Chapter 3

Frameworks for Inventory Management and Production Planning & Control

3.1 The Diversity of Stock Keeping Units

A typical medium-sized manufacturing company → keep 10,000 types of raw materials, parts, and finished goods in inventory

diversity → cost, weight, volume, color, or physical shape

perishable → deterioration over time, theft, pilferage, obsolescence (style or technology)

Demands for items also can occur in many ways.

(cf.) substitute vs. complement

decision making in production planning and inventory management → problem of coping with large numbers and with a diversity of factors external and internal to the organization → must be consistent with the overall objectives of management

♣ 3 Basic Issues

(1) How often the inventory status should be determined → review period

(2) When a replenishment order should be placed → reorder interval

(3) How large the replenishment order should be → order quantity

3.2 The Bounded Rationality of a Human Being

All decision makers are forced to ignore some relevant aspects of a complex problem and base their decisions on a smaller number of carefully selected factors → personal biases, abilities, perceptions, decision technology available

The decisions must be viewed *simultaneously* from the point of view of: the individual item in its relation to other similar items (i.e. interactions), the total aggregate inventory investment, the master plan of the organization, the production-distribution systems of suppliers and customers, and the economy as a whole

Existing theory is insufficient to do the whole job → The authors of this book weave their own brand of personalized approaches with theory → From an intellectual point of view, inventory management and production decisions are both challenging and exciting

3.3 Decision Aids for Managing Diverse Individual Items

♣ Conceptual Aids

(1) Decisions in an organization can be considered as a hierarchy → strategic, tactical, operational

(2) A related type of hierarchy can be conceptualized with respect to decision making (eg. highest → choose a particular type of control system, middle → specific parameters such as service level, low → data collection, calculations)

(3) grouping into a smaller number of categories (eg. ABC analysis)

(4) identify the most important variables for explicit consideration

♣ Physical Aids

(1) Decision makers can employ spreadsheets and other computer programs.

(2) principle of management by exception

3.4 Frameworks for Inventory Management

3.4.1 Functional Classifications of Inventories

(1) Cycle Stock

order or produce in batches instead of one unit at a time

(i) economies of scale

(ii) quantity discounts (purchase price or freight cost)

(iii) technological restrictions

Determination of appropriate cycle stocks $\rightarrow$ Chapters 5, 6, 8, and 11

(2) Congestion Stock

inventories due to items competing for limited capacity (eg. ELSP)

(3) Safety Stock

inventory for the uncertainty of demand and supply $\propto$ customer service level

(4) Anticipation Inventory

stock accumulated in advance of an expected peak in sales $\rightarrow$ production smoothing

(5) Pipeline (WIP) Inventory

goods in transit (eg. in physical pipelines, on trucks, or in railway cars) between levels of a multiechelon distribution systems or between adjacent work stations in a factory

(6) Decoupling Stock

permit the separation of decision making at the different echelons

Zipkin (1995) notes that inventory is a boundary phenomenon

3.4.2 The A-B-C Classification as a Basis for Designing Individual Item Decision Models

SKU (Stock Keeping Unit): specific unit of stock to be controlled $\rightarrow$ an SKU will be defined as an item of stock that is completely specified as to function, style, size, colour, and usually, location

A useful statistical regularity in the usage rates of different items $\rightarrow$ About 20% of the SKUs account for 80% of the total annual dollar usage

♣ DBV (Distribution by Value) Curve (Figure 1.2 and Table 3.1)

It is *common* to use three priority ratings: A (most important), B (intermediate in importance), C (least important)

(1) Class A items should receive the most personalized attention from management. The first 5 to 10% of the SKUs account for 50% or more of the total annual dollar usage ($\sum_i D_i v_i$) of the population of items under consideration

(2) Class B items are of secondary importance in relation to class A. $\rightarrow$ Usually more than 50% of total SKUs account for most of the remaining 50% of the annual dollar usage. $\rightarrow$ monitor using computer-based system with management-by-exception rules

(3) Class C items are the relatively numerous remaining SKUs that make up only a minor part of total dollar investment $\rightarrow$ Decision systems must be kept as simple as possible (eg. two-bin system)

(cf.) Some inexpensive SKUs may be classified as 'A' simply because they are crucial to the operation of the firm.

3.5 A Framework for Production Planning & Control

3.5.1 A Key Marketing Concept: The Product Life Cycle

Figure 3.2 The Fundamental Stages of Product/Market Evolution

(i) Products have a limited life.

(ii) Product profits tend to follow a predictable course through the life cycle.

(iii) Products require a different marketing, production planning, inventory management, and financial strategy in each stage.

3.5.2 Different Types of Production Processes

job shop, batch flow, assembly line, and continuous process

Table 3.2 Differences in Product and Market Characteristics

3.5.3 The Product-Process Matrix

Figure 3.3 The Product-Process Matrix

3.6 Performance Measures

3.6.1 Cost Factors

(1) The Unit Value or Unit Variable Cost, v (dollars/unit)

The price (including freight) paid to the supplier, plus any cost incurred to make it ready for sale. $\rightarrow$ quite difficult to determine $\rightarrow$ A good starting point is the cost figure given by accounting, adjusted for obvious errors.

(Ex.) supervising cost

(i) total acquisition (or production) costs per year clearly depend on its value

(ii) carrying (inventory holding) cost depends on v.

(2) The Cost of Carrying Items in Inventory

opportunity cost of the money invested, warehouse cost, handling and counting costs, the costs of special storage requirements, deterioration of stock, damage, theft, obsolescence, insurance, and taxes

$$\text{carrying cost per year} = \overline{I}vr$$

where $\overline{I}$ is the average inventory in units and r is the cost in dollars of carrying one dollar of inventory for one year.

In practice, the opportunity cost of capital can range from the bank's prime leading rate to 50% and higher.

A single value of r is usually assumed to apply for most items

(cf.) r itself could depend on the total size of the inventory $\rightarrow$ master thesis by Mr. S. Ryu

(3) The Ordering or Setup Cost, A

fixed cost (independent of the size of the replenishment) associated with a replenishment

ordering cost: cost of order forms, postage, telephone calls, authorization, typing of orders, receiving inspection, following up, and handling of vendor invoices

(cf.) learning effects or stabilization period $\rightarrow$ After setup, there often follows a period of time during which the facility produces at lower quality or slower speed while the equipment is fine tuned and the operator adjusts to the new part.

With Electronic Data Interchange (EDI), which facilitates computer-to-computer transactions, the cost decreases to as low as \$5 (Business Week, 1996).

(4) The Cost of Insufficient Capacity in the Short Run

costs of avoiding stockouts and the costs incurred when stockouts take place

expenses related to the emergency orders, expediting costs, rescheduling, split lots, emergency shipments, substitution of a less profitable item $\rightarrow$ can be estimated well

cost related to the loss of customer $\rightarrow$ difficult to estimate $\rightarrow$ can be estimated empirically through an actual study for only a limited number of SKUs

(5) System Control Cost

costs associated with the operation of the particular decision system selected

costs of data acquisition, data storage and maintenance, and computation, human interpretation of results, training

3.6.2 Other Key Variables

Figure 3.5 Inventory Planning Decision Variables

(1) Replenishment Lead Time, L

the time that elapses from the moment at which it is decided to place an order, until it is physically on the shelf ready to satisfy customer demands

♣ 5 distinct components

(i) administrative time at the stocking point (order preparation time)

(ii) transit time to the supplier

(iii) time at the supplier (most variable)

(iv) transit time back to the stocking point

(v) time from order receipt until it is available on the shelf

(cf.) lead time reduction

(2) Production vs. Nonproduction

decisions in a production context are more complicated $\rightarrow$ capacity constraints as well as an interdependency of demand among finished products and their components

(3) Demand Pattern

The nature of item can influence the demand pattern (eg. demand for spare parts is less predictable)

3.7 Three Types of Modeling Strategies

A proper problem diagnosis is often more important than the subsequent analysis.

(1) Detailed Modeling and Analytical Selection of the Values of a Limited Number of Decision Variables eg. EOQ

(2) Broader Scope Modeling with Less Optimization

eg. MRP

(3) Minimization of Inventories with Little Modeling

eg. JIT manufacturing philosophy, OPT

3.8 The Art of Modeling

Incorrect modeling can lead to costly, erroneous decisions.

Little (1970) recommends that decision models should be understandable to the decision maker, complete, evolutionary, easy to control, easy to communicate with, robust, and adaptive.

♣ Helpful Tips for Modeling Complex Production/Inventory Systems

(1) The measures of effectiveness used in a model must be consistent with the objectives of the organization.

(2) Heuristic decision rules, which are based on sound logic, that are designed to yield reasonable (not necessarily optimal) answers to complex real problems, are advocated.

(3) A model should permit results to be presented in a form suitable for management review.

(4) One should start with as simple a model as possible, only adding complexities as necessary.

(5) In most cases in the text we advocate modeling that leads to analytic decision rules that can be implemented through the use of formulas and spreadsheets.

(6) Where it is known a priori that the solution to a problem will possess a certain property, this information should be used, if possible, to simplify the modeling or the solution process.

(7) When facing a new problem, one should at least attempt to show an equivalence with a different problem for which a solution method is already known.

3.9 Explicit Measurement of Costs

♣ 3 fundamental purposes for cost accounting systems

(i) valuing inventory for financial statements

(ii) providing feedback to production managers

(iii) measuring the cost of individual SKUs

Table 3.4 Costing and Control Alternatives

3.10 Implicit Cost Measurement and Exchange Curves

The inventory planning decision deals with the design of an entire system: consisting of an ordering function, a warehousing system, and the servicing of customer demand – all to top management specification.

As A/r increases, the total average inventory (in dollars) increases and the total number of replenishments per year decreases $\rightarrow$ exchange curve

Figure 3.7 Example of Exchange Curve

3.11 The Phases of a Major Study of an Inventory Management or Production Planning and Control System

(1) Phase I: Consideration

This first phase focuses on conceptualizing the problem and covers a number of strategic and organizational issues as well as some detailed modeling concerns.

♣ What are the important operations objectives for this firm? (cost, quality, delivery, and flexibility)

♣ Who has overall responsibility for inventory management and production planning and control?

♣ What is the relationship between operations and marketing?

♣ What are the annual sales of the organization?

♣ What is the current average aggregate inventory level in dollars?

♣ What are the current inventory management and production planning and control procedures?

♣ What are the computer resources and skills available?

♣ What is the stock keeping unit?

♣ Is the inventory necessary?

♣ What modeling strategy seems appropriate?

(2) Phase II: Analysis

The second phase is one of the two modeling stages, and focuses on data collection and a detailed understanding of the uncontrollable and controllable variables.

♣ How many items are to be studied?

♣ Are the items independent of one another?

♣ What does the supply chain look like?

♣ Are customer transaction sizes in single units or batches?

♣ Is demand deterministic or variable?

♣ Do customers arrive at a constant or variable rate?

♣ Is the average demand seasonal, or is it somewhat constant over the year?

♣ What historical data are available?

♣ How many customers are served?

♣ How many, and how powerful, are the firm's competitors?

♣ What is the typical customer promise time?

♣ How is customer service measured?

♣ How many suppliers serve the firm?

♣ What is the replenishment lead time for each item studied?

♣ Is there concern about obsolescence or spoilage?

♣ How many components go into each end item?

♣ How many processing stages do the products go through?

♣ What are the setup times for each part?

♣ How reliable is the production equipment?

♣ What is the layout of the factory?

♣ How frequently should inventory levels be reviewed?

♣ How much should be reordered or produced?

♣ What costs, lead times, transportation modes, and other factors should be changed?

(3) Phase III: Synthesis

The second modeling stage, which is the third phase of the process, attempts to bring together the vast amount of information gathered in the previous phase.

A mathematical objective function stated in terms of controllable variable is the most common result of this phase.

(4) Phase IV: Choosing Among Alternatives

The model should accommodate sensitivity analysis so that managers can get a feel for the result of changes in the input data.

(5) Phase V: Control

implementation of the decision rules

♣ Training of the staff who will use the new system.

♣ How often should the values of the control parameters be recomputed?

♣ How will the uncontrollable variables be monitored?

♣ How will the firm keep track of inventory levels?

♣ How will exceptions be handled?

(6) Phase VI: Evaluation

(cf.) General Comments

Gradual implementation, accompanied by extensive education, is essential.

Where possible, pilot approach should be first utilized.

(cf.) cycle counting

Chapter 5

An Order Quantity Decision System for the Case of Approximately Level Demand

♣ Demand Properties

Inventory problems exist only because there are demands; otherwise, we have no inventory problems. Inventory systems in which the demand size is known will be referred to as *deterministic systems*. Demand rate is the demand size per unit time.

5.1 Assumptions Leading to the Basic EOQ

(1) The demand rate is constant and deterministic.

(2) The order quantity need not be an integral number of units.

(3) The unit variable cost is independent of the replenishment quantity.

(4) The cost factors do not change appreciably with time (i.e. no inflation).

(5) The item is treated entirely independently of other items.

(6) The replenishment lead time is of zero duration.

(7) No shortages are allowed.

(8) The entire order quantity is delivered at the same time.

5.2 Derivation of the EOQ

♣ Data

D = demand rate in units per year

m = production rate in units per year (For the models that have finite production rates)

A = fixed cost of a replenishment order

v = unit variable cost of production (or purchase)

h = inventory carrying cost per unit per year, usually expressed as $h = rv$, where r is the annual inventory carrying cost rate

π = shortage cost per unit short per year (For the models that allow backorders)

♣ Decision Variable

Q = replenishment order quantity

b = maximum backorder level permitted (For the models that allow backorders)

T = cycle length, the length of time between placement of replenishment orders

$TRC(Q)$ = total relevant costs per unit time

The average inventory carrying cost per cycle is the area under the inventory triangle.

$$\frac{1}{2}hQT = \frac{1}{2}h\frac{Q^2}{D}$$

The average cost per cycle is the sum of procurement and inventory carrying cost.

$$A + \frac{1}{2}h\frac{Q^2}{D}$$

To obtain the average annual cost, $TRC(Q)$, we multiply the cost per cycle by the number of cycles per year, D/Q. Doing this and writing $h = vr$, we get

$$TRC(Q) = \frac{AD}{Q} + \frac{rvQ}{2}$$

The optimum value of Q can be found by solving

$$\frac{\partial TRC(Q)}{\partial Q} = -\frac{AD}{Q^2} + \frac{vr}{2} = 0$$

since $TRC(Q)$ is convex function in Q.

(note) A differentiable function $f(x)$ is convex in x, if the second derivative is nonnegative. For the above model,

$$\frac{\partial^2 TRC}{\partial Q^2} = \frac{2AD}{Q^3} \geq 0$$

Consequently,

$$EOQ = \sqrt{\frac{2AD}{vr}}$$

and the minimum average annual cost will be

$$TRC(EOQ) = \sqrt{2ADvr}$$

5.3 Sensitivity Analysis

♣ Penalty for using a wrong EOQ

Let $Q' = (1+p)EOQ$. That is, $100p$ is the percentage deviation of Q' from the EOQ.

$$PCP = \frac{TRC(Q') - TRC(EOQ)}{TRC(EOQ)} \times 100$$

$$PCP = 50\left(\frac{p^2}{1+p}\right)$$

(proof)

(cf.) penalty for wrong estimation of cost parameters

5.5 Quantity Discounts

We assume 'all units discount' which is the most common type of discount structure.

$$v = \begin{cases} v_0 & \text{if } 0 \leq Q < Q_b \\ v_0(1-d) & \text{if } Q_b \leq Q \end{cases}$$

$$TRC(Q) = \frac{Qv_0r}{2} + \frac{AD}{Q} + Dv_0, \quad 0 \leq Q < Q_b$$

$$TRC(Q) = \frac{Qv_0(1-d)r}{2} + \frac{AD}{Q} + Dv_0(1-d), \quad Q \geq Q_b$$

♣ tradeoff between extra carrying cost vs. a reduction in the acquisition costs

Figure 5.6 TRC under All Units Discount

♣ Algorithm

(Step 1)

$$\text{Compute} \quad \text{EOQ(discount)} = \sqrt{\frac{2AD}{v_0(1-d)r}}$$

(Step 2) If $EOQ(d) \geq Q_b$, then EOQ(d) is optimal (case (c)).

If $EOQ(d) < Q_b$, go to Step 3.

(Step 3) Compute TRC(EOQ) and TRC(Q_b).

If TRC(EOQ) $\leq$ TRC(Q_b), EOQ is optimal (case (b)).

If TRC(EOQ) $>$ TRC(Q_b), Q_b is optimal (case (a)).

5.6 Accounting for Inflation

r = continuous discount rate

i = inflation rate

$$\begin{aligned}
PV(Q) &= (A+Qv) + (A+Qv)e^{\frac{iQ}{D}}e^{-\frac{rQ}{D}} + (A+Qv)e^{\frac{2iQ}{D}}e^{-\frac{2rQ}{D}} + \cdots \\
&= (A+Qv)\left(1 + e^{-\frac{(r-i)Q}{D}} + e^{-\frac{2(r-i)Q}{D}} + \cdots\right) \\
&= (A+Qv)\frac{1}{1-e^{-\frac{(r-i)Q}{D}}}
\end{aligned}$$

The optimal Q satisfies

$$e^{\frac{(r-i)Q}{D}} = 1 + \left(\frac{A}{v} + Q\right)\left(\frac{r-i}{D}\right)$$

(proof)

Approximating e^x by $1 + x + \frac{x^2}{2}$ gives

$$Q^* = \sqrt{\frac{2AD}{v(r-i)}} = EOQ\sqrt{\frac{1}{1-\frac{i}{r}}}$$

5.7 Limits on Order Sizes

5.7.1 Maximum Time Supply or Capacity Restriction

(1) shelf life of the commodity

If $T_{EOQ} = \frac{EOQ}{D} = \sqrt{\frac{2A}{Dvr}} > SL$, $Q_{SL} = D(SL)$.

(2) Even without a shelf life limitation, an EOQ that represents a very long time supply may be unrealistic for other reasons.

(3) There may be a storage capacity limitation on the replenishment.

5.7.2 Minimum Order Quantity

5.7.3 Discrete Units

The best integer value of Q has to be one of the two integers surrounding the real value of Q.

5.8 Finite Replenishment Rate

Here there is a finite production rate m rather than infinite replenishment rate.

Note that in the above figure

$$\frac{Q}{T_m + T_D} = D,\ I_{max} = (m-D)T_m,\ Imax = DT_D.$$

Consequently,

$$I_{max} = \left(1 - \frac{D}{m}\right) Q$$

The average inventory carrying cost per cycle is the area under the inventory triangle.

$$\frac{1}{2}hTI_{max} = \frac{1}{2}h\frac{Q}{D}(1 - \frac{D}{m})Q = \frac{1}{2}h\frac{Q^2}{D}(1 - \frac{D}{m})$$

The average cost per cycle is the sum of procurement and inventory carrying cost.

$$A + \frac{1}{2}h\frac{Q^2}{D}(1 - \frac{D}{m})$$

To obtain the average annual cost, $TRC(Q)$, we multiply the cost per cycle by the number of cycles per year, D/Q. Doing this and writing $h = vr$, we get

$$TRC(Q) = \frac{AD}{Q} + \frac{vrQ}{2}(1 - \frac{D}{m})$$

The optimum value of Q can be found by solving

$$\frac{\partial TRC}{\partial Q} = -\frac{AD}{Q^2} + \frac{vr}{2}(1 - \frac{D}{m}) = 0$$

since $TRC(Q)$ is convex function in Q. Consequently,

$$FREOQ = \sqrt{\frac{2AD}{vr(1 - \frac{D}{m})}}$$

and the minimum average annual cost will be

$$TRC(FREOQ) = \sqrt{2ADvr(1 - \frac{D}{m})}$$

Note that if $m \to \infty$, $FREOQ = \sqrt{\frac{2AD}{vr}}$. (Reduces to EOQ)

5.9 Incorporation of Other Factors

5.9.1 Nonzero Constant Lead Time that is known with Certainty

When the inventory level hits DL, an order is placed and it arrives exactly L time units later just as the inventory hits zero.

5.9.2 Different Type of Carrying Charge

Suppose that there is an additional charge (in addition to the usual inventory carrying charge r) of w dollars per unit time per cubic foot of space allocated to an item $\rightarrow$ Derivation!

5.9.3 Multiple Setup Costs: Freight Discounts

$$\text{Setup Cost} = \begin{cases} A + Qv & \text{if } 0 < Q \leq Q_0 \\ 2A + Qv & \text{if } Q_0 < Q \leq 2Q_0 \\ 3A + Qv & \text{if } 2Q_0 < Q \leq 3Q_0 \end{cases}$$

Aucamp (1982, EJOR) has shown that the best solution is either the standard EOQ or one of the two surrounding integer multiples of Q_0.

5.9.4 Joint Replenishment Problem

Suppose a company carry more than one item. If the purchasing manager purchase those items from a same vendor, it may be a good idea to order those items together so that he/she can save ordering cost. For the simplicity, we study two items case.

Data

D_1 = demand rate for item 1 in units per year

D_2 = demand rate for item 2 in units per year

A = fixed cost of a replenishment order

v_1 = unit cost of purchasing item 1

v_2 = unit cost of purchasing item 2

r = annual inventory carrying cost rate

Decision Variables

Q_1 = order quantity for item 1

Q_2 = order quantity for item 2

T = common cycle length, the length of time between placement of replenishment orders

Since T is same for both items, we get

$$T = \frac{Q_1}{D_1} = \frac{Q_2}{D_2}.$$

The average inventory carrying cost for item 1 per cycle is the area under the inventory triangle for item 1.

$$\frac{1}{2} r v_1 Q_1 T = \frac{1}{2} r v_1 \frac{Q_1^2}{D_1}$$

The average inventory carrying cost for item 2 per cycle is the area under the inventory triangle for item 2.

$$\frac{1}{2} r v_2 Q_2 T = \frac{1}{2} r v_2 \frac{Q_2^2}{D_2}$$

The average cost per cycle is the sum of procurement and inventory carrying cost.

$$A + \frac{1}{2} r v_1 \frac{Q_1^2}{D_1} + \frac{1}{2} r v_2 \frac{Q_2^2}{D_2}$$

To obtain the average annual cost, $TRC(Q_1, Q_2)$, we multiply the cost per cycle by the number of cycles per year, $D_1/Q_1 (\equiv D_2/Q_2)$. Doing this, we get

$$TRC(Q_1, Q_2) = \frac{AD_1}{Q_1} + \frac{r v_1 Q_1}{2} + \frac{r v_2 Q_2}{2}$$

By substituting $Q_2 = \frac{D_2}{D_1} Q_1$, we get

$$TRC(Q_1) = \frac{AD_1}{Q_1} + \frac{rv_1 Q_1}{2} + \frac{rv_2 D_2 Q_1}{2D_1}$$

The optimum value of Q_1 can be found by solving

$$\frac{\partial TRC}{\partial Q_1} = -\frac{AD_1}{Q_1^2} + \frac{r(v_1 D_1 + c_2 D_2)}{2D_1} = 0$$

since $TRC(Q_1)$ is convex function in Q_1.

For the above model,

$$\frac{\partial^2 TRC}{\partial Q_1^2} = \frac{2AD_1}{Q_1^3} \geq 0$$

Consequently,

$$Q_1^* = \sqrt{\frac{2AD_1^2}{r(v_1 D_1 + v_2 D_2)}}, \quad Q_2^* = \sqrt{\frac{2AD_2^2}{r(v_1 D_1 + v_2 D_2)}}$$

And the optimal common replenishment interval is

$$T^* = \sqrt{\frac{2A}{r(v_1 D_1 + v_2 D_2)}}$$

5.9.5 Different Order Arrivals

In the classical EOQ (Economic Order Quantity) Model,

assume that when we order Q units, we receive our order in two parts. The first part arrives immediately and contains $\alpha Q (0 < \alpha \leq 1)$ and the second part arrives T units of time after the order and contains the rest of our order, i.e., $(1 - \alpha)Q$. We assume no shortages are allowed. See the following figure for understanding. Note that here T is not a variable but a parameter (given data).

Since we assume that there are no shortages,

$$\alpha Q \geq DT \tag{$*$}$$

Since $\frac{\alpha Q - x}{T} = D$, we get

$$x = \alpha Q - DT$$

Since $\frac{(1-\alpha)Q + x}{T_f} = D$, we get

$$T_f = \frac{Q - DT}{D}$$

Consequently,

$$T_c = T + T_f = Q/D$$

The average inventory carrying cost per cycle is the area under the inventory triangle plus the area of trapezoid.

$$\frac{1}{2}hT_f[x + (1-\alpha)Q] + \frac{1}{2}hT[\alpha Q + x] = \frac{1}{2}hT(2\alpha Q - DT) + \frac{1}{2}h\frac{(Q - DT)^2}{D}$$

The average cost per cycle is the sum of procurement and inventory carrying cost.

$$A + \frac{1}{2}hT(2\alpha Q - DT) + \frac{1}{2}h\frac{(Q - DT)^2}{D}$$

To obtain the average annual cost, $TRC(Q)$, we multiply the cost per cycle by the number of cycles per year, D/Q. Doing this, we get

$$TRC(Q) = \frac{AD}{Q} + h\alpha DT - \frac{hD^2T^2}{2Q} + \frac{h(Q - DT)^2}{2Q}$$

The optimum value of Q can be found by solving

$$\frac{\partial TRC}{\partial Q} = -\frac{AD}{Q^2} + \frac{h}{2} = 0$$

since $TRC(Q)$ is convex function in Q. Consequently,

$$Q^* = \sqrt{\frac{2AD}{h}}$$

There are 2 possible cases.

(i) If $Q^* = \sqrt{\frac{2AD}{h}}$ satisfies $(*)$, i.e., $\sqrt{\frac{2AD}{h}} \geq \frac{DT}{\alpha}$, then

$$Q^* = \sqrt{\frac{2AD}{h}}$$

(ii) If $Q^* = \sqrt{\frac{2AD}{h}}$ doesn't satisfy $(*)$, i.e., $\sqrt{\frac{2AD}{h}} < \frac{DT}{\alpha}$, then

$$Q^* = \frac{DT}{\alpha}$$

5.9.6 A Special Opportunity to Procure

Chapter 6

Lot Sizing for Individual Items with Time-varying Demand

6.1 The Complexity of Time-Varying Demand

In the basic inventory models, deterministic and level demand rates are assumed. Here we allow the average demand rate to vary with time, thus encompassing a broader range of practical situations such as :

(a) Multi-echelon assembly operations where a firm schedule of finished products exploded back through the various assembly stages leads to production requirements at these earlier levels, which are relatively deterministic but almost always vary appreciably with time.

(b) Production to contract, where the contract requires that certain quantities have to be delivered to the customer on specified dates.

(c) Items having a seasonal demand pattern.

6.2 The Choice of Approaches

There are 3 approaches that try to solve this time-varying demand case.

(a) straight-forward use of the economic order quantity

(b) an exact optimal procedure (Wagner-Whitin Algorithm)

(c) an approximate heuristic method (Silver-Meal Heuristic)

You may have a question that why we need to seek a heuristic algorithm even though we have an exact optimal algorithm to solve the problem. The answer is owing to the complexity of the optimal algorithm. The complexity of the exact optimal algorithm is exponential. That is, we cannot solve large problems using the algorithm. Also, it requires an additional assumption which will be explained later.

6.3 General Assumptions and A Numerical Example

(a) The demand rate is given in the form of D_j to be satisfied in period j ($j = 1, \ldots, N$) where the planning horizon is at the end of period N. Of course, the demand rate may vary from one period to the next, but it is assumed known.

(b) The entire requirements of each period must be available at the beginning of that period.

(c) The unit variable cost does not depend on the replenishment quantity.

(d) Inflation is at a negligibly low level.

(e) The item is treated entirely independently of other items.

(f) The replenishment lead time is known with certainty.

(g) No shortages are allowed.

(h) The entire order quantity is delivered at the same time.

(i) The carrying cost is only applicable to inventory that is carried over from one period to the next.

♣ A Numerical Example

The MIDAS company uses the following simple decision rule for ascertaining production run quantities : "Each time a production run is made, a quantity sufficient to satisfy the total demand in the next three months is produced." The requirements for the seasonal product PSF-007 in the following upcoming year are:

Table 6.1 Monthly Requirements

It is seen that the demand pattern has two peaks, one in the late spring, the other in the autumn season. The company estimates the fixed setup cost (A) per replenishment to be \$54, and the carrying charge (r) has been set by management at \$0.02/\$,month. The unit variable cost (v) of the product is \$20/box.

If we use the company's "three-month decision rule", we get the replenishment schedule as follows : The total relevant costs are \$663.20.

Table 6.2 Results of Using the Company's Three-Month Rule on the Numerical Example

6.4 Use of A Fixed Economic Order Quantity

One possible approach to the case of a time-varying rate is to simply ignore the time-variability, thus continuing to use the economic order quantity. To be more precise, the average demand rate ($\overline{D}$) out to the horizon (N periods) is evaluated and the economic order quantity

$$EOQ = \sqrt{\frac{2A\overline{D}}{vr}}$$

is used anytime a replenishment is needed. To account for the discrete opportunities to replenish, at the time of a replenishment, the EOQ should be adjusted to exactly satisfy the requirements of an integer number of periods. A simple way to do this is to keep accumulating periods of requirements until the closest total to the EOQ is found.

To illustrate, for our numerical example,

$$\overline{D} = \frac{\text{total requirements}}{12} = 100 \text{ boxes/month}$$

Therefore,

$$EOQ = \sqrt{\frac{2 \times 54 \times 100}{0.02 \times 20}} = 164$$

Consider the selection of the replenishment quantity at the beginning of January. See the following table.

Month	January	February	March	April
Demand	10	62	12	130
Cumulative Demand	10	72	84	214

The EOQ of 164 boxes lies between 84 and 214, and 214 is closer to 164 than is 84. Therefore, the first replenishment quantity is 214 boxes, lasting through the end of April. The detailed results of applying the fixed EOQ approach to the numerical example are as shown in the following table.

Table 6.3 Results of Using the Fixed EOQ Approach on the Numerical Example

The fixed EOQ approach, compared to the company's "three-month rule", reduces the total costs from \$663.20 to \$643.20.

6.5 The Wagner-Whitin Method : An Optimal Solution under an Additional Assumption

Wagner and Whitin (1958) developed an algorithm that guarantees an optimal solution of replenishment quantities under one additional assumption : Either the demand pattern terminates at the horizon or else the ending inventory must be prespecified.

The algorithm is an application of dynamic programming, a mathematical procedure for solving sequential decision problems. Suppose we define $F(t)$ as the total cost of the best replenishment strategy that satisfies the demand requirements in periods $1, 2, \ldots, t$. To illustrate the procedure for finding $F(t)$, we again use the example.

$F(1)$ is the total cost of a replenishment of size 10 at the start of January, simply the setup cost A or \$54.

To determine $F(2)$, we have two possible options to consider :

(Option 1) Replenish enough (72 boxes) at the start of January to cover the requirements of both January and February.

Costs : Setup cost for January replenishment + Carrying costs for February's requirements $= 54 + 62 \times 0.40 \times 1\text{month} = \$78.80$

(Option 2) Replenish 10 boxes at the start of January and 62 boxes at the start of February

Costs : $F(1)$ + Cost of a replenishment at the start of February to meet February's requirements $= \$54 + \$54 = \$108$

Consequently, $F(2) = \$78.80$.

To satisfy requirements through to the end of March there are three options **where we position the last replenishment** :

(Option 1) Have a single replenishment of 84 boxes at the start of January.

Costs : A + Carrying costs for February's requirements + Carrying costs for March's requirements = $54 + 62 \times 0.4 \times 1 + 12 \times 0.4 \times 2 = \$88.40$

(Option 2) Cover to the end of January in the best possible fashion and replenish 74 boxes at the start of February

Costs : $F(1) + A+$ Carrying cost for March's requirements $= 54 + 54 + 12 \times 0.40 \times 1 = \$112.80$

(Option 3) Cover to the end of February in the best possible fashion and replenish 12 boxes at the start of March

Costs : $F(2) + A = 78.80 + 54 = \$132.80$

Consequently, $F(3) = \$88.40$. That is, a single replenishment at the start of January is best in terms of meeting requirements through to the end of March.

We continue forward in this fashion until we complete period N. For any specific month t there are t possible options to evaluate. Note that the method requires an ending point where it is known that the inventory level is to be at zero or some other specified value.

Mathematically, we can represent the preceding procedures as follows :

$$F(t) = \text{Min}_{0 \leq j \leq t-1}[F(j) + c_{jt}]$$

where c_{jt} = cost in period $j+1$ to satisfy demands in period $j+1, \ldots, t$.

Using this formulation, let us find $F(4)$ together.

.

The computational details of the best strategy for the 12 month period are shown in the following table.

Table 6.5 Results of Using the W-W Algorithm or the S-M Heuristic on the Numerical Example

There are 7 replenishments and the total costs amount to \$501.20.

♣ Potential Drawbacks

(i) Relatively complex nature of the algorithm

(ii) Need for a well-defined ending point for the demand

(iii) Rolling horizon problem

(iv) The restricted assumption that replenishments can be made only at discrete intervals

6.6 Heuristic Approaches

6.6.1 The Silver-Meal or Least Period Cost Heuristic

The Wagner-Whitin algorithm has some drawbacks from the practitioner's standpoint. For example, the considerable computational effort, complex nature of the algorithm, additional assumption, etc. Therefore, the natural question to ask is "Is there a simpler approach that will capture most of the potential savings?" Silver and Meal (1973) have developed a simple variation of the basic EOQ which accomplishes exactly what we desire. Moreover, in numerous test examples the Silver-Meal heuristic has performed extremely well when compared with the other rules encountered in the literature.

• The Criterion Used for Selecting a Replenishment Quantity

The heuristic selects the replenishment quantity in order to replicate a property that the basic EOQ possesses when the demand rate is constant with time, namely, the *total relevant costs per unit time for the duration of the replenishment quantity are minimized.* If a replenishment arrives at the beginning of the first period and it covers requirements through to the end of the Tth period, then the criterion function can be written as follows :

$$\frac{\text{Setup Cost} + \text{Total Carrying Cost to end of period } T}{T}$$

- The Essence of the Heuristic

Because we constrained to replenishing at the beginning of periods, the best strategy must involve replenishment quantities that last for an integer number of periods. The replenishment quantity Q, associated with a particular value of T is

$$Q = \sum_{j=1}^{T} D_j \tag{$*$}$$

According to the chosen criterion, we wish to pick the T value that minimizes the total relevant costs per unit time over the time period T.

Let the total relevant costs associated with a replenishment that lasts for T periods be denoted by TRC(T). We wish to select T to minimize the total relevant costs per unit time, TRCUT(T), where

$$\text{TRCUT}(T) = \frac{\text{TRC}(T)}{T} = \frac{A + \text{ carrying costs}}{T}$$

If $T = 1$, there are no carrying costs, i.e.

$$\text{TRCUT}(1) = A$$

If the setup cost is large, this may be unattractive when compared with including the second period's requirements in the replenishment.

With $T = 2$ the carrying costs are D_2vr, the cost of carrying the requirement D_2 for one period. Therefore,

$$\text{TRCUT}(2) = \frac{A + D_2vr}{2}$$

With $T = 3$ we still carry D_2 for one period, but now we also carry D_3 for two periods. Thus,

$$\text{TRCUT}(3) = \frac{A + D_2vr + 2D_3vr}{3}$$

In this case the setup charge is apportioned across three periods, but this may not be attractive because of the added carrying costs.

The basic idea of the heuristic is to evaluate TRCUT(T) for increasing values of T until, for the first time,

$$\text{TRCUT}(T+1) > \text{TRCUT}(T)$$

that is, the total relevant costs per unit time start increasing. When this happens the associated T is selected as the number of periods that the replenishment should cover. The corresponding replenishment quantity Q is given by Eq. $*$.

This method guarantees only a local minimum in the total relevant costs per unit time, for the current replenishment. It is possible that still larger values of T would yield still lower costs per unit time since we stop testing with the first increase in costs per unit time.

To illustrate the application of the heuristic, let us again use the same example. The calculations for the first replenishment quantity are shown in the following table. The heuristic selects a t value of 3 with an associated $Q = 84$ boxes.

T	A	D_2vr	$2D_3vr$	$3D_4vr$		Row Sum	TRC(T)	TRCUT(T)
1	54					54.00	54.00	54.00
2		62×0.4				24.80	78.80	39.40
3			$2 \times 12 \times 0.4$			9.60	88.40	29.47
4				$3 \times 130 \times 0.4$		156.00	244.40	61.10

Let us perform the next iteration together.

It turns out for this numerical example that this simple heuristic gives the same solution as the Wagner-Whitin algorithm. Thus, the solution has already been shown before.

Example Consider the following multi-echelon inventory distribution system. Demand for items at the warehouse is as follows:

period	1	2	3	4
demand	30	25	20	40

Holding cost of items in the warehouse is \$2 per item for items carried over from one period to the next. Fixed order cost is \$100 per order. The warehouse places orders to a factory that produces the items. Production time and lead time are assumed to be negligible. Each time the factory starts to produce items, it costs the factory \$20 per setup, and holding cost in the factory is \$1.5 per item for items carried over from one period to the next. As a consultant for the factory, suggest a production plan for the factory using *Silver-Meal Heuristic*.

6.6.2 The EOQ Expressed as a Time Supply (POQ)

$$T_{EOQ} = \frac{EOQ}{D} = \sqrt{\frac{2A}{\overline{D}vr}}$$

Round T_{EOQ} to the nearest integer greater than zero, then, any replenishment of the item is made large enough to cover exactly the requirements of this integer number of periods.

In our example, T_{EOQ}=1.64 $\simeq 2 \rightarrow$ Total cost=\$553.60

6.6.3 Lot-for-Lot (L4L)

Order the exact amount needed for each time period $\rightarrow$ inventory holding costs become zero

In our example, total cost $=12 \times \$54 = \$648$.

6.6.4 Least Unit Cost (LUC)

Identical to the Silver-Meal Heuristic except that it accumulates requirements until the cost per unit increases.

Table 6.7 Computations for the First Replenishment Quantity using the LUC Heuristic

In our example, total cost =\$558.80

6.6.5 Part-Period Balancing (PPB)

Select the number of periods covered by the replenishment such that the total carrying costs are made as close as possible to the setup cost, A.

Table 6.8 Results of Using Part-Period Balancing on the Numerical Example

6.6.6 Performance of the Heuristics

The average penalty of using Silver-Meal Heuristic is less than 1%.

6.7 Handling of Quantity Discounts

Chapter 7

Individual Items with Probabilistic Demand

7.1 Some Important Issues and Terminology

7.1.1 Different Definitions of Stock Level

1. On-hand Stock: stock that is physically on the shelf

2. Net Stock = On-hand Stock − Backorders

3. Inventory Position = On-hand Stock + On-order Stock − Backorders − Committed
$\rightarrow$ key quantity in deciding when to replenish

4. Safety Stock: average level of the net stock just before a replenishment arrives

7.1.2 Backorders vs. Lost Sales

Complete Backordering $\rightarrow$ captive market, exclusive dealerships

Complete Lost Sales $\rightarrow$ retail-consumer link

(cf.) stockout $\rightarrow$ stockout occasion or event

7.1.3 Three Key Issues to be Resolved by a Control System under Probabilistic Demand

(1) How often the inventory status should be determined?

(2) When a replenishment order should be placed?

(3) How large the replenishment order should be?

♣ To respond to these issues managers can use the following questions.

(1) How important is the item?

(2) Can, or should, the stock status be reviewed continuously or periodically?

(3) What form should the inventory policy take?

(4) What specific cost or service objectives should be set?

7.2 The Importance of the Item: A, B, C Classification

A item (20%, 80%) $\rightarrow$ Chapter 8

B item (30%, 15%) $\rightarrow$ Chapter 7

C item (50%, 5%) $\rightarrow$ Chapter 9

7.3 Continuous vs. Periodic Review

(1) Continuous Review

In reality, continuous surveillance is usually not required; instead, each transaction triggers an immediate updating of the status $\rightarrow$ transactions reporting

eg. manual stock card system, POS data collection systems

Advantages: less safety stock (lower carrying costs)

Disadvantages: the workload is less predictable, more expensive in terms of reviewing costs and reviewing errors (eg. POS equipment is quite expensive)

(2) Periodic Review

The stock status is determined only every R time units.

eg. soda machine

Advantages: coordination replenishments, reasonable prediction of the level of the workload on the staff involved

7.4 The Form of the Inventory Policy: Four Types of Control Systems

7.4.1 Order-Point, Order Quantity (s, Q) System

continuous review: A fixed quantity Q is ordered whenever the **inventory position** drops to the reorder point s or lower.

(cf.) If net stock was used for ordering purposes, we might unnecessarily place another order today even though a large shipment was due in tomorrow. (eg. aspirin $\rightarrow$ the relief is on order)

(cf. two bin system: amount in the 2nd bin $\rightarrow$ (re)order point)

Advantages: quite simple, the production requirements for the supplier are predictable

Disadvantage: not able effectively cope with the situation where individual transactions are large

7.4.2 Order-Point, Order-up-to Level (s, S) System

continuous review: Order to raise the inventory position to the order-up-to level S (min-max system)

Figure 7.1 Two Types of Continuous Review Systems

Advantage: the best (s, S) system costs no more than the best (s, Q) system (even at the computational effort) $\rightarrow$ A items

Disadvantage: variable order quantity

7.4.3 Periodic Review, Order-up-to Level (R, S) System

periodic review: every R units of time, enough is ordered to raise the inventory position to the level S

Figure 7.2 The (R, S) System

widely used in companies not utilizing computer control, or ordered from the same supplier

Advantage: coordination

Disadvantage: higher carrying costs

7.4.4 (R, s, S) System

Scarf (1960) shows that the best (R, s, S) system produces a lower total cost than does any other system (but more computational time) $\rightarrow$ A item

7.5 Specific Cost and Service Objectives

♣ Four Methods to Balance Cost and Service Objectives

(i) Safety Stock Established through the use of a Simple-Minded Approach

Assigning a common safety factor as the safety stock of each item $\rightarrow$ we will find that there is a logical flaw in the use of this method

(ii) Safety Stocks based on Minimizing Cost

These approaches involve specifying (explicitly or implicitly) a way of costing a shortage and then minimizing total cost

(iii) Safety Stocks based on Customer Service

Recognizing the severe difficulties associated with costing shortages, an alternative approach is to introduce a control parameter known as the service level. The service level becomes a constraint in establishing the safety stock of an item; for example, minimize the carrying costs on an item subject to satisfying, routinely from stock, 95% of all demands. Again, there is considerable choice in the selection of a service measure.

(iv) Safety Stocks based on Aggregate Measure

Establish the safety stocks of individual items, using a given budget, to provide the best possible aggregate service across a population of items.

7.5.1 Choosing the Best Approach

Which one to use depends on the competitive environment of the particular company (eg.) new product $\rightarrow$ delivery performance may have significant implications for capturing market share

Table 7.2 Summary of Different Methods of Selecting the Safety Stocks

7.5.2 Safety Stock Established through the use of a Simple-Minded Approach

(1) Equal Time Supplies

The safety stocks of a broad group of items are set equal to the same time supply $\rightarrow$ fails to take account of the difference in the uncertainty of forecasts from item to item.

(2) Equal Safety Factors

$$SS = k\sigma_L$$

Use a common value of k for a broad range of items.

7.5.3 Safety Stocks based on Minimizing Cost

(1) Specified Fixed Cost (B_1) per Stockout Occasion

(2) Specified Fractional Charge (B_2) per Unit Short

(3) Specified Charge (B_3) per Unit Short per Unit Time

(4) Specified Charge (B_4) per Customer Line Item Short

7.5.4 Safety Stocks based on Customer Service

(1) Specified Probability (P_1) of No Stockout per Replenishment Cycle: Cycle Service Level

fraction of cycles in which a stockout does not occur

(2) Specified Fraction (P_2) of Demand to be Satisfied Routinely from the Shelf: Fill Rate

$$P_2 = \frac{B_3}{B_3 + r}$$

(3) Specified Fraction of Time (P_3) during which Net Stock is Positive: Ready Rate

Under Poisson demand, this measure is equivalent with the P_2 measure.

(4) Specified Average Time (TBS) between Stockout Occasions

Reciprocal of TBS: Desired average number of stockout occasions per year

7.5.5 Safety Stocks based on Aggregate Considerations

(1) Allocation of a Given Total Safety Stock among Items to Minimize the Expected Total Stockout Occasions per Year (ETSOPY)

(2) Allocation of a Given Total Safety Stock among Items to Minimize the Expected Total Value of Shortages per Year (ETVSPY)

7.6 Two Examples of Finding s in a (s, Q) System

7.6.1 Protection over the Replenishment Lead Time

Figure 7.3 The Occurrence of a Stockout in an (s, Q) System

No stockout occurs $\longleftrightarrow$ sum of the undershoot and the total demand in the replenishment lead time is < reorder point (s)

7.6.2 An Example Using a Discrete Distribution

Table 7.3 Lead Time Demand

expected demand per week = 2.2 units, annual demand=50$\times$ 2.2=110 units

L= 1 week, Q=20 units, A=\$18, h=\$10, B_2v =\$20, (P_1 =90% $\rightarrow$ s = 4units)

(1) Ordering Cost

number of orders per year=110/20=5.5 orders $\rightarrow$ total annual ordering cost=\$99

(2) Holding Cost (depends on s)

Figure 7.5 Behavior of Inventory Level with Time: Probabilistic Demand

$$h \times \left(\frac{Q}{2} + \sum_{x=0}^{s} (s-x) Pr(X=x) \right) = \$10 \times (10 + 0.4) = \$104$$

(3) Shortage Cost

$$\text{Cost per unit short} \quad \times \sum_{x=s+1}^{\infty} (x-s) Pr(X=x) \quad \times \quad \text{Number of cycles per year}$$

$= \$20 \times 0.6 \times 5.5 = \$66$

(4) Total Cost

\$99+\$104+\$66=\$269 (for s=2) $\rightarrow$ need to find an optimal s!

Table 7.6 The Optimal Reorder Point

7.7 Decision Rules for (s, Q) Control Systems

7.7.1 Common Assumptions and Notation

(1) Although demand is probabilistic, the average demand rate changes very little with time.

(2) A replenishment order of size Q is placed when the inventory position is exactly at the order point s.

(3) Crossing of orders is not permitted.

(4) Average level of backorders is negligibly small.

(5) Forecast errors have a normal distribution with no bias and a known standard deviation σ_L for forecasts over a lead time L.

Figure 7.6 Normally Distributed Forecast Errors

(6) Where a value of Q is needed, it is assumed to have been predetermined.

(7) The costs of the control system do not depend on the specific value of s selected.

♣ Change the Givens!

Optimize inventory levels given the parameters as they are; and then devote resources to changing the givens.

(i) Choose a supplier that is closer to your facility.

(ii) Ship via a faster transportation mode.

(iii) Improving forecast accuracy and providing customer incentives for specific purchase times and quantities.

♣ Notation

D = demand per year in units/year

$G_u(k) = \int_k^\infty (u_0 - k) \frac{1}{\sqrt{2\pi}} exp(-u_0^2/2) du_0$

a special function of the unit normal variable used in finding ESPRC

k = safety factor

L = replenishment lead time, in years

$p_{u\geq}(k) = 1 - \Phi(k)$

Q = prespecified order quantity, in units

r = inventory carrying charge, in \$/\$/year

s = (re)order point, in units

SS = safety stock, in units

v = unit variable cost, in \$/unit

$\hat{x}_L$ = forecast demand over a replenishment lead time, in units

σ_L = standard deviation of errors of forecasts over a replenishment lead time, in units

7.7.2 General Approach to Establishing the Value of s

$$s = \hat{x}_L + \text{ safety stock} = \hat{x}_L + k\sigma_L$$

Figure 7.7 General Decision Logic Used in Computing the Value of s

7.7.3 Common Derivation

1. Safety Stock (SS) = E(Net stock just before the replenishment arrives)

$$\int_0^\infty (s-x)f(x)dx = s - \hat{x}_L$$

2. Prob{stockout in a replenishment lead time} = $Prob\{x \geq s\} = \int_s^\infty f(x)dx$

$$(cf.)\ \text{If } X \sim N(\hat{x}_L, \sigma_L^2),\ \text{then } Prob\{x \geq s\} = 1 - \Phi(k) = p_{u\geq}(k)$$

(proof)

3. Expected Shortage Per Replenishment Cycle (ESPRC)

$$ESPRC = \int_s^{\infty} (x-s)f(x)dx$$

$$(cf.)\ \text{If } X \sim N(\hat{x}_L, \sigma_L^2),\ \text{then } ESPRC = \sigma_L G_u(k)$$

(proof)

$$E(OH) = \frac{1}{2}\{(s-\hat{x}_L)+(s-\hat{x}_L+Q)\} = \frac{Q}{2} + (s-\hat{x}_L) = \frac{Q}{2} + k\sigma_L$$

7.7.4 Decision Rule for a Specified Safety Factor (k)

Step 1. Safety stock, SS=$k\sigma_L$.

Step 2. Reorder point, $s = \hat{x}_L + SS$, increased to the next higher integer.

7.7.5 Decision Rule for a Specified Cost (B_1) per Stockout Occasion

$$ETRC(k) = C_r + C_c + C_s = \frac{AD}{Q} + \left(\frac{Q}{2} + k\sigma_L\right)vr + \frac{DB_1}{Q}p_{u\geq}(k)$$

$$\frac{dETRC(k)}{dk} = \sigma_L vr + \frac{DB_1}{Q}\frac{dp_{u\geq}(k)}{dk} = 0$$

$$f_u(k) = \frac{Qv\sigma_L r}{DB_1} \rightarrow k = \sqrt{2ln\left(\frac{DB_1}{\sqrt{2\pi}Qv\sigma_L r}\right)}$$

Step 1. Is

$$\frac{DB_1}{\sqrt{2\pi}Qv\sigma_L r} < 1?$$

If yes, then go to Step 2.
If no, then continue with

$$k = \sqrt{2ln\left(\frac{DB_1}{\sqrt{2\pi}Qv\sigma_L r}\right)}$$

Step 2. Set k at its lowest allowable value.

Step 3. Reorder point $s = \hat{x}_L + k\sigma_L$.

(Example)

7.7.6 Decision Rule for a Specified Fractional Charge (B_2) per Unit Short

$$TC(k) = \frac{AD}{Q} + \left(\frac{Q}{2} + k\sigma_L\right) vr + \frac{B_2 v \sigma_L G_u(k) D}{Q}$$

$$\frac{dTC(k)}{dk} = \sigma_L vr - \frac{B_2 v \sigma_L D p_{u\geq}(k)}{Q} = 0$$

$$p_{u\geq}(k) = \frac{Qr}{DB_2}$$

(Example)

7.7.9 Decision Rule for a Specified Probability (P_1) of No Stockout per Replenishment Cycle: Cycle Service Level

$$\text{Prob}\{\text{stockout in a lead time}\} = p_{u\geq}(k) = 1 - P_1$$

(Example)

7.7.10 Decision Rule for a Specified Fraction (P_2) of Demand Satisfied Directly from Shelf: Fill Rate

We assume complete backordering.

$$P_2 = 1 - \text{ Fraction backordered } = 1 - \frac{ESPRC}{Q} = 1 - \frac{\sigma_L G_u(k)}{Q}$$

$$G_u(k) = \frac{Q}{\sigma_L}(1 - P_2)$$

(cf.) If we assume complete lost sales, the above decision rule should be modified as follows:

$$P_2 = 1 - \frac{ESPRC}{Q + ESPRC} = 1 - \frac{\sigma_L G_u(k)}{Q + \sigma_L G_u(k)}$$
$$G_u(k) = \frac{Q}{\sigma_L}\left(\frac{1 - P_2}{P_2}\right)$$

(Example)

7.7.11 Decision Rule for a Specified Average Time (TBS) between Stockout Occasions

$$\text{desired average number of stockout occasions per year } = \frac{1}{TBS} = \frac{D}{Q} p_{u\geq}(k)$$
$$p_{u\geq}(k) = \frac{Q}{D(TBS)}$$

7.7.12 Decision Rule for the Allocation of a Total Safety Stock to Minimize the Expected Total Stockout Occasions per Year (ETOSPY)

$$\text{Minimize } \sum_{i=1}^{n} \frac{D_i}{Q_i} p_{u\geq}(k_i)$$
$$\text{subject to } \sum_{i=1}^{n} k_i \sigma_{L_i} v_i \leq Y$$

We form the Lagrangian function:

$$L(k_1, \cdots, k_n, \lambda) = \sum_{i=1}^{n} \frac{D_i}{Q_i} p_{u\geq}(k_i) + \lambda\left(\sum_{i=1}^{n} k_i \sigma_{L_i} v_i - Y\right)$$

$$\frac{\partial L}{\partial k_i} = -\frac{D_i}{Q_i} f_u(k_i) + \lambda \sigma_{L_i} v_i = 0 \rightarrow f_u(k_i) = \lambda \frac{Q_i v_i \sigma_{L_i}}{D_i}$$

(cf.) exactly the same decision rule in B_1 shortage costing method

Step 1. Start from an arbitrary $\lambda < \frac{D_i}{Q_i v_i \sigma_{L_i}}$.

Step 2. Compute k_i's from

$$f_u(k_i) = \lambda \frac{Q_i v_i \sigma_{L_i}}{D_i}$$

Step 3. If $\sum_{i=1}^{n} k_i \sigma_{L_i} v_i < Y$, increase λ and go to Step 2.

If $\sum_{i=1}^{n} k_i \sigma_{L_i} v_i > Y$, decrease λ and go to Step 2.

If $\sum_{i=1}^{n} k_i \sigma_{L_i} v_i = Y$, stop. The k_i's are optimal.

7.7.13 Decision Rule for the Allocation of a Total Safety Stock to Minimize the Expected Total Value of Shortages per Year (ETVSPY)

$$\text{Minimize } \sum_{i=1}^{n} \frac{D_i}{Q_i} \sigma_{L_i} G_u(k_i) v_i$$
$$\text{subject to } \sum_{i=1}^{n} k_i \sigma_{L_i} v_i \le Y$$

We form the Lagrangian function:

$$L(k_1, \cdots, k_n, \lambda) = \sum_{i=1}^{n} \frac{D_i}{Q_i} \sigma_{L_i} G_u(k_i) v_i + \lambda\left(\sum_{i=1}^{n} k_i \sigma_{L_i} v_i - Y\right)$$

$$\frac{\partial L}{\partial k_i} = -\frac{D_i}{Q_i} \sigma_{L_i} p_{u\ge}(k_i) v_i + \lambda \sigma_{L_i} v_i = 0 \;\rightarrow\; p_{u\ge}(k_i) = \lambda \frac{Q_i}{D_i}$$

(cf.) exactly the same decision rule in B_2 shortage costing method

Step 1. Start from an arbitrary $\lambda < \frac{D_i}{Q_i}$.

Step 2. Compute k_i's from

$$p_{u\ge}(k_i) = \lambda \frac{Q_i}{D_i}$$

Step 3. If $\sum_{i=1}^{n} k_i \sigma_{L_i} v_i < Y$, decrease λ and go to Step 2.

If $\sum_{i=1}^{n} k_i \sigma_{L_i} v_i > Y$, increase λ and go to Step 2.

If $\sum_{i=1}^{n} k_i \sigma_{L_i} v_i = Y$, stop. The k_i's are optimal.

7.7.14 Nonnormal Lead Time Distribution

♣ Distribution-Free Approach

"Much work has been done on distribution-free approaches, the original work by Scarf (1958) proposed finding the worst possible distribution for each decision variable and then finding the optimal inventory policy for that distribution. Thus, it is a conservative approach; and it requires only that the mean and variance of the lead time demand are known. Gallego (1992), Bulinskaya (1990), Gallego and Moon (1993), Moon and Choi (1994, 1995), and Moon and Gallgeo (1994) have advanced the research, and have shown that, in some cases, applying a distributional form incorrectly can lead to large errors. The distribution-free approach could generate significant savings. If the distribution appears to be very different from a known form, one should consider the distribution-free formulas, or those described in the next paragraph."

7.8 Implied Costs and Performance Measures

$$\text{input performance objective } (B_1, B_2, P_1, \cdots)$$

$$\longrightarrow k \longrightarrow \text{ reorder point \& implied values of any of the other input performance objectives}$$

(Example) D=4,000 units/yr, A=\$20.25, r=0.03\$/\$/yr, v=\$6/unit, L=1 week, $\hat{x}_L$=80 units, σ_L=20 units, P_2=0.98

$$EOQ = 300 \text{ units}, G_u(k) = 0.30 \rightarrow k = 0.22 \rightarrow s = 84.4 \rightarrow s = 85$$

$$s = 85 \rightarrow k = 0.25 \rightarrow p_{u\geq}(k) = 0.4013 \rightarrow P_1 = 1 - 0.4013 = 0.5987$$

$$\text{fill rate} = 98\% \rightarrow \text{ cycle service level} = 59.87\%$$

7.9 Decision Rules for Periodic-Review, Order-Up-To-Level (R, S) Control Systems

No need to repeat all the detail since there is a simple analogy between (R, S) and (s, Q) systems.

(s, Q)	(R, S)
s	S
Q	DR
L	$R + L$

♣ (main idea of proving the analogy)

For (R, S) system, a stockout will occur $\longleftrightarrow$ the total demand in an interval of duration $R + L$ exceeds S

For (s, Q) system, a stockout will occur $\longleftrightarrow$ the total demand in an interval of duration L exceeds s

7.9.1 The Review Interval (R)

We assume that a value of R has been predetermined $\rightarrow$ Determination of R is equivalent to the determination of an EOQ expressed as a time supply except

(i) cost of reviewing the inventory status must be included as part of A.

(ii) avoid to implement certain senseless review intervals, eg. 2.36 days $\rightarrow$ motivation for my paper with Prof. Silver (paper # 10)

7.9.2 The Order-Up-To Level (S)

The key time period over which protection is required is now of duration $R + L$, instead of just a replenishment lead time L.

Figure 7.12 The Time Period of Protection in an (R, S) System (cf.) The order-up-to level at time t_0 must be sufficient to cover demand through a period of duration $R + L$.

7.9.3 Common Assumptions and Notation

♣ Assumptions (in addition to the assumptions for (s, Q) system)

(i) A replenishment order is placed at every review.

(ii) The value of R is assumed to be predetermined.

♣ Notation

R = prespecified review interval (years)

S = order-up-to level (units)

$\hat{x}_{R+L}$ = expected demand over $R + L$ (units)

σ_{R+L} = standard deviation of demand over $R + L$ (units)

7.9.4 Common Derivation

0. number of reviews per year $= \frac{1}{R}$

1. Safety Stock (SS) = E(net stock just before order Y arrives)

$$\int_0^\infty (S - x) f(x) dx = S - \hat{x}_{R+L} = k\sigma_{R+L}$$

2. Prob{stockout in a replenishment cycle} $= Prob\{x \geq S\} = \int_S^\infty f(x)dx$

3. Expected Shortage Per Replenishment Cycle (ESPRC)

$$ESPRC = \int_S^\infty (x - S) f(x) dx = \sigma_{R+L} G_u(k)$$

$$E(OH) = \frac{1}{2}\{(S - \hat{x}_{R+L}) + (S - \hat{x}_{R+L} + DR)\} = \frac{DR}{2} + (S - \hat{x}_{R+L})$$

7.10 Variability in the Replenishment Lead Time Itself

♣ Two Possible Actions

(i) Try to ascertain the distribution of total demand over the lead time.

(ii) Measure the distribution of the lead time (or $R+L$), and the distribution of demand per unit period, separately. Then, combine them.

7.10.1 Approach 1: Use of the Total Demand over the Full Lead Time

♣ Lordahl and Bookbinder (1994, NRQ)'s Distribution-Free Method

Try to find the reorder point, s, when the parameters, and form, of the lead time demand distribution are unknown and P_1 is given (The idea is simple). They show that this procedure is better than using the normal distribution in many cases.

(Step 1) Rank order the observed lead time demand.

$$x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$$

(Step 2) Let $(n+1)P_1 = y + w$, where $0 \leq w \leq 1$, and y is an integer.

(Step 3) If $(n+1)P_1 > n$, set $s = x_{(n)}$. Otherwise, set $s = (1-w)x_{(y)} + wx_{(y+1)}$.

(Example) p282

7.10.2 Approach 2: Use of the Distribution of Demand Rate per Unit Time Combined with the Lead Time Distribution

Assume that lead time (L) and the demand (D) in each unit time period are independent random variables.

$$\text{lead time demand } x = \sum_{i=1}^{L} D_i \quad \text{where } E(D_i) = E(D)$$

$$E(x) = E(L)E(D)$$

$$Var(x) = E(L)Var(D) + [E(D)]^2 Var(L)$$

(proof)

$$Var(x) = Var(\sum_{i=1}^{L} D_i) = E\left[\left(\sum_{i=1}^{L} D_i\right)^2\right] - \left(E\left[\sum_{i=1}^{L} D_i\right]\right)^2$$

$$\begin{aligned}
E\left[\left(\sum_{i=1}^{L} D_i\right)^2\right] &= \sum_{l=0}^{\infty} E\left[\left(\sum_{i=1}^{L} D_i\right)^2 \mid L = l\right] P[L = l] \\
&= \sum_{l=0}^{\infty} E\left[\left(\sum_{i=1}^{l} D_i\right)^2\right] P[L = l] \\
&= \sum_{l=0}^{\infty} \left(Var\left(\sum_{i=1}^{l} D_i\right) + \left(E\left[\sum_{i=1}^{l} D_i\right]\right)^2\right) P[L = l] \\
&= \sum_{l=0}^{\infty} \left(lVar(D_i) + l^2[E(D_i)]^2\right) P[L = l] \\
&= E(L)Var(D) + [E(D)]^2 E(L^2)
\end{aligned}$$

$$\begin{aligned}
Var(x) &= E(L)Var(D) + [E(D)]^2 E(L^2) - [E(L)E(D)]^2 \\
&= E(L)Var(D) + [E(D)]^2 (E(L^2) - [E(L)]^2) \\
&= E(L)Var(D) + [E(D)]^2 Var(L)
\end{aligned}$$

(Example) p283

7.11 Exchange Curves Involving Safety Stocks for (s, Q) Systems

7.11.1 Single Item Exchange Curve-Inventory versus Service

(Q) Determine the best customer service possible for the given investment.

Figure 7.13 A Single Item Exchange Curve

♣ Consider the possibility of changing the givens! → lead time reduction by implementing EDI, reduction of demand variability by closer relationship to customers

7.11.2 An Illustration of the Impact of Moving Away from Setting Reorder Points as Equal Time Supplies

(cf.) The following contents have been extracted from the paper by Moon and Silver (2001, International Journal of Production Economics)

Our problem can be represented as follows:

$$\textbf{(ETVSPY)} \qquad Min\ \text{ETVSPY} = \sum_{i=1}^{n} \frac{D_i}{Q_i} \sigma_i v_i G_u\left(\frac{D_i t_i - \hat{x}_i}{\sigma_i}\right)$$

$$\text{subject to} \qquad \sum_{i=1}^{n} D_i v_i t_i \le Y' \tag{1}$$

$$t_i \in \{T_1, T_2, \cdots, T_m\} \quad \forall i \tag{2}$$

$$\text{where} \quad Y' = Y + \sum_{i=1}^{n} \hat{x}_i v_i$$

We derive a lower bound on (ETVSPY) for two reasons. First, it represents the optimal value of the objective function when the t_i's are not restricted to a discrete set of values. Thus, it will provide an indication of the degradation (increase in the expected total value short per year) caused by the introduction of the pragmatic constraint of restricting the t_i's to the discrete set T. The second reason is that the lower bound solution will be used as a starting point in our heuristic approach.

If we relax constraint (2), we obtain a relaxed version of (ETVSPY) as follows.

$$\textbf{(LB)} \qquad Min\ \text{ETVSPY} = \sum_{i=1}^{n} \frac{D_i}{Q_i} \sigma_i v_i G_u\left(\frac{D_i t_i - \hat{x}_i}{\sigma_i}\right)$$

$$\text{subject to} \qquad \sum_{i=1}^{n} D_i v_i t_i \le Y'$$

If we can find a Karush-Kuhn-Tucker (KKT) solution, it will be a global minimum due to the fact that the objective function is convex and constraint (1) is a convex set. The Lagrangian function is as follows:

$$L(t_1, \cdots, t_n, \lambda) = \sum_{i=1}^{n} \frac{D_i}{Q_i} \sigma_i v_i G_u\left(\frac{D_i t_i - \hat{x}_i}{\sigma_i}\right) + \lambda\left(\sum_{i=1}^{n} D_i v_i t_i - Y'\right)$$

The Karush-Kuhn-Tucker conditions become as follows:

$$\frac{\partial L}{\partial t_i} = \left(\frac{D_i \sigma_i v_i}{Q_i}\right) \left\{ -\frac{D_i}{\sigma_i} p_{u\ge} \left(\frac{D_i t_i - \hat{x}_i}{\sigma_i}\right) \right\} + \lambda D_i v_i = 0$$

$$\text{where} \quad p_{u\geq}(k) = \int_k^\infty \frac{1}{\sqrt{2\pi}} exp(-u^2/2)du$$

is the right hand tail of the unit normal distribution and represents the probability of a stockout during a replenishment lead time. Simplifying we obtain

$$\lambda = \frac{D_i}{Q_i} p_{u\geq} \left(\frac{D_i t_i - \hat{x}_i}{\sigma_i} \right) \tag{3}$$

$$\sum_{i=1}^{n} D_i v_i t_i = Y' \tag{4}$$

Note that equation (3)implies the same expected number of stockout occasions per year for all items, namely the optimal value of the Lagrangian multiplier. We can use the following line search algorithm to find a solution which satisfies (3) and (4).

(Step 1) Start from an arbitrary $\lambda > 0$.

(Step 2) Using the λ and (3), find t_is.

(Step 3) If $\sum_{i=1}^{n} D_i v_i t_i = Y'$, stop. We find an associated optimal solution for (LB), say t_i^0s.

If $\sum_{i=1}^{n} D_i v_i t_i < Y'$, decrease λ and go to (Step 2).

If $\sum_{i=1}^{n} D_i v_i t_i > Y'$, increase λ and go to (Step 2).

We shall now illustrate the above algorithm using an example in Silver *et al* reproduced in Table 7.8. The three items are produced and stocked by a company. It is not uncommon for organizations to use the following type of rule for setting reorder points of a broad range of items: reorder when the inventory position has dropped to a specific time supply. We assume that the current reorder points of the company are each based on a two-month time supply (that is, $D/6$).

Assuming normally distributed lead time demand, it can be shown that the safety stock and ETVSPY using the current policy are as listed under the title of 'Equal Time Supply' in Table 7.9. If we use the algorithm to find the lower bound solution presented above, which is equivalent to the optimal solution under continuous possible time supplies, we can achieve 70% savings compared to the strategy based on equal probabilities of stockout per cycle.

Table 7.8 Data for the 3 item example

Item	D(units/yr)	v($/unit)	$\hat{x}$(units)	σ(units)	Q (units)	s(units)
PSP-001	6,000	20	750	125	6,000	1,000
PSP-002	3,000	10	375	187.5	1,000	500
PSP-003	2,400	12	300	62.5	1,200	400

s: reorder point

Table 7.9 (modified) Results for the 3 item example

Item	Equal Time Supply		Optimal	
No.	SS	ETVSPY	SS	ETVSPY
1	$5,000	$21	$2,972	$143.4
2	$1,250	$845	$3,302	$88.5
3	$1,200	$35	$1,175	$37.7
Total	$7,450	$901/yr	$7,450	$269.6/yr

SS: Safety Stock, ETVSPY: Expected Total Value Short per Year

Chapter 8

Managing the Most Important (Class A) Inventories

8.1 The Nature of Class A Items

A Items $\rightarrow$ high annual dollar usage (large Dv), essential spare part item

♣ Trade-off
control system costs (costs of collecting data, performing the computations, providing action reports, etc) vs. other costs (total costs of replenishment, carrying stock, and shortages)

(Example) (justification of using sophisticated control system)

other costs of A item = \$900/year, other costs of B item = \$70/year

Using complex control system (\$40/year/item higher than that of simple system)

♣ the type of control to use within the A category $\propto D$ and v

(i) a low D and a high v (slow moving item) $\rightarrow$ section 8.3

(ii) a high D and a low v (fast moving item) $\rightarrow$ section 8.4−8.6

8.2 Guidelines for Control of A Items

1. Inventory records should be maintained on a perpetual basis, particularly for the more expensive items. $\rightarrow$ This need not be through the use of a computer.
2. Keep top management informed.
3. Estimate and influence demand.
   (i) Provide manual input to forecasts.
   (ii) Ascertain the predictability of demand. $\rightarrow$ no need to carry protection stock or use a pool of spare parts shared among several companies within the same industry.
   (iii) Manipulate a given demand pattern.
4. Estimate and influence supply. $\rightarrow$ Negotiations with suppliers (eg. freeze periods)
5. Use conservative initial provisioning. $\rightarrow$ risk of overstocking
6. Review decision parameters frequently. $\rightarrow$ monthly or bimonthly
7. Determine precise values of control quantities. $\rightarrow$ Restricting attention to a limited number of possible time supplies (eg. 1 week, 2 weeks, 1 months, 2 months, etc.) results in a small cost penalties for B items (papers #10 & 11). This is not the case for A items.
8. Confront shortages as opposed to setting service levels.

8.3 Order-Point, Order Quantity (s, Q) Systems for Slow-Moving A Items

♣ distribution of lead time demand

(i) normal distribution (if $\hat{x}_L \geq 10$)

(ii) Poisson distribution (if $\hat{x}_L < 10$ the observed $\sigma_L \simeq \sqrt{\hat{x}_L}$)

♣ Sequential Approach in Section 7

$$ETRC(k) = C_r + C_c + C_s = \frac{AD}{Q} + \left(\frac{Q}{2} + SS\right) vr + \frac{DB_1}{Q} \times (\text{Probability of a stockout in a cycle})$$

Select Q that minimizes $C_r + C_c$; then choose the best value of safety stock (or $k\sigma_L$ in the case of normal demand). (percentage penalty using the sequential approach tends to be quite small!) $\rightarrow$ However, for A items, the small percentage may cause a lot of extra money!

8.3.1 The B_2 Cost Measure for Very Slow-Moving, Expensive Items ($Q = 1$)

$$TRC(Q) = \frac{Qvr}{2} + \frac{AD}{Q}$$

$$TRC(1) < TRC(2) \longleftrightarrow D < \frac{vr}{A}$$

(ex.) If $\frac{A}{r} = 13.11 \rightarrow$ Use $Q = 1$ if $D < 0.0763v$

♣ Assumptions Behind the Derivation of the Decision Rule

(i) Continuous-review, order-point, order-quantity system with $Q = 1$.

(ii) Poisson demand.

(iii) The replenishment lead time is a constant L.

(iv) There is complete backordering of demands when out of stock.

(v) There is a fixed cost, $B_2 v$, per unit backordered.

♣ Decision Rule

$p_{NS}(n_0)$=probability that the net stock at a random point in time takes on the value n_0

x= total demand in the replenishment lead time

$p_x(x_0)$=probability that total lead time demand is x_0

$\overline{I}$ = expected on-hand inventory (i.e. expected positive net stock)

Prob{a demand is not satisfied} $= p_{NS\leq}(0) = p_{x\geq}(S)$

$$p_{NS}(n_0) = Prob\{x = S - n_0\}$$

$$\overline{I} = \sum_{n_0=0}^{S} n_0 p_{NS}(n_0) = \sum_{n_0=0}^{S} n_0 p_x(S-n_0) = \sum_{j=0}^{S}(S-j)p_x(j)$$

$$C_s = B_2 v D p_{x\geq}(S)$$

$$ETRC(S) = \overline{I}vr + C_s = vr\sum_{j=0}^{S}(S-j)p_x(j) + B_2 v D p_{x\geq}(S)$$

$$ETRC(S) = ETRC(S+1) \longleftrightarrow \frac{p_x(s+1)}{p_{x\leq}(s+1)} = \frac{r}{DB_2}$$

(Numerical Illustration) p322.

8.4 Simultaneous Determination of s and Q for Faster-Moving Items

We assume normal distribution for the lead time demand.

$$ETRC(k,Q) = \frac{AD}{Q} + \left(\frac{Q}{2} + k\sigma_L\right)vr + \frac{DB_1}{Q}p_{u\geq}(k)$$

$$\frac{\partial ETRC(k,Q)}{\partial Q} = -\frac{AD}{Q^2} + \frac{vr}{2} - \frac{DB_1}{Q^2}p_{u\geq}(k) = 0$$

$$Q = \sqrt{\frac{2(AD + DB_1 p_{u\geq}(k))}{vr}} = \sqrt{\frac{2AD}{vr}\left(1 + \frac{B_1}{A}p_{u\geq}(k)\right)} = EOQ\sqrt{1 + \frac{B_1}{A}p_{u\geq}(k)} \quad 8.11$$

$$\frac{\partial ETRC(k,Q)}{\partial k} = \sigma_L vr + \frac{DB_1}{Q}\frac{dp_{u\geq}(k)}{dk} = 0$$

$$f_u(k) = \frac{Qv\sigma_L r}{DB_1} \rightarrow k = \sqrt{2ln\left(\frac{DB_1}{\sqrt{2\pi}Qv\sigma_L r}\right)} \quad 8.12$$

♣ Iterative Algorithm

(Step 0) Start from an arbitrary Q (eg. EOQ).

Repeat the following steps until either $Q_i \simeq Q_{i-1}$ or $k_i \simeq k_{i-1}$.

(Step 1) Compute k using equation 8.12 and Q value obtained in (Step 2) (At first iteration, use EOQ).

(Step 2) Compute Q using equation 8.11 and k value obtained in (Step 1).

(cf.) Hadley and Whitin (1963) showed that the algorithm is converging.

(Example) p327.

8.4.1 Cost Penalties

Figure 8.2 Percent Cost Penalty Associated with Using the Sequential Approach

(Numerical Illustration) p328.

♣ (p325) Similar results have been developed for P_2 (See Yano (1995) for the case of normal demand, and Moon and Choi (1994) for the case where we know only the mean and variance of demand).

♣ The Distribution Free Continuous Review Inventory System with a Service Level Constraint (Moon and Choi (1994, Computers & IE))

The problem objective involves minimizing the average annual ordering cost and inventory carrying costs subject to a constraint on the level of service. Service is measured here as the fraction of demand satisfied directly from stock.

The data and decision variables are as follows:

Q = order quantity (decision variable),

r = reorder point (decision variable),

D = average demand per year,

h = inventory carrying cost per item per year,

K = fixed ordering cost per order,

x = demand during the lead time (random variable),

$f(x)$ = density of demand during the leadtime,

$F(x)$ = cumulative distribution of leadtime demand,

$1-\beta$ = proportion of demands which are met from stock, i.e. service level.

The average annual cost can be written as follows:

$$Min\, G^F(Q,r) = \frac{KD}{Q} + h(\frac{Q}{2} + r - \mu)$$

subject to

$$n(r) \leq \beta Q \tag{1}$$

where $n(r) = \int_r^\infty (x-r)f(x)dx$ is the expected number of stockouts per cycle.

In this model, the inventory position of an item is reviewed continuously, and the policy is to order a lot size Q when the inventory position (on hand plus on order minus backorder) drops to a reorder point r.

The Lagrangian function of the annual cost function of the above is as follows:

$$L(Q,r,\lambda) = \frac{KD}{Q} + h(\frac{Q}{2} + r - \mu) + \lambda[n(r) - \beta Q]$$

where λ is a Lagrangian multiplier associated with the service constraint. Upon using Leibniz's rule and set $\partial L(Q,r,\lambda)/\partial Q = 0, \partial L(Q,r,\lambda)/\partial r = 0, \partial L(Q,r,\lambda)/\partial \lambda = 0$, we get the following first order necessary conditions:

$$\frac{\partial L}{\partial Q} = -\frac{KD}{Q^2} + \frac{h}{2} - \lambda\beta = 0 \tag{2}$$

$$\frac{\partial L}{\partial r} = h - \lambda[1 - F(r)] = 0 \tag{3}$$

$$\frac{\partial L}{\partial \lambda} = n(r) - \beta Q = 0 \tag{4}$$

Combining $(2), (3)$, and (4) , we get the following equation:

$$\frac{KD}{Q^2} = \frac{h}{2} - \frac{h}{1-F(r)}\frac{n(r)}{Q}$$

Solving above equation for Q, we obtain

$$Q^F = \frac{hn(r) + \sqrt{h^2n^2(r) + 2KDh[1-F(r)]^2}}{h[1-F(r)]} = \frac{n(r)}{1-F(r)} + \sqrt{\left[\frac{n(r)}{1-F(r)}\right]^2 + \frac{2KD}{h}} \tag{5}$$

From (4), we obtain

$$n(r) = \beta Q \tag{6}$$

Using the following iterative algorithm, we can find the optimal reorder point r^F, and optimal order quantity Q^F.

Algorithm

(Step 0) We use $EOQ = \sqrt{\frac{2KD}{h}}$ as the initial estimate for Q. Call this value Q_0.

(Step 1) Use equation (6) with $Q = Q_0$ to find the reorder point r. Call this value r_0.

(Step 2) Use equation (5) with $r = r_0$ to find Q_1.

(Step 3) Repeat (Step 1) and (Step 2) with $Q = Q_1$, etc. Convergence occurs when $Q_i = Q_{i-1}$ or $r_i = r_{i-1}$ for some i.

Example 1. The average demand per year is 200 units, the ordering cost is \$50, the inventory carrying cost per unit per year is \$2, and the service level is set as 0.98. The lead time demand is normally distributed with a mean (μ) of 100 units and standard deviation (σ) of 25 units. We obtain the optimal order quantity and reorder point as follows: Note that we use

$$n(r) = \sigma L'(\frac{r-\mu}{\sigma})$$

where $L'(\mu) = \int_u^\infty (z-\mu)\phi(z)dz$ which is a unit normal linear-loss integrals.

(iter. 1) $Q_0 = \sqrt{\frac{2KD}{h}} = 100$. Since $L'(\frac{r_0-100}{25}) = 0.08, r_0 = 151$.

(iter. 2) Since $F(r_0) = 0.9783$, $Q_1 = \sqrt{\left(\frac{2}{0.0217}\right)^2 + \frac{2*50*200}{2}} + \frac{2}{0.0217} = 228$. Since $L'(\frac{r_1-100}{25}) = 0.182, r_1 = 114$.

(iter. 3) Since $F(r_1) = 0.7123$, $Q_2 = \sqrt{\left(\frac{4.56}{0.2877}\right)^2 + \frac{2*50*200}{2}} + \frac{4.56}{0.2877} = 117$. Since $L'(\frac{r_2-100}{25}) = 0.094, r_2 = 124$.

(iter. 4) Since $F(r_2) = 0.8315$, $Q_3 = \sqrt{\left(\frac{2.34}{0.1685}\right)^2 + \frac{2*50*200}{2}} + \frac{2.34}{0.1685} = 115$. Since $L'(\frac{r_3-100}{25}) = 0.092, r_3 = 124$.

Using the above algorithm, we get $(Q^N, r^N) = (115, 124)$, and average annual cost of using (Q^N, r^N) is \$251 where $N \in \mathcal{F}$ represents the normal distribution. ∎

Now we consider the distribution free approach. We make no assumption on the distribution F of x other than saying that it belongs to the class $\mathcal{F}$ of cumulative distribution functions with mean μ and variance σ^2. If we replace $n(r)$ in equation (1) by its worst case upper bound, we can keep the service level against the worst possible distribution in $\mathcal{F}$. To this end, we need the following proposition.

Proposition 1.

$$E[x-r]^{+} \leq \frac{\sqrt{\sigma^2+(r-\mu)^2}-(r-\mu)}{2} \tag{7}$$

Moreover, the upper bound (7) is tight. That is, for every r, there exists a distribution $F^{*} \in \mathcal{F}$ where the bound (7) is tight.

Our problem is now to solve the following problem:

$$Min\, G^{W}(Q,r) = \frac{KD}{Q} + h(\frac{Q}{2} + r - \mu)$$

subject to

$$\frac{\sqrt{\sigma^2+\Delta^2}-\Delta}{2} \leq \beta Q \tag{8}$$

where $\Delta \equiv r - \mu$

The Lagrangian function of the above problem is

$$L(Q,\Delta,\lambda) = \frac{KD}{Q} + h(\frac{Q}{2} + \Delta) + \lambda(\frac{\sqrt{\sigma^2+\Delta^2}-\Delta}{2} - \beta Q)$$

where λ is a Lagrangian multiplier associated with the service constraint. Upon using Leibniz's rule and set $\partial L(Q,\Delta,\lambda)/\partial Q = 0, \partial L(Q,\Delta,\lambda)/\partial r = 0, \partial L(Q,\Delta,\lambda)/\partial\lambda = 0$, we get the following first order necessary conditions:

$$\frac{\partial L}{\partial Q} = -\frac{KD}{Q^2} + \frac{h}{2} - \lambda\beta = 0 \tag{9}$$

$$\frac{\partial L}{\partial \Delta} = h + \frac{\lambda}{2}\left(\frac{\Delta}{\sqrt{\sigma^2+\Delta^2}} - 1\right) = 0 \tag{10}$$

$$\frac{\partial L}{\partial \lambda} = \frac{\sqrt{\sigma^2+\Delta^2}-\Delta}{2} - \beta Q = 0 \tag{11}$$

Combining $(9), (10)$, and (11) , we get the following equation:

$$\frac{KD}{Q^2} = \frac{h}{2} - \frac{h}{Q}\sqrt{\delta^2+\sigma^2}$$

Solving above equation for Q, we obtain

$$Q^{W} = \sqrt{\sigma^2+\Delta^2} + \sqrt{\sigma^2+\Delta^2+\frac{2KD}{h}} \tag{12}$$

From (11), we obtain

$$\Delta = \frac{\sigma^2}{4\beta Q} - \beta Q \tag{13}$$

Using the following iterative algorithm, we can find the optimal reorder point r^W, and optimal order quantity Q^W. The convergence of the following algorithm can be proven by adopting a similar technique used in Yano (1985, NRL), and can be found in Choi (1999, Ph.D Thesis).

Algorithm

(Step 0) Start from $EOQ = \sqrt{\frac{2KD}{h}} = Q_0$.

(Step 1) Use equation (13) with $Q = Q_0$ to find Δ. Call this value Δ_0.

(Step 2) Use equation (12) with $\Delta = \Delta_0$ to find Q_1.

(Step 3) Repeat (Step 1) and (Step 2) with $Q = Q_1$, etc. Convergence occurs when $Q_i = Q_{i-1}$ or $\Delta_i = \Delta_{i-1}$ for some i. The optimal reorder point is $r = \Delta_i + \mu$.

Example 2. We continue Example 1. From the above algorithm, we can find the optimal order quantity and reorder point, (Q^W, r^W) against the worst distribution. We compare the procedure for the worst case distribution with that for the normal distribution. The results are $(Q^W, r^W) = (164, 145)$ and $(Q^N, r^N) = (115, 124)$ where $N \in \mathcal{F}$ represents the normal distribution. The annual average cost using (Q^W, r^W) is \$315 which is about 25% increase compared with using (Q^N, r^N) for the normal distribution. However, if we use (Q^N, r^N) for the worst case distribution, the expected number of shortages per cycle is 5.38. Consequently, the service level results in 0.953 which is much smaller than the prescribed service level 0.98. In other words, we pay additional \$64 to keep up with the service level against the worst distribution ∎

♣ (p325) Finally, see Moon and Gallego (1994, JORS) for the \$/unit short case where we know only the mean and variance of demand.

Chapter 9

Managing Routine (Class C) Inventories

9.1 The Nature of C Items

C Items (so-called *cats and dogs*) $\rightarrow$ low annual dollar usage (low Dv)

♣ A low-dollar-usage item which should not be C items.

(i) A slow-moving product that rounds out a service line provided to an important customer

(ii) A product that is the "pride and joy" of the president because he or she was instrumental in its development $\rightarrow$ a high *implicit* shortage cost.

(iii) An inexpensive product that has a high *explicit* shortage cost.

Use simple procedures that keep the control costs per SKU quite low!

An item's classification can change over time!

9.2 Control of C Items Having Steady Demand

9.2.1 Inventory Records

most appropriate not to maintain any inventory record of a C item → rely on an administrative mechanism for reordering (eg. two-bin system) → a record should be kept of when orders were placed and received for demand estimation and order control purposes

♣ Two Choices for Selecting a Review Interval for C Item

(1) periodic review with a relatively long interval

(2) continuous review but with a mechanism for triggering orders that requires neither a physical stock count nor the manual updating of the stock status

9.2.2 Selecting the Reorder Quantity (or Reorder Interval)

One of at most a few possible time supplies should be assigned to each class C item.

$$TRC(\text{using T months}) = \frac{DTvr}{24} + \frac{12A}{T}$$

Let two adjacent allowable values of the months of supply be T_1 and T_2. The value of Dv at which we are indifferent to using T_1 and T_2

$$\frac{DT_1vr}{24} + \frac{12A}{T_1} = \frac{DT_2vr}{24} + \frac{12A}{T_2}$$

$$(Dv)_{indifference} = \frac{288A}{T_1T_2r}$$

(Example) A=\$3.20, r=0.24/year, T_1=6 months, T_2=12 months

$$\frac{288 \times 3.20}{6 \times 12 \times 0.24} = \$53/year$$

Table 9.1 Suggested Reorder Time Supplies for a Sample Firm's C Items

♣ D would not be estimated through a forecasting system, but rather through knowledge of the staring and ending inventories over a convenient time period.

Two consecutive orders A and B. Let A be received at time t_A and B be placed at time t_B. Let I_A be the inventory level just before A is received.

$$\text{Demand Rate } \simeq \frac{I_A + Q - s}{t_B - t_A} \tag{9.1}$$

9.2.3 Selecting the Reorder Point (or Order-up-to Level)

♣ Select the safety factor to provide a specified expected time, TBS, between stockout occasions.

(i) Thinking in terms of an average time between stockouts is apparently more straightforward than dealing with probabilities or fractions.

(ii) Large values of TBS are not unreasonable when we recognize the small added expense of carrying a high safety stock.

$$p_{u\geq}(k) = \frac{Q}{D(TBS)} \tag{9.2}$$

$$s = \hat{x}_L + k\sigma_L \tag{9.3}$$

$$\hat{x}_L = DL \tag{9.4}$$

(Example) TBS=20 years, Time Supply in Months = 6 months

$$p_{u\geq}(k) = \frac{Q}{D(TBS)} = \frac{0.5}{20} = 0.025 \rightarrow k = 1.96 \text{ Table B.1 on page 724}$$

Table 9.2 Table to Select Safety Factor, k_i for a Sample Firm

(cf.) An estimate of σ_L is required? $\rightarrow$ Use Poisson distribution rather than forecasting!

$$\sigma_L = \sqrt{\hat{x}_L} = \sqrt{DL} \tag{9.5}$$

(Example) D=48 units/year, L=2 months, $\frac{Q}{D}$ = 12 months, TBS=20 years $\rightarrow$ s =?

$$\hat{x}_L = DL = 8 \text{ units}, \sigma_L = \sqrt{8} = 2.83 \text{ units}$$

$$s = 8 + 1.64 \times 2.83 = 12.6 \simeq 13 \text{ units}$$

9.2.4 The Two-Bin System Revisited

capacity of the reserve bin = reorder point

(Opening the reserve bin is a signal to place a replenishment order! $\rightarrow$ When the order arrives the reserve bin is refilled and sealed. The remainder of the order is placed in the other bin.)

(cf.) To facilitate ordering, a bin reserve tag should be attached to the reserve bin (similar to Kanban!)

9.2.5 A Simple Form of the (R, S) System

♣ A Simple (R, S) System developed by J. C. Penney

Periodically (eg. each quarter) management specifies a desired value of S and at only that time the on-hand stock is counted and an order is placed to raise the inventory position to S. Then, at each regular review instant (eg. each week), the computer simply orders enough to replace sales since the last review.

(Numerical Illustration) (p363)

9.2.6 Grouping of Items

Coordinated control may very well be in order to reduce replenishment costs.

Coordination does not rule out the use of a two-bin system. $\rightarrow$ Reserve tags of opened reserve bins are held centrally between designated periodic times at which that group is ordered.

9.3 Control of Items with Declining Demand Patterns

9.3.2 The Sizing of the Final Replenishment under Probabilistic Demand

♣ decision rule based on satisfying a certain fraction of the remaining demand (eg. meeting 95% of all request for spare parts during a ten-year period after the last sale of a new unit)

$$\text{total remaining demand} = y \sim N(\hat{y}, \sigma_y^2)$$

$$\text{Expected Shortage over the Lifetime} = ES = \int_S^\infty (y - S) f(y) dy = \sigma_y G_u(k)$$

$$\frac{ES}{\hat{y}} = 1 - P_2$$

$$G_u(k) = \frac{\hat{y}(1 - P_2)}{\sigma_y}$$

$$S = \hat{y} + k\sigma_y$$

(Numerical Illustration) (p366)

9.4 Reducing Excess Inventories

(cf.) In some industries the percentage of stocked items that have had no usage in the previous 52 weeks can be as high as 47% → dead items

♣ reasons for excess inventories

(1) errors associated with replenishments

 (i) production overruns

 (ii) unjustified quantity purchases

 (iii) errors in transmission of an order request

 (iv) inaccurate inventory records

(2) overestimation of the demand rate

 (i) inaccurate forecasts

(ii) deliberate changes in sales/marketing efforts

(iii) technological obsolescence

(iv) customer cancellation

♣ Identify the associated items! Then, decide on what remedial action to take!

9.4.1 Review of the Distribution by Value (DBV)

$$\text{Coverage in months} = CO = \frac{12I}{D}$$

Table 9.3 Items Listed By Coverage

(cf.) 4.2% of the total inventory is tied up in stock of zero-movers and some 6.9% is included in items having a coverage of five years or more.

9.4.2 A Rule for the Disposal Decision

benefits of disposal: benefits of salvage revenue (possibly negative) and reduced inventory carrying costs, reduced storage area

However, we need to replenish at an earlier time than if no stock disposal was made.

W= amount of stock to dispose (decision variable)

I = current inventory level

Compare costs out to time $\frac{I}{D}$

(i) Under the option of no disposal

$$C_{ND} = \frac{I^2 vr}{2D}$$

(ii) Under the option of disposal

$$C_D = -gW + \left(\frac{I-W}{D}\right) \times \left(\frac{I-W}{2}\right) vr + \frac{W}{D}(\sqrt{2ADvr} + Dv)$$

The last part is derived from multiplying the time between $\frac{I-W}{D}$ and $\frac{I}{D}$ by the cost per unit time $\sqrt{2ADvr} + Dv$.

$$\text{Maximize}_W \; \{C_{ND} - C_D\} = \; \text{Minimize}_W \; C_D$$

$$\frac{\partial C_D}{\partial W} = -g - \left(\frac{I-W}{D}\right) vr + \frac{1}{D}(\sqrt{2ADvr} + Dv) = 0$$

$$W = I - EOQ - \frac{D(v-g)}{vr} \qquad 9.13$$

(Numerical Example) (p370)

9.4.3 Options for Disposing of Excess Stock

(1) Use for other purposes.

(2) Shipment of the material to another location.

(3) Use of stock for promotional purposes.

(4) Mark-downs or special sales.

(5) Returns to suppliers at a unit value likely lower than the initial acquisition cost.

(6) Auctions.

(7) Disposal for scrap dealer.

9.5 Stocking vs. Not Stocking An Item

Should we make a special purchase from the supplier (or production run) to satisfy each customer-demand transaction or should we purchase (or produce) to stock?

9.5.1 The Relevant Factors

♣ factors that influence the decision to stock or not stock the item

(1) the system cost (file maintenance, forecasting, etc.) per unit time of stocking an item

(2) the unit variable cost of the item

(3) the cost of a temporary backorder associated with each demand when the item is not stocked

(4) the fixed setup cost associated with a replenishment in each context

(5) the cost of carrying each unit of inventory per unit time

(6) the frequency and magnitude of demand transactions

(7) the replenishment lead time

9.5.2 A Simple Decision Rule

♣ Assumptions

(i) The unit variable cost is the same under stocking and nonstocking.

(ii) The fixed setup cost is the same under stocking and nonstocking.

(iii) We allow the order quantity to be a noninteger.

(iv) The replenishment lead time is negligible.

♣ Notations

c_s = system cost, in dollars per unit time, of having the item stocked

A = fixed setup cost, in dollars

$E(i)$ = expected interval between demand transactions

$E(t)$ = expected size of a demand transaction in units

v = unit variable cost of the item, in \$/unit

r = carrying charge, in \$/\$/unit time

$$D = \frac{E(t)}{E(i)} \tag{9.21}$$

$$EOQ = \sqrt{\frac{2AE(t)}{vrE(i)}}$$

(i) total relevant costs per unit time, if the item is stocked

$$TRC_s(EOQ) = \sqrt{\frac{2AE(t)vr}{E(i)}} + c_s \qquad 9.22$$

(ii) total relevant costs per unit time, if the item is not stocked

$$TRC_{ns} = \frac{A}{E(i)} \qquad 9.23$$

♣ Do not stock the item if either of the following two conditions holds:

$$c_s > \frac{A}{E(i)} \qquad 9.15$$

$$TRC_{ns} < TRC_s \rightarrow \frac{A}{E(i)} < \sqrt{\frac{2AE(t)vr}{E(i)}} + c_s \rightarrow E(t)vr > \frac{E(i)}{2A}\left[\frac{A}{E(i)} - c_s\right]^2 \qquad 9.16$$

(Numerical Illustration) (p374)

Chapter 10

Style Goods and Perishable Items

♣ decision situations where style goods and perishable items are relevant

(1) newsvendor

(2) garment manufacturer

(3) Christmas tree vendor

(4) cafeteria manager

(5) supermarket manager

(6) administrator of a regional blood bank

(7) supplies manager in a remote region

(8) farmer

(9) toy manufacturer

10.1 The Style Goods Problem

♣ main features of the style goods problem

(1) There is a relatively short selling season with a well-defined beginning and end.

(2) Buyers or producers have to commit themselves the order quantity or production quantity prior to the start of the selling season.

(3) There may be one or more opportunities for replenishment after the initial order is placed.

(4) Forecasts prior to the season include considerable uncertainty stemming from the long period of inactivity (no sales) between seasons.

(5) When the total demand in the season exceeds the stock made available, there are associated underage costs.

(6) When the total demand in the season turns out to be less than the stock made available, overage costs result. (a special case of perishability where there is not physical deterioration but a marked deterioration in the economic value of the goods as of a particular point in time)

(7) Style goods products are often substitutable.

(8) Sales of style goods are usually influenced by promotional activities and space allocation in the store, among other things.

10.2 The Simplest Case: The Unconstrained, Single-Item, Newsvendor Problem

10.2.2 An Equivalent Result Obtained through Profit Maximization

♣ Notation

v= acquisition cost, in dollars/unit

p= selling price, in dollars/unit

B (or B_2v)= penalty for not satisfying demand, in dollars/unit

g= salvage value, in dollars/unit

X= demand (random variable)

$\hat{x}$= expected demand

σ_x= standard deviation of demand

$p_{x<}(x_0)$= cumulative distribution of total demand

$X^+ = \max\{X, 0\}$

Q=quantity to be stocked, in units (decision variable)

♣ Derivation (much simpler than that in the text)

The expected profit can be written as

$$\pi^F(Q) = pE\ \min(Q, X) + gE\ (Q - X)^+ - BE\ (X - Q)^+ - vQ$$

since $\min(Q, X)$ units are sold, $(Q-X)^+$ units are salvaged, $(X-Q)^+$ units are unsatisfied, and Q units are purchased. Noting that

$$\min(Q, X) = X - (X - Q)^+$$

and that

$$(Q - X)^+ = (Q - X) + (X - Q)^+,$$

we can write the expected profits as

$$\begin{aligned} \pi^F(Q) &= p\hat{x} - pE\ (X - Q)^+ + gQ - g\hat{x} + gE\ (X - Q)^+ - BE\ (X - Q)^+ - vQ \\ &= (p - g)\hat{x} - (v - g)Q - (p - g + B)E\ (X - Q)^+ \qquad 10.3 \end{aligned}$$

By applying Leibnitz's rule, we get the following result:

$$\frac{d\pi^F(Q)}{dQ} = -(v - g) - (p - g + B)[-p_{x>}(Q)] = -(v - g) - (p - g + B)[p_{x<}(Q) - 1] = 0$$

$$p_{x<}(Q^*) = \frac{p - v + B}{p - g + B} \qquad 10.2$$

(cf.) We can prove that $\pi^F(Q)$ is concave by showing that $\frac{d^2\pi^F(Q)}{dQ^2}$.

10.2.3 The Case of Normally Distributed Demand

$$k = \frac{Q - \hat{x}}{\sigma_x} \qquad 10.5$$

$p_{u\geq}(k)$= probability that a unit normal variable takes on a value of k or larger

$$p_{u<}(k) = \frac{p - v + B}{p - g + B} \longrightarrow 1 - p_{u\geq}(k) = \frac{p - v + B}{p - g + B} \longrightarrow p_{u\geq}(k) = \frac{v - g}{p - g + B} \qquad 10.6$$

$$Q = \hat{x} + k\sigma_x \qquad 10.7$$

(Numerical Illustration)

$p = \$50.30,\ v = \$35.10,\ g = \$25.00,\ B = \$9.70,\ X \sim N(900, 122^2)$

$$p_{u\geq}(k) = \frac{v-g}{p-g+B} = \frac{35.10-25}{50.30-25+9.70} = 0.288 \longrightarrow k = 0.56 \ \ \text{(p724)}$$

$$Q = \hat{x} + k\sigma_x = 900 + 0.56 \times 122 = 968.3 \simeq 968$$

10.2.4 The Case of a Fixed Charge to Place the Order

Let $I \geq 0$ denote the initial inventory and suppose a fixed cost, say A, is charged for placing an order. Let $S = I + Q$, then the expected profit can be written as

$$\pi^F(S) = -A\mathbf{1}_{\{S>I\}} + (p-g)\hat{x} + (v-g)I - (v-g)S - (p-g+B)E\ (X - S + I)^+$$

where $\mathbf{1}$ denotes the indicator function.

The problem reduces to

$$\min_{S\geq I}\ \left[A\mathbf{1}_{\{S>I\}} + K(S)\right]$$

where

$$K(S) = -(p-g)\hat{x} - (v-g)I + (v-g)S + (p-g+B)E\ (X - S + I)^+$$

Let S^* denote the unconstrained minimizer of $K(S)$. From the result of the previous section we know that (for normal distribution case)

$$S^* = \hat{x} + k\sigma_x$$

$$\text{where } \ p_{u\geq}(k) = \frac{v-g}{p-g+B} \qquad 10.6$$

Clearly an order should be placed if $I < S^*$ and $K(I) > A + K(S^*)$. Since $K(S)$ is strictly convex and is not bounded from above, there exists a unique $s^* < S^*$ satisfying

$$K(s^*) = A + K(S^*).$$

The ordering rule is: Order up to S^* ($Q^* = S^* - I$) units if $I < s^*$ and do not order otherwise.

10.2.5 Distribution Free Newsboy Problem

$$C^F(Q) = -(p-g)\hat{x} + (v-g)Q + (p-g+B)E\ (X-Q)^+$$

$$\min_{Q\geq 0} \max_{F\in\mathcal{F}} C^F(Q) \qquad (*)$$

To this end, we need the following Lemma.

Lemma 1.

$$\max_{F\in\mathcal{F}} E\ (X-Q)^+ = \frac{\sqrt{\sigma^2 + (Q-\hat{x})^2} - (Q-\hat{x})}{2}$$

Using Lemma 1, our problem (*) reduces to

$$\min_{Q\geq 0} C^W(Q) = -(p-g)\hat{x} + (v-g)Q + (p-g+B)\left[\frac{\sqrt{\sigma^2 + (Q-\hat{x})^2} - (Q-\hat{x})}{2}\right]$$

$$\frac{dC^W(Q)}{dQ} = (v-g) + \frac{p-g+B}{2}\left[\frac{Q-\hat{x}}{\sqrt{\sigma^2+(Q-\hat{x})^2}} - 1\right] = 0$$

Please try to derive equation 10.11 from the above equation (It seems not easy)!

$$Q^* = \hat{x} + \frac{\sigma}{2}\left(\sqrt{\frac{p-v+B}{v-g}} - \sqrt{\frac{v-g}{p-v+B}}\right) \qquad 10.11$$

(cf.) Equation 10.11 in the text does not contain B term.

(Numerical Illustration (continued))

$$Q^* = 900 + \frac{122}{2} \times \left(\sqrt{\frac{50.3-35.1+9.7}{35.1-25}} - \sqrt{\frac{35.1-25}{50.3-35.1+9.7}}\right) = 962.88 \simeq 963$$

(cf.) 963 (not 925 in the text) is the true distribution-free solution, and the percent of best possible profit is larger than 99.2% reported in Table 10.2 on p390.

Example 1. This example is taken from Silver and Peterson. The unit cost is \$35.10, the unit selling price is \$50.30, and the unit salvage value is \$25.00. The mean and standard deviation of the demand are 900 and 122, respectively (Note that we assume B=0).

We compare the result for the worst case distribution against the normal distribution. Let the optimal order quantity for the normal distribution be Q^N where $N \in \mathcal{F}$ represents the normal distribution. Using equation 10.6, we get $Q^N \simeq 931$ units.

The maximum expected profit can be computed as follows:

$$\pi^N(Q^N) = \$12,488.15$$

We also get $Q^W \simeq 925$ units from equation 10.11 and a worst case expected profit of \$12,168.38.

Now we compute the penalty of using Scarf's ordering rule even though the actual distribution is normal:

$$\pi^N(Q^N) - \pi^N(Q^W) = \$12,488.15 - \$12,486.70 = \$1.45$$

This is the largest amount that we would be willing to pay for the knowledge of the distribution of demand (normal distribution in this example). This quantity can be regarded as the *value of the distributional information.*

Example 2. The unit cost is \$40, the unit selling price is \$60, and the unit salvage value is \$5.00. The mean and standard deviation of the demand are 300 and 200, respectively.

Again, we compare the result for the worst case distribution against the normal distribution. We get $Q^N \simeq 230$ units, and the maximum expected profit \$1,870.56. We also get $Q^W \simeq 243$ units, and a worst case expected profit of \$708.50.

The *value of the distributional information* is

$$\pi^N(Q^N) - \pi^N(Q^W) = \$1,870.56 - \$1,862.06 = \$8.50$$

10.3 The Single-Period, Constrained, Multi-Item Situation

♣ Examples

(i) several different newspapers sharing a limited space or budget

(ii) a buyer for style goods department of a retail outlet who has a budget limitation

(iii) provisioning of supplies or spare parts on a spacecraft or a submarine

(iv) repair kit taken by a maintenance crew on a routine visit to an operating facility

Suppose that the cost of purchasing all the items cannot exceed a predetermined budget of W dollars. We want to find the order quantities that maximize the expected profit without exceeding the budget constraint. The problem can be formulated as follows:

$$\min_{Q_1,\ldots,Q_n} \sum_{i=1}^{n} \left[(p_i - g_i)\hat{x}_i - (v_i - g_i)Q_i - (p_i - g_i + B_i)E\,(X_i - Q_i)^+\right]$$

$$\text{subject to } \sum_{i=1}^{N} v_i Q_i \leq W \qquad 10.24$$

Dualizing the budget constraint and letting λ denote the dual variable (Lagrangian multiplier) we see that the solution is of the form:

$$p_{x<}(Q_i^*) = \frac{p_i - (\lambda + 1)v_i + B_i}{p_i - g_i + B_i}$$

The problem is to find the smallest nonnegative λ such that $Q_i(\lambda)$ satisfies 10.24. The following algorithm is essentially a line search to find the optimal value of λ.

Algorithm

(Step 1) Check if $Q_i(0)$ ($\lambda = 0$) satisfies the budget constraint 10.24. If it satisfies the constraint, the solution is optimal, stop. Else go to Step 2.

(Step 2) Start from an arbitrary $\lambda > 0$, set $\epsilon > 0$.

(Step 3) Determine each Q_i from

$$p_{x<}(Q_i^*) = \frac{p_i - (\lambda + 1)v_i + B_i}{p_i - g_i + B_i}$$

(Step 4) If $\sum_{i=1}^{n} v_i Q_i < W - \epsilon$, decrease λ and go to Step 3.

If $\sum_{i=1}^{n} v_i Q_i > W + \epsilon$, increase λ and go to Step 3.

If $-\epsilon \leq \sum_{i=1}^{n} v_i Q_i - W \leq \epsilon$, stop.

(Numerical Illustration) (p395)

10.4 More Than One Period In Which To Prepare For The Selling Season

There may be an extended length of time in which replenishment commitments are made before the actual selling season begins. There are likely to be production constraints on the total amounts that can be acquired in each time period. Furthermore, forecasts of total demand are almost certain to change during these preseason periods.

10.5 The Multiperiod Newsvendor Problem

♣ myopic policy: one that selects the production quantities for the current period to minimize expected costs in the current period alone. Ignall and Veinott (1969) have developed conditions under which a myopic policy is optimal for the sequence of periods. → multiperiod newsvendor problem does not satisfy the conditions for which a myopic policy is optimal.

10.6 Other Issues Relevant to the Control of Style Goods

10.6.1 Updating of Forecasts

(1) Exploitation of the properties of the forecasts made by decision makers. (When the committee members independently forecasted demand, the average of the forecasts tended to be much more accurate.)

(2) Taking advantage of the observation that sales at the retail level tend to be proportional to inventory displayed. (cf. balking phenomenon)

(3) Simple extrapolation methods using a particular mathematical form for the cumulative sales as a function of time.

$$Y_{ult} = lnY_t + ab^{-bt} \qquad 10.15$$

Y_{ult}= the ultimate (total) sales of the item

Y_t= cumulative sales as of time t

(4) Bayesian procedures (As demands are observed in the early part of the season, the probability distributions are appropriately modified to take account of the additional information)

(5) Use of information about patterns of past product demand in conjunction with estimates of the total life cycle sales of the current product.

10.6.2 Reorders and Markdowns

♣ Wolfe's (1968) Method (assumption: sales are proportional to the inventory level)

(1) The expected time T_F to sell a fraction F of the initial inventory if a fraction f has been sold by time t ($f < F$).

(2) The associated order-up-to level if a reorder is to be placed at a specific time.

(3) The timing of a markdown as a function of the fraction of initial inventory sold to date, the current price, the markdown price, the salvage value of leftover material, and the assumed known ratio of the sales rate after the markdown to that before it.

Feng and Gallego (1995) → They determine when to lower or raise the price as a function of the time until the end of the season, if there is but one chance to change the price → yield management (eg. airline tickets, hotel rooms, etc.)

Khouza (1995) → Multiple markdowns provide higher expected profit than a single markdown

10.6.3 Reserving Capacity Ahead of Time

To ensure that enough production capacity is available to meet peak season demand, the firm agrees to buy a certain number of products from a supplier over the year.

Jain and Silver (1995) → capacity reservation problem for an item with uncertainty in its demand and uncertainty in the capacity of the supplier

10.6.4 Operations Reversal

By changing the order of production, in conjunction with extensive use of point-of-sale information, Benetton was able to dramatically reduce lead times and costs. Prior to the change, Benetton dyed yarn, then knitted the dyed yarns into garments. Their revised system switched the order from dye then knit, to knit then dye.

10.6.5 Inventory Policies for Common Components

In a manufacturing firm that can assemble finished products to customer order, it is often desirable to hold safety stock of components, rather than of end-items. In addition, if there is significant commonality among components, total safety stocks may be reduced by holding components and then assembling them to order.

Eynan and Rosenblatt (1995) & Moon and Choi (1997) → Make-In-Advance strategy vs. Make-To-Order Strategy

10.7 Inventory Control of Perishable Items

Perishability refers to the physical deterioration of units of a product. Perishable items can be divided into two categories, fixed or random, depending on the lifetime of a unit of the item.

See Nahmias (1982) for excellent review.

Chapter 11

Coordinated Replenishments at a Single Stocking Point

11.1 Advantages and Disadvantages of Coordination

♣ Advantages of Coordination

1. Savings on unit purchase costs.
2. Savings on unit transportation costs.
3. Savings on ordering costs.
4. Ease of scheduling.

♣ Disadvantages of Coordination

1. A possible increase in the average inventory level.
2. An increase in system control costs.
3. Reduced flexibility.

11.2 The Deterministic Case: Selection of Replenishment Quantities in a Family of Items

11.2.1 Assumptions

(i) The demand rate of each items is constant and deterministic.

(ii) The replenishment quantity of an item need not be an integer.

(iii) The unit variable cost of any of the items does not depend on the quantity.

(iv) The replenishment lead time is of zero duration.

(v) No shortages are allowed.

(vi) The entire order quantity is delivered at the same time.

11.2.2 The Decision Rule

♣ Notation

A= major setup cost for the family, in dollars

a_i= minor setup cost for item i, in dollars

D_i = demand rate of item i, in units/unit time

v_i= unit variable cost of item i, in \$/unit

n = number of items in the family

m_i= the integer number of T intervals that the replenishment quantity of item i will last (decision variable)

T= time interval between replenishments of the family (decision variable)

♣ Derivation

$$Q_i = D_i m_i T \tag{11.18}$$

$$TRC(T, m_1, \cdots, m_n) = \frac{A + \sum_{i=1}^{n} \frac{a_i}{m_i}}{T} + \sum_{i=1}^{n} \frac{D_i m_i T v_i r}{2} \tag{11.19}$$

$$\frac{\partial TRC}{\partial T} = -\frac{A + \sum_{i=1}^{n} \frac{a_i}{m_i}}{T^2} + \sum_{i=1}^{n} \frac{D_i m_i v_i r}{2} = 0$$

$$T^*(m_1, \cdots, m_n) = \sqrt{\frac{2(A + \sum_{i=1}^{n} \frac{a_i}{m_i})}{r \sum_{i=1}^{n} m_i D_i v_i}} \qquad 11.20$$

$$TRC^*(m_1, \cdots, m_n) = \sqrt{2\left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right) r \sum_{i=1}^{n} m_i D_i v_i} \qquad 11.21$$

$$\begin{aligned} & Minimize_{m_1,\cdots,m_n}\ TRC^*(m_1, \cdots, m_n) \\ \equiv\ & Minimize_{m_1,\cdots,m_n}\ F(m_1, \cdots, m_n) = \left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right) r \sum_{i=1}^{n} m_i D_i v_i \end{aligned} \qquad 11.22$$

If we ignore the integrality of m_i's, then

$$\frac{\partial F(m_1, \cdots, m_n)}{\partial m_j} = -\frac{a_j}{m_j^2} \sum_{i=1}^{n} m_i D_i v_i + D_j v_j \left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right) = 0$$

$$m_j^2 = \frac{a_j \sum_{i=1}^{n} m_i D_i v_i}{D_j v_j \left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right)} \quad \forall j \qquad 11.23$$

$$\frac{m_j^2}{m_k^2} = \frac{a_j}{D_j v_j} \frac{D_k v_k}{a_k} \longrightarrow \frac{m_j}{m_k} = \sqrt{\frac{a_j}{D_j v_j} \frac{D_k v_k}{a_k}} \quad j \neq k$$

$$\frac{a_j}{D_j v_j} < \frac{a_k}{D_k v_k} \longleftrightarrow m_j < m_k$$

The item i having the smallest value of $a_i/D_i v_i$ should have the lowest value of m_i, namely, 1. Without loss of generality, we number the items such that item 1 has the smallest value of $a_i/D_i v_i$.

$$m_1 = 1 \qquad 11.24$$

$$\text{Equation 11.23} \rightarrow m_j = \sqrt{\frac{a_j}{D_j v_j}} \sqrt{\frac{\sum_{i=1}^{n} m_i D_i v_i}{\left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right)}} \quad j = 2, 3 \cdots, n \qquad 11.25$$

$$\sqrt{\frac{\sum_{i=1}^{n} m_i D_i v_i}{\left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right)}} = C \qquad 11.26$$

$$\text{Equation 11.25} \rightarrow m_j = C\sqrt{\frac{a_j}{D_j v_j}} \quad j = 2, 3 \cdots, n \qquad 11.27$$

$$\sum_{i=1}^{n} m_i D_i v_i = D_1 v_1 + \sum_{i=2}^{n} C\sqrt{\frac{a_i}{D_i v_i}} D_i v_i = D_1 v_1 + C \sum_{i=2}^{n} \sqrt{a_i D_i v_i} \qquad 11.28$$

$$\sum_{i=1}^{n} \frac{a_i}{m_i} = a_1 + \frac{1}{C} \sum_{i=2}^{n} \sqrt{a_i D_i v_i} \qquad 11.29$$

Substituting equations 11.28 & 11.29 into equation 11.26, we get

$$\frac{D_1 v_1 + C \sum_{i=2}^{n} \sqrt{a_i D_i v_i}}{A + a_1 + \frac{1}{C} \sum_{i=2}^{n} \sqrt{a_i D_i v_i}} = C^2$$

$$C = \sqrt{\frac{D_1 v_1}{A + a_1}}$$

$$m_j = \sqrt{\frac{a_j}{D_j v_j} \frac{D_1 v_1}{A + a_1}} \quad j = 2, 3 \cdots, n \qquad 11.30$$

Algorithm

(Step 1) Number the items such that $a_i / D_i v_i$ is smallest for item 1. Set m_1=1.

(Step 2) Evaluate

$$m_i = \sqrt{\frac{a_i}{D_i v_i} \frac{D_1 v_1}{A + a_1}} \qquad 11.3$$

rounded to the nearest integer.

(Step 3) Evaluate T^* using the m_i's of Step 2.

$$T^*(m_1, \cdots, m_n) = \sqrt{\frac{2(A + \sum_{i=1}^{n} \frac{a_i}{m_i})}{r \sum_{i=1}^{n} m_i D_i v_i}} \qquad 11.2$$

(Step 4) Determine

$$Q_i = D_i m_i T^* \quad \forall i \qquad 11.4$$

(Numerical Illustration) (p428)

11.2.3 A Bound on the Cost Penalty of the Heuristic Solution

In order to find a lower bound on the minimum cost, we use $m_1 = 1$ and m_j's $(j = 2, \cdots, n)$ in equation 11.30 which are not necessarily integer values.

$$TRC_{LB} = \sqrt{2(A + a_1)D_1 v_1 r} + \sum_{j=2}^{n} \sqrt{2a_j D_j v_j r} \qquad 11.5$$

The cost of heuristic is

$$TRC_H = \frac{A + \sum_{i=1}^{n} \frac{a_i}{m_i}}{T} + \sum_{i=1}^{n} \frac{D_i m_i T v_i r}{2} \qquad 11.6$$

(Example) TRC_{LB}=\$2,054.14/year vs. TRC_H=\$2,067.65/year $\rightarrow$ ratio=1.007

11.4 The Case of Probabilistic Demand and No Quantity Discounts

11.4.1 (S, c, s) or Can-Order System

• A special type of continuous review system for controlling coordinated items $\leftarrow$ savings in setup costs are of primary concern

• Whenever item i's inventory position drops to or below s_i (must-order point), it triggers a replenishment action that raises item i's level to its order-up-to level S_i. At the same time any other item j (within the associated family) with its inventory position at or below its can-order point c_j is included in the replenishment.

Figure 11.2 Behavior of an Item under (S, c, s) Control

11.6 The Production Environment

11.6.1 The Case of Constant Demand and Capacity: The Economic Lot Scheduling Problem (ELSP)

• Find a cycle length, a production sequence, production times, and idle times, so that the production sequence can be completed in the chosen cycle, the cycle can be repeated over time, demand can be fully met, and annual inventory and setup costs can be minimized.

• NP-hard problem → the need to satisfy a production capacity constraint and the need to have only one product in production at a time (synchronization constraint)

♣ Notation

T= common order interval, or time supply, for each product, in units of time

p_i = production rate of item i, in units/unit time

A_i= setup cost for item i, in dollars

K_i= setup time for item i, in dollars

D_i = demand rate of item i, in units/unit time

v_i= unit variable cost of item i, in \$/unit

n = number of items in the family

$$\text{Minimize} \quad \sum_{i=1}^{n} TRC_i(T) = \left[\frac{A_i}{T} + \frac{rv_iD_i(p_i - D_i)T}{2p_i}\right]$$

$$\text{subject to} \quad \sum_{i=1}^{n} \left(K_i + \frac{D_iT}{p_i}\right) \leq T \qquad 11.12$$

$$T^* = Max\left(\sqrt{\frac{\sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} \frac{rv_iD_i(p_i-D_i)}{2p_i}}}, \frac{\sum_{i=1}^{n} K_i}{1 - \sum_{i=1}^{n} \frac{D_i}{p_i}}\right)$$

(Numerical Illustration) (p446)

♣ Relevant Literature

Hahm and Yano (1992, 1995a, 1995b): ELDSP (ELSP+ delivery schedule)

Gallego and Moon (1992): In the realm of changing the givens, they examine a multiple product factory that employs a cyclic schedule to minimize holding and setup costs. When setup times can be reduced, at the expense of setup costs, by externalizing setup operations, they show that dramatic savings are possible for highly utilized facilities.

See also Moon, Gallego, and Simchi-Levi (1991), Hwang, Kim, and Kim (1993), Gallego (1993), Gallego and Moon (1995), and Moon (1994).

11.7 Shipping Consolidation

♣ Shipment Consolidation Decisions (Higginson and Bookbinder (1994))

(i) Which orders will be consolidated and which will be shipped individually?

(ii) When will orders be released for possible shipping? Immediately, or after some time or quantity-trigger?

(iii) Where will the consolidation take place? At the factory or at an off-site warehouse or terminal?

(iv) Who will consolidate? The manufacturer, customer, or a third party?

♣ Three Possible Policies (Higginson and Bookbinder (1994))

(i) a time policy that ships at a prespecified time

(ii) a quantity policy that ships when a given quantity is achieved

(iii) a time/quantity policy that ships at the earliest of the time and quantity values

• The shipper must trade off cost per unit with customer service in deciding on which policy to use.

Chapter 12

Supply Chain Management and Multiechelon Inventories

12.1 Supply Chain Management (SCM)

Supply Chain Management (SCM): management of materials and information across the entire supply chain, from suppliers to component producers to final assemblers to distribution (warehouse and retailers), and ultimately to the consumer. In fact, it often includes after-sales service, and returns or recycling.

Figure 12.1 A Schematic of a Supply Chain

(cf.) recent boom on the SCS $\leftarrow$ the realization that actions taken by one member of the chain can influence the profitability of all other in the chain

(cf.) Efficient Consumer Response (ECR) $\equiv$ Just-In-Time Distribution $\equiv$ Continuous Replenishment $\rightarrow$ an aspect of SCM

♣ Bullwhip Effect

(i) the variability increases in moving up the supply chain from consumer to grocery store to distribution center to central warehouse to factory

(ii) (Example) In the Italian pasta industry, demand is quite steady throughout the year. However, because of trade promotions, volume discounts, long lead times, full-truckload discounts, and end-of-quarter sales incentives the orders seen at the manufacturers are highly variable.

(iii) The costs of this variability are high- insufficient use of production and warehouse resources, high transportation costs, and high inventory costs, etc.

(iv) (Example) Acer America, Inc. sacrificed \$20 million in profits by paying \$10 million for air freight to keep up with surging demand, and then paying \$10 million more later when that inventory became obsolete.

(v) One of the main causes is that retailers and distributors often overact to shortages by ordering more than they need.

Figure 12.2 An Illustration of the Bullwhip Effect

♣ Four rational factors that create the bullwhip effect

(i) demand signal processing (if demand increases, firms order more in anticipation of further increases, thereby communicating an artificially high level of demand)

(ii) rationing game (there is, or might be, a shortage so a firm orders more than the actual forecast in the hope of receiving a larger share of the items in short supply)

(iii) order batching (fixed costs at one location lead to batching of orders)

(iv) manufacturer price variations (which encourage bulk orders)

(cf.) Some recent innovations, such as increased communication about consumer demand via electronic data interchange (EDI) and everyday lower pricing (EDLP) (to eliminate forward buying of bulk orders), can mitigate the bullwhip effect.

(cf.) Some firms are even considering how the product will be handled after its useful life ends and are designing the product accordingly. → Design for Recycling or Design for Disassembly → Dr. Mok's main research area → closely related to inventory management

(cf.) this chapter → supply chain management from the perspective of inventory management → models of multiechelon inventory systems that can be used to optimize the deployment of inventory in a supply chain and to evaluate a change in the supply chain 'givens' → the difficulty comes from the dependent demand situation

Figure 12.3 A Multiechelon Inventory Situation

12.2 Structure and Coordination

♣ Structural Decisions → a network of facilities designed to produce and distribute the products under consideration

- Where to locate factories, warehouses, and retail sites?
- How many of these facilities to have?
- What capacity should each of these facilities have?
- When, and by how much, should capacity be expanded or contracted?
- Which facilities should produce and distribute which products?
- What modes of transportation should be used for which products, and under which circumstances?

♣ Coordination Decisions (take the structure of the multiechelon network as given and focus on the short-term)

- Should inventory stocking and replenishment decisions be made centrally or in a decentralized fashion?
- Should inventory be held at central warehouses or should these simply be used as break-bulk facilities?
- Where should inventory be deployed? In other words, should most inventory be held at a central location, or should it be pushed "forward" to the retail level?
- How should a limited and insufficient amount of stock be allocated to different locations that need it?

12.3 Deterministic Demand

12.3.1 Sequential Stocking Points with Level Demand

• the simplest of multiechelon situations where the stocking points are serially connected. (eg.) one central warehouse, one retailer warehouse, and one retail outlet.

Figure 12.4 A Serial Production Process

♣ Notation

D= deterministic, constant demand rate at the retailer, in units/unit time

A_W= fixed (setup) cost associated with a replenishment at the warehouse, in dollars

A_R= fixed (setup) cost associated with a replenishment at the retailer, in dollars

v_W= unit variable cost or value of the item at the warehouse, in \$/unit

v_R= unit variable cost or value of the item at the retailer, in \$/unit

r= carrying charge, in\$/\$/unit time

Q_W = replenishment quantity at the warehouse, in units (decision variable)

Q_R = replenishment quantity at the retailer, in units (decision variable)

♣ Derivation

$$Q_W = nQ_R \qquad n = 1, 2, 3, \cdots \tag{12.1}$$

Figure 12.5 Behavior of the Inventory Levels in a Deterministic Two-Stage process

(cf.) The inventory at the warehouse does not follow the usual sawtooth pattern $\leftarrow$ withdrawals from the warehouse inventory are of size Q_R which occurs intermittently

♣ Echelon Stock Concept by Clark and Scarf (1960)

• echelon stock of echelon j: the number of units in the system that are at, or have passed through, echelon j but have as yet not been specifically committed to outside customers $\rightarrow$ each echelon stock has a sawtooth pattern with time $\rightarrow$ the same physical units of stock can appear in more than one echelon inventory $\rightarrow$ value any specific echelon inventory at only the value added at that particular echelon

$$v'_W = v_W \quad v'_R = v_R - v_W$$

$$v'_i = v_i - \sum_{j \in P} v_j \tag{12.2}$$

where P ={all immediate predecessors of i}

$$TRC(Q_W, Q_R) = \frac{A_W D}{Q_W} + \overline{I}'_W v'_W r + \frac{A_R D}{Q_R} + \overline{I}'_R v'_R r \qquad 12.3$$

where

$\overline{I}'_W$ = average value of the warehouse echelon inventory, in units

$\overline{I}'_R$ = average value of the retailer echelon inventory, in units

Substituting equation 12.1 into equation 12.3, we get

$$TRC(n, Q_R) = \frac{D}{Q_R}\left(A_R + \frac{A_W}{n}\right) + \frac{Q_R r}{2}\left(nv'_W + v'_R\right)$$

$$\frac{\partial TRC}{\partial Q_R} = -\frac{D}{Q_R^2}\left(A_R + \frac{A_W}{n}\right) + \frac{r}{2}\left(nv'_W + v'_R\right) = 0$$

$$Q_R^*(n) = \sqrt{\frac{2\left(A_R + \frac{A_W}{n}\right)D}{(nv'_W + v'_R)r}} \qquad 12.39$$

Substituting equation 12.39 into equation 12.38, we obtain the lowest cost possible for the given value of n.

$$TRC^*(n) = \sqrt{2\left(A_R + \frac{A_W}{n}\right)D(nv'_W + v'_R)r}$$

Minimize $TRC^*(n) \equiv$ Minimize $F(n)$ where

$$F(n) = \left[A_R + \frac{A_W}{n}\right](nv'_W + v'_R) \qquad 12.40$$

$$\frac{dF(n)}{dn} = (nv'_W + v'_R)\left(-\frac{A_W}{n^2}\right) + \left[A_R + \frac{A_W}{n}\right]v'_W = 0 \longrightarrow n^* = \sqrt{\frac{A_W v'_R}{A_R v'_W}} \qquad 12.41$$

♣ Decision Rule

(Step 1) Compute

$$n^* = \sqrt{\frac{A_W v'_R}{A_R v'_W}} \qquad 12.5$$

If n^* is exactly an integer, go to (Step 4) with $n = n^*$. Also, if $n^* < 1$, go to (Step 4) with n=1. Otherwise, proceed to (Step 2).

(Step 2) Ascertain the two integer values, n_1 and n_2 that surround n^*.

(Step 3) Evaluate

$$F(n_1) = \left[A_R + \frac{A_W}{n_1}\right](n_1 v'_W + v'_R)$$

$$F(n_2) = \left[A_R + \frac{A_W}{n_2}\right](n_2 v'_W + v'_R) \qquad 12.6$$

If $F(n_1) \leq F(n_2)$, use $n = n_1$.

If $F(n_1) > F(n_2)$, use $n = n_2$.

(Step 4) Evaluate

$$Q_R = \sqrt{\frac{2\left(A_R + \frac{A_W}{n}\right)D}{(nv'_W + v'_R)r}} \qquad 12.7$$

(Step 5) Calculate

$$Q_w = nQ_R$$

(Numerical Illustration) (p481)

12.3.2 Other Results for the Case of Level Demand

♣ Pure Assembly System (Schwarz and Schrage (1975))

• each node feeds into, at most, one other node

• A myopic strategy where each node and its successor are treated in isolation by much the same procedure as for the two-stage serial case

Figure 12.7 A "Pure" Assembly System

♣ General System (Maxwell and Muckstadt (1985))

(i) nested policy

(ii) stationary policy

(iii) base planning period

(iv) powers-of-two restriction

12.4 Probabilistic Demand

Figure 12.8 A Multiechelon Situation with Single-Stage Information Flow

♣ Each stocking level would independently make replenishments decision based on its own

(i) cost factors and service considerations

(ii) predicted demand from the next stocking point downstream

(iii) replenishment lead time from the next stocking point upstream

♣ Three serious flaws of the above decisions

(i) The lead time observed at, say, the retailer is dependent on whether the branch warehouse has sufficient stock to fill the order.

(ii) It ignores the cost implications at one echelon of using certain ordering logic at another level.

(iii) Even if end-customer demand is fairly smooth, the orders placed farther up the line become progressively larger and less frequent.

♣ Other complicating factors

(i) How do we define service in a multiechelon situation?

(ii) Is a partial shipment made, or does the system wait until the entire order can be shipped?

(iii) What about the possibility of an emergency shipment directly from the central warehouse to the retailer?

(iv) In more complicated multiechelon structures, transshipments between points at the same echelon may be possible.

(v) The central facility is likely to adopt a rationing policy when it faces multiple requests with insufficient stock to meet them all.

Table 12.1 Information and Control

12.4.1 The Base Stock Control System

Make end-item demand information available for decision making at all stocking points $\leftarrow$ EDI

Each stocking point makes replenishments based on actual end-item customer demands rather than on replenishments orders from the next level downstream.

♣ (s, S) system

(i) Q is established independently using end-item demand forecasts

(ii) reorder point s is established by one of the procedures of Chapter 7, using end-item demand forecasts over the replenishment lead time appropriate to the echelon under consideration

$$S \text{ (order-up-to level or base stock level) } = s + Q \qquad 12.13$$

$$\text{echelon inventory position} = \text{echelon stock} + \text{on order} \qquad 12.14$$

(Example) physical stocks at branch warehouse=50 units, physical stocks at retail outlet = 20 units, unsatisfied demand = 5 units, amount in transit between the branch and the retail outlet = 10 units, no outstanding order

$$\text{inventory position} = (50+10+20\text{-}5) + (0) = 75 \text{ units}$$

Whenever the echelon inventory position is at or lower than s, enough is ordered from the preceding echelon to raise the position to the base stock level S.

12.4.2 The Serial Situation

♣ Assumptions

(i) External demand occurs only at the retailer and is a stationary process.

(ii) There is a deterministic replenishment lead time associated with each stage.

(iii) (s, Q) policy

(iv) Q_R and $Q_W = nQ_R$ have been predetermined by the procedure of Section 12.3.1

♣ Decision Rules

1. Select s_R

$$s_R = \hat{x}_{L_R} + k_R \sigma_{L_R} \quad 12.15$$

$$p_{\mu\geq}(k_R) = \frac{Q_R(v_R - v_W)r}{B_2 v_R D} \quad 12.16$$

2. Select s_W

$$s_W = \hat{x}_{L_W+L_R} + k_W \sigma_{L_W+L_R} \quad 12.17$$

$$p_{\mu\geq}(k_W) = \frac{Q_R[(v_R + (n-1)v_W]r}{B_2 v_R D} \quad 12.18$$

From equations 12.16 and 12.18, we know that

$$k_R \geq k_W$$

It means that the safety factor at the retailer is always larger than the safety factor at the warehouse level.

(Numerical Illustration) (p.493)

12.5 Remanufacturing and Product Recovery

consumable items vs. repairable or recoverable items

(Examples) vehicles, telephones, military equipment, computers, copying machines, glass bottles, etc.

♣ product recovery: handling of all used and discarded products, components, and materials.

This emerging area of research and practice is generating much interest, particularly due to new and proposed laws that assign responsibility to manufacturers for the ultimate disposal of their products. Because the issues are varied and complex, the field is quite broad, ranging from studies of the logistics of reusable containers to the process of designing products for disassembly.

Figure 12.11 Product Recovery Options

Today, firms are beginning to consider design for the environment (DFE) and design for disassembly (DFD).

♣ Five decision variables at a single location

(i) How often to review the stock status?

(ii) When to recover returned units?

(iii) How many to recover at a time?

(iv) When to order new units?

(v) How many to order?

12.6 Additional Insights

12.6.1 Economic Incentives to Centralize Stocks

♣ Assumptions

(i) n retail outlets

(ii) (s_i, EOQ_i) policy at each retailer

(iii) Use the same safety factor k at each retailer

(iv) Independent demand across retailers

(v) $A_i = A$ for all i

Let demand at retailer i be X, and assume that the mean is D_i and the variance is σ_i^2.

$$\begin{aligned} ETRC(decentralized) &= \sum_{i=1}^{n} ETRC_i = \sum_{i=1}^{n}[\sqrt{2A_iD_ivr} + k\sigma_i vr] \\ &= \sqrt{2Avr}\sum_{i=1}^{n}\sqrt{D_i} + kvr\sum_{i=1}^{n}\sigma_i \qquad 12.34 \end{aligned}$$

If the stocking was done at a single centralized location, then

$$\sum_{i=1}^{n} X_i \sim (D_c = \sum_{i=1}^{n} D_i, \sigma_c^2 = \sum_{i=1}^{n}\sigma_i^2)$$

$$ETRC(centralized) = \sqrt{2AD_c vr} + kvr\sigma_c \qquad 12.35$$

$$\clubsuit \quad ETRC(centralized) \le ETRC(decentralized)$$

(proof) $\left(\sum_{i=1}^{n}\sqrt{D_i}\right)^2 - \left(\sqrt{D_c}\right)^2 = \left(\sum_{i=1}^{n} D_i + 2\sum_{i=1}^{n}\sum_{j\neq i}\sqrt{D_i D_j}\right) - \sum_{i=1}^{n} D_i = 2\sum_{i=1}^{n}\sum_{j\neq i}\sqrt{D_i D_j} \ge 0$

$$\left(\sum_{i=1}^{n}\sigma_i\right)^2 - \sum_{i=1}^{n}\sigma_i^2 = 2\sum_{i=1}^{n}\sum_{j\neq i}\sigma_i\sigma_j \ge 0$$

(cf.) portfolio effect

12.6.2 Where to Deploy Stock

The general question is whether the warehouse should hold back substantial inventories so that they can be allocated to retailer demands as needed, or whether most inventory should be pushed forward to the retailers.

Firms in higher demand often ship in full truckloads, and the clear choice for deployment of inventory is to push at least some of it to the retail level. The reason is that if the product does not sell today, it will almost surely sell tomorrow, and the savings from shipping full truckloads outweigh any small inventory savings that could be gained by holding back inventory at the warehouse level. On the other hand, some stocks, say one week's demand, should be held back to account for emergency requirements. The exact amount to hold back will depend on demand rates, transportation costs, lead times, holding costs, variability of demand, and the service objective.

12.6.3 Lateral Transshipments

The most common assumption in multiechelon research is that shipments among retailers are not allowed.

Karmarkar and Patel (1977) have shown that costs can decrease, and service can improve, if lateral transshipments are used in emergencies. If, on the other hand, transshipments are used in anticipation of stock imbalances among retailers, costs can go up due to excessive unnecessary movement of product.

Inventory Theory: Supplement

Dependent Demand Inventory

Learning Objectives

1. Contrast dependent and independent demand, and trace the development of material requirements planning (MRP).
2. Explain the inputs to an MRP system.
3. Compute single-level MRP records.
4. Compute multiple-level MRP records and explain the outputs generated.
5. Describe the evolution of MRP to enterprise resource planning (ERP) and identify ways in which ERP is utilized to integrate all the functions of an organization.
6. Explain how dependent demand is handled in service organizations and describe the use of technology.
7. Define three critical features for success with ERP.

Dependent Demand at Kellogg’s

- Kellogg’s employs dependent demand planning techniques, including material requirements planning.
- Every 2 months, a plan is developed for all production items in a given group of plants.
- For the morning foods division, Kellogg’s develops a plan for three plants that produce Pop-Tarts.
- The plan calls for 61,500 boxes of Hot Fudge Sundae Pop-Tarts and 54,000 boxes of Strawberry Pop-Tarts, along with other varieties during one week.

History of Dependent Demand Planning

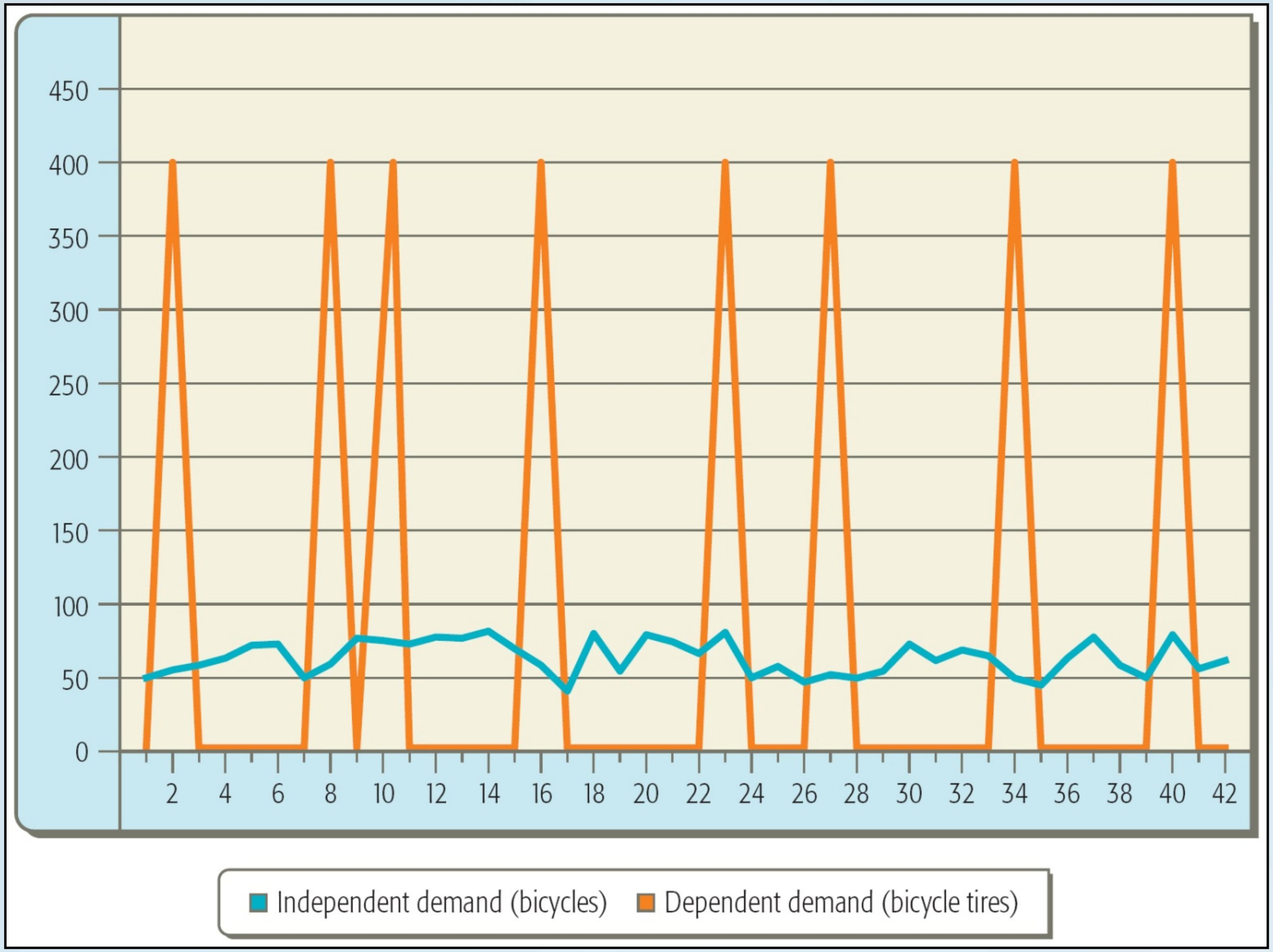
- **Independent demand**: demand for items that are considered end items that go directly to a customer, and for which demand is influenced by market conditions and not related to inventory decisions for any other item
- **Dependent demand**: demand for items that are used to make another item or are considered to be parts of another item

Material Requirements Planning (MRP)

- **MRP**: a computer-based system that develops plans for ordering and producing dependent demand items.

- MRP utilizes two basic principles:
  1. Requirements for dependent demand items are *derived* from the production schedule for their parents (the items that are assembled from component parts).
  2. The production order is *offset* to account for the lead time.

Figure 7.3: Demand Pattern for Independent versus Dependent Items



Material Requirements Planning (MRP)

- MRP is a technique that has been employed since the 1940s and 1950s.
- Joe Orlicky is known as the Father of MRP
- The use and application of MRP grew through the 1970s and 1980s as the power of computer hardware and software increased.
- MRP gradually evolved into a broader system called manufacturing resource planning (MRP II).

Source: © Image Source/Corbis

Material Requirements Plan

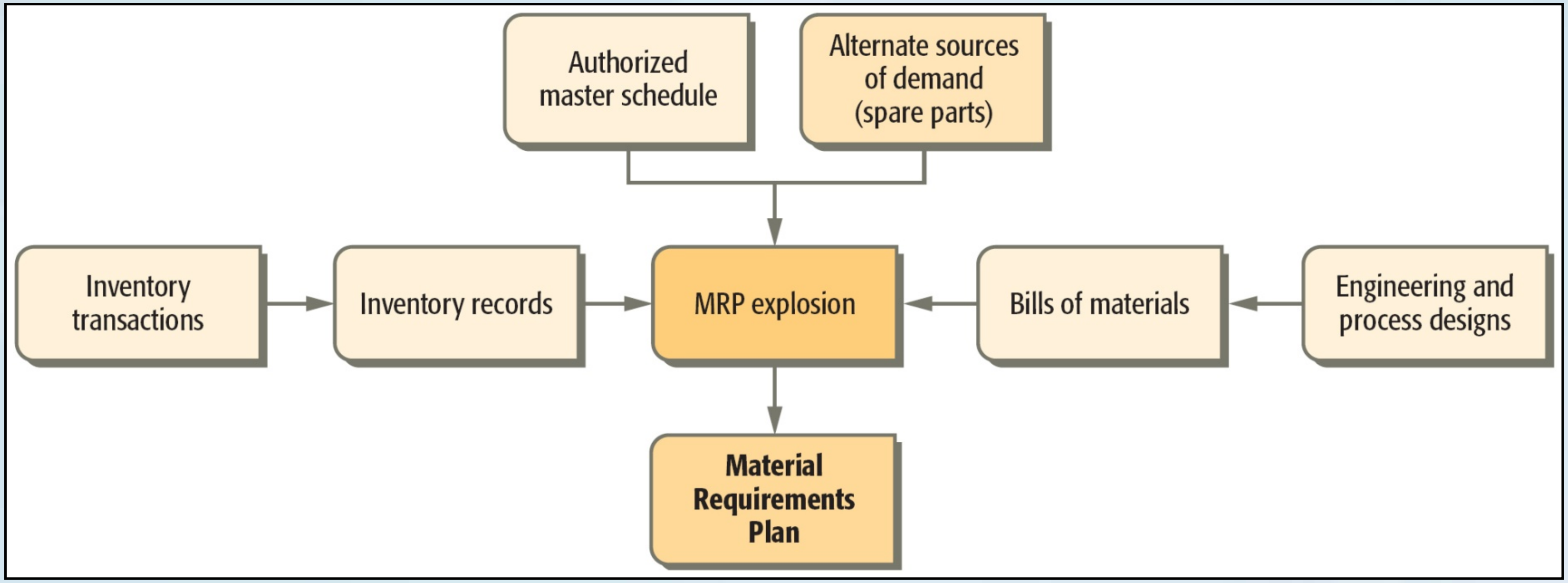
- **Material requirements plan:** a plan that specifies the timing and size of new production orders, adjustments to existing order quantities, and expediting or delay of late/early orders.
- The process of developing the material requirements plan is called MRP explosion; it is a technique for converting the requirements of final products into a material requirements plan that specifies the production/order quantities and timing for all subassemblies, components, and raw materials needed by final products.

MRP Inputs

- **MRP Inputs**

  - Developed through a combination of three inputs:
    1. The Master Schedule
    2. The Bill of Materials
    3. Inventory Records

Figure 7.4: Material Requirements Plan Inputs



The Master Schedule

- **Master schedule (MS)**: a document that details the quantity of end items to be produced within a specified period of time
- Objectives:
  - The MS must balance the workload for a given company in terms of not only total capacity, but also capacity at each workstation and for each worker.
  - The MS seeks to minimize total cost and provides a way of assessing the impact of new orders and providing delivery dates for accepted orders.
  - The planned production quantities in the MS are intended to satisfy demand, which is estimated based on computer orders and forecasts.
  - The MS is usually frozen or unchangeable in the near term.

The goal is to plan production but allow some flexibility to change orders as demand or customer requirements change.

Discussion Starter

What may require you to change the MPS?

Table 7.1: Master Schedule for a Family of Bicycles

TABLE 7.1 Master Schedule for a Family of Bicycles

	February				March			
	Feb. 1	**Feb. 8**	**Feb. 15**	**Feb. 22**	**Mar. 1**	**Mar. 8**	**Mar. 15**	**Mar. 22**
Aggregate production plan for bicycle family	600				500			
Mountain bike	200		100			80		80
Road bike		50		100	100		100	
Tandem bike		75		75		70		70

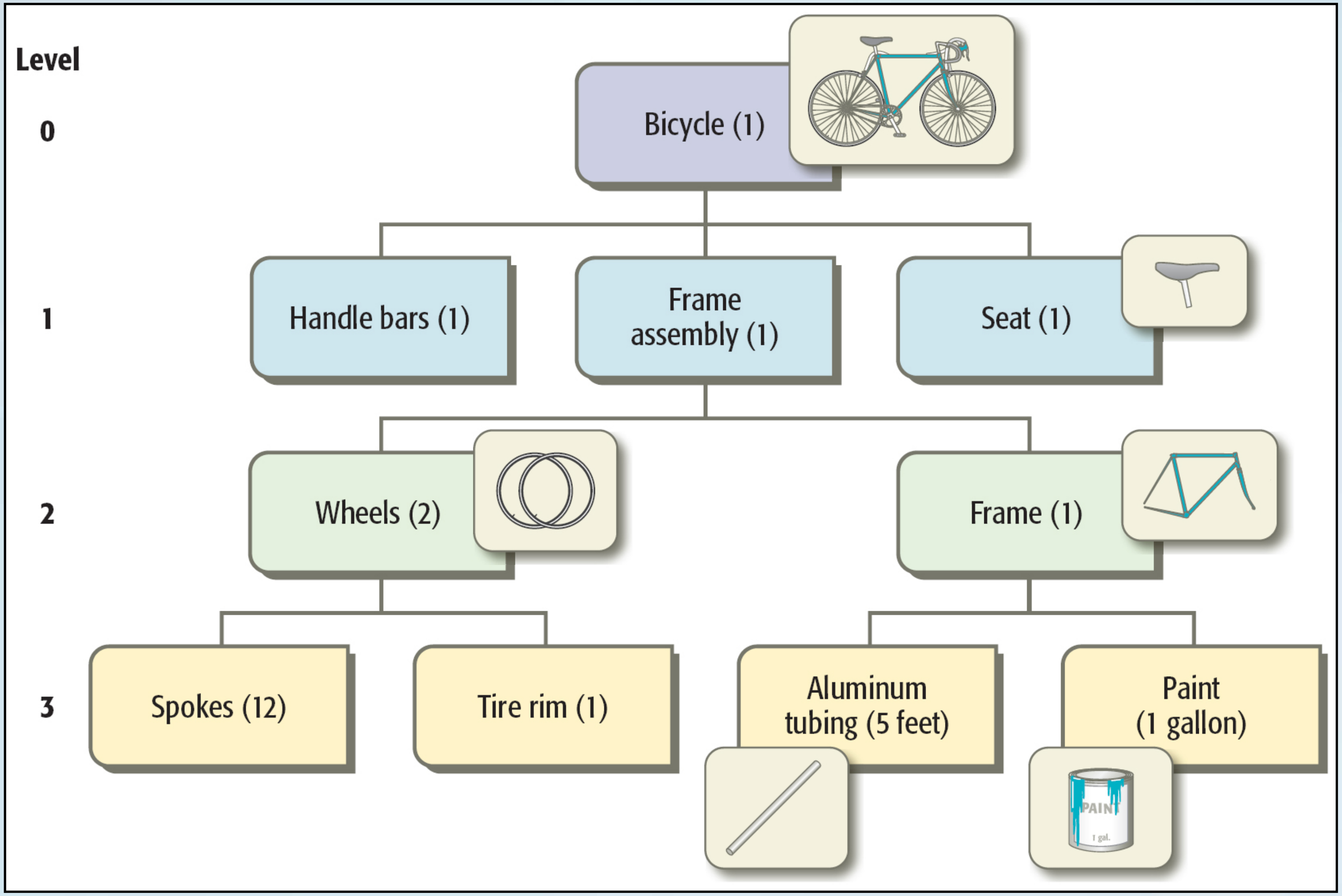
Key Aspects of Master Scheduling

- The sums of the quantities in the MS must equal those in the aggregate production plan.
- Aggregate production quantities should be planned efficiently over time in order to minimize setup, production, and inventory costs.
- Capacity limitations must be considered before finalizing the MS, including labor and machine capacity, storage space, transportation equipment, and other factors.

The Bill of Materials

- **Bill of materials (BOM)**: a document that specifies all assemblies, subassemblies, parts, and raw materials that are required to produce one unit of the finished product

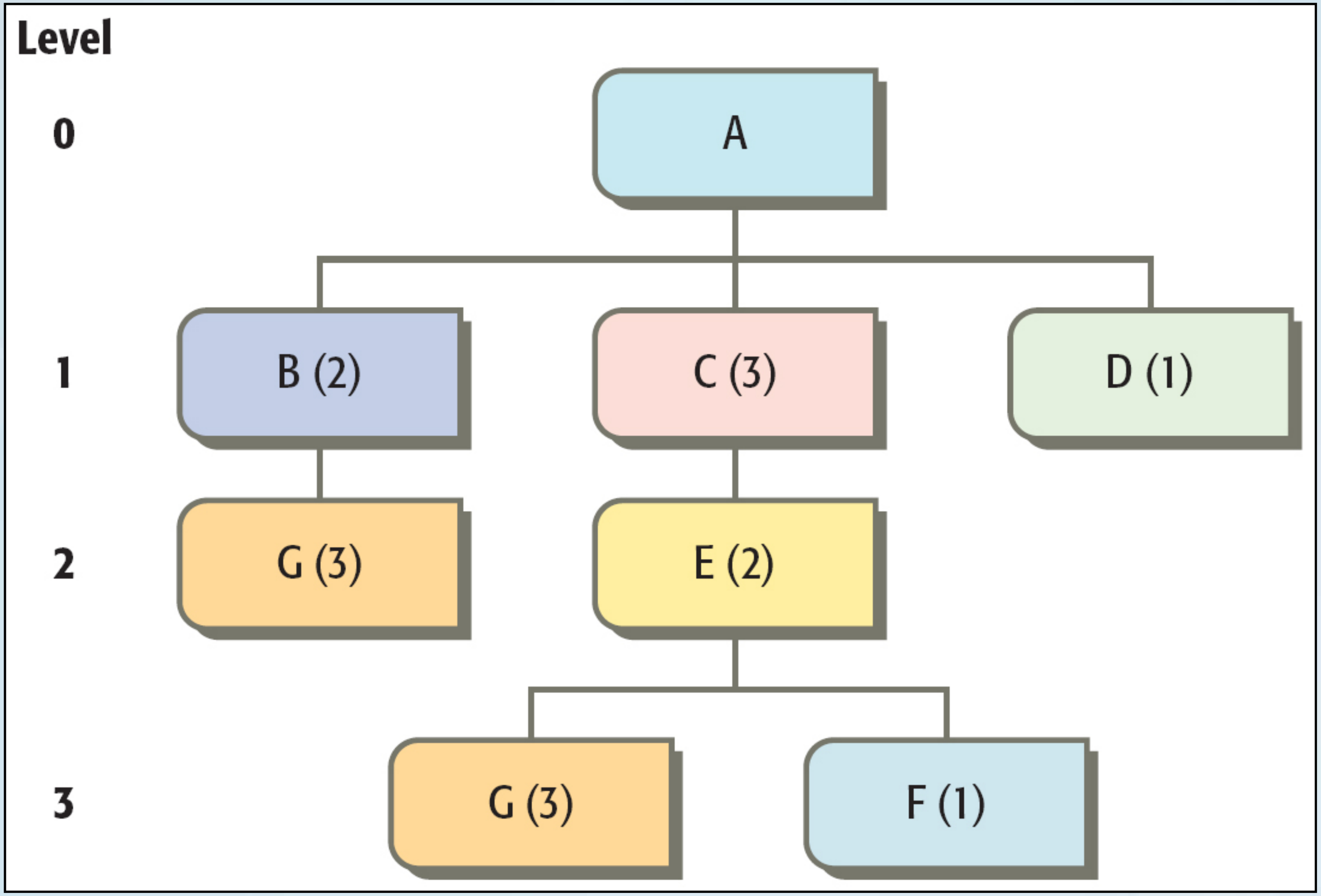
Figure 7.5: Partial Bill of Materials for a Bicycle



Bill of Materials for a Bicycle

- Every part in a bill of materials is assigned a level.
- End items or finished products that are sold directly to an end customer are Level 0.
- The handle bars, frame assembly and seat are Level 1 parts that are components of a complete bicycle.
- The wheels and frame are Level 2 parts that are components of the frame assembly.
- The spokes and tire rim are Level 3 components of a wheel.

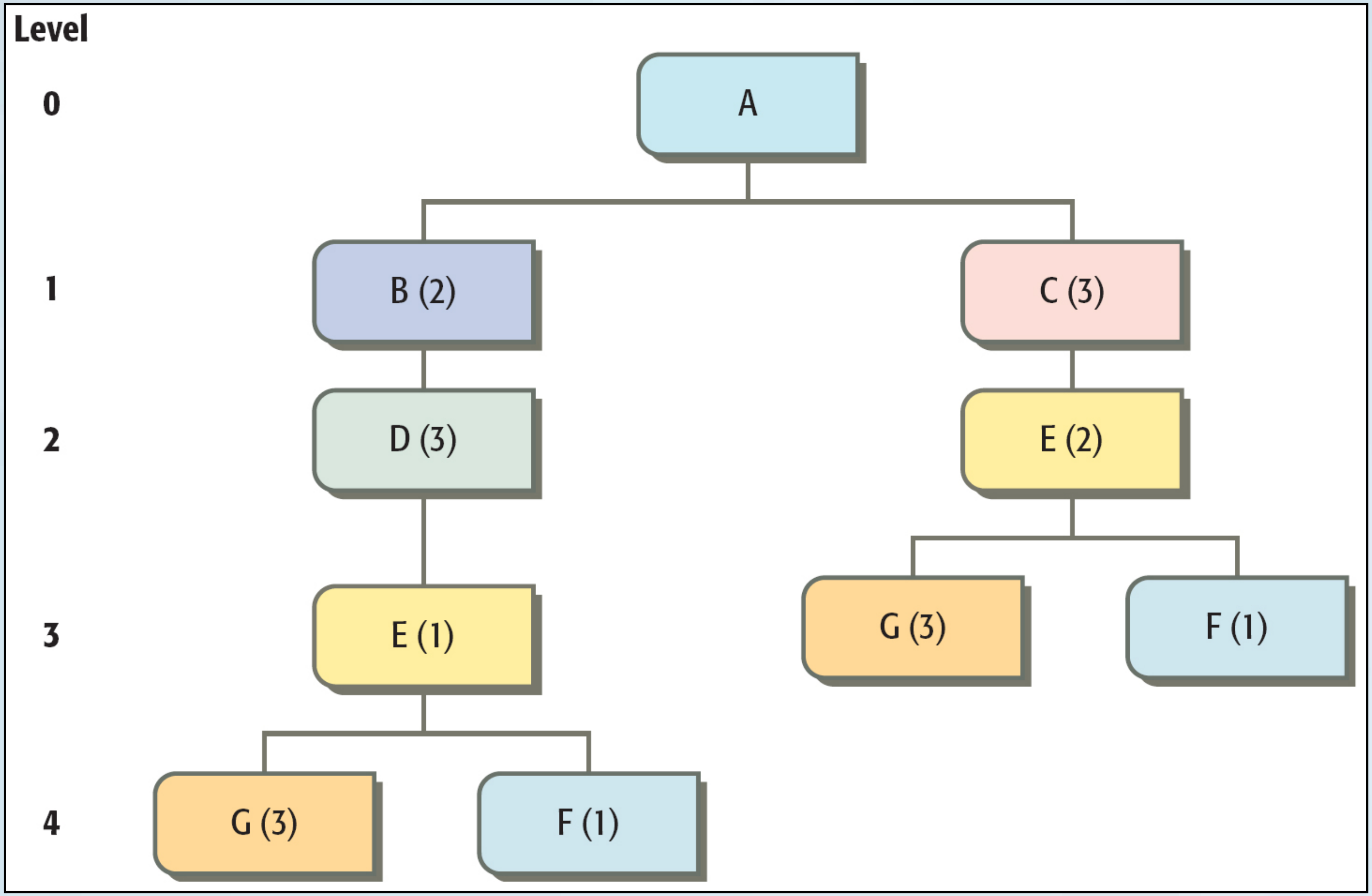
Figure 7.6: Product Structure Tree for Item A



BOMs

- **Common parts**: parts that are used in more than one place in a single product or in more than one product
- **Low-level coding**: involves assigning a part to the lowest level at which it appears anywhere in the BOM
- It is critical that the bill of materials is an accurate representation of the parts required to produce a product, since errors at one level are magnified when they are multiplied by parts requirements at lower levels.
- Attention to detail and accuracy, combined with periodic updates and checks of BOMs, are essential if an MRP system is to work effectively.

Figure 7.7: Bill of Materials with a Buried Component



Inventory Records

- **Inventory record**: a document that specifies order/lot size policy and lead time and records all transactions made for parts, assemblies, and components
  - Includes: transactions made for parts, assemblies, and components both from manufacturing within an organization and from purchasing items from external suppliers
- **Inventory transaction**: any change in the quantity of a specific part or material
  - Includes: receipt of new orders, shipment of complete orders, scrapping of defective parts, release of new orders, adjustment of due dates for scheduled receipts, cancellation of orders, and confirmation of scrap losses and returns.

MPR Processing—Creating an Inventory Record for a Single Item

- Developing Inventory Records for Single Items
- Determining Planning Factors

Developing Inventory Records for Single Items

- **Planning factors**: three parameters—lot size, lead time, and safety stock—that are chosen by managers utilizing the MRP system
- **Lot size**: the quantity of a part to be produced or ordered when additional inventory is required
- **Lead time**: the time between when an order is placed and when it is expected to arrive or be finished
- **Safety stock**: excess inventory that a company holds to guard against uncertainty in demand, lead time, and supply

Planning Factors for an MRP Record

- The planning factors for an MRP record are fairly constant—they are entered into the system once and may not be updated for months or years.
- **Time buckets**: the periods of time into which an MRP record is divided
- **Planning horizon**: the time period in the future that the MRP system plans for
- **Beginning inventory**: the amount of inventory that was physically in stock at the end of the most recent time bucket

Gross Requirements/Scheduled Receipts

- **Gross requirements**: the total number of units of a part or material derived from all parent production plans
- **Scheduled receipts**: orders that have been placed but not yet received or completed

Projected On-hand Inventory

- **Projected on-hand inventory**: the estimated inventory that will be available after the gross requirements have been satisfied, plus any planned or scheduled receipts for that time bucket
  - Abbreviated: projected OH inventory
  - Is adjusted according to every inventory transaction

Table 7.3: Illustration of Projected On-hand Inventory

TABLE 7.3 Illustration of Projected On-Hand Inventory

Week	Starting Inventory	Scheduled Receipts	Gross Requirements	Projected On-Hand Inventory
February 1	40	+0	–0	= 40
February 8	40	+200	–124	= 116
February 15	116	+0	–0	= 116
February 22	116	+0	–176	= –60
March 1	–60	+0	–100	= –160
March 8	–160	+0	–70	= –230
March 15	–230	+0	–100	= –330
March 22	–330	+0	–70	= –400
March 29	–400	+0	–0	= –400

Planned Receipts

- **Planned receipts**: future orders that which have not yet been released but are planned in order to avoid a shortage or backlog of inventory

Planned Order Release

- **Planned order release**: when an order must be released in order to offset for the lead time so that the order will be received when planned
- The difference between a planned and a scheduled receipt: a planned receipt is not firmly committed to and can be changed relatively easily up until the time the order is released.
- As soon as the order is released, it becomes a scheduled order, which is much harder to change.

Determining Planning Factors

- Every MRP record includes three planning factors:
  - Lead time
  - Lot size
  - Safety stock

These are called planning factors because the decisions managers make regarding these quantities have a large impact on how well the MRP system, and by extension the entire inventory system and supply chain, functions.

Lead Time

- Lead time is an estimate of the time between releasing an order and receiving that order.
- Accuracy in lead times is very important, since early or late orders can greatly affect other items and production schedules through excessive inventory holding costs or shortage, stock-out, and expediting costs.

Lead Time

- For items that are manufactured or produced within the company, the lead time must take into account a number of factors, including:
  - Set up time
  - Processing time
  - Materials handling time
  - Waiting time

Discussion Starter

Lead Time = Inventory

What do we mean by this?

Lot Size

- Lot size rules determine:
  - the size of the order placed, and by extension the timing of orders,
  - the frequency of set-ups, and
  - the inventory holding costs for an item.

- Three types:
  - Fixed order quantity
  - Periodic order quantity
  - Lot for lot

Fixed Order Quantity

- **Fixed order quantity (FOQ)**: a lot size rule with a constant order size where the same quantity is ordered every time
- The FOQ can be determined by a desire to:
  – work with equipment capacity, such as when a certain machine has a capacity limit.
  – mimic the EOQ
  – make planning consistent
  – receive a quantity discount
  – minimize shipping costs
  – reach a minimum purchase quantity

Periodic Order Quantity

- **Periodic order quantity (POQ)**: a lot size rule with a variable lot size designed to order exactly the amount required for a specified period of time

- Equation:

POQ Lot Size to Arrive in Period *t* =

(Gross Requirements for *P* Periods, Including Period *t*) – (Projected On-Hand Inventory at End of Period *t* – 1) + (Safety Stock) of time of time

Lot for Lot (L4L)

- **Lot for lot (L4L)**: a lot size rule that is a special case of the periodic order quantity with the period equal to 1

- Equation:

L4L Lot Size to Arrive in Period t =

(Gross Requirements in Period t) – (Projected On-Hand Inventory at End of Period t – 1) + (Safety Stock)

Table 7.8: MRP Record with L4L Order

TABLE 7.8 **MRP Record with L4L order**

Item:	B100	Lot size:	L4L
Description:	Bicycle wheel	Lead time:	2 weeks
Beg. inv.:	40	Safety stock:	15

Week	Feb. 1	Feb. 8	Feb. 15	Feb. 22	Mar. 1	Mar. 8	Mar. 15	Mar. 22	Mar. 29
Gross requirements	0	124	0	176	100	70	0	70	100
Scheduled receipts	0	200	0	0	0	0	0	0	0
Planned OH inventory	40	116	116	15	15	15	15	15	15
Planned receipts				75	100	70		70	100
Planned order release		75	100	70		70	100		

Lot Size Rules Summary

1. The FOQ rule has the highest average inventory because its fixed nature creates inventory remnants.
2. The POQ rule reduces the amount of OH inventory by matching gross requirements with planned receipts.
3. The L4L rule always minimizes inventory, but also requires more frequent setups/orders.

Safety Stock

- It would seem that an MRP inventory system should not require safety stock.

Why is safety stock necessary?

1. There may be bottlenecks or blockages that prevent orders from being complete on a timely basis.
2. Quality problems often arise where an order will be only 95 percent filled.
3. Humans may enter incorrect information into the system.
4. There is variability in demand, and the master schedule is made to match forecasts.

MRP Nervousness

- **MRP nervousness**: a situation in MRP planning where a change at one part level ripples down to affect lower-level parts

MRP Explosion

- **MRP explosion**: the process of translating MRP inputs into a plan that specifies required quantities and timing of all subassemblies, components, and raw materials required to produce parent items

Action Notices

- **Action notice**: a notice that is generated when an order needs to be released or placed or when the quantity or timing of an order needs to be changed

MRP as a Dynamic System

- Two approaches to updating:
  - **Periodic update**: an approach to updating that involves collecting all new or updated information and processing it once a week or once a day
  - **Net change update**: an approach to updating that makes changes as soon as they occur

Figure 7.10: Illustration of a Rolling MRP Schedule

Master Production Schedule – Speedy Road Bicycle

45	0	62	0	88	50	35	50	35	100

Item: A130 — Description: Frame assembly — Beginning inventory: 20

1 frame assembly for each bicycle

Lot size: L4L — Lead time: 1 week — Safety stock: 20

Week	1	2	3	4	5	6	7	8	9	10
Gross requirements	45	0	62	0	88	50	35	50	35	100
Scheduled receipts	45									
Planned OH inventory	20	20	20	20	20	20	20	20	20	20
Planned receipts			62		88	50	35	50	35	100
Planned order release		62		88	50	35	50	35	100	

Item: B100 — Description: Bicycle wheel — Beginning inventory: 40

2 wheels for each frame assembly = 2 x 62

Lot size: 200 — Lead time: 2 weeks — Safety stock: 15

Week	1	2	3	4	5	6	7	8	9	10
Gross requirements	0	124	0	176	100	70	100	70	200	
Scheduled receipts	0	200	0	0	0	0	0	0	0	
Planned OH inventory	40	116	116	140	40	170	70	200	200	
Planned receipts				200		200		200	200	
Planned order release		200		200		200	200			

Item: Z125 — Description: Spokes — Beginning inventory: 200

12 spokes for each wheel = 12 x 200

Lot size: 2500 — Lead time: 3 weeks — Safety stock: 100

Week	1	2	3	4	5	6	7	8	9	10
Gross requirements		2400		2400		2400	2400			
Scheduled receipts		2500		2500						
Planned OH inventory	200	300	300	400	400	500	600	600	600	600
Planned receipts						2500	2500			
Planned order release	2500		2500	2500						

Item: D200 — Description: Tire rim — Beginning inventory: 220

1 tire rim for each wheel

Lot size: P = 3 — Lead time: 2 weeks — Safety stock: 10

Week	1	2	3	4	5	6	7	8	9	10
Gross requirements		200		200		200	200			
Scheduled receipts				390						
Planned OH inventory	220	20	20	210	210	10	10	10	10	
Planned receipts							200			
Planned order release	390				200					

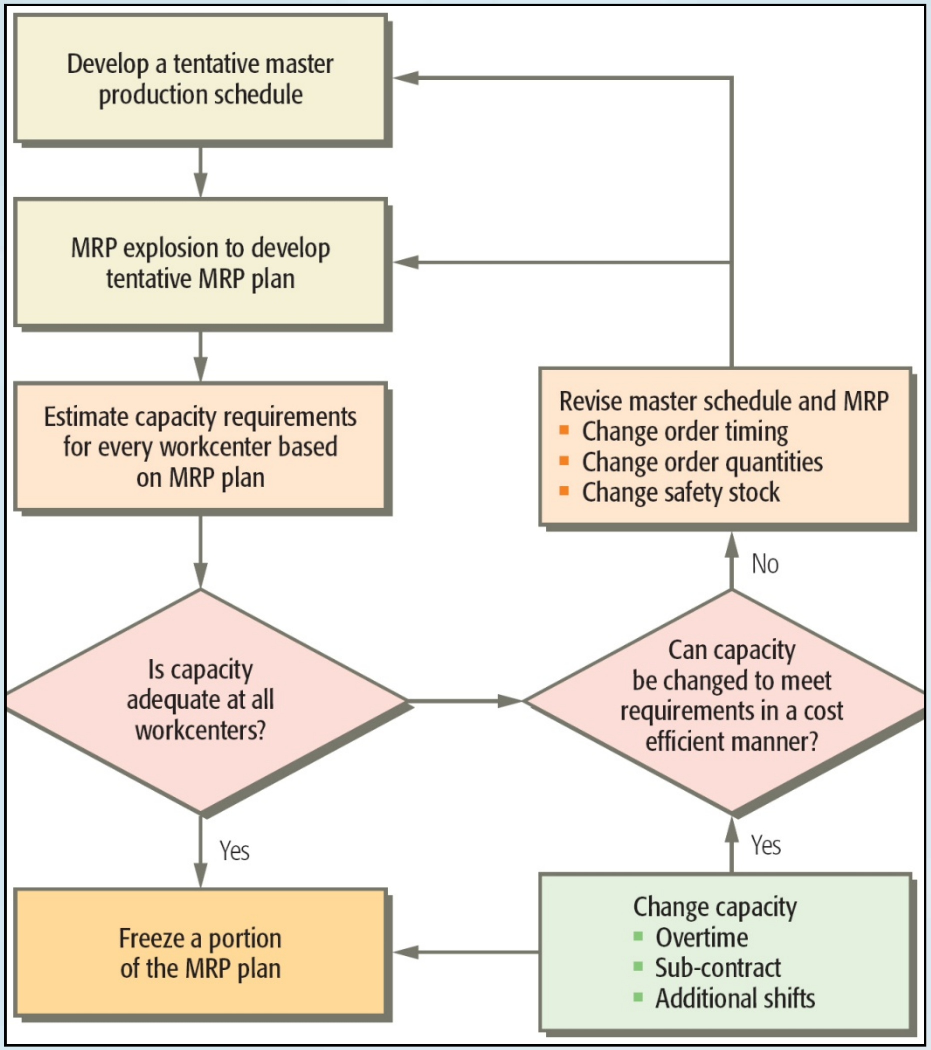
Capacity Planning

- There are three approaches for managing capacity and ensuring that the MRP plan is feasible:
  1. Capacity requirements planning
  2. Finite capacity scheduling
  3. Input/Output reports

Capacity Requirements Planning

- **Capacity requirements planning**: the process of determining short-range capacity requirements based on a tentative MRP plan
  - Short range generally refers to the next one to three months.
- Inputs include the planned order releases generated from the MRP system, workloads at each work center, routing information, and job setup/processing times.
- The master schedule and the MRP plan are usually generated by looking at what is needed to support sales, rather than what is possible.
- **Load report**: a report for a department or work center that projects already scheduled and expected future capacity requirements against capacity availability

Figure 7.11: MRP with Capacity Planning Requirements



Strategies for Dependent Demand Inventory

- There are strategic keys to making MRP work effectively.
  1. Evolution of MRP to Enterprise Resource Planning
  2. Service Resource Planning
  3. Making MRP/ERP Work

Evolution of MRP to Enterprise Resource Planning

- **Manufacturing resource planning (MRP II)**: a system that links the basic MRP system to other company systems, including finance, accounting, purchasing, and logistics
- **Enterprise resource planning (ERP)**: a system that provides a complete linkage of all functional areas of a business
  - Allows manufacturing to see new orders as soon as marketing or sales enters them into the system.

Making MRP/ERP Work

Dependent demand planning and material requirements planning are critical components for manufacturing businesses.

Three key factors contribute to success:

1. The hardware and software have to be carefully set up to fit with the organization's method of doing business.
2. The users of the system (employees) need to be thoroughly trained in the system.
3. The input data need to be close to 100 percent accurate because MRP will magnify any inconsistencies.