Fatigue Analysis and Design: Theory

2014 Fall

School of Mechanical and Aerospace Engineering
Seoul National University

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5. Stress-Life(S-N) Method

Fatigue Strength and Analysis (2-2)

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- 8. Variable Amplitude Loading

5.1 Stress-Life(S-N) Diagram

5.1.1 Stress-Life (S-N) Method

- The first approach used in an attempt to understand and quantify metal fatigue.
- It is still widely used in design applications where the applied stress is primarily within the elastic range of the material and the resultant lives(cycles to failure) are long.
- Wohler or S-N diagram.

August Wohler(1819~1914)

5.1.2 Rotating Bending Test

- The most common procedure for generating the S-N data.

Ex) R. R. Moore Test

- Four-point loading to apply a constant moment to a rotating(1750rpm) cylindrical hourglass-shaped specimen.
- This loading produces a fully reversed uniaxial state of stress.
- The S-N approach ignores true stress-strain behavior and treats all strains as elastic.

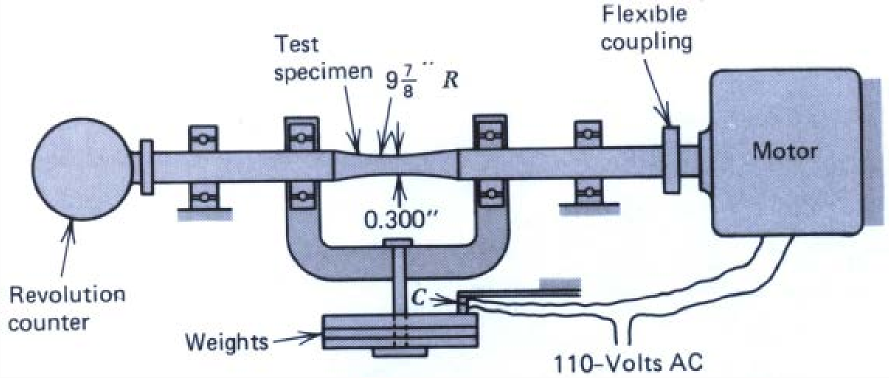


Figure 5.1 Reversed Bending Test

5.1 Stress-Life(S-N) Diagram

5.1.3 S-N Test Data

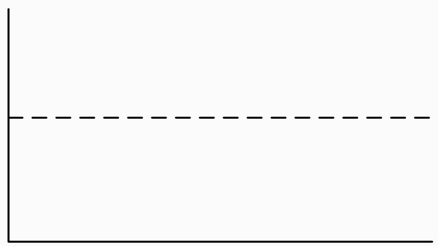
- To be presented on a log-log plot with the actual S-N line representing the mean of the data.
- Endurance or fatigue limit(S_e) is a stress level below which the material has an infinite life.
- For engineering purposes, this infinite life is usually considered to be 10^6 cycles (Fig. 5.2).
- Most nonferrous alloys have no endurance limit and the S-N line has a continuous slope (Fig. 5-3).
- A pseudo-endurance limit or fatigue strength for these materials is taken as the stress value corresponding to a life of 5×10^8 cycles.

Figure 5.2 S-N curve for 1045 steel

Figure 5.3 S-N curve for Al 2024-T4

5.2 Mean Stress Effects

5.2.1 Definition



$$\Delta\sigma = \sigma_{max} - \sigma_{min} = \text{stress range}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \text{stress amplitude}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \text{mean stress}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \text{stress ratio}$$

$$A = \frac{\sigma_a}{\sigma_m} = \text{amplitude ratio}$$

5.2.2 The stress ratio and amplitude ratio values corresponding to several common loadings

- Fully reversed R=-1 A = ∞
- Zero to max R=0 A=1
- Zero to min R=∞ A=-1

5.2.3 Haigh Diagram

- The results of a fatigue test using a nonzero mean stress are plotted on a Haigh diagram with lines of constant life drawn through the data points.

5.2.4 Empirical relationships

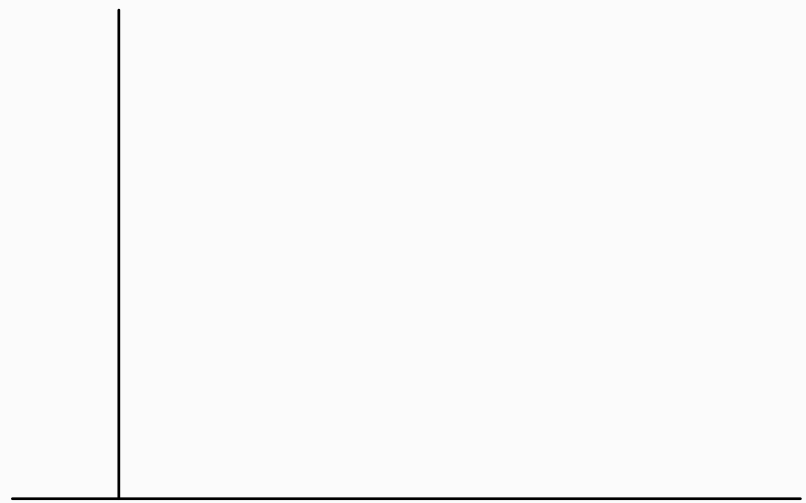
- Since the tests required to generate a Haigh diagram can be expensive, several empirical relationships have been developed to generate the line defining the infinite-life design region.
- These methods use various curves to connect the endurance limit on the alternating stress axis to either the yield strength, ultimate strength, true fracture stress.

Soderberg (USA, 1930) : $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$

Goodman (England, 1899) : $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1$

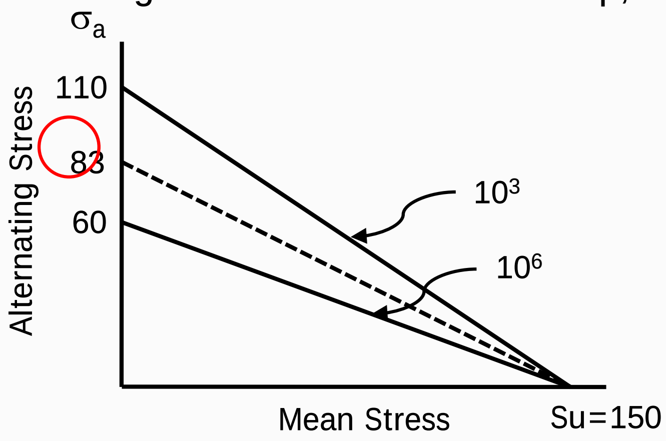
Gerber (Germany, 1874) : $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = 1$

Morrow (USA, 1960s) : $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma_f} = 1$



5.2.5 Example

(Ex) A component undergoes a cyclic stress with a maximum value of 110 ksi and a minimum value of 10 ksi. The component is made from a steel with an ultimate strength, $S_{u,}$ of 150 ksi, an endurance limit, S_e, of 60 ksi, and a fully reversed stress at 1000 cycles, S_{1000}, of 110 ksi. Using the Goodman relationship, determine the life of the component.



(Sol) (1) To determine the stress amplitude and mean stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{110-10}{2} = 50 \text{ ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{110+10}{2} = 60 \text{ ksi}$$

(2) To generate a Haigh diagram with constant life lines at 10^6 and 10^3

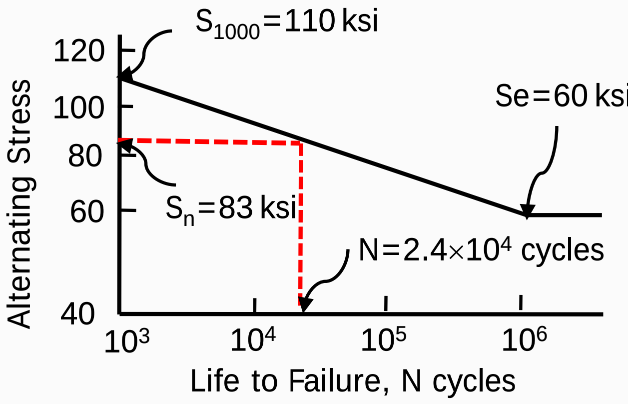
(Connect the endurance limit, S_e, and S_{1000} values)

(3) Calculate S_n by using Goodman relationship

$$\frac{50}{S_n} + \frac{60}{150} = 1\,, \quad S_n = 83 \text{ ksi}$$

(4) The value for S_n can now be entered on the S-N diagram to determine the life of the component, N_f

The resulting life to failure, N_f is 2.4×10^4 cycles.



5.3 Modifying Factors

5.3.1 Modifying Factors

(1) Size

(2) Type of Loading

(3) Surface Finish

(4) Surface Treatments

(5) Temperature

(6) Environment

$$S_e = C_L C_G C_S C_T C_R S'_e \text{ (Moore endurance limit)}$$

5.3.1.1 Size Effects

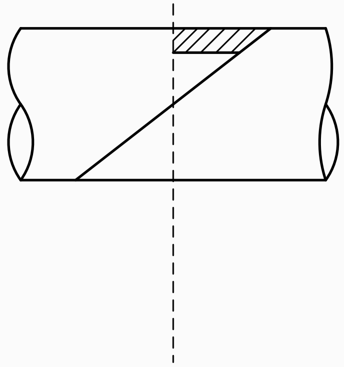
- Influence of Size on Endurance Limit

Diameter(in)	Endurance Limit(S_e , ksi)
0.3	33.0
1.5	27.6
6.75	17.3

- Fatigue is controlled by the weakest link of the material.
- The probability of having a weak link increasing with material volume.

5.3 Modifying Factors

- A large component will have a less steep stress gradient and hence a larger volume of material subjected to this high stress.
- There will be a greater probability of initiating a fatigue crack in large component.



5.3.1.2 Load Effects

- Since the axial specimen has no gradient, it has a greater volume of material subjected to the high stress.
- The ratio of endurance limits for a material found using axial and rotating bending tests ranges from 0.9~1.0.

- The ratio of torsional endurance limit to a rotational bending stress ranges from 0.5~0.6.

5.3 Modifying Factors

5.3.1.3 Surface Finish

- The correlation factor for surface finish is sometimes presented on graphs that use a qualitative description of surface finish such as "polished" or "machined".
- The condition of the surface is more important for higher strength steels.
- The residual surface stress caused by a machining operation can be important.
- In the case of shorter lives, where crack propagation dominates, the condition of surface finish has less effect on the fatigue life.
- Localized surface irregularities such as stamping marks can serve as very effective stress concentrations and should not be ignored.

Figure 5.7 Surface finish factor: steel parts

5.3 Modifying Factors

5.3.1.4 Surface Treatment

- Effect of plating

: Chrome and nickel plating on steels can cause up to a 60% reduction in endurance limits.

: This is mainly because of tensile stress (residual stress) developed during plating.

Figure 5.8 Effect of chrome planting on S-N curve of 4140 steel

* Alleviation of the residual tensile stress problem

(1) Thermal process 1 – Nitride the part before plating

(2) Thermal process 2 – Bake or anneal the part after plating

(3) Surface hardening – Shot peen the part before or after plating

5.3 Modifying Factors

- Thermal treatment

: Carburizing or nitriding diffusion is very effective for increasing fatigue strength by causing volumetric changes through the development of residual compressive surface stresses.

Geometry	Endurance Limit (ksi)	
	Nitrided	Not Nitrided
Without notch	90	45
Half-circle notch	87	25
V notch	80	24

- Mechanical treatment

: Several methods can be used to produce a residual compressive stress: Cold rolling, Shot peening

5.3 Modifying Factors

5.3.1.5 Temperature

- There is a tendency for the endurance limits of steels to increase at low temperatures.
- At high temperatures the endurance limit for steels disappears due to the mobilizing of dislocations.
- At temperatures beyond approximately one-half of the melting point of the material, creep becomes important. In this range the S-N approach is no longer applicable.

5.3.1.6 Environment

- Effect of corrosion-fatigue

  : The curves generated in room air and a vacuum show that even the humidity and oxygen in room air can slightly reduce fatigue strength.

  : The reduction of fatigue properties for this curve is due to the rough surface caused by corrosion pitting.

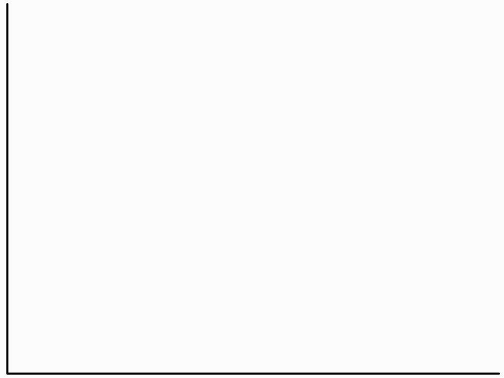


Figure 5.10 Effect of various environments on the S-N curve of steel

6. Strain-Life(ε -N) Method

Fatigue Strength and Analysis (2-2)

- 5. Stress-Life
- 6. Strain-Life
  - 6.1 Concept of Strain-Life Method
  - 6.2 Material behavior
  - 6.3 Stress-plastic strain power law relation
  - 6.4 Stress-life curve
  - 6.5 Mean stress effects
- 7. Fracture Mechanics
- 8. Variable Amplitude Loading

6.1 Concept of Strain-Life Method

6.1.1 Concept of Strain-Life Method

- The strain-life method is based on the observation that the response of the material in critical locations (i.e., notch) is strain or deformation.
- In the regime of low cycle fatigue (LCF) or high loading, this method works well to describe the cyclic stress-strain response and the material behavior.
- This method can simulate fatigue damage of notched part with smooth specimen under strain-controlled test.
- Assume that both notched and smooth specimens have equivalent fatigue damage under same stress-strain histories.

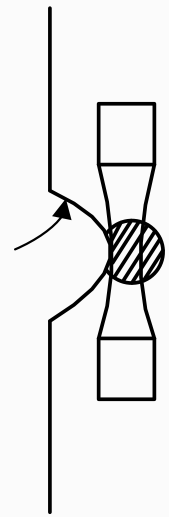


Figure 6.1 Equally stressed volume of material

6.2 Material behavior

6.2.1 Cyclic Stress-Strain Behavior

6.2.1.1 Hysteresis Loop

- Cyclic stress-strain curves are useful for assessing the durability of structures.
- The material response subject to cyclic inelastic loading is in the form of a hysteresis loop.
- The area within the loop is the dissipated energy per cycle and volume.

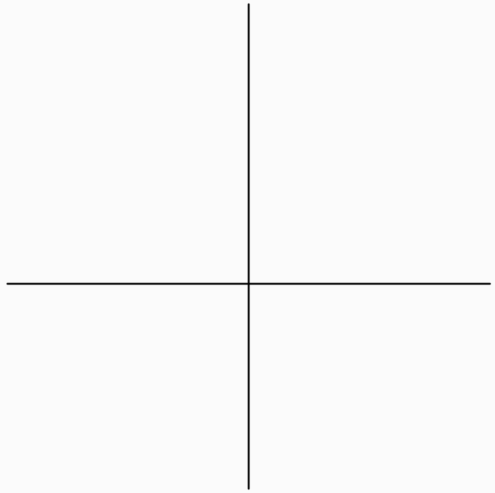


Figure 6.2 Hysteresis Loop

6.2 Material behavior

6.2.1.2 Bauschinger effect

- Fig. 6.3(a) shows the material behavior of a bar under the load that produces σ_{max} via the yield strength σ_y.

- Fig. 6.3(b) depicts the material behavior upon unloading and then compressive loading to $-\sigma_{max}$.

- Under compressive loading, inelastic (plastic) strains develop before $-\sigma_y$ reaches.

figure 6.3 Bauschinger effect

6.2 Material behavior

6.2.1.3 Transient behavior: Cyclic Strain Hardening and Softening

- Cyclically harden : The maximum stress obtained increases with each cycle of strain.

- Cyclically soften : The maximum stress obtained decreases with each cycle of strain.

- Generally, transient behavior (strain hardening or softening) occurs only during the early fatigue life. After this, the material achieves a cyclically stable condition.

- This is usually achieved after approximately 20 to 40% of the fatigue life.

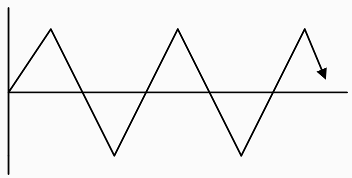


Figure 6.4 Cyclic hardening/softening

(a) constant strain amplitude (b) stress response (increasing/decreasing stress level);

(c) cyclic stress-strain response.

6.2 Material behavior

6.2.1.4 Cyclic Stress-Strain Curve Determination

- Cyclic stress-strain curves can be obtained from tests by several methods

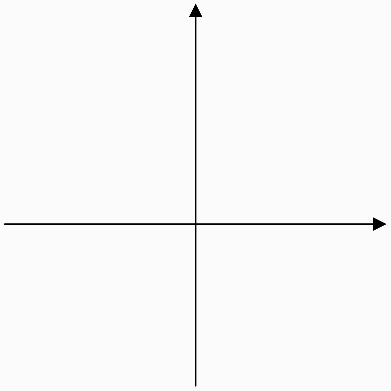


Figure 6.5 Cyclic stress-strain curve obtained by connecting tips of stabilized hysteresis loops.

- Massing hypothesis : The stabilized hysteresis loop can be obtained by doubling the cyclic stress-strain curve.

6.3 Stress-plastic strain power law relation

6.3 Cyclic Stress-Strain Curve Determination

- Power law function

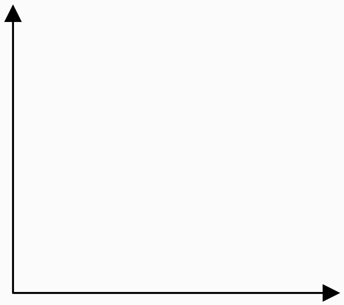
$$\sigma = K'\left(\varepsilon_p\right)^{n'}$$

$\sigma =$ cyclically stable stress amplitude

$\varepsilon_p =$ cyclically stable plastic strain amplitude

$K' =$ cyclic strength coefficient

$n' =$ cyclic strain hardening exponent



- The total strain is the sum of the elastic and plastic components

$$\varepsilon = \left(\frac{\sigma}{E}\right) + \left(\frac{\sigma}{K'}\right)^{1/n'}$$

- Recall that Massing's hypothesis

- From Massing hypothesis a point corresponding to P1 can be located on the hysteresis curve

- Rearranging these equations, we obtain

6.4 Strain-Life Curve

6.4.1 Strain-life equations

- Basquin(1910)'s equation: stress-life data could be plotted linearly on a log-log scale

$$\frac{\Delta\sigma}{2} = \sigma'_f \left(2N_f\right)^b$$

$\frac{\Delta\sigma}{2}$ = ture stress amplitude

$2N_f$ = reversals to failure(1rev= $\frac{1}{2}$ cycle)

σ'_f = fatigue strength coefficient

b = fatigue strength exponent (Basquin's exponenet)

- Coffin, Manson(1955)'s equation: plastic strain-life data could be linearized on log-log coordinates

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f \left(2N_f\right)^c$$

$\frac{\Delta\varepsilon}{2}$ = ture strain amplitude

$2N_f$ = reversals to failure(1rev= $\frac{1}{2}$ cycle)

ε'_f = fatigue ductility coefficient

c = fatigue ductility exponent

6.4 Strain-Life Curve

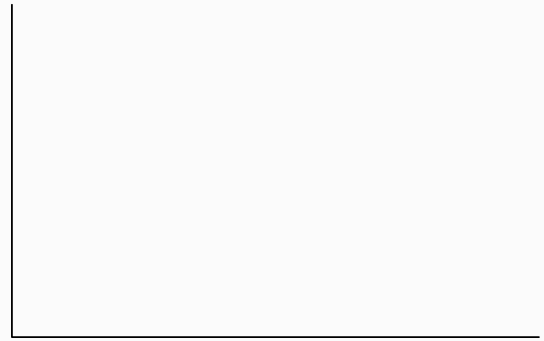
- Total strain is the sum of the elastic and plastic strains

- The elastic term can be written as

- The total strain can now be rewritten as

6.5 Mean Stress Effects

6.5.1 Effect of mean stress on strain-life curve



6.5.2 Modifications to the strain-life equation by Morrow

6.5.3 Modification to the strain-life equation by Manson and Halford

6.5.4 Modification to the strain-life equation by Smith, Watson, and Topper (SWT)

Fatigue Strength and Analysis (2-2)

- 5. Stress-Life
- 6. Strain-Life
- 7. Fracture Mechanics
  - 7.1 Linear elastic fracture mechanics
  - 7.2 Fatigue crack growth
- 8. Variable Amplitude Loading

7.1 Linear Elastic Fracture Mechanics

7.1.1 Historical Overview

- Griffith (1920s)

"A crack in a component will propagate if the total energy of the system is lowered with crack propagation."

: That is, if the change in elastic strain energy due to crack extension is larger than the energy required to create new crack surfaces, crack propagation will occur.

Alan Arnold Griffith (1893~1963)

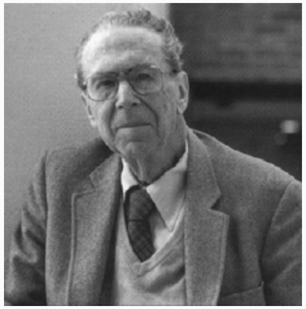
- Irwin (1940s)

: Griffith's theory was extended for ductile materials.

"For ductile materials, the surface energy term is often negligible compared to the energy associated with plastic deformation."

* Strain energy release rate, Crack driving force "G"

: The total energy absorbed during cracking per unit increase in crack length and per unit thickness.



Dr. Geroge R.Irwin (1907~1998)

- Irwin (1950s)

: The local stresses near the crack tip are of the general form.

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \cdot \cdot \cdot \cdot$$

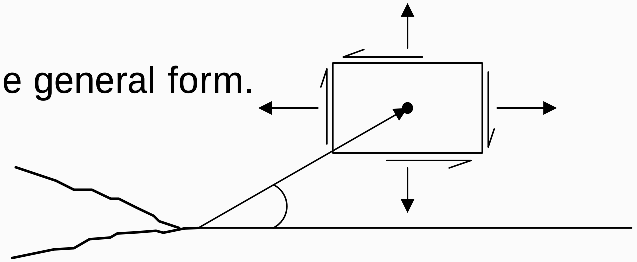


Figure 7.1 Location of local stresses near a crack tip in cylindrical coordinates

7.1 Linear Elastic Fracture Mechanics

7.1.2 Linear Elastic Fracture Mechanics(LEFM) assumptions

- LEFM is based on the application of the theory of elasticity to bodies containing cracks or defects
- The general form of the LEFM

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \cdot \cdot \cdot$$

: A singularity exists such that as r, the distance from the crack tip, tends toward zero, the stresses go to infinity

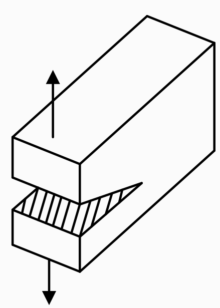
- Since materìals plastically deform as the yield stress is exceeded, a plastic zone will form near the crack tip. The basis of LEFM remains valid, though, if this regíon of plasticity remains small in relation to the overal1 dimensions of the crack and cracked body

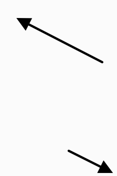
7.1.3 Loading Modes

(1) Mode - Ⅰ : Opening Mode

(2) Mode - Ⅱ : Sliding Mode

(3) Mode - Ⅲ : Tearing Mode





7.1.4 Stress Intensity Factor

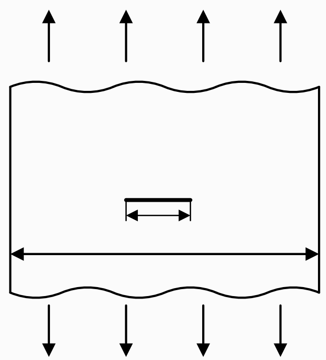
- The magnitude of the local stresses around the crack tip
- Depends on loading, crack size, crack shape, and geometric boundaries
- The general form

$$K = f(g)\sigma\sqrt{\pi a}$$

σ = remote stress applied to component

a = crack length

$f(g)$ = correction factor that depends on specimen and crack geometry



7.1.5 Plastic Zone Size

- Materials develop plastic strains as the yield stress is exceeded in the region near the crack tip
- The amount of plastic deformation is restricted by the surrounding material, which remains elastic

Figure 7.4 Yielding near crack tip

- Monotonic Plastic Zone Size

7.1 Linear Elastic Fracture Mechanics

-Cyclic Plastic Zone Size

Figure 7.6 Reversed plastic zone size

7.1.6 Fracture Toughness

- K_{IC} , critical value of the stress intensity factor
- The plane strain fracture toughness is dependent on specimen geometry(e.g. thickness) and metallurgical factors

7.2 Fatigue Crack Growth

7.2.1 Fatigue Crack Growth Curves

- Typical constant amplitude crack propagation data(Fig. 7.8)
- The crack length(a) is plotted versus the corresponding number of cycles(N)
- Most of the life of the component is spent while the crack length is relatively small
- The larger force is applied, the larger crack growth rate is happen

7.2 Fatigue Crack Growth

7.2.2 Fatigue crack growth curve

- crack growth(da/dN) is obtained by taking derivative of the crack length(a) versus cycles(N) curve

- Values of log da/dN can be plotted versus log ΔK

$$\Delta K = K_{\max} - K_{\min} = f(g)\Delta\sigma\sqrt{\pi a}$$

Δσ=remote stress applied to the component

- log da/dN versus log ΔK curve

(1) Region Ⅰ

(2) Region Ⅱ

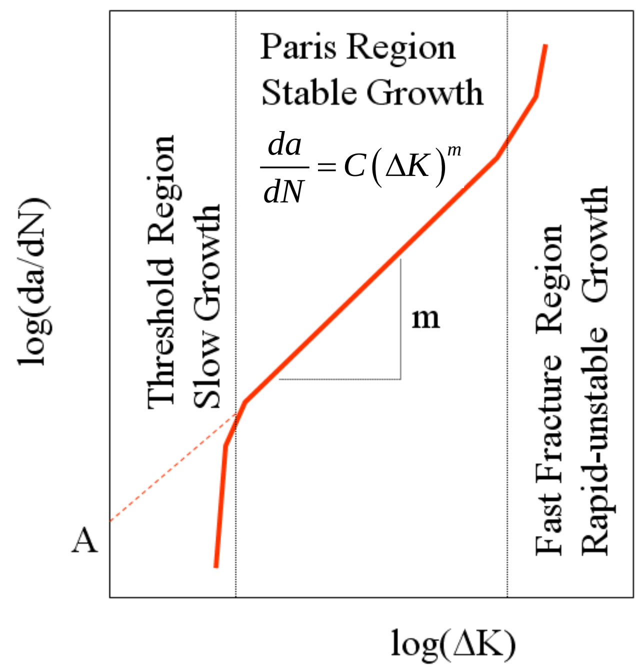
(3) Region Ⅲ

Figure 7.9 Three regions of crack growth rate curve

7.2.2.1 Region Ⅱ

- Most of the current applications of LEFM to describe crack growth behavior are associated with Region Ⅱ
- The slope of the log da/dN versus log ΔK is approximately linear and lies roughly between 10^{-6} and 10^{-3} in./cycle
- Paris equation(1960) $da/dN = C(\Delta K)^m$

  C , m : material constant, ΔK=stress intensity range (K_{max}-K_{min})



- Cycles to failure, N_f using Paris equation

  (Fatigue life calculations for a small edge-crack in a large plate)

$$N_f = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m}$$

: N_f=cycles to failure

: a_i = initial crack length , a_f = final crack length

$$\Delta K = 1.12 \Delta\sigma \sqrt{\pi a}$$

: stress intensity range

$$da/dN = C\left(1.12\Delta\sigma\sqrt{\pi a}\right)^m$$

: Substituting into the Paris equation

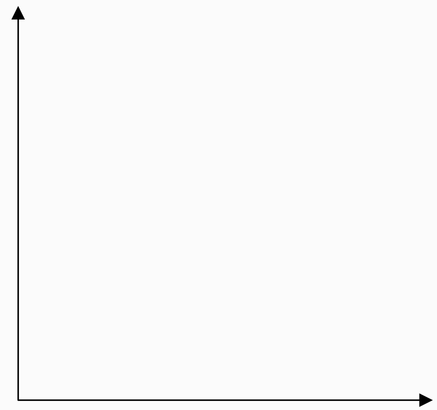
$$N_f = \int_{a_i}^{a_f} \frac{da}{C\left(1.12\Delta\sigma\sqrt{\pi a}\right)^m} = \frac{2}{(m-2)C\left(1.12\Delta\sigma\sqrt{\pi}\right)^m}\left(\frac{1}{a_i^{(m-2)/2}} - \frac{1}{a_f^{(m-2)/2}}\right)$$

: Sperating variables and integrating

7.2 Fatigue Crack Growth

7.2.2.2 Region Ⅰ

- Region Ⅰ of the sigmoidal crack growth rate curve is associated with threshold effect
- Below the value of the threshold stress intensity factor, ΔK_{th}, fatigue crack growth does not occur or occurs at a rate too slow to measure
- Steel: 5~15 ksi$\sqrt{in}$, Aluminum alloys: 3~6 ksi $\sqrt{in}$
- The fatigue threshold is dependent to the stress ratio, R(stress ratio, $R=\sigma_{max} / \sigma_{min}$)



7.2.2.3 Region Ⅲ

- In Region Ⅲ, rapid, unstable crack growth occurs
- Forman's equation: developed to model Region Ⅲ behavior

$$\frac{da}{dN}=\frac{C\left(\Delta K\right)^{m}}{\left(1-R\right)K_{c}-\Delta K}$$

: The sharp upturn in the da/dN versus ΔK curve as fracture toughness is approached

7.2 Fatigue Crack Growth

7.2.3 Factors influencing fatigue crack growth

7.2.3.1 Stress ratio effects

- In general, for a constant ΔK, the more positive the stress ratio, R, the higher the crack growth rates
- Forman's equation : predict stress ratio effects -> As R increases, the crack growth rate, da/dN, increases

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K}$$: Forman's equation

- Another method used to compensate for stress ratio effect: Walker's equation

$$\frac{da}{dN} = C\left[(1-R)^m K_{max}\right]^n$$

7.2.3.2 Environmental effects

(1) Frequency of Loading

(2) Temperature Effect

(3) Waveform of Loading Cycle

Fatigue Strength and Analysis (2-2)

- 5. Stress-Life
- 6. Strain-Life
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- 8. Variable Amplitude Loading
  - 8.1 Fatigue Damage Accumulation
  - 8.2 Cycle Counting

8.1 Fatigue Damage Accumulation

8.1.1 Fatigue Damage Accumulation – "Miner Law"

- Cycle ratio

$$\frac{n}{N} = cycle\ ratio$$

n : Number of loading cycles applied to achieve a crack length "a"

N : Number of cycles applied to achieve the crack length "a_f"

- Damage fraction (D)

: Defined as the fraction of life used up by an event or a series of events

: Failure in any of the cumulative damage theories is assumed to occur when the summation of damage fractions equals 1, or

$$\sum D_i = 1$$

- Linear Damage Rule

"The damage fraction, D_i, at stress level S_i is equal to the cycle ratio, n_i/N_i.

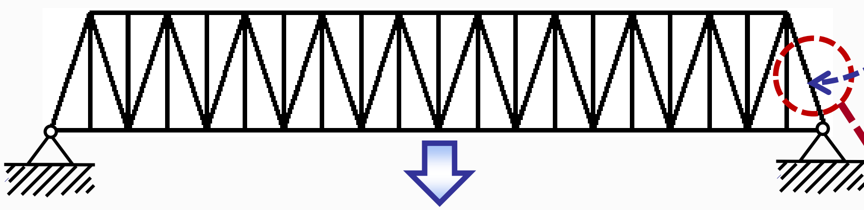
$$D_i = \frac{n}{N} \geq 1 \quad \text{: The failure criterion for variable amplitude loading}$$

(ex) The damage fraction, D, due to one cycle of loading is 1/N. In other words, the application of one cycle of loading consumes 1/N of the fatigue life.

8.1.2 Example : Fatigue Damage Accumulation

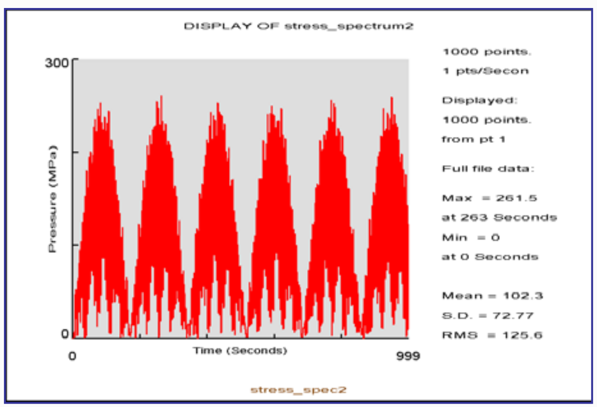
- Plane Truss Structure (MANTEN Steel)

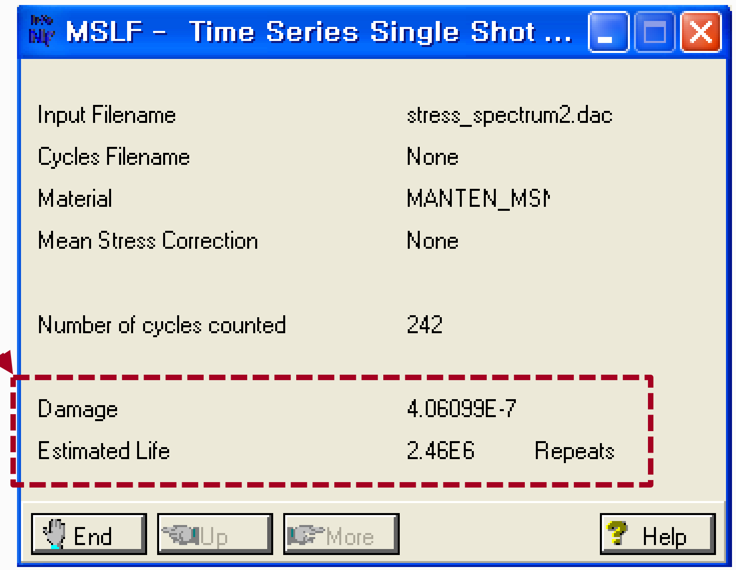
Fatigue Critical Location



F= 40000N×sin(ω ×T), ω =782.3 rad/s

- Stress Spectrum





Fatigue Analysis using MSC.Fatigue Advanced Utilities (Single Location S-N Method)

- Fatigue Analysis Results (S-N Method)

Damage : **4.0699E-7**

Fatigue Life : 2.46E6 Repeats

$$D_1 + D_2 + D_3 + ... + D_{i-1} + D_i \geq 1$$

$$\mathbf{(4.0699 \times 10^{-7}) \times (2.46 \times 10^{6}) = 1.0012 > 1}$$

8.2 Cycle Counting

8.2.1 Cycle Counting

To predict the life of a component subjected to a variable load history, it is necessary to reduce the complex history into a number of events which can be compared to the available constant amplitude test data. This process of reducing a complex load history into a number of constant amplitude events involves what is termed cycle counting.

8.2.2 Cycle Counting Method – Rainflow Counting

- Matsuishi & Endo (1968)
- Drawing the strain-time history so that the time axis is oriented vertically, with increasing time downward.
- The strain history forms a number of “pagoda roofs.”
- The rules specifying the manner in which rain falls are as follows:

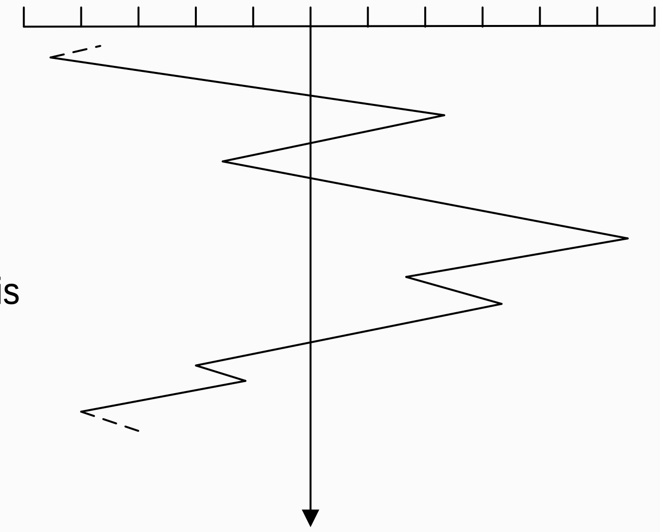
1. To eliminat the counting of half cycles, the strain-time history is drawn so as to begin and end at the strain value of greatest magnitude.
2. A flow of rain is begun at each strain reversal in the history and is allowed to continue to flow unless;

   a. The rain began at a local maximum point (peak) and falls opposite a local maximum point greater than that from which it came.

   b. The rain began at a local minimum point (valley) and falls opposite a local minimum point greater (in magnitude) than that from which it came.

   c. It encounters a previous rainflow.

8.2.3 Rainflow Counting



A. Rain flows from point A over points B and D and continues to the end of the history since none of the conditions for stopping rainflow are satisfied.

B. Rain flows from point B over point C and stops opposite point D, since both B and D are local maximums and the magnitude of D is greater than B (rule 2a above).

C. Rain flows from point C and must stop upon meeting the rain flow from point A (rule 2c).

D. Rain flows from point D over points E and G and continues to the end of the history since none of the conditions for stopping rainflow are satisfied.

E. Rain flows from point E over point F and stops opposite point G, since both E and G are local minimums and the magnitude of G is greater than E (rule 2b).

F. Rain flows from point F and must stop upon meeting the flow from point D

G. Rain flows from point G over point H and stops opposite point A, since both G and A are local minimums and the magnitude of A is greater than G (rule 2b).

H. Rain flows from point H and must stop upon meeting the rainflow from point D (rule 2c).

8.2.4 Formation of hysteresis and calculation of fatigue life using rainflow counting

- A-D, D-A : A full cycle
- B-C form additional cycle combined with C-B
- build E-F, G-H cycle

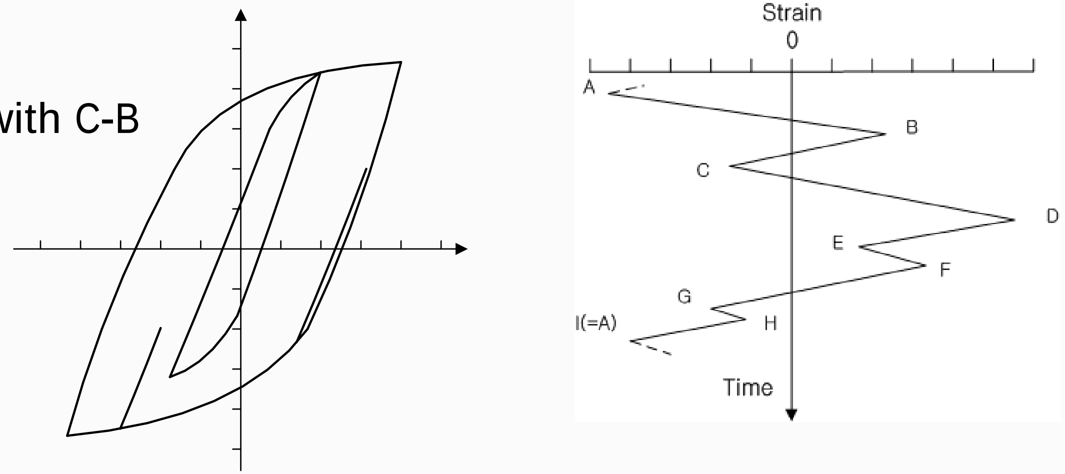


Figure 8.2 Stress – strain response for a given strain

- Fatigue Life Analysis

: Once the closed hysteresis loops have been determined, a fatigue life analysis can be performed on a variable amplitude history by using a strain-life equation that incorporates mean stress effects, such as suggested by Morrow

$$\frac{\Delta\varepsilon}{2} = \left(\frac{\sigma_f^{'} - \sigma_0}{E}\right)(2N_f)^b + \varepsilon_f^{'}(2N_f)^c$$

: If the value of strain range, $\Delta\varepsilon$, and mean stress, σ_0 , for the hysteresis loop are input into the equation, it can be solved for life to failure, N_f .

: If Miner's linear damage rule is used, this value, $1/N_f$, corresponds to the damage fraction for the hysteresis loop.

: Life to failure will be predicted when the sum of the damage fractions of the individual hysteresis loops is greater than or equal to 1. Or, $\sum \frac{1}{N_f} \geq 1$