Biofilm kinetics II

Today's lecture

- Biofilm analysis: biomass
- Steady state biofilm analysis
- Steady state biofilm solution

Biofilm analysis

- The active biomass mass balance at a position inside the biofilm with a thickness of *dz*:

$$\frac{d(X_f dz)}{dt} = Y \frac{\hat{q} S_f}{K + S_f} \left(X_f dz\right) - b' X_f dz$$

b' = overall biofilm specific loss rate [T^{-1}]

: *b* (decay coeff.) + detachment

Biofilm analysis

- Integrating over the entire biofilm depth:

$$\int_0^{L_f} \frac{d(X_f dz)}{dt} = \int_0^{L_f} Y \frac{\hat{q} S_f}{K + S_f} X_f dz - \int_0^{L_f} b' X_f dz$$

Steady state assumption

- (Pseudo) Steady state assumption:
  - The biofilm thickness, L_f, and the active biomass density, X_f, do not change with time
  - Then, the biomass per unit surface area, $X_f L_f$, do not change with time
  - Steady state as a whole: at any given point of the biofilm steady state is not achieved, the whole biofilm is at steady state
  - Dynamic steady state: near the outer surface, the substrate concentrations are high, and active biomass growth is positive; near the attachment surface, substrate concentrations are low, and active biomass growth is negative. The active biomass exchanges within the biofilm to maintain uniform X_f

Steady state biofilm analysis

- By steady state assumption:

$$\int_0^{L_f} \frac{d(X_f dz)}{dt} = \int_0^{L_f} \frac{d(X_f L_f)}{dt} = 0$$

- The active biomass mass balance over the whole biofilm depth at steady state:

$$0 = \int_0^{L_f} Y \frac{\hat{q} S_f}{K + S_f} X_f dz - \int_0^{L_f} b' X_f dz$$

Steady state biofilm analysis

- The growth term:

$$\int_0^{L_f} Y \frac{\hat{q} S_f}{K + S_f} X_f \, dz = Y \underbrace{\int_0^{L_f} r_{ut} \, dz}_{\text{substrate utilization rate over the entire depth}} = Y\, J$$

Substrate transport from bulk liquid to the biofilm

- The loss term:

$$\int_0^{L_f} b' X_f \, dz = b' X_f L_f$$

(the loss process is averaged across the biofilm)

Steady state biofilm analysis

Now, the equation reduces to:

$$0 = YJ - b'X_f L_f$$

Biomass per unit area,

$$X_f L_f = \frac{YJ}{b'}$$

The biofilm thickness,

$$L_f = \frac{YJ}{X_f b'}$$

Steady state biofilm solution

- Here, our goal is to use the equations for substrate and active biomass mass balance to calculate the substrate flux, *J* and the biofilm accumulation, $X_f L_f$., at a given substrate concentration, *S*, in the bulk liquid.

- We have: $$0 = D_f \frac{d^2 S_f}{dz^2} - \frac{\hat{q} X_f S_f}{K + S_f}$$

$$0 = \left. \frac{dS_f}{dz} \right|_{z=L_f} \qquad \frac{D}{L}(S - S_s) = D_f \left. \frac{dS_f}{dz} \right|_{z=0}$$

$$0 = YJ - b' X_f L_f$$

Steady state biofilm solution

- Key features of the solution (see [Fig. 4.3])
  1) At $S < S_{min}$, $J = 0$ and $X_f L_f = 0$ (cannot maintain the steady state). S_{min} is defined as: $$S_{min} = K \frac{b'}{Y\hat{q} - b'}$$
  2) J and $X_f L_f$ increases very sharply as S increases slightly above S_{min}.
  3) At some value of $S > S_{min}$, the slope of J vs. S approaches 1.
  4) For a sufficiently large S, the flux becomes equal to that of a deep biofilm. The slope of J vs. S approaches 0.5.