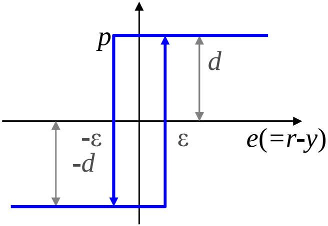


- Negative gain process: direct-acting relay



Advantage of Relay Tuning Method over Continuous Cycling

- The amount of oscillation can be controlled by the engineer (d).
- No risk of making the process go unstable
- Deadzone (in the relay) can be adjusted to control the sensitivity to noise. However, too large a deadzone may bias the result.



- Commercial auto-tuner based on this idea.