CSTR performance analysis – Ex 3.1

1) S_{min}

$$S_{min} = K\frac{b}{Y\hat{q}-b} = (20\ mg\ BOD_L / L)\frac{0.15/d}{(0.42\ g\ VSS_a / g\ BOD_L)(20\ g\ BOD_L / g\ VSS_a - d) - 0.15/d}$$
$$= 0.36\ mg\ BOD_L / L$$

2) Θ_x^{min}

$$\theta_x^{min} = \frac{K+S^0}{S^0(Y\hat{q}-b)-bK} = \frac{(20+500)mg\ BOD_L / L}{(50\ mg\ BOD_L / L)(0.42\cdot 20/d - 0.15/d) - (20\ mg\ BOD_L / L)(0.15/d)}$$

$$= 0.126\ d$$

3) Θ_x

$$\theta_x = \theta = V/Q = \frac{2000\,m^3}{1000 m^3 / d} = 2d$$

$$\frac{\theta_x}{\theta_x^{min}} = SF = \frac{2d}{0.126d} = 16 \qquad \text{(SF of 16 for washout)}$$

4) Effluent VSS (X_v)

Let's calculate X_a & X_i separately:

$$X_a = Y(S^0 - S)\frac{1}{1+b\theta_x} = (0.42 g\ VSS_a / g\ BOD_L)(500 - 1.7\ mg\ BOD_L / L)\frac{1}{1+(0.15/d)(2d)}$$

$$= 161\ mg\ VSS_a / L$$

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x = 50\ mg\ VSS_i / L + (161\ mg\ VSS_a / L)(1-0.8\ g\ VSS_i / g\ VSS_a)(0.15/d)(2d)$$
$$= 60\ mg\ VSS_i / L$$

$$X_v = X_a + X_i = 161 + 60 = 221\ mg\ VSS / L$$

5) Effluent COD & BOD_L

Effluent COD = remaining substrate + SMP + all biomass (active+inert)

Effluent BOD_L = remaining substrate + SMP + active <u>and</u> biodegradable biomass

i) substrate

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

$$= (20mg\ BOD_L / L)\frac{1 + (0.15/d)(2d)}{(0.42\,g\ VSS_a / g\ BOD_L)(20\ g\ BOD_L / g\ VSS_a - d)(2d)\quad 1 + (0.15/d)(2d)}$$

$$= 1.7\,mg\,BOD_L / L$$

ii) SMP

Let's first calculate individual terms for Eqs [3.38] & [3.39]

$$r_{ut} = -\frac{\hat{q}S}{K + S}X_a = \frac{dS}{dt} = -\frac{S^0 - S}{\theta} = -\frac{(500 - 1.7)mg\ BOD_L / L}{2d} = -249\ mg\ BOD_L / L - d$$

$$\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta = 1.8\ \ 161\ \ 2 + 100 + 0.12\ \ (\quad 250)\ \ 2 = 620\ mg\ BOD_L / L$$

$$4K_{UAP}k_1 r_{ut}\theta = \quad 4\ \ 100\ \ 0.12\ \ (\quad 250)\ \ 2 = 24000\ (mg\ BOD_L / L)^2$$

$$K_{BAP} + (\hat{q}_{BAP}\quad k_2)X_a\theta = 85 + (0.1\quad 0.09)\ \ 161\ \ 2 = 88.2\ mg\ BOD_L / L$$

$$4K_{BAP}k_2 X_a\theta = 4\ \ 85\ \ 0.09\ \ 161\ \ 2 = 9850\ (mg\ BOD_L / L)^2$$

$$UAP = \frac{620 + \sqrt{(620)^2 + 23900}}{2} = 9.5\ mg\ BOD_L / L$$

$$BAP = \frac{88.2 + \sqrt{(88.2)^2 + 9850}}{2} = 22.3\ mg\ BOD_L / L$$

$$SMP = UAP + BAP = 9.5 + 22.3 = 31.8\ mg\ BOD_L / L$$

iii) Biomass

Recall that we calculated the COD for the cell formula, $C_5H_7O_2N$, as 1.42 g COD/g cells (lecture #3)

In sum,

$$Effluent\ COD = Substrate + SMP + Biomass\ COD = 1.7 + 31.8 + (1.42\ g\ COD/g\ VSS)X_v$$

$$= 1.7 + 31.8 + 1.42 \cdot 221 = 1.7 + 31.8 + 313.8 = 347\ mg\ COD/L$$

* Biomass accounts for most of COD – this COD can be removed by settling (but good settling property should be guaranteed)

* SMP accounts for most of soluble COD

$$Effluent\ BOD_L = Substrate + SMP + \text{biodegradable biomass}$$
$$= 1.7 + 31.8 + (1.42\ g\ COD/g\ VSS) \cdot X_a \cdot f_d = 216\ mg\ BOD_L/L$$