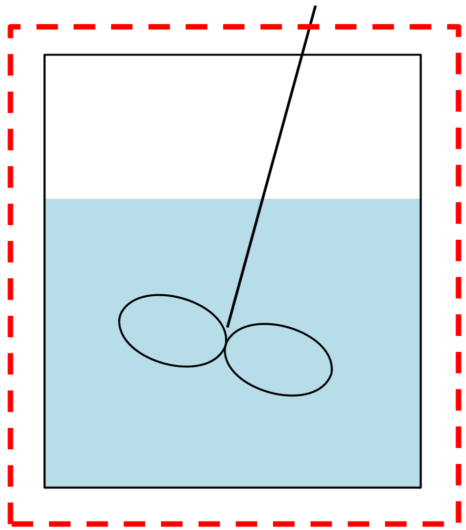
Reactors II

Today's lecture

- Reactor analysis (continued)
  - Plug flow reactor
  - Completely mixed flow reactor
  - Detention time

Review – CMBR, 1st order reaction

- **Completely mixed batch reactor (CMBR)**
  - Fill-and-draw type
  - No flow in or flow out



1) define control volume

2) write a mass balance eq.

$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + R$$

$$V\frac{dC}{dt} \qquad -kCV$$

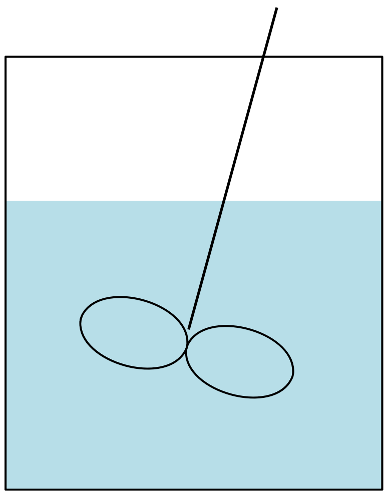
Review - CMBR, 1st order reaction

3) solve the equation

$$\frac{dC}{dt} = -kC$$

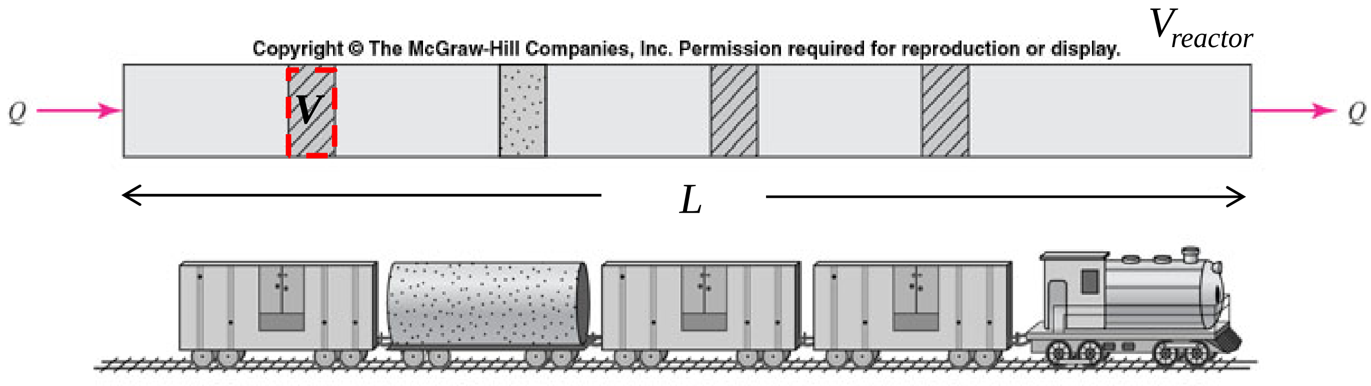
integrating over $t=0$ to t_{final}:

$$\frac{C_{final}}{C_{initial}} = e^{-kt_{final}}$$



Reactor analysis – PFR, 1st order reaction

- Plug-flow reactor (PFR)



1) define control volume:

the moving "plug": a very thin, homogeneous plate moving in the direction of flow

Reactor analysis – PFR, 1st order reaction

2) write a mass balance eq.:

$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + R$$

$$\frac{dM}{dt} = V\frac{dC}{dt} \qquad R = -kCV \quad (1^{st} \text{ order})$$

3) solve the equation: $$\frac{dC}{dt} = -kC$$

integrating over $t=0$ to t_0 (= L/v_{flow} = $V_{reactor}/Q$):

$$\frac{C_{out}}{C_{in}} = e^{-kt_0}$$

→ same form as the batch reactor! (why??)

Reactor analysis – PFR, 1st order reaction

- Examples of PFRs

Disinfection

Rivers and streams

Reactor analysis - PFR

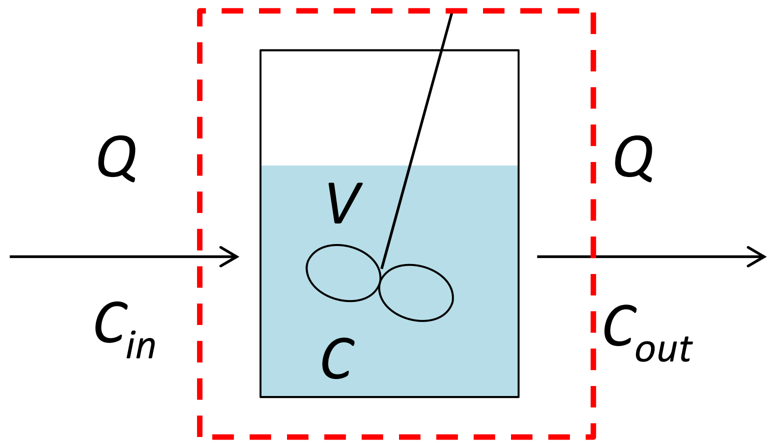
Q: In the U.S., a wastewater treatment plant must disinfect its effluent before discharging the wastewater to a stream. The wastewater contains 4.5×10^5 CFU/L of fecal coliform. The effluent standard for fecal coliform is 2×10^3 CFU/L. Assuming that the disinfection facility is a PFR, determine the length of pipe required if the velocity of the wastewater in the PFR is 0.75 m/s. Assume that the PFR is at steady state and the first-order reaction rate constant for destruction of the fecal coliforms is 0.23 min^{-1}.

Reactor analysis – CMFR, 1st order reaction

- Completely mixed flow reactor (CMFR) (continuous-flow stirred tank reactor, CSTR)

  1) define control volume

  2) write a mass balance eq.

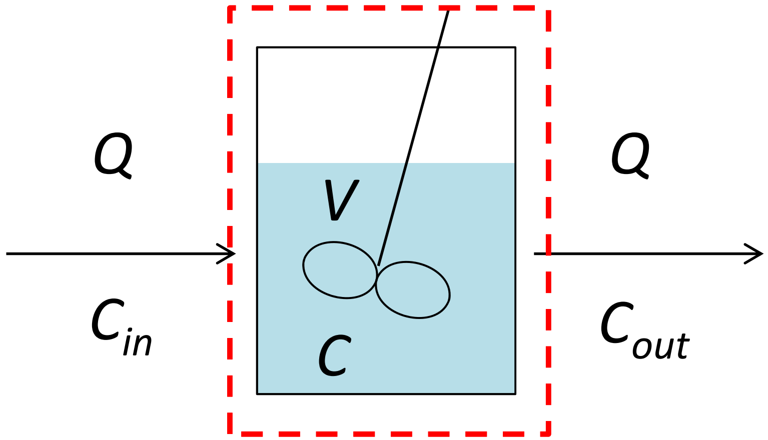


$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + R$$

$$V\frac{dC}{dt} \qquad Q \cdot C_{in} \qquad Q \cdot C_{out} \qquad -kCV \quad (1^{st}\ \text{order})$$

Reactor analysis – CMFR, 1st order reaction

3) solve the equation:



Because of homogeneous mixing, $C = C_{out}$

$$V\frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

Reactor analysis – CMFR, 1st order reaction

$$V\frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

Special case I: No reaction, initial concentration = C_0

$$V\frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

$$\frac{dC_{out}}{dt} = \frac{1}{t_0}\left(C_{in} - C_{out}\right) \qquad (t_0 = V/Q)$$

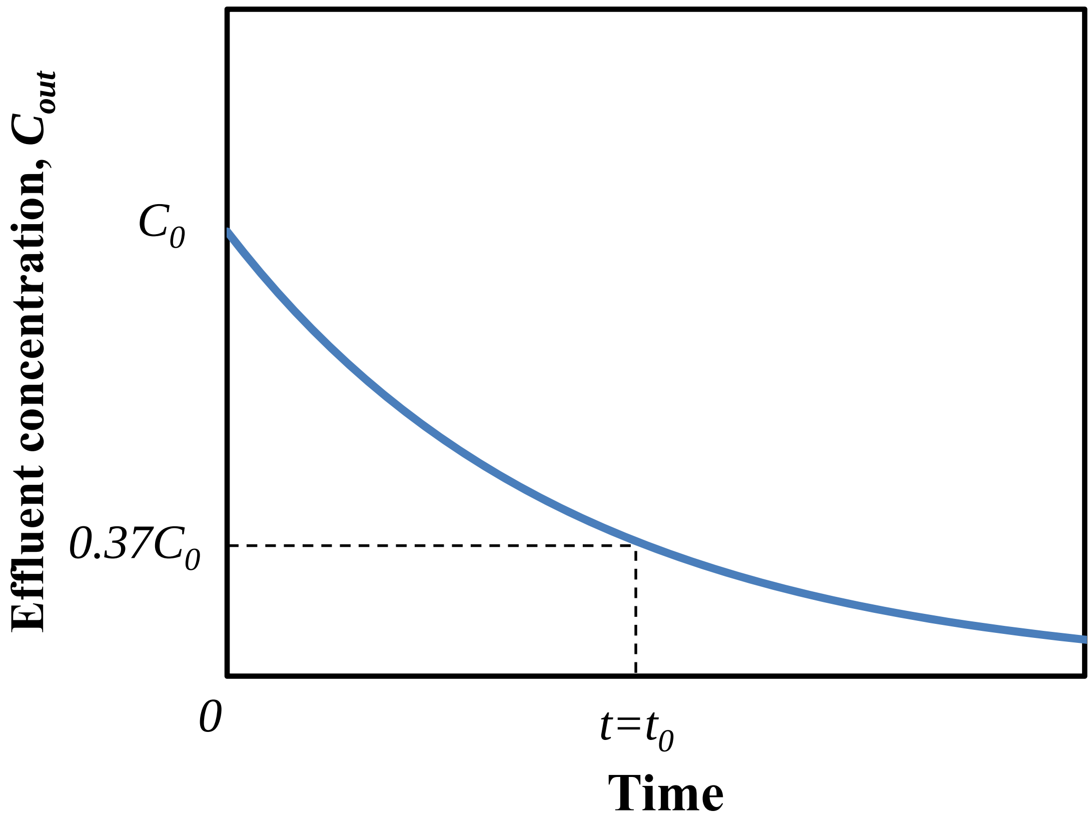
integrating over t=0 to t:

$$C_{out,t} = C_0\left[exp\left(-\frac{t}{t_0}\right)\right] + C_{in}\left[1 - exp\left(-\frac{t}{t_0}\right)\right]$$

Reactor analysis – CMFR, 1st order reaction

when C_{in} = 0,

$$C_{out,t} = C_0\left[exp\left(-\frac{t}{t_0}\right)\right]$$



Reactor analysis – CMFR, 1st order reaction

$$V\frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

Special case II: Steady state

$$V\frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

$$C_{out} = \frac{C_{in}}{1+kt_0} \qquad (t_0 = V/Q)$$

→ influent concentration (C_{in}) is reduced in the effluent by a factor of ($1+kt_0$)

Reactor analysis – CMFR, 1st order reaction

- Examples of CMFRs

Biological wastewater treatment

Lake

Reactor analysis - CMFR

Q: Activated sludge is a key process for most wastewater treatment facilities. The process is often run as a CMFR. Assume a 400 m^3-sized CMFR for an activated sludge process receiving 2000 m^3/d of wastewater. If a terrorist dumped 10 kg of a non-biodegradable toxic chemical to the reactor, how long will it take for the toxic chemical concentration in the reactor to a safe level (1 mg/L)?

Reactor analysis - CMFR

Q: A chemical degrades in a steady-state CMFR according to first-order reaction kinetics. The upstream concentration of the chemical is 10 mg/L and the downstream concentration is 2 mg/L. Water is being treated at a rate of 29 m^3/min. The volume of the tank is 580 m^3. What is the rate of decay? What is the rate constant?

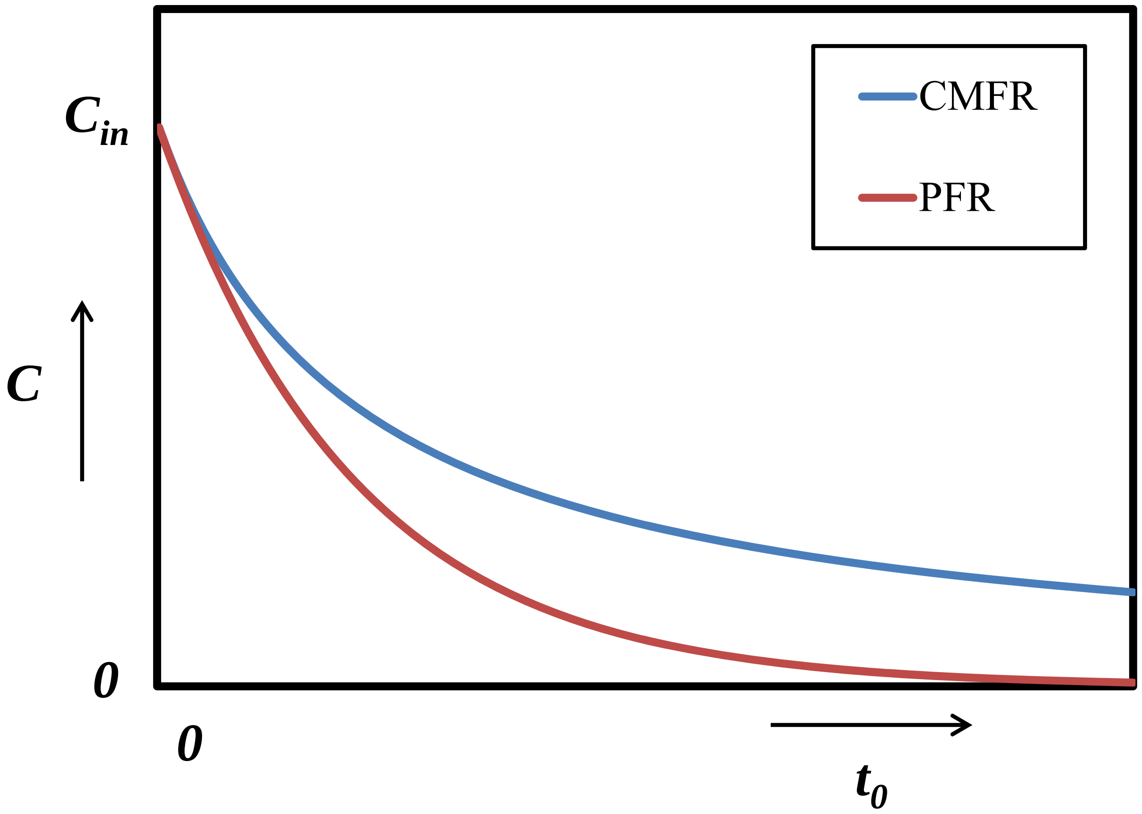
Retention time

- Retention time (detention time), t_0

$$t_0 = V / Q$$

  – The average time the fluid particles spend in the reactor
  – PFR: the time that fluid particles spend in a reactor is the same (=t_0) for all particles
  – CMFR: the time that fluid particles spend in a reactor is different

PFR vs. CMFR



- At the same t_0, PFR shows better performance
- Advantage of using CMFR: less sensitive to shock loads and toxic compounds

Reading assignment

- Textbook Ch4 p. 162-168