Chapter 11 Frequency Response

Chapter Outline

Fundamental Concepts

- Bode's Rules
- Association of Poles with Nodes
- Miller's Theorem

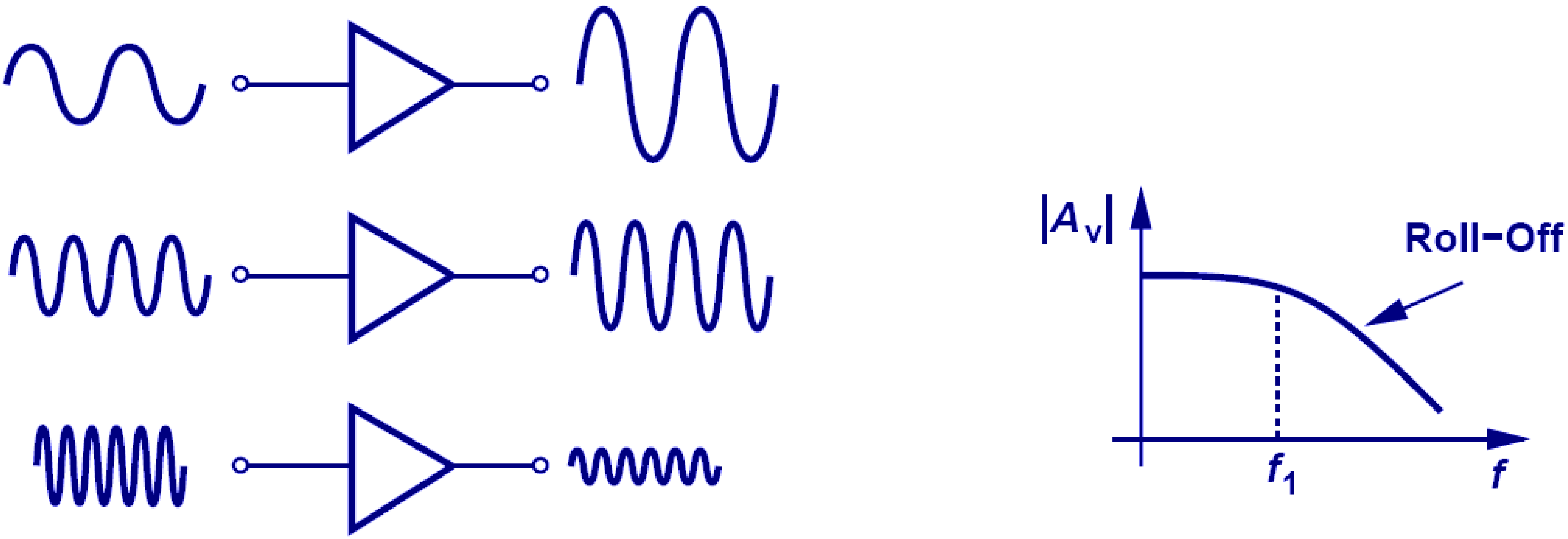
High-Frequency Models of Transistors

- Bipolar Model
- MOS Model
- Transit Frequency

Frequency Response of Circuits

- CE/CS Stages
- CB/CG Stages
- Followers
- Cascode Stage
- Differential Pair

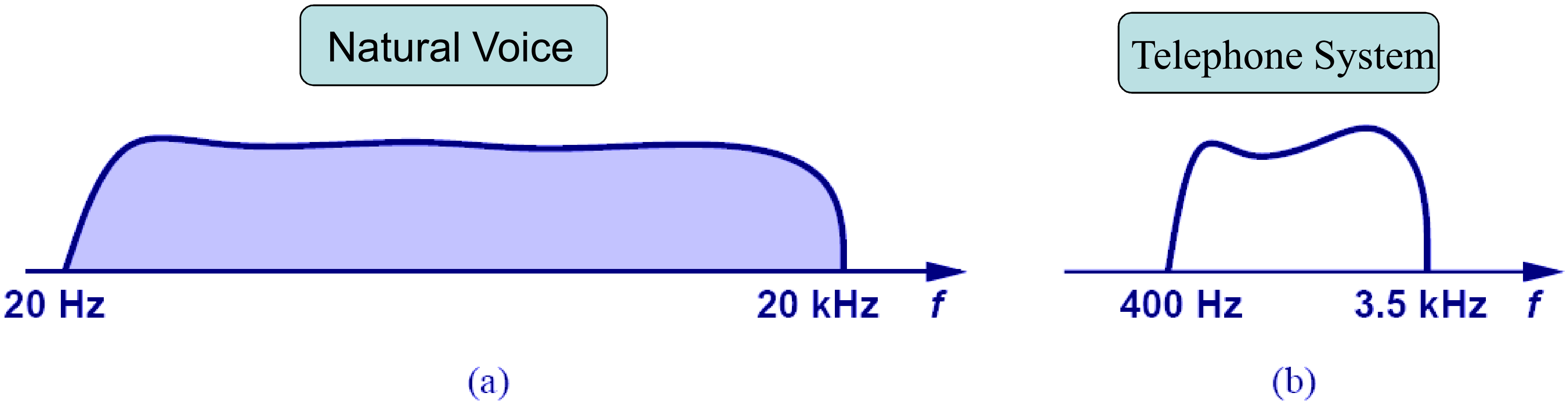
High Frequency Roll-off of Amplifier



(a) (b)

- As frequency of operation increases, the gain of amplifier decreases. This chapter analyzes this problem.

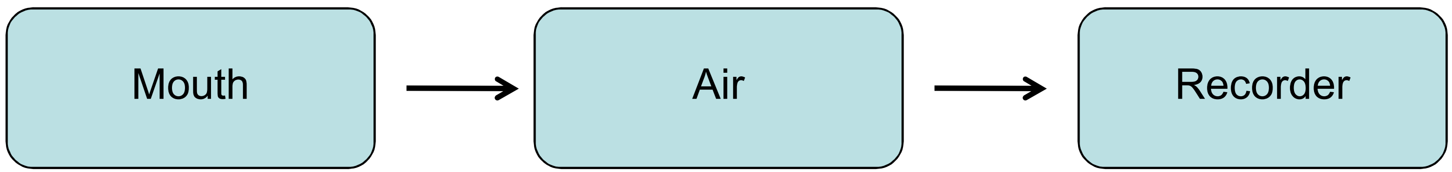
Example: Human Voice I



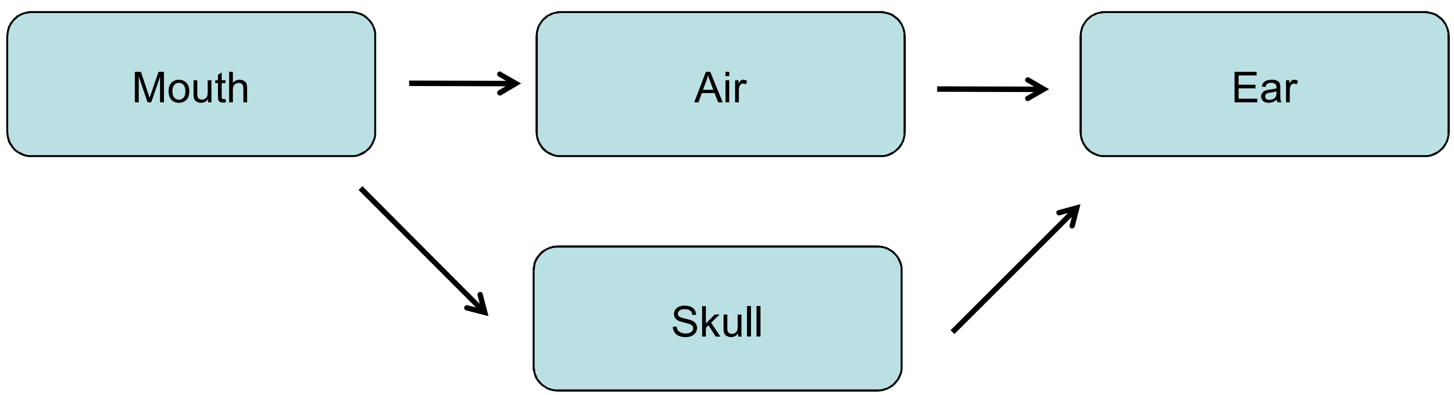
- Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.

Example: Human Voice II

Path traveled by the human voice to the voice recorder

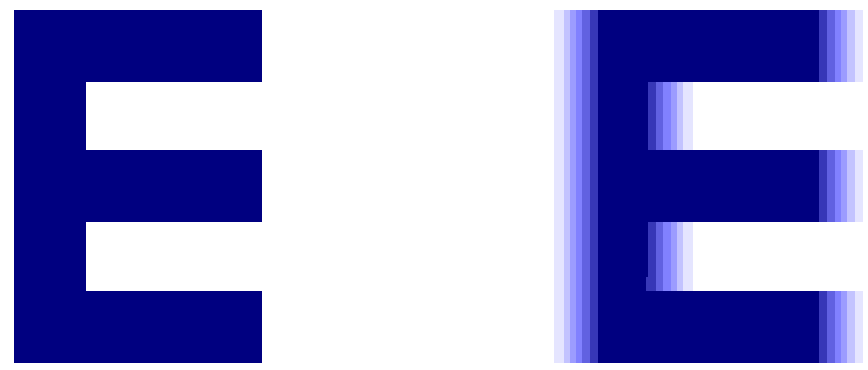


Path traveled by the human voice to the human ear



- **Since the paths are different, the results will also be different.**

Example: Video Signal

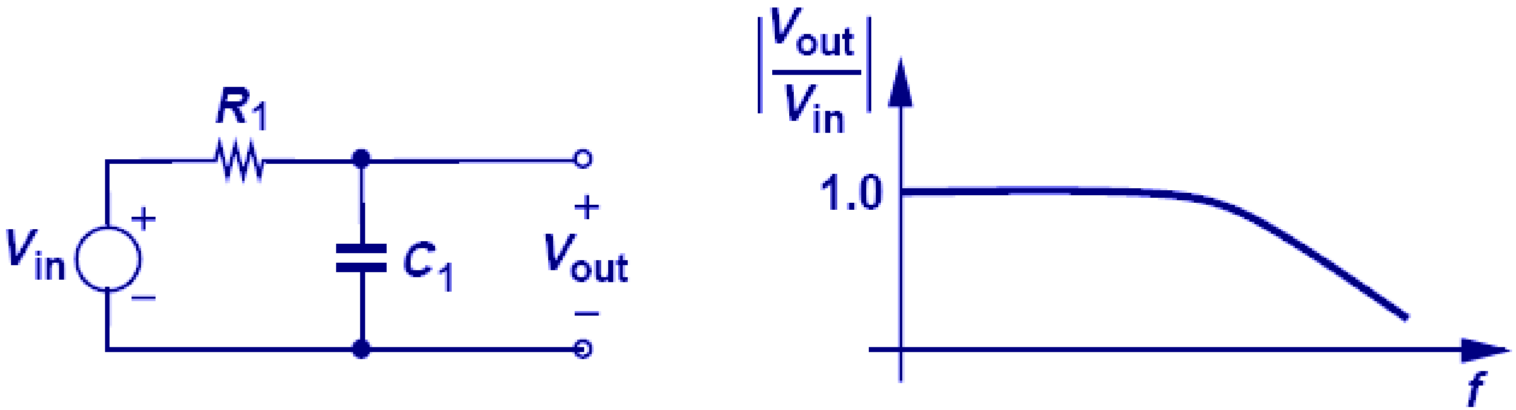


(a) (b)

High Bandwidth | Low Bandwidth

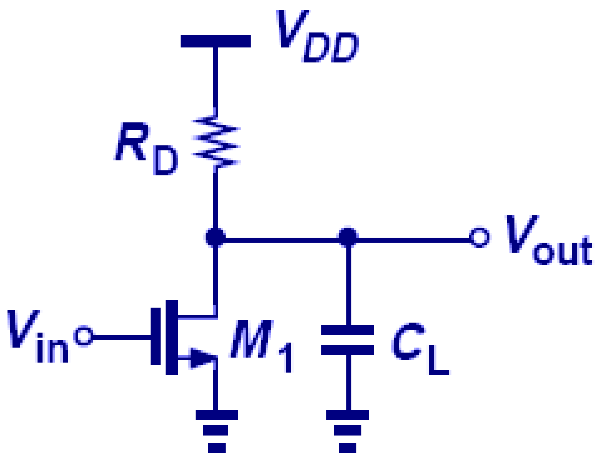
- Video signals without sufficient bandwidth become fuzzy as they fail to abruptly change the contrast of pictures from complete white into complete black.

Gain Roll-off: Simple Low-pass Filter

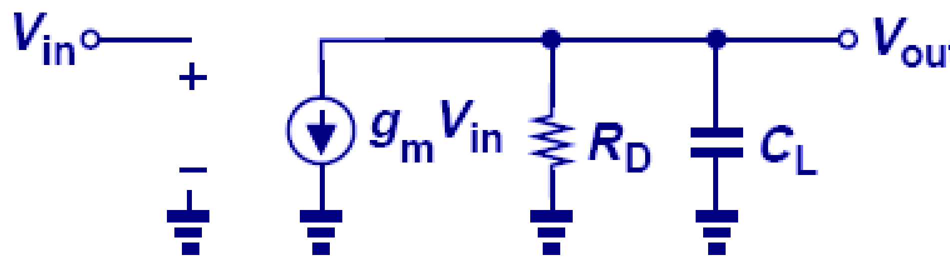


- **In this simple example, as frequency increases the impedance of C_1 decreases and the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.**

Gain Roll-off: Common Source



(a)

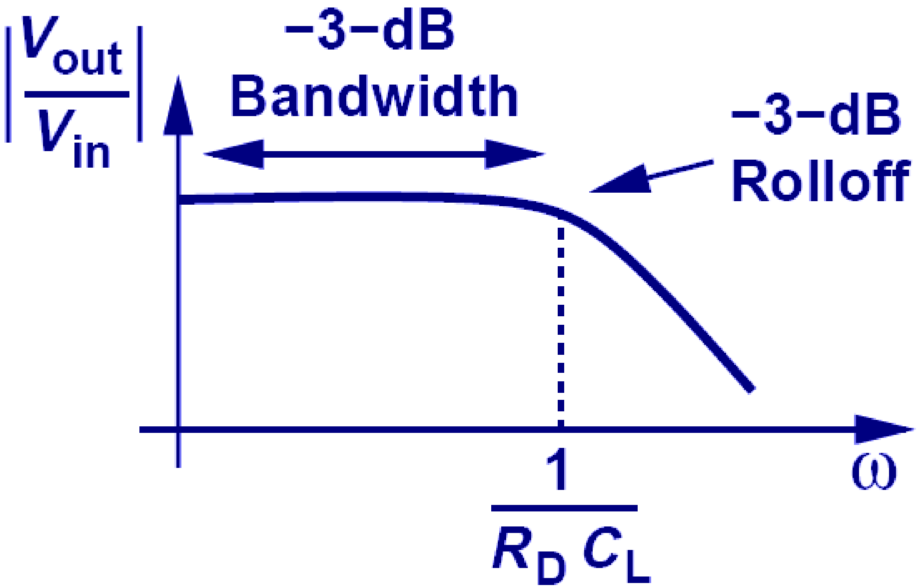


(b)

$$V_{out} = -g_m V_{in}\left(R_D \,\|\, \frac{1}{C_L s}\right)$$

- **The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will "steal" away some signal current and shunt it to ground.**

Frequency Response of the CS Stage



$$H(s) = \frac{V_{out}}{V_{in}}(s)$$

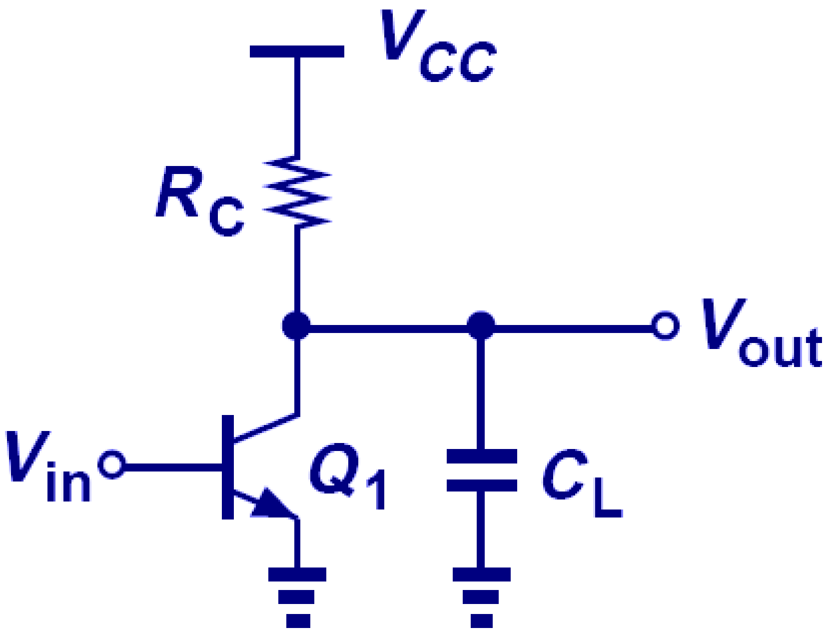
$$= -g_m \left(R_D \,\Big\|\, \frac{1}{C_L s} \right)$$

$$= \frac{-g_m R_D}{R_D C_L s + 1}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

- At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease. A special frequency is $\omega=1/(R_D C_L)$, where the gain drops by 3dB.

Example: Figure of Merit



$$F.O.M. = \frac{Gain \times Bandwidth}{Power\ \ Consumption}$$

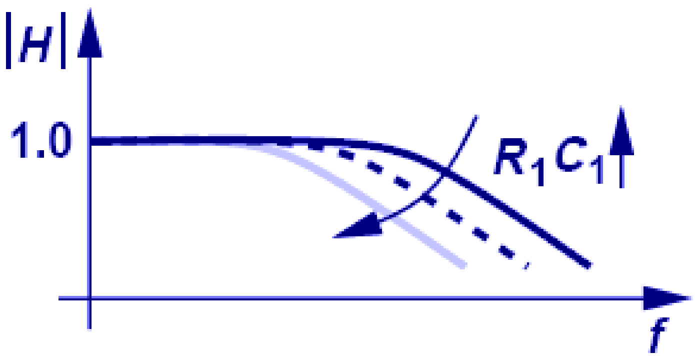
$$= \frac{g_m R_C \times \frac{1}{R_C C_L}}{I_C V_{CC}}$$

$$= \frac{\frac{I_C}{V_T} R_C \times \frac{1}{R_C C_L}}{I_C V_{CC}}$$

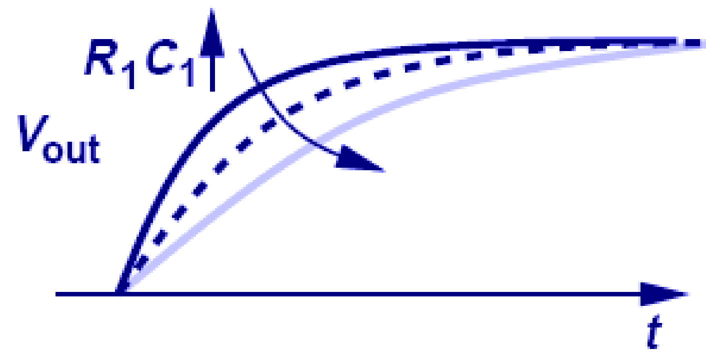
$$\boxed{= \frac{1}{V_T V_{CC} C_L}}$$

- This metric quantifies a circuit's gain, bandwidth, and power dissipation. In the bipolar case, low temperature, supply, and load capacitance mark a superior figure of merit.

Example: Relationship between Frequency Response and Step Response



(a)



(b)

$$\left|H\left(s=j\omega\right)\right|=\frac{1}{\sqrt{R_1^2C_1^2\omega^2+1}}$$

$$V_{out}\left(t\right)=V_0\left(1-\exp\frac{-t}{R_1C_1}\right)u\left(t\right)$$

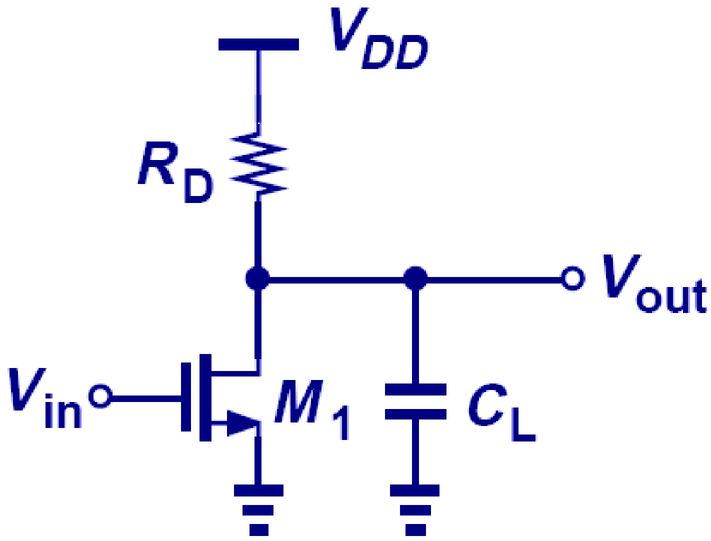
- **The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower.***

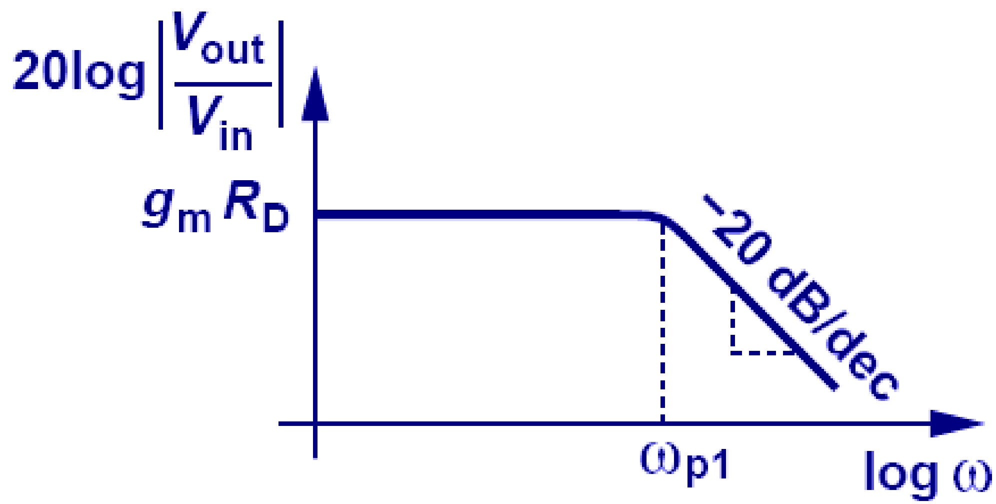
Bode Plot

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

- **When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.**
- **When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec**

Example: Bode Plot

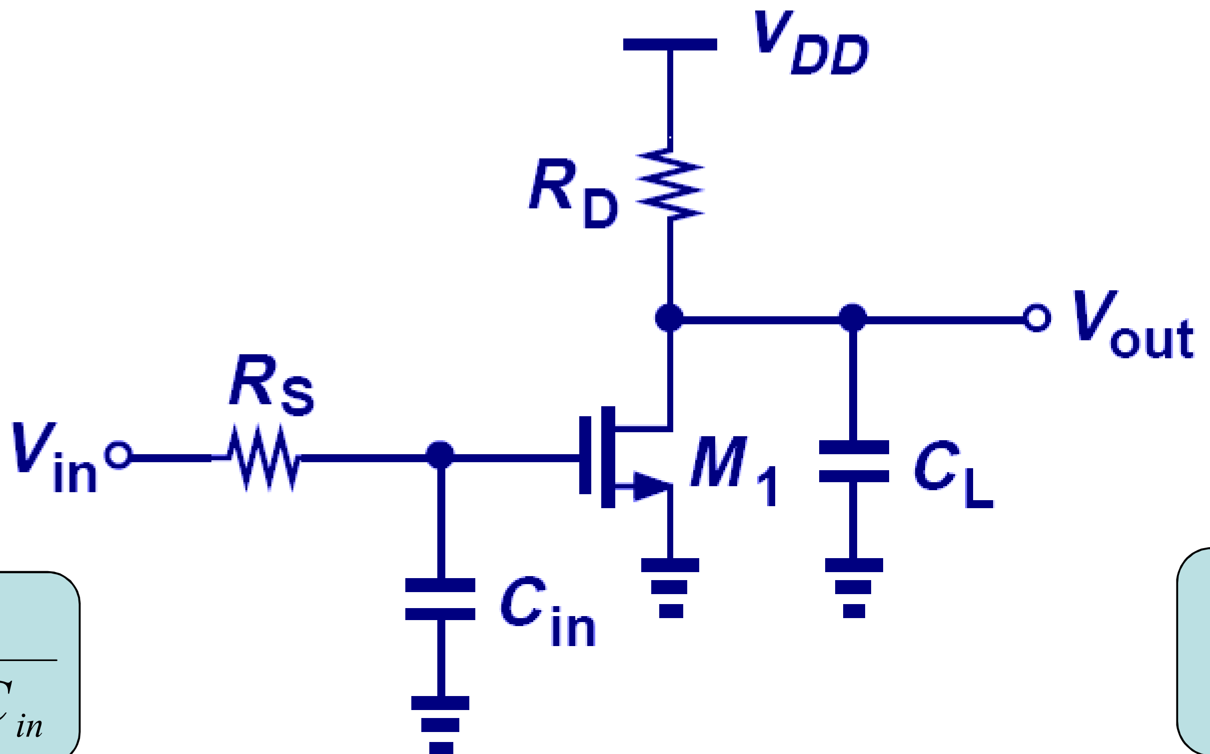




$$\left|\omega_{p1}\right| = \frac{1}{R_D C_L}$$

- The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}.

Pole Identification Example I

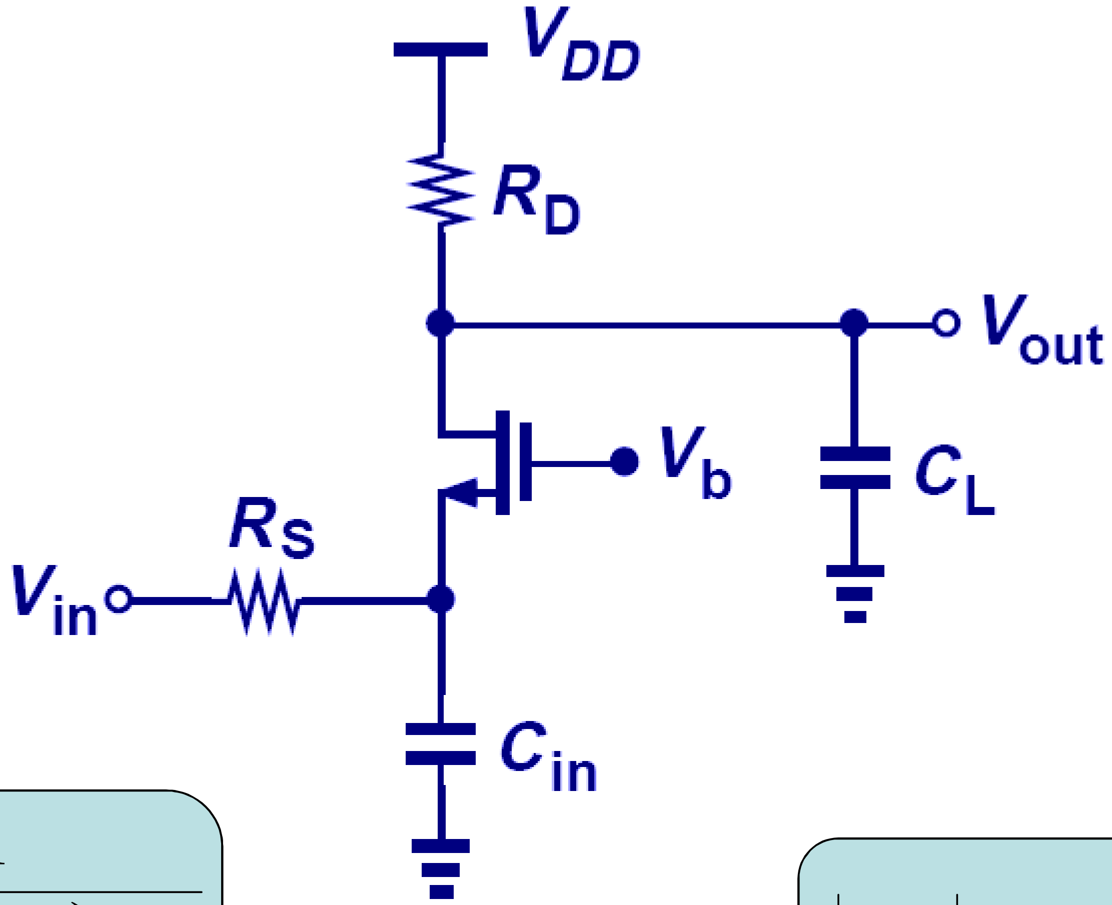


$$|\omega_{p1}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

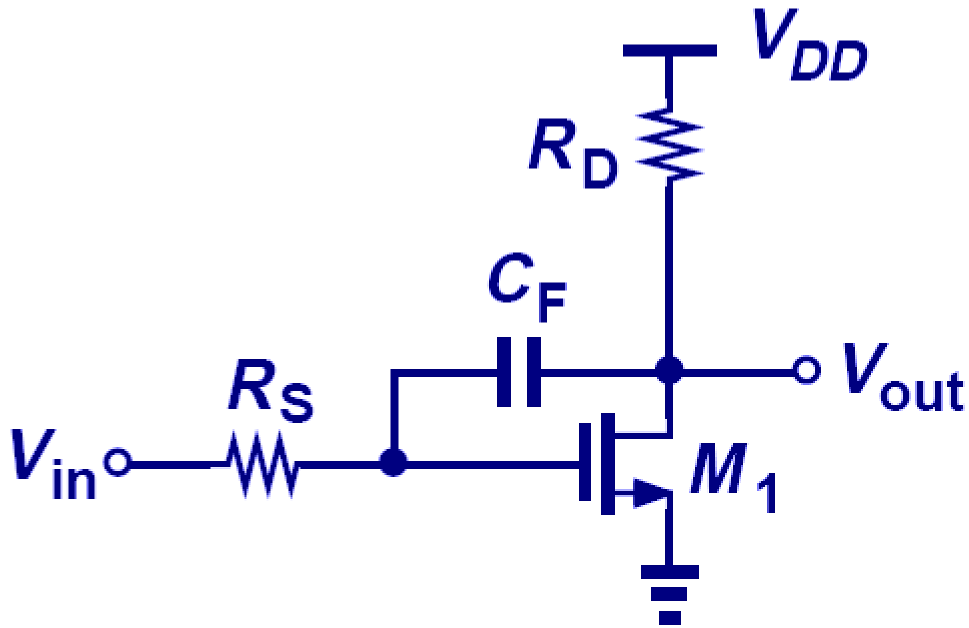
Pole Identification Example II



$$\left|\omega_{p1}\right| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right)C_{in}}$$

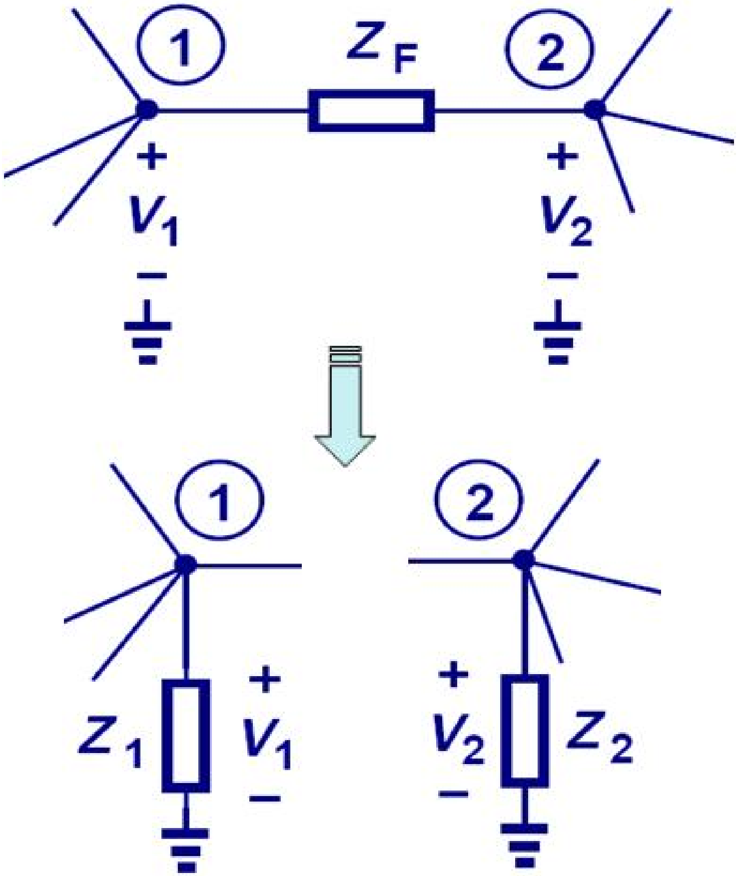
$$\left|\omega_{p2}\right| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



- The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.

Miller's Theorem



$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}, \qquad \frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}$$

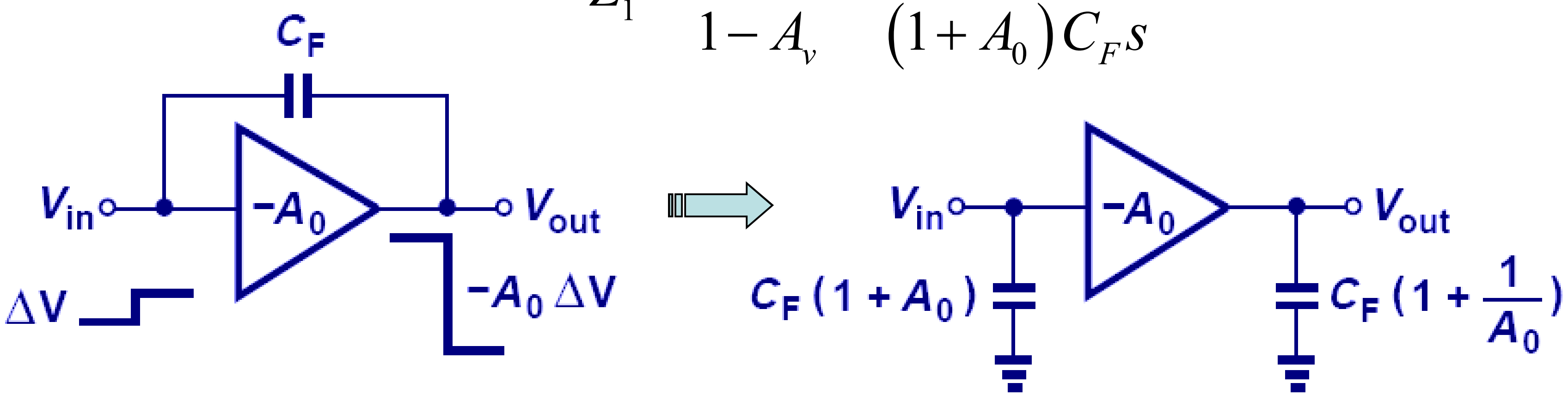
$$\Rightarrow \boxed{Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v}}$$

$$\boxed{Z_2 = Z_F \frac{-V_2}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}}$$

- **If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.**

Miller Multiplication

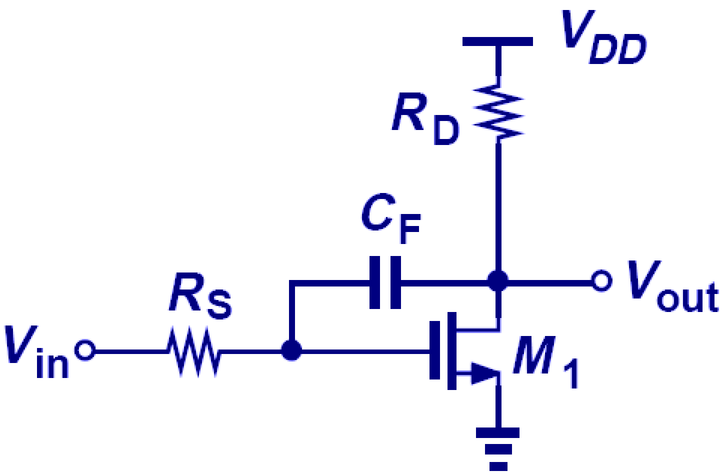
$$Z_1 = \frac{Z_F}{1 - A_v} = \frac{1}{(1 + A_0) C_F s}$$

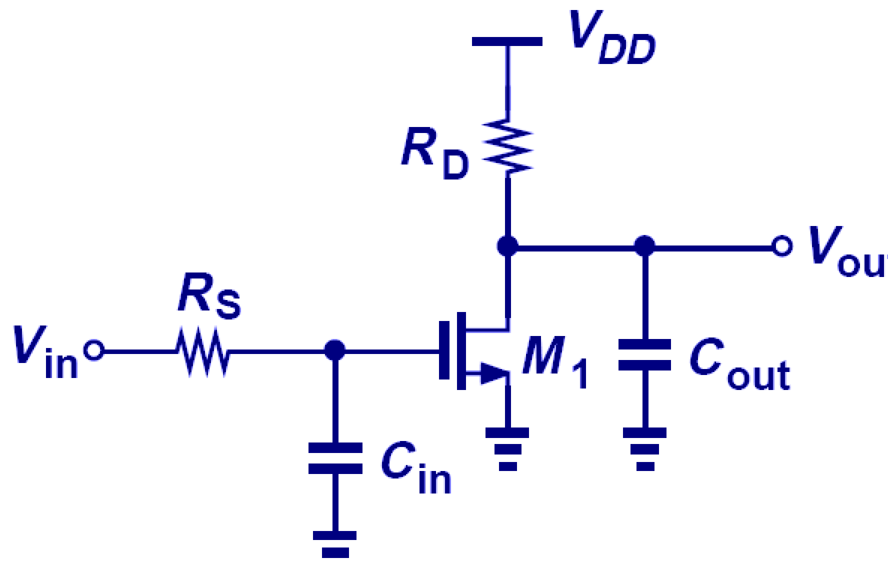


$$Z_2 = \frac{Z_F}{1 - \frac{1}{A_v}} = \frac{1}{\left(1 + \frac{1}{A_0}\right) C_F s}$$

- With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Example: Miller Theorem

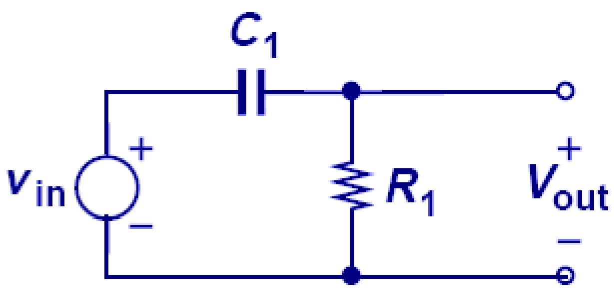




$$\omega_{in} = \frac{1}{R_S(1+g_m R_D)C_F}$$

$$\omega_{out} = \frac{1}{R_D\left(1+\frac{1}{g_m R_D}\right)C_F}$$

High-Pass Filter Response



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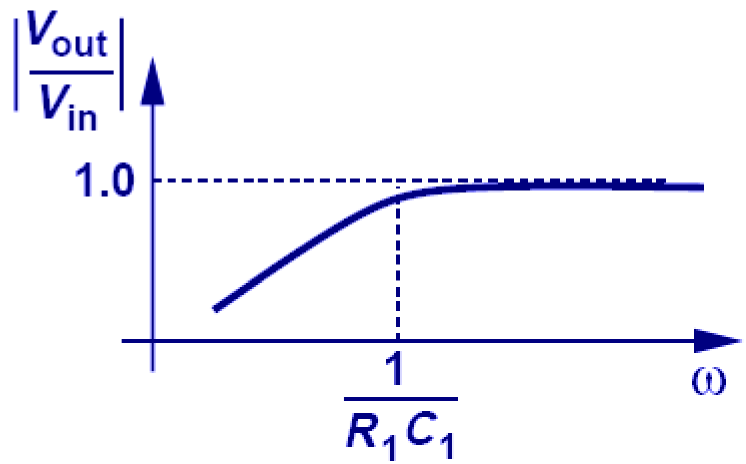
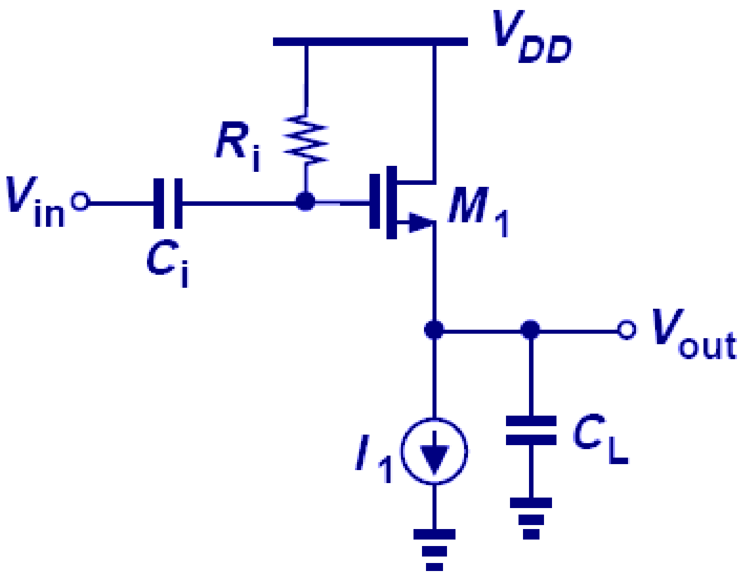


(b)

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1C_1\omega}{\sqrt{R_1^2C_1^2\omega_1^2+1}}$$

- **The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.**

Example: Audio Amplifier



$$R_i = 100\ \text{k}\Omega$$
$$g_m = 1/200\ \Omega$$

$$\frac{1}{R_i C_i} \le 2\pi \times (20\text{Hz})$$

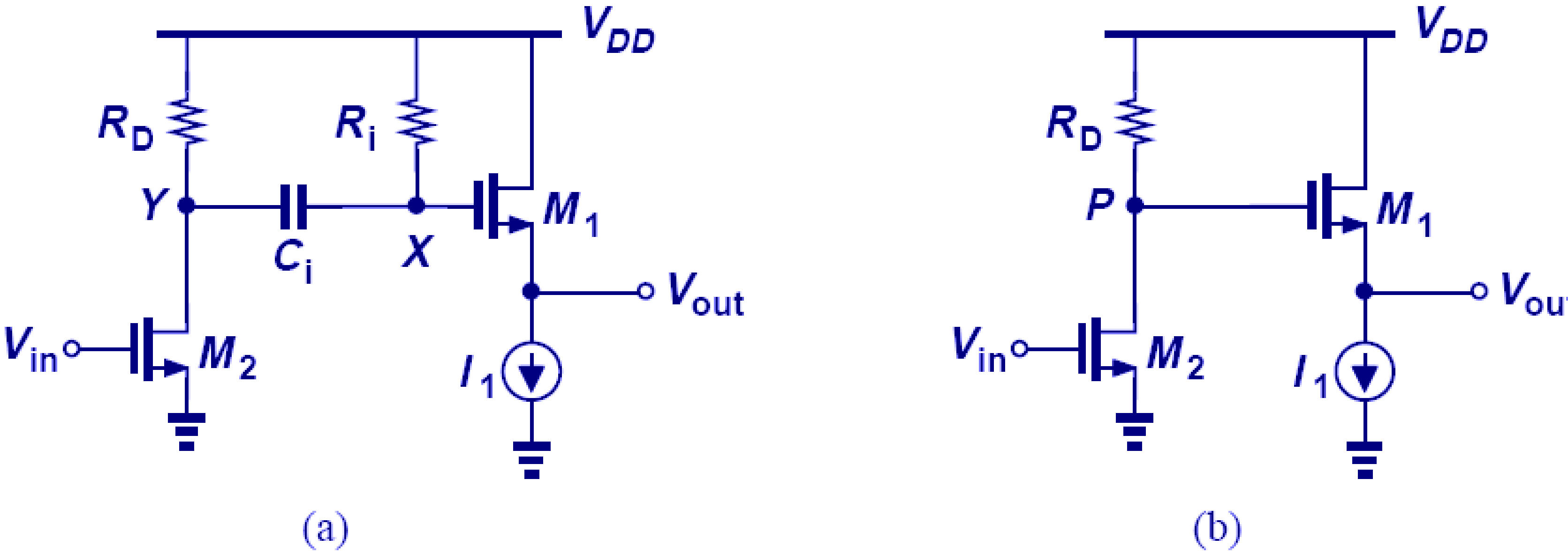
$$\Rightarrow C_i \ge \frac{1}{100k \times 2\pi \times 20} = 79.6\text{nF}$$

$$\omega_{p,out} = \frac{g_m}{C_L} \ge 2\pi \times (20\text{kHz})$$

$$\Rightarrow C_L \le \frac{1}{200 \times 2\pi \times 20\text{k}} = 39.8\text{nF}$$

- **In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and small output capacitances are needed.**

Capacitive Coupling vs. Direct Coupling

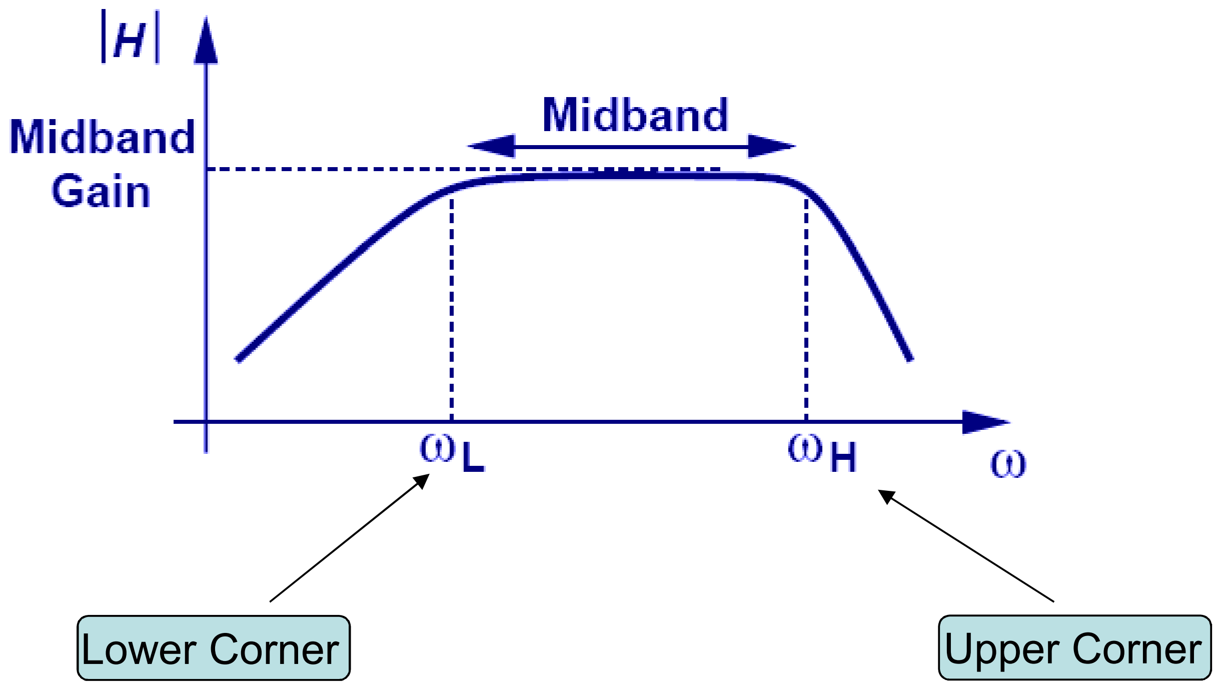


Capacitive Coupling

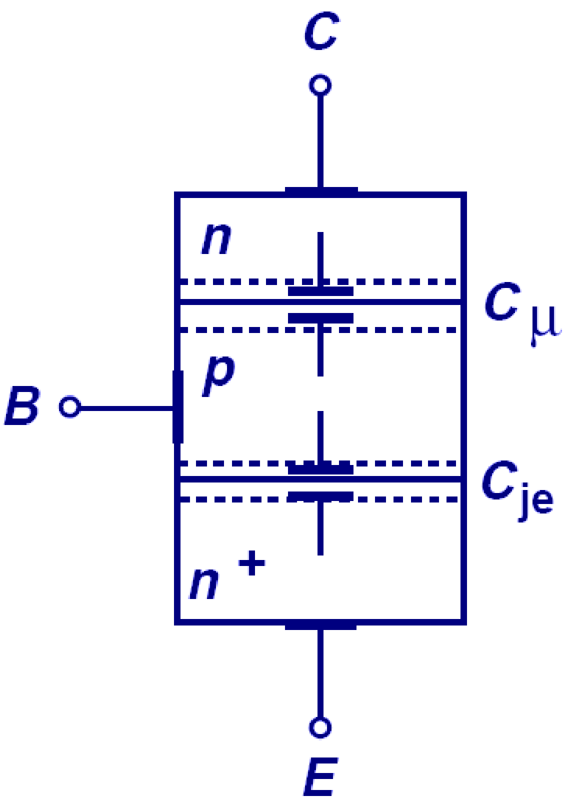
Direct Coupling

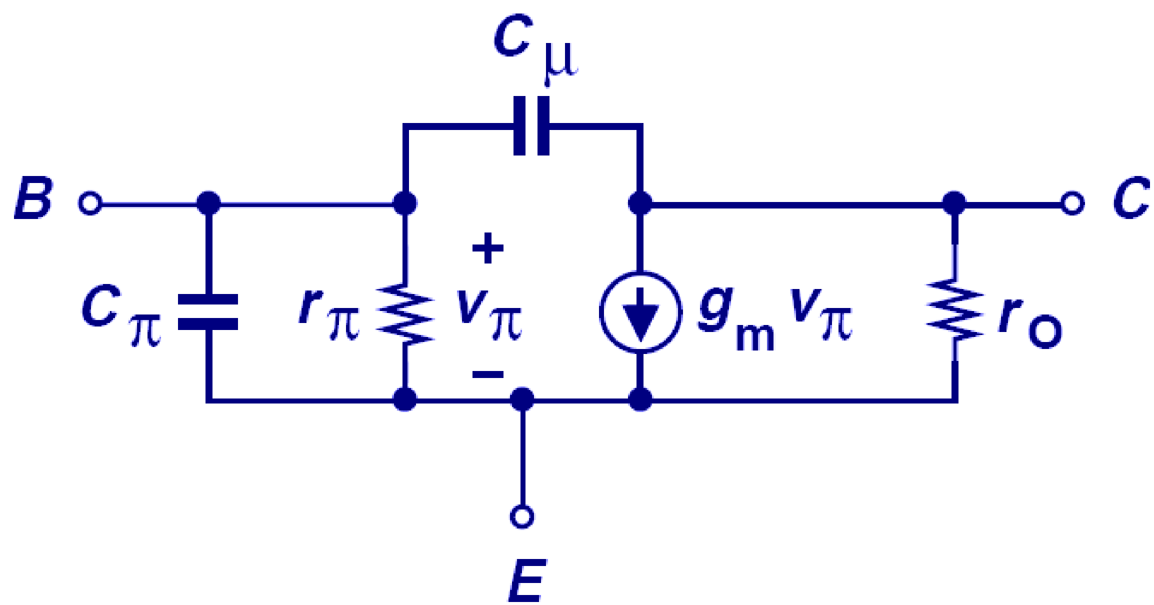
- Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.
- This technique allows independent bias conditions between stages. Direct coupling does not.

Typical Frequency Response



High-Frequency Bipolar Model

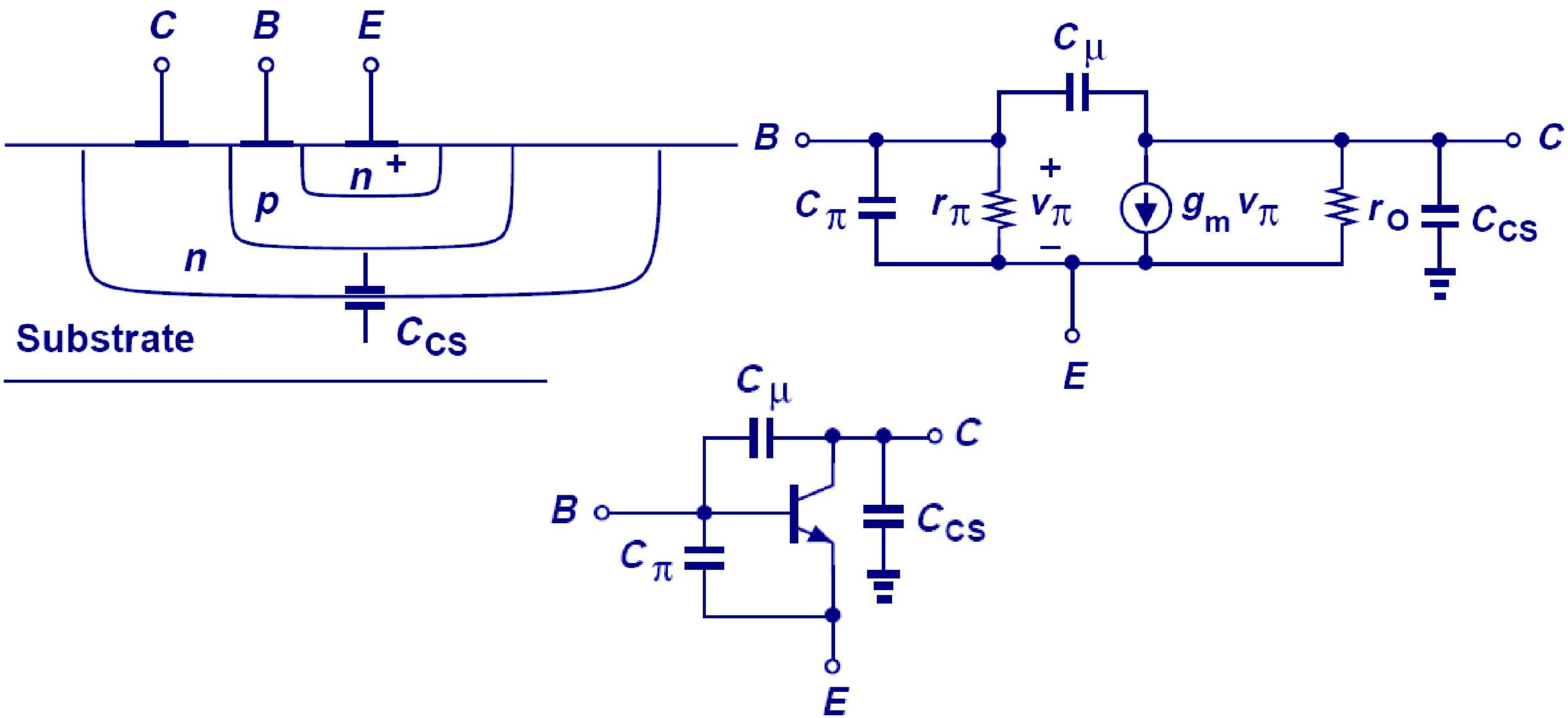




$$C_\pi = C_b + C_{je}$$

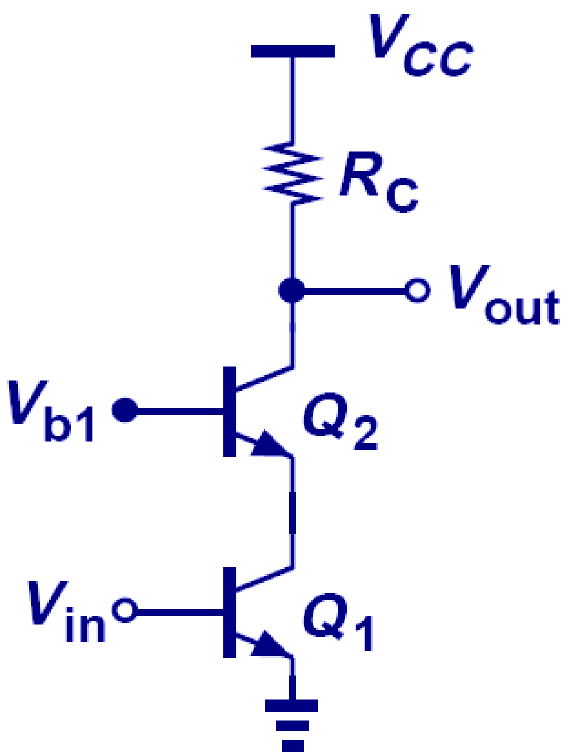
- **At high frequency, capacitive effects come into play. C_b represents the base charge, whereas C_μ and C_{je} are the junction capacitances.**

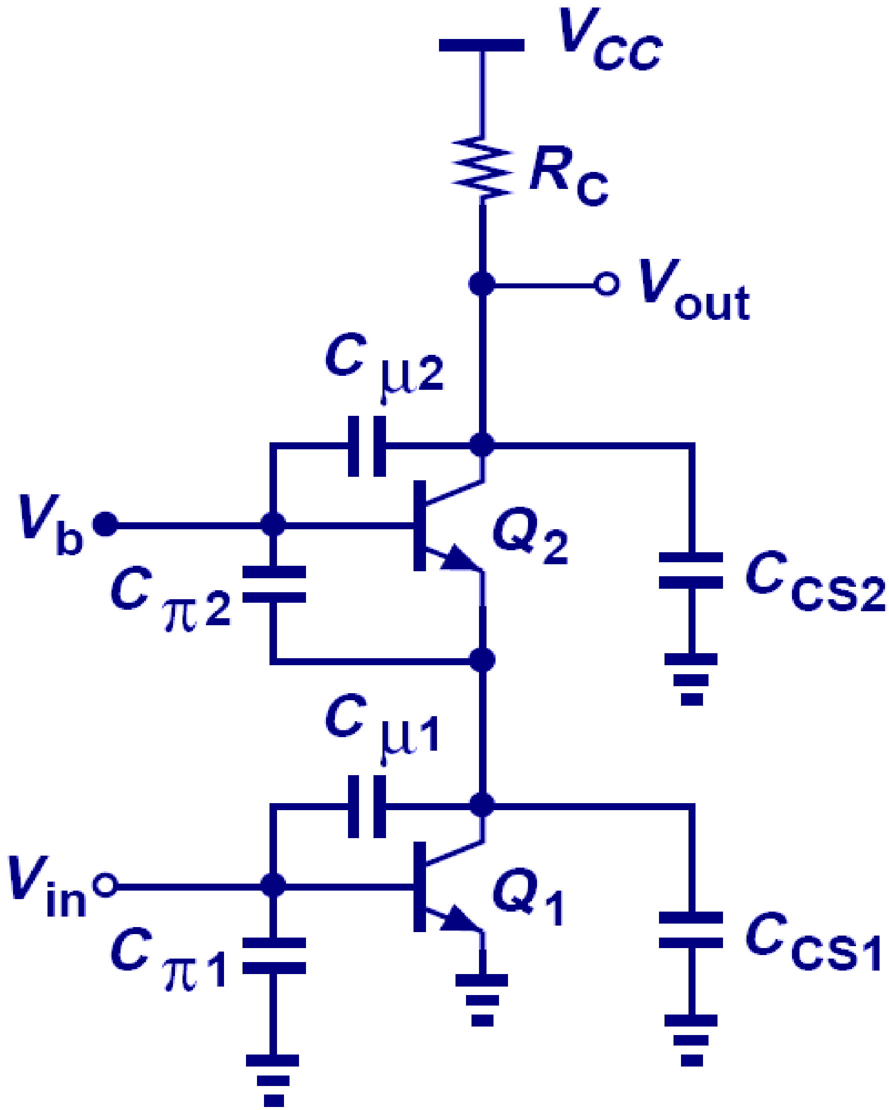
High-Frequency Model of Integrated Bipolar Transistor



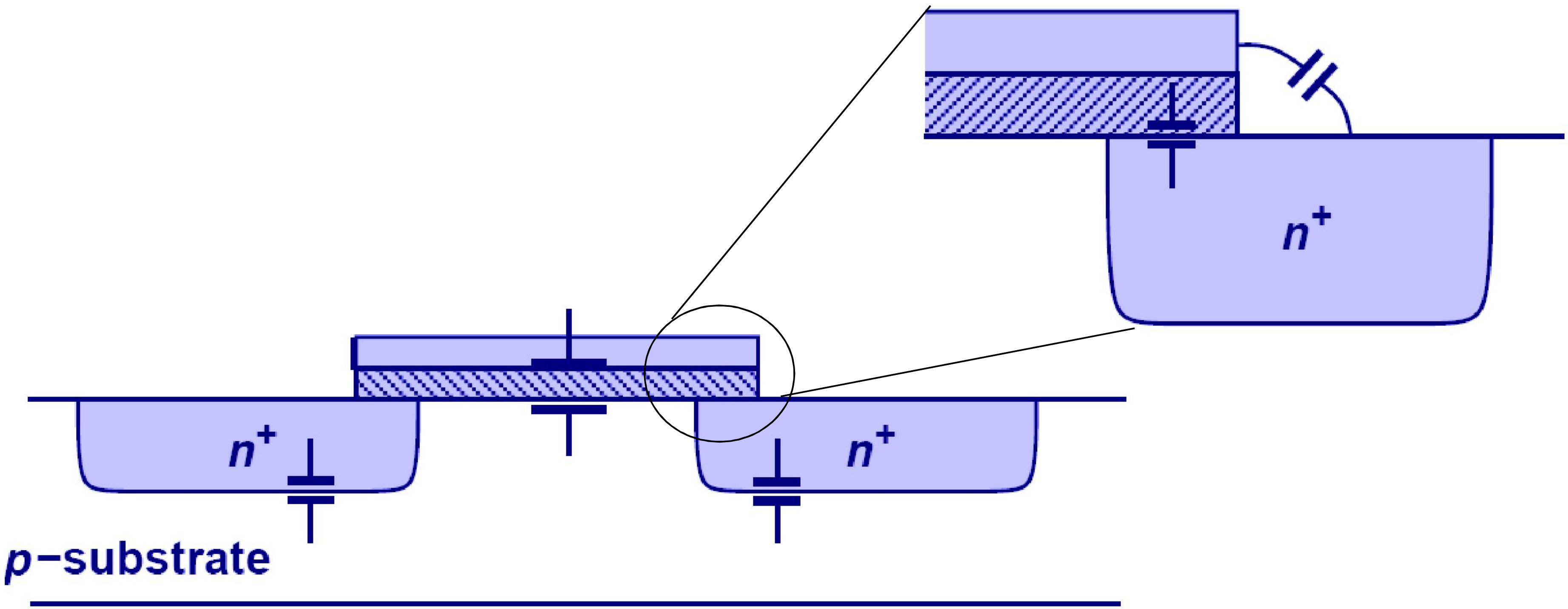
- **Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely C_{CS}.**

Example: Capacitance Identification



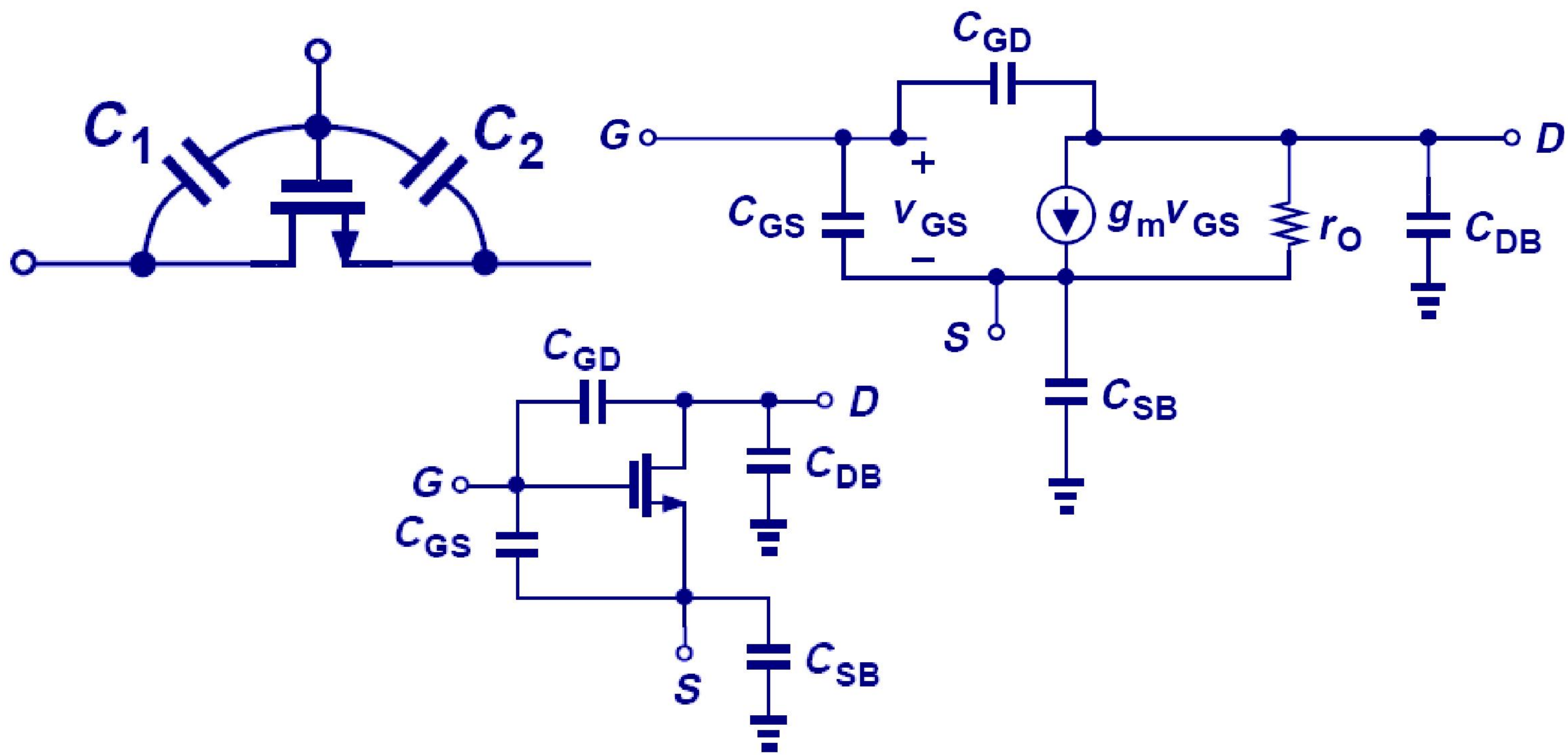


MOS Intrinsic Capacitances



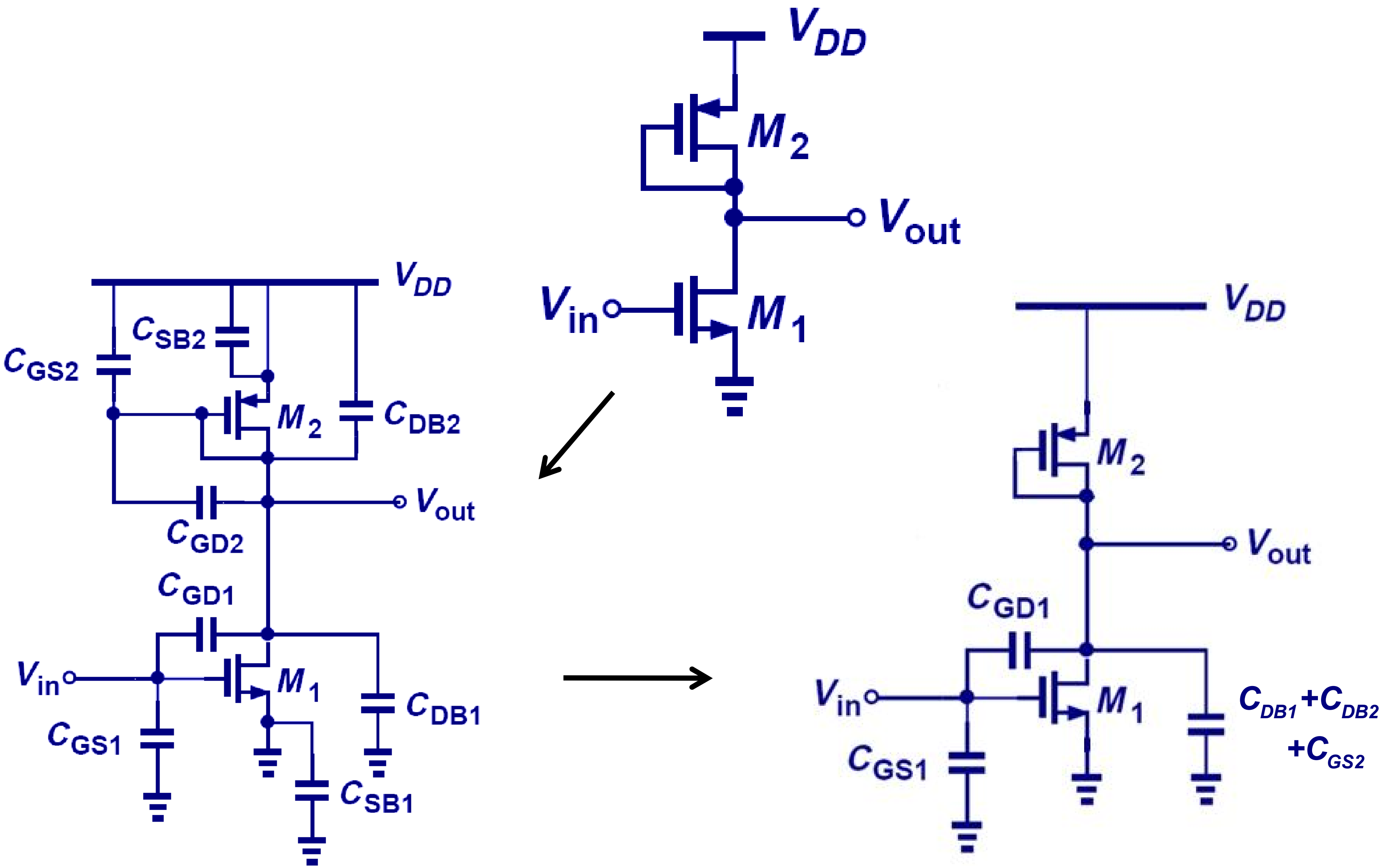
- For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model

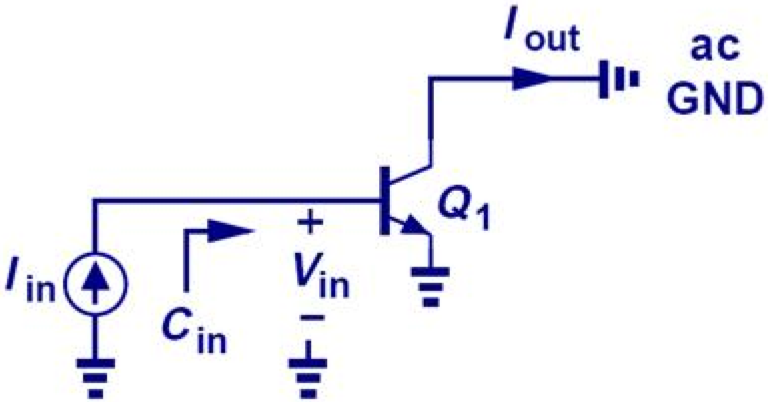


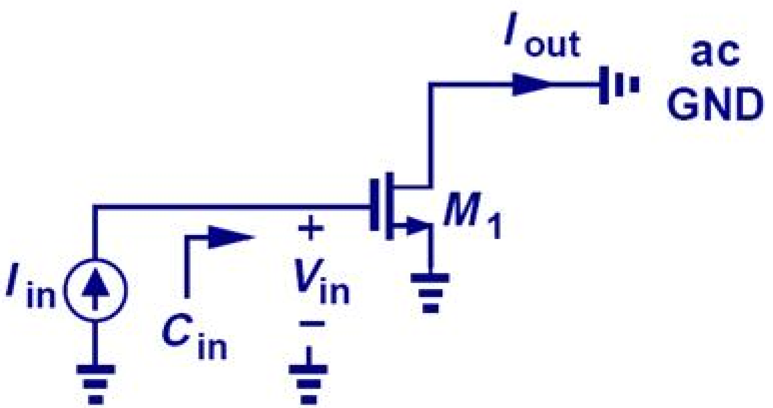
- The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_2 \sim C_{gate}$, and $C_1 \sim 0$. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD}.

Example: Capacitance Identification



Transit Frequency





$$Z_{in} = \frac{1}{C_\pi s} \| r_\pi, \quad I_{out} = g_m I_{in} Z_{in}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = \frac{g_m r_\pi}{r_\pi C_\pi s + 1} = \frac{\beta}{r_\pi C_\pi s + 1}$$

$$\left|\frac{I_{out}}{I_{in}}\right| = 1 \quad \Rightarrow \quad r_\pi^2 C_\pi^2 \omega_T^2 = \beta^2 - 1 \approx \beta^2$$

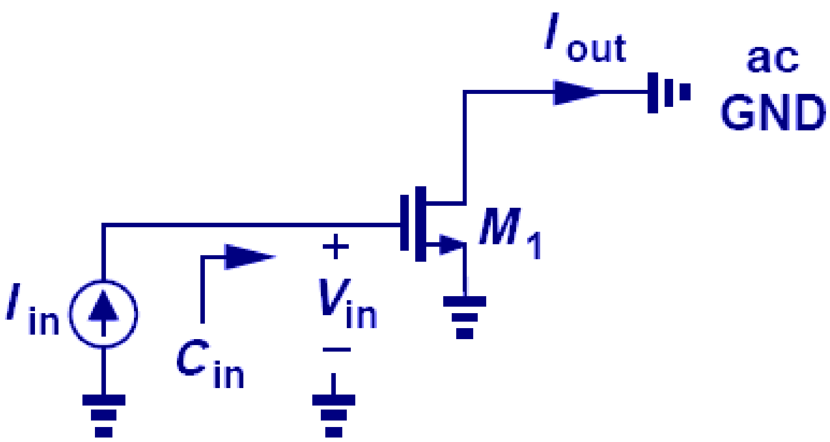
$$\Rightarrow \omega_T = 2\pi f_T \approx \frac{g_m}{C_\pi}$$

The transit frequency of MOSFETs is obtained in a similar fashion.

$$\omega_T = 2\pi f_T \approx \frac{g_m}{C_{GS}}$$

- **Transit frequency, f_T, is defined as the frequency where the current gain from input to output drops to 1.**

Example: Transit Frequency Calculation



From Problem 11.28,

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$\Rightarrow \frac{f_{T,today}}{f_{T,1980s}} = \frac{100}{\left(65\times10^{-9}\right)^2} \bigg/ \frac{400}{\left(1\times10^{-6}\right)^2} \approx 59$$

If $\mu_n = 400\ cm^2/(V\cdot s)$,

$$f_{T,today} \approx 226\ GHz$$

- **The minimum channel length of MOSFETs has been scaled from 1μm in the late 1980s to 65nm today. Also, the inevitable reduction of the supply voltage has reduced the gate-source overdrive voltage from about 400mV to 100mV. By what factor has the f_T of MOSFETs increased?**

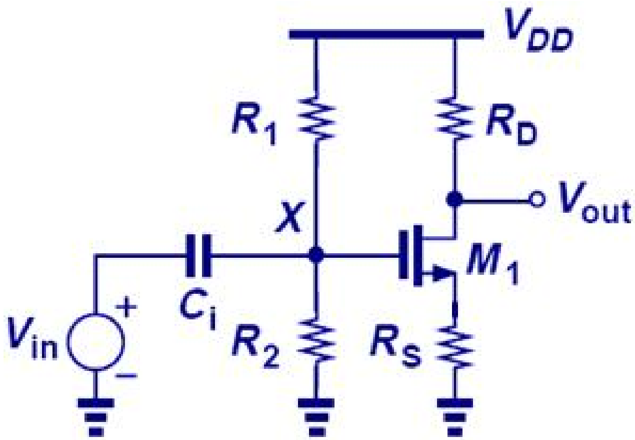
Analysis Summary

- **The frequency response refers to the magnitude of the transfer function.**
- **Bode's approximation simplifies the plotting of the frequency response if poles and zeros are known.**
- **In general, it is possible to associate a pole with each node in the signal path.**
- **Miller's theorem helps to decompose floating capacitors into grounded elements.**
- **Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.**

High Frequency Circuit Analysis Procedure

- Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).
- Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).
- Include transistor capacitances.
- Merge capacitors connected to AC grounds and omit those that play no role in the circuit.
- Determine the high-frequency poles and zeros.
- Plot the frequency response using Bode's rules or exact analysis.

Frequency Response of CS Stage

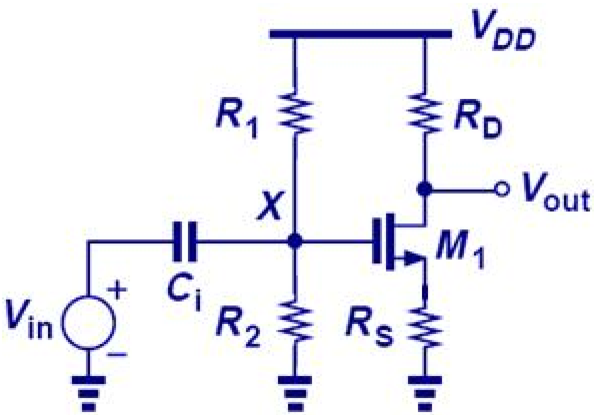


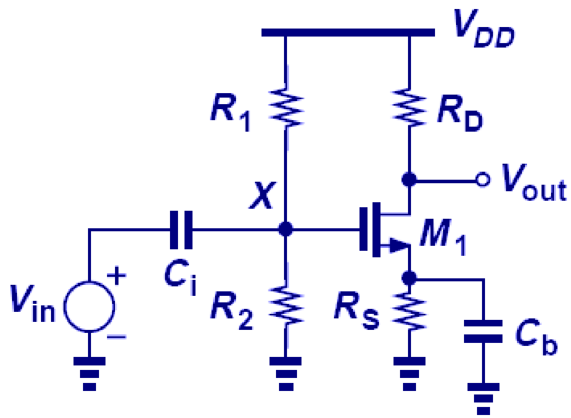
$$\frac{V_X}{V_{in}}(s) = \frac{R_1 \| R_2}{R_1 \| R_2 + \frac{1}{C_i s}} = \frac{(R_1 \| R_2) C_i s}{(R_1 \| R_2) C_i s + 1}$$

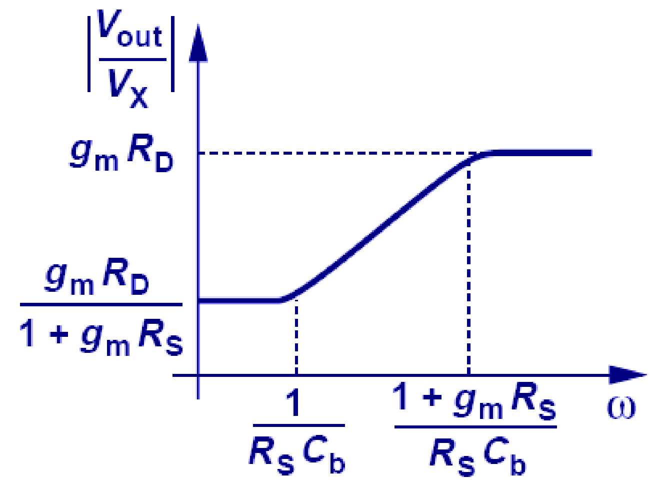
$$\text{Thus,} \quad \frac{1}{2\pi\left[(R_1 \| R_2) C_i\right]} < f_{sig,\min}$$

- *Ci* acts as a high pass filter.
- Lower cut-off frequency must be lower than the lowest signal frequency $f_{sig,min}$ (20 Hz in audio applications).

Frequency Response of CS Stage with Bypassed Degeneration





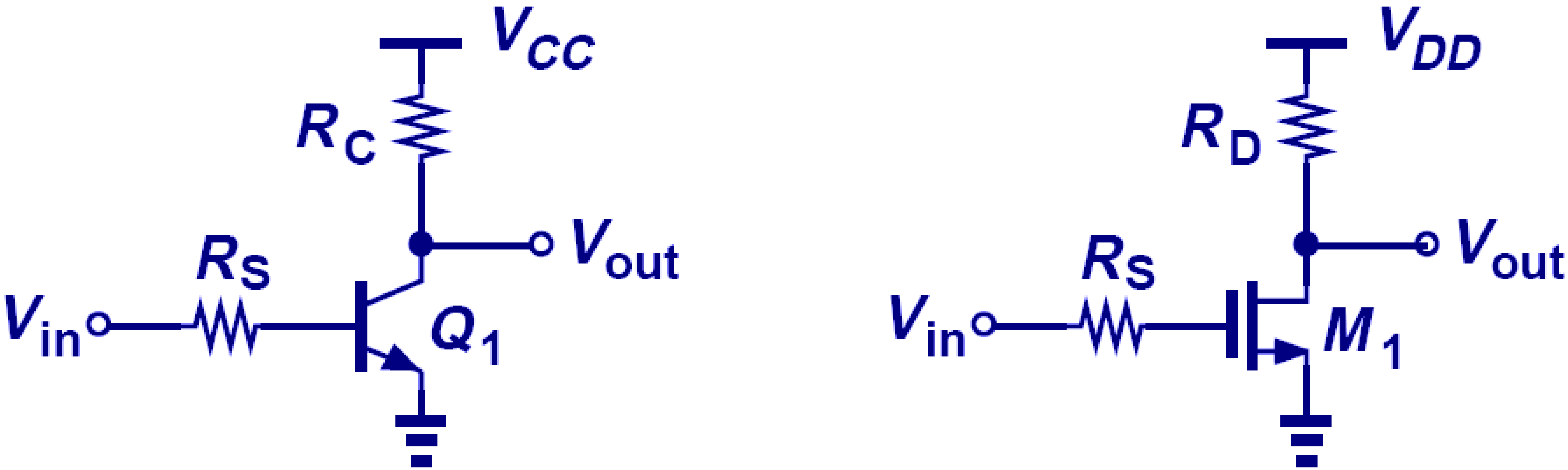


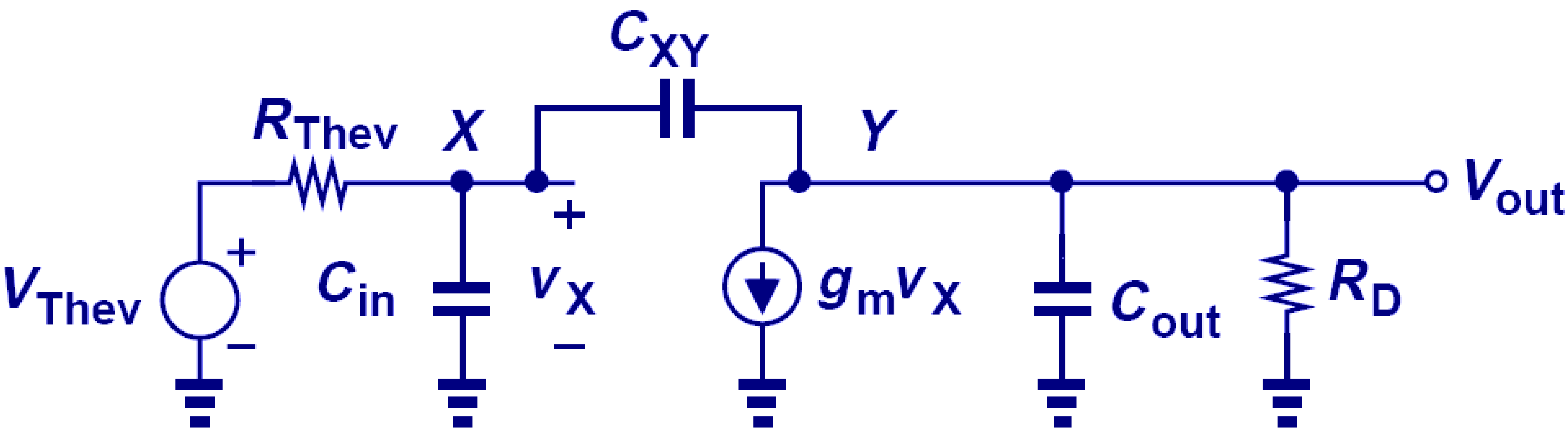
$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S + \frac{1}{g_m}}$$

$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S \parallel \frac{1}{C_b s} + \frac{1}{g_m}} = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

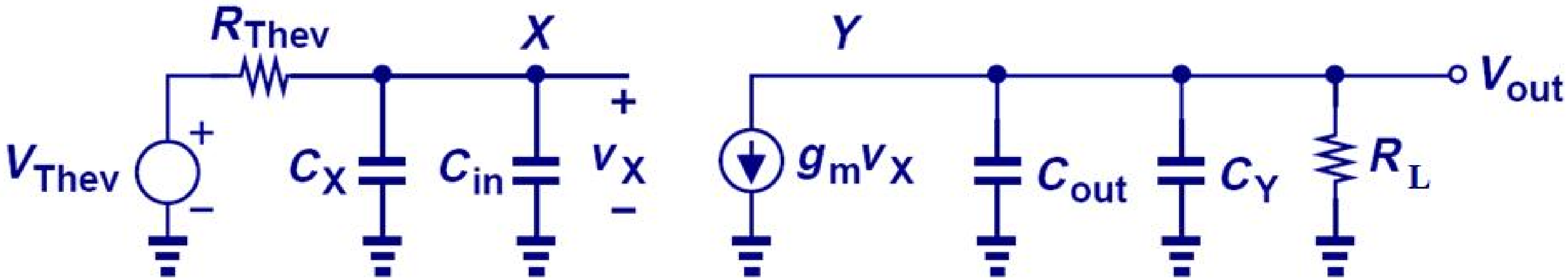
- **In order to increase the midband gain, a capacitor C_b is placed in parallel with R_s.**
- **The pole frequency must be well below the lowest signal frequency to avoid the effect of degeneration.**

Unified Model for CE and CS Stages





Unified Model Using Miller's Theorem



$$\left|\omega_{p,in}\right| = \frac{1}{R_{Thev}\left[C_{in} + \left(1 + g_m R_L\right) C_{XY}\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[C_{out} + \left(1 + \frac{1}{g_m R_L}\right) C_{XY}\right]}$$

CE Stage

$$V_{Thev} = V_{in}\frac{r_\pi}{r_\pi + R_S}$$

$$R_{Thev} = R_S \parallel r_\pi$$

$$C_X = C_\mu (1 + g_m R_L)$$

$$C_Y = C_\mu \left(1 + \frac{1}{g_m R_L}\right)$$

CS Stage

$$V_{Thev} = V_{in}$$

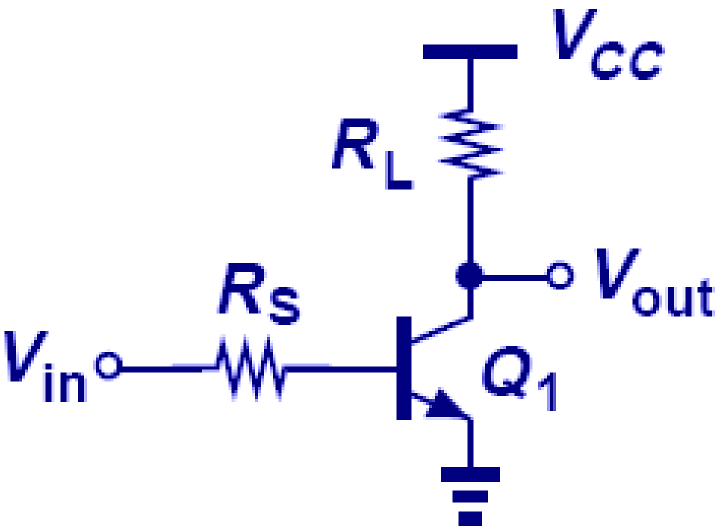
$$R_{Thev} = R_S$$

$$C_X = C_{GD} (1 + g_m R_L)$$

$$C_Y = C_{GD} \left(1 + \frac{1}{g_m R_L}\right)$$

Example: CE Stage

- (a) Calculate the input and output poles if R_L=2 kΩ. Which node appears as the speed bottleneck?



$$|\omega_{p,in}| = \frac{1}{(R_S \| r_\pi)[C_\pi + (1 + g_m R_L) C_\mu]}$$

$$|\omega_{p,out}| = \frac{1}{R_L\left[C_{CS} + \left(1 + \frac{1}{g_m R_L}\right) C_\mu\right]}$$

$$R_S = 200\ \Omega,\quad I_C = 1\text{ mA}$$
$$\beta = 100,\quad C_\pi = 100\text{ fF}$$
$$C_\mu = 20\text{ fF},\quad C_{CS} = 30\text{ fF}$$

$$|\omega_{\text{p,in}}| = 2\pi \times (516\text{ MHz})$$
$$|\omega_{\text{p,out}}| = 2\pi \times (1.59\text{ GHz})$$

Example: CE Stage – cont'd

- **(b) Is it possible to choose R_L such that the output pole limits the bandwidth?**

$$\left|\omega_{p,in}\right| > \left|\omega_{p,out}\right|$$

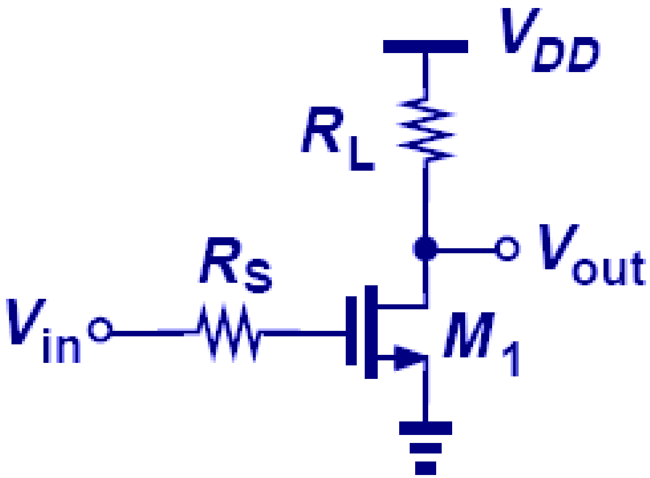
$$\Rightarrow \frac{1}{\left(R_S \| r_\pi\right)\left[C_\pi + \left(1+g_m R_L\right)C_\mu\right]} > \frac{1}{R_L\left[C_{CS} + \left(1+\frac{1}{g_m R_L}\right)C_\mu\right]}$$

If $g_m R_L \gg 1$,

$$\Rightarrow \left[C_{CS} + C_\mu - g_m\left(R_S \| r_\pi\right)C_\mu\right]R_L > \left(R_S \| r_\pi\right)C_\pi$$

With the values assumed in this example, the left-hand side is negative, implying that no solution exists. Thus, the input pole remains the speed bottleneck.

Example: Half Width CS Stage



$$W \downarrow 2X$$

$$\text{bias current} \downarrow 2X$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \downarrow 2X$$

$$\text{capacitances} \downarrow 2X$$

$$\left|\omega_{p,in}\right| = \frac{1}{R_S\left[\frac{C_{in}}{2} + \left(1 + \frac{g_m R_L}{2}\right)\frac{C_{XY}}{2}\right]}$$

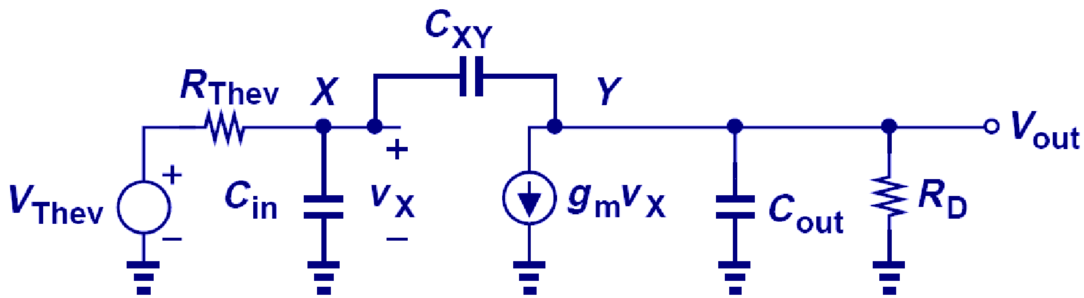
$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[\frac{C_{out}}{2} + \left(1 + \frac{2}{g_m R_L}\right)\frac{C_{XY}}{2}\right]}$$

$$\text{bandwidth} \uparrow 2X$$

$$\text{gain} \downarrow 2X$$

$$\text{gain} \cdot \text{bandwidth} \rightarrow \text{constant}$$

Direct Analysis of CE and CS Stages



$$\text{At Node Y: } \left(V_X - V_{out}\right) C_{XY} s = g_m V_X + V_{out}\left(\frac{1}{R_L} + C_{out} s\right) \Rightarrow V_X = V_{out}\frac{C_{XY}s + \dfrac{1}{R_L} + C_{out}s}{C_{XY}s - g_m}$$

$$\text{At Node X: } \left(V_{out} - V_X\right) C_{XY} s = V_X C_{in} s + \frac{V_X - V_{Thev}}{R_{Thev}}$$

$$\Rightarrow V_{out} C_{XY} s - \left(C_{XY}s + C_{in}s + \frac{1}{R_{Thev}}\right)\frac{C_{XY}s + \dfrac{1}{R_L} + C_{out}s}{C_{XY}s - g_m} V_{out} = \frac{-V_{Thev}}{R_{Thev}}$$

$$\Rightarrow \frac{V_{out}}{V_{Thev}}(s) = \frac{\left(C_{XY}s - g_m\right) R_L}{as^2 + bs + 1} \quad \text{where } a = R_{Thev} R_L \left(C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out}\right),$$
$$b = \left(1 + g_m R_L\right) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L \left(C_{XY} + C_{out}\right)$$

Direct Analysis of CE and CS Stages – cont'd

$$|\omega_z| = \frac{g_m}{C_{XY}}$$

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

$$if\ \ \omega_{p2} \gg \omega_{p1} \quad \Rightarrow \quad \omega_{\mathrm{p1}}^{-1} + \omega_{\mathrm{p2}}^{-1} \approx \omega_{\mathrm{p1}}^{-1}$$

Dominant-pole approximation

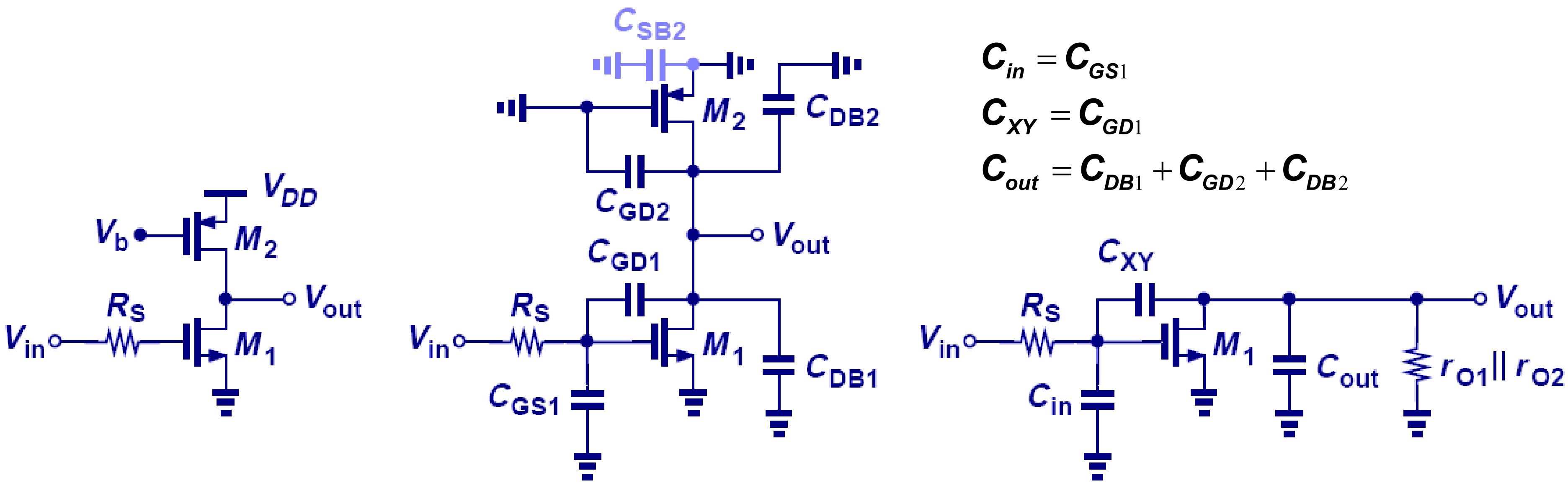
$$\Rightarrow b = \frac{1}{\omega_{p1}}$$

$$|\omega_{p1}| = \frac{1}{(1+g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})}$$

$$|\omega_{p2}| = \frac{b}{a} = \frac{(1+g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})}{R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

- **Direct analysis yields different pole locations and an extra zero.**

Example: Dominant-pole approximation

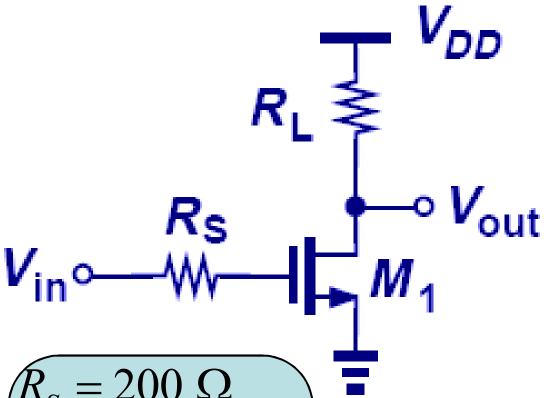


$$C_{in} = C_{GS1}$$
$$C_{XY} = C_{GD1}$$
$$C_{out} = C_{DB1} + C_{GD2} + C_{DB2}$$

$$\omega_{p1} \approx \frac{1}{[1+g_{m1}(r_{O1} \| r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \| r_{O2})(C_{XY} + C_{out})}$$

$$\omega_{p2} \approx \frac{[1+g_{m1}(r_{O1} \| r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \| r_{O2})(C_{XY} + C_{out})}{R_S(r_{O1} \| r_{O2})(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

Example: Comparison Between Different Methods



$R_S = 200\ \Omega$

$C_{GS} = 250\ \text{fF}$

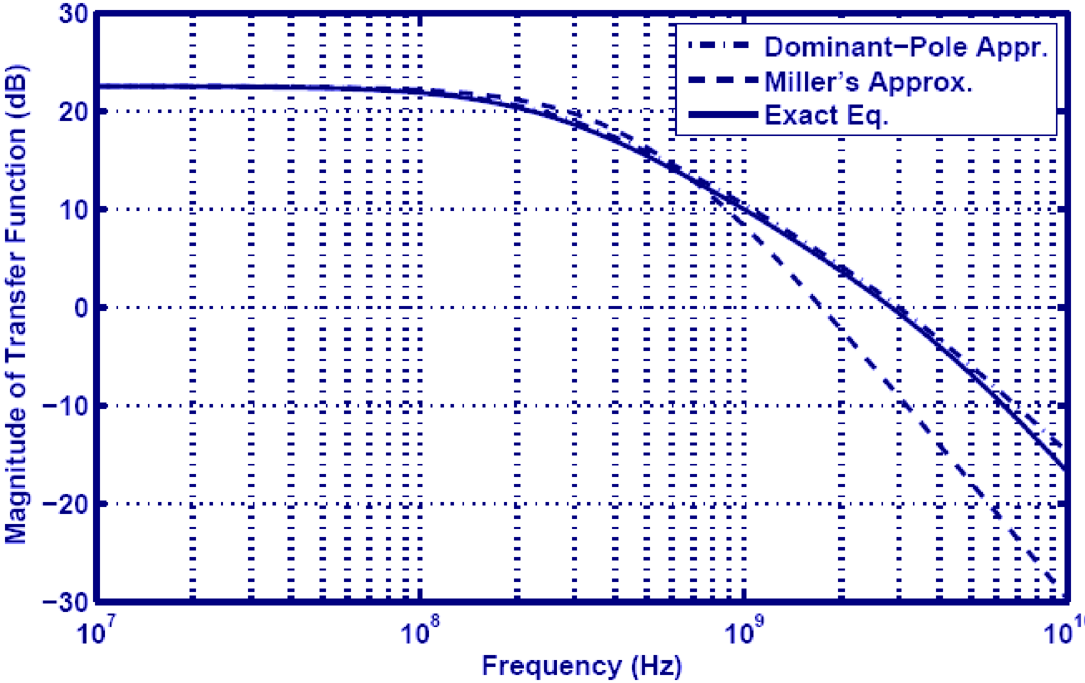
$C_{GD} = 80\ \text{fF}$

$C_{DB} = 100\text{fF}$

$g_m = (150\ \Omega)^{-1}$

$\lambda = 0$

$R_L = 2\ \text{k}\Omega$



This error arises because we have multiplied C_{GD} by the midband gain $(1+g_mR_L)$ rather than the gain at high frequencies.

Miller's

$$|\omega_{p,in}| = 2\pi \times (571\ \text{MHz})$$
$$|\omega_{p,out}| = 2\pi \times (428\ \text{MHz})$$

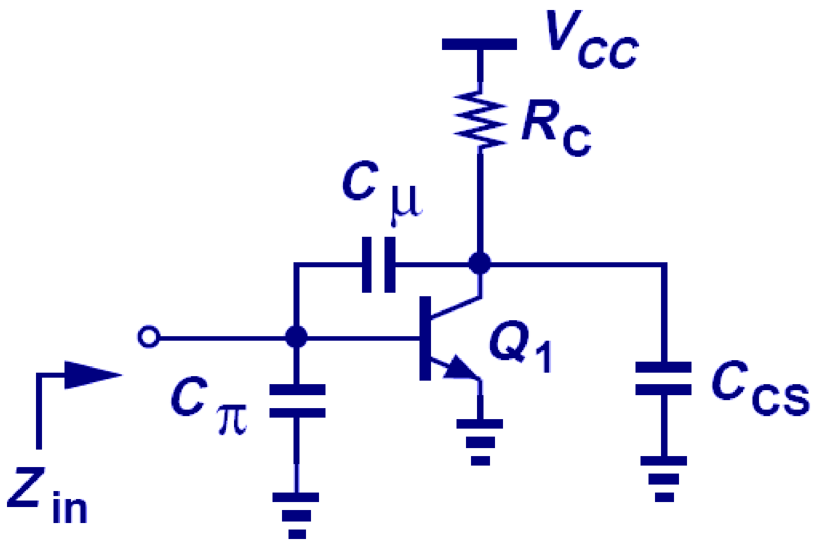
Exact

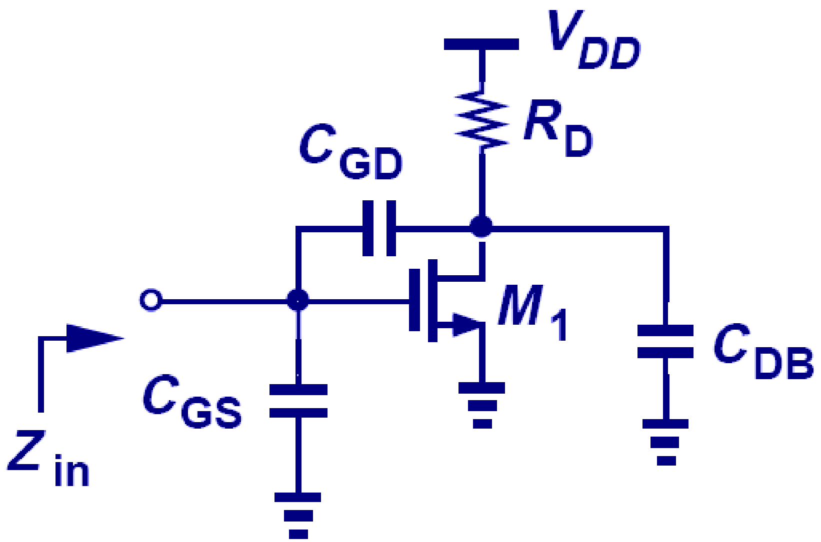
$$|\omega_{p,in}| = 2\pi \times (264\ \text{MHz})$$
$$|\omega_{p,out}| = 2\pi \times (4.53\ \text{GHz})$$

Dominant Pole

$$|\omega_{p,in}| = 2\pi \times (249\ \text{MHz})$$
$$|\omega_{p,out}| = 2\pi \times (4.79\ \text{GHz})$$

Input Impedance of CE and CS Stages

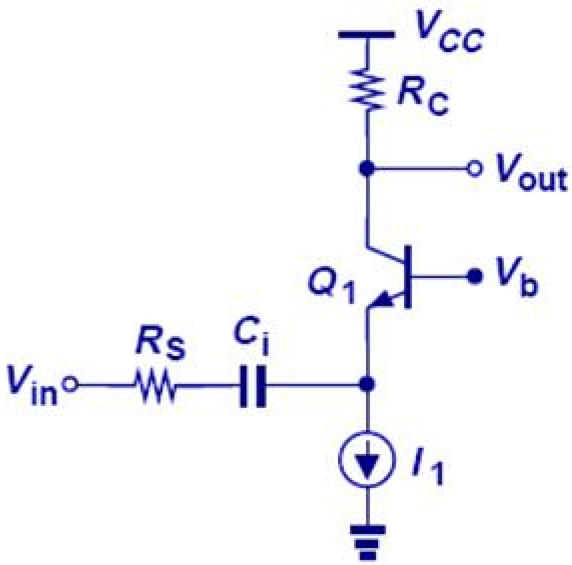


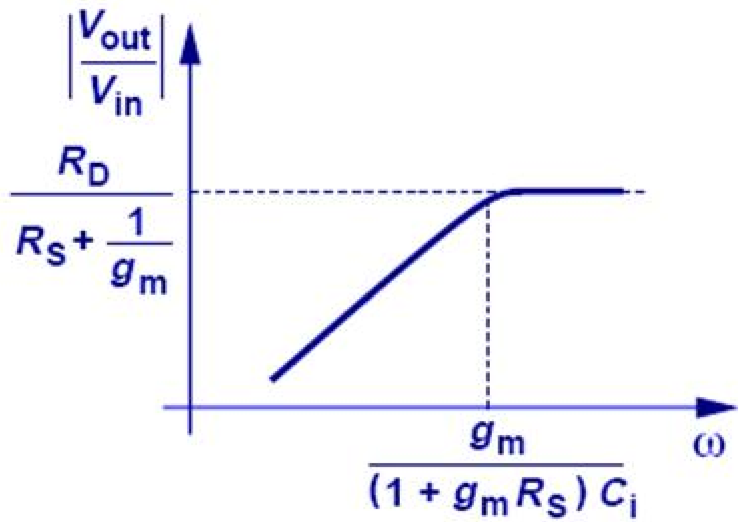


$$Z_{in} \approx \frac{1}{\left[C_\pi + (1+g_m R_C)C_\mu\right]s} \,||\, r_\pi$$

$$Z_{in} \approx \frac{1}{\left[C_{GS} + (1+g_m R_D)C_{GD}\right]s}$$

Low Frequency Response of CB and CG Stages

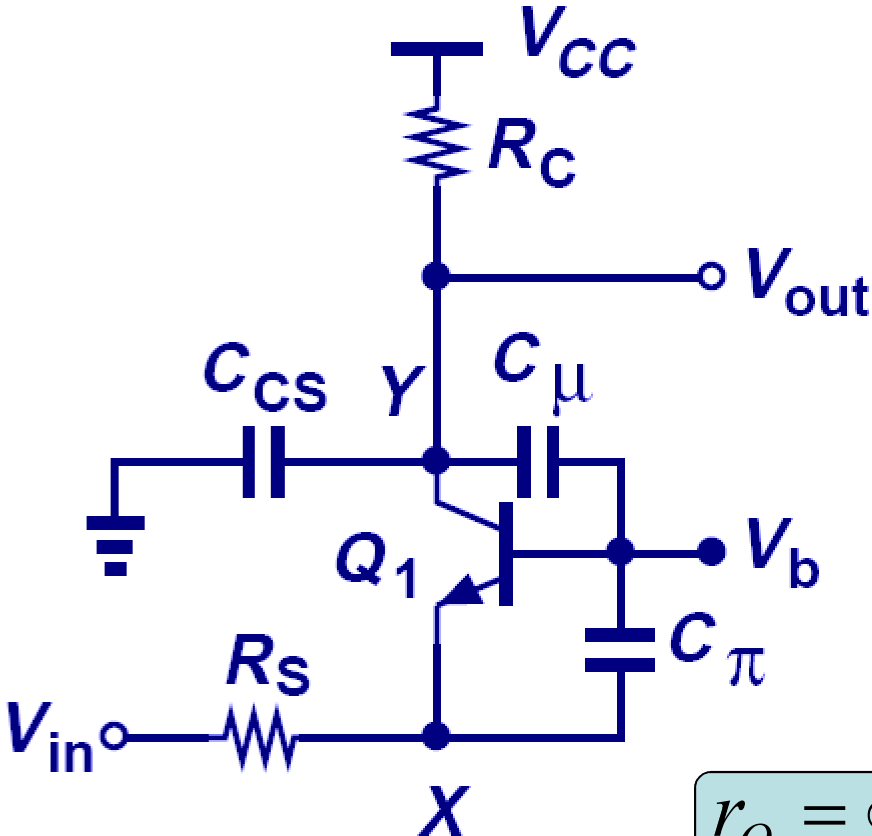




$$\frac{V_{out}}{V_{in}}(s) = \frac{R_C}{R_S + (C_i s)^{-1} + 1/g_m} = \frac{g_m R_C C_i s}{(1 + g_m R_S) C_i s + g_m}$$

- As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).

Frequency Response of CB Stage



$$r_O = \infty$$

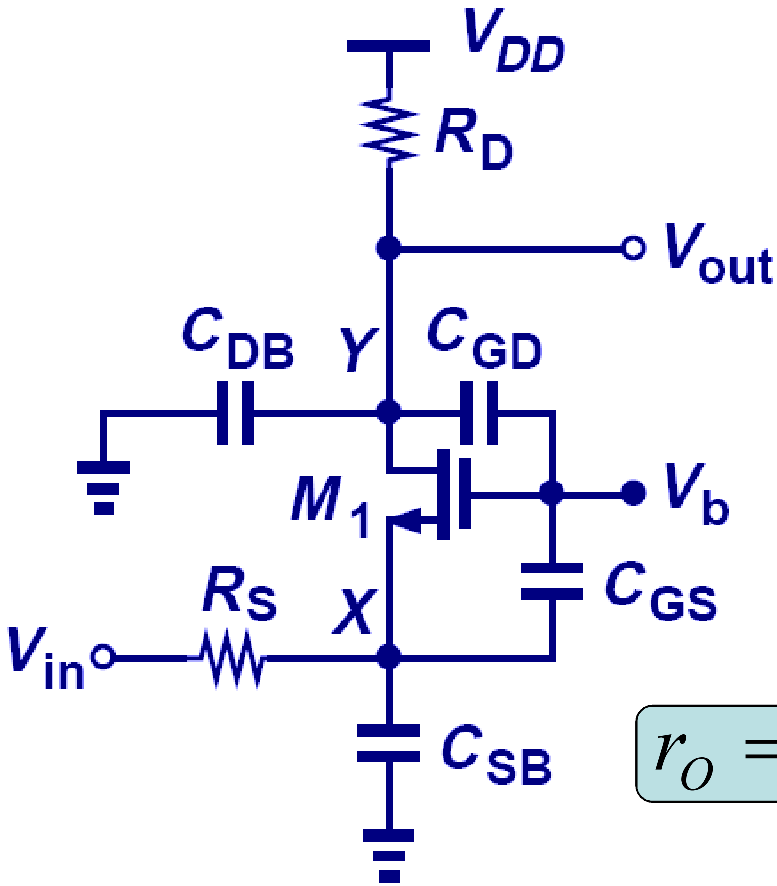
$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

$$C_X = C_\pi$$

$$\omega_{p,Y} = \frac{1}{R_C C_Y}$$

$$C_Y = C_\mu + C_{CS}$$

Frequency Response of CG Stage



$$r_O = \infty$$

$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

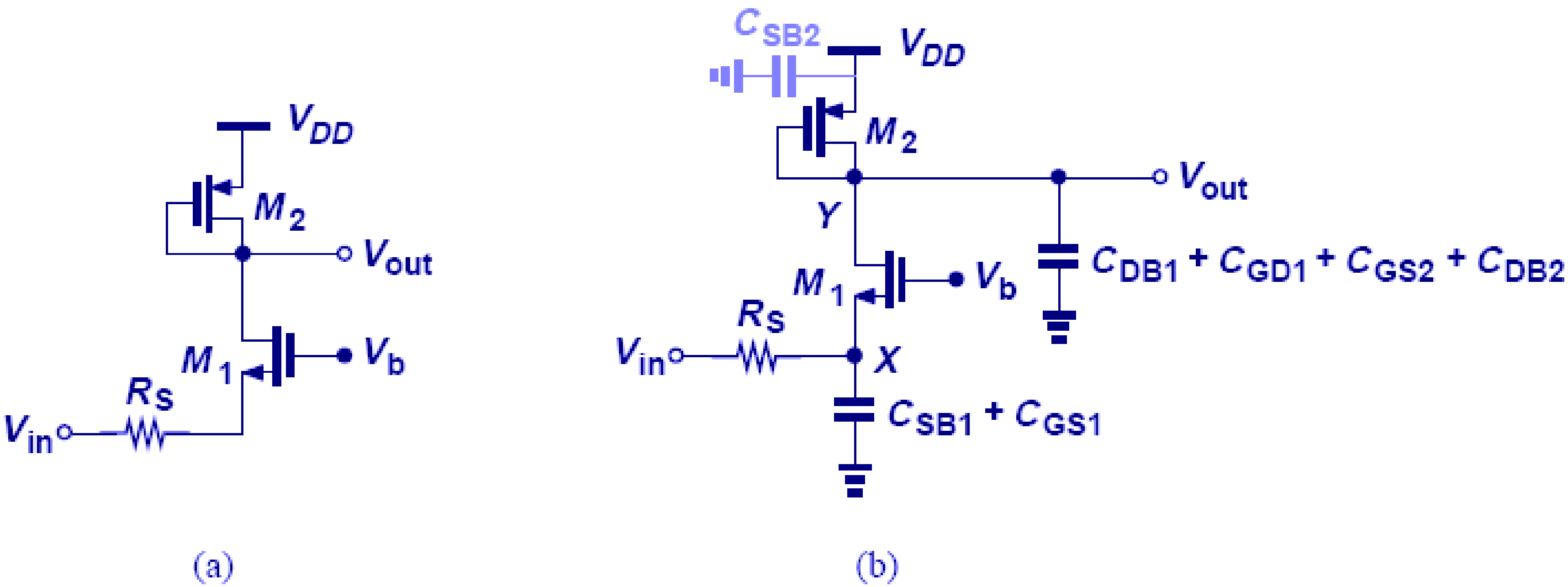
$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_D C_Y}$$

$$C_Y = C_{GD} + C_{DB}$$

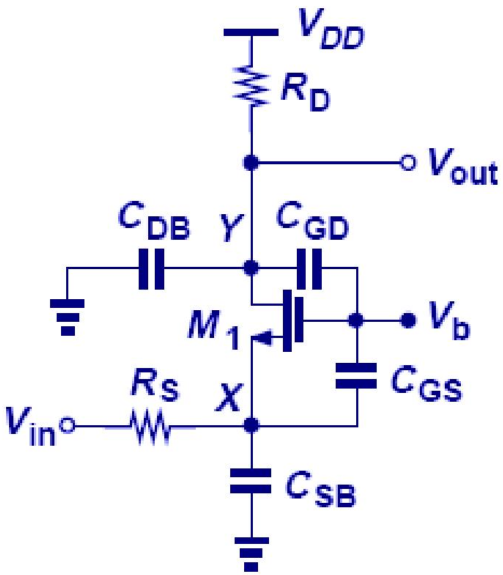
- Similar to a CB stage, the input pole is on the order of f_T, so rarely a speed bottleneck.

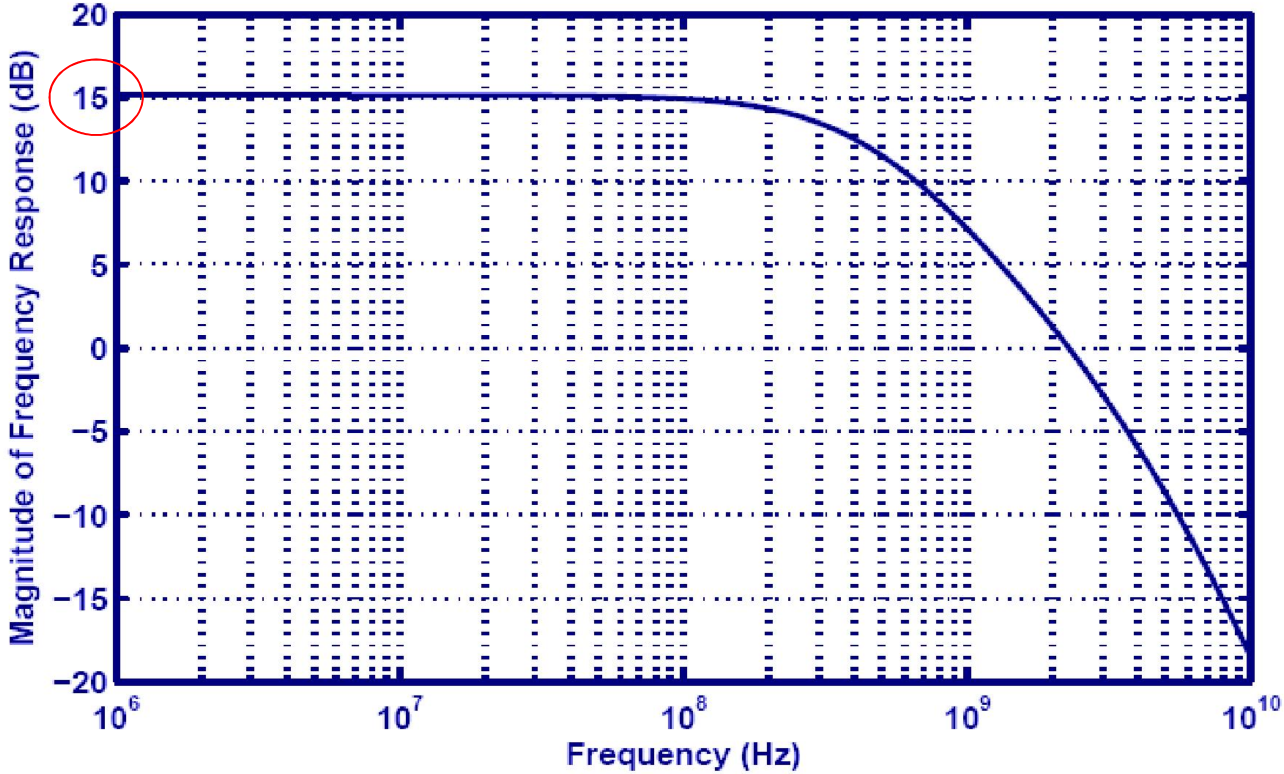
Example: CG Stage Pole Identification



$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_{m1}}\right)\left(C_{SB1} + C_{GS1}\right)} \qquad \omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left(C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2}\right)}$$

Example: Frequency Response of CG Stage





$R_S = 200\ \Omega$

$C_{GS} = 250\ \text{fF}$

$C_{GD} = 80\ \text{fF}$

$C_{DB} = 100\ \text{fF}$

$g_m = (150\ \Omega)^{-1}$

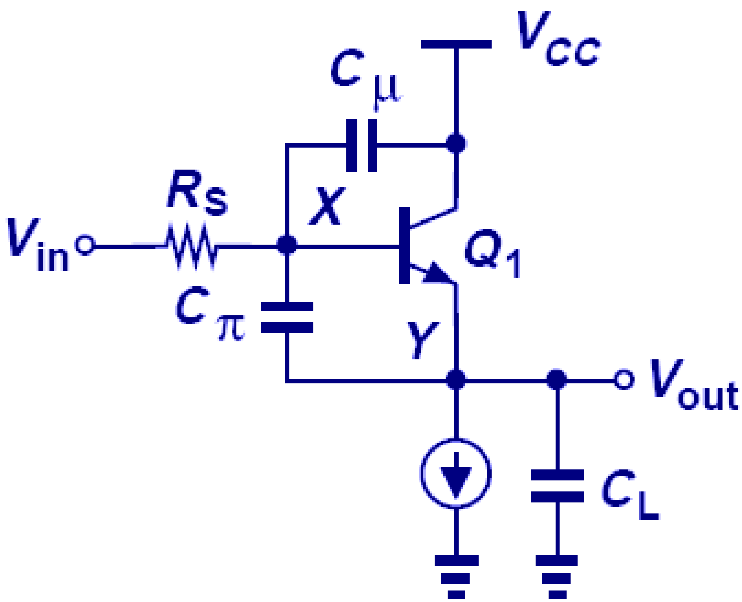
$\lambda = 0$

$R_D = 2\ \text{k}\Omega$

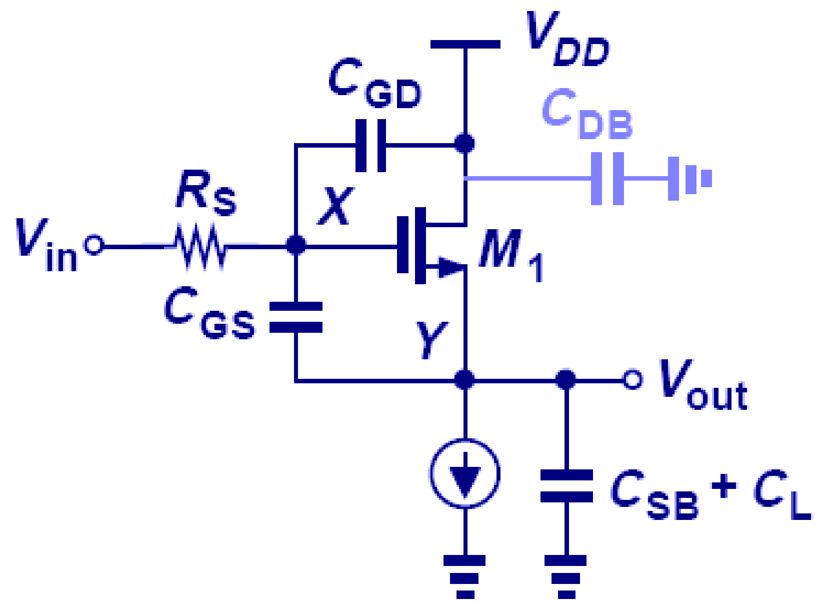
$$\left|\omega_{p,X}\right| = 1/\left(R_S \parallel \frac{1}{g_m}\right)C_X = 2\pi\times(5.31\ \text{GHz})$$

$$\left|\omega_{p,Y}\right| = R_L / C_Y = 2\pi\times(442\ \text{MHz})$$

Emitter and Source Followers



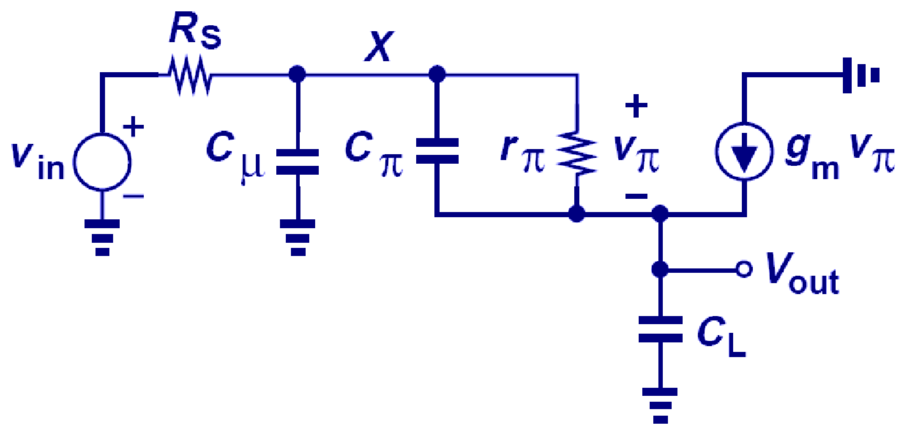
(a)



(b)

- The following will discuss the frequency response of emitter and source followers using direct analysis.
- Emitter follower is treated first and source follower is derived easily by allowing r_π to go to infinity.

Direct Analysis of Emitter Follower



At node X: $$\frac{V_{out}+V_\pi-V_{in}}{R_S}+\left(V_{out}+V_\pi\right)C_\mu s+\frac{V_\pi}{r_\pi}+V_\pi C_\pi s=0$$

At output node: $$\frac{V_\pi}{r_\pi}+V_\pi C_\pi s+g_m V_\pi=V_{out}C_L s \quad\Rightarrow\quad V_\pi=\frac{V_{out}C_L s}{\dfrac{1}{r_\pi}+C_\pi s+g_m}$$

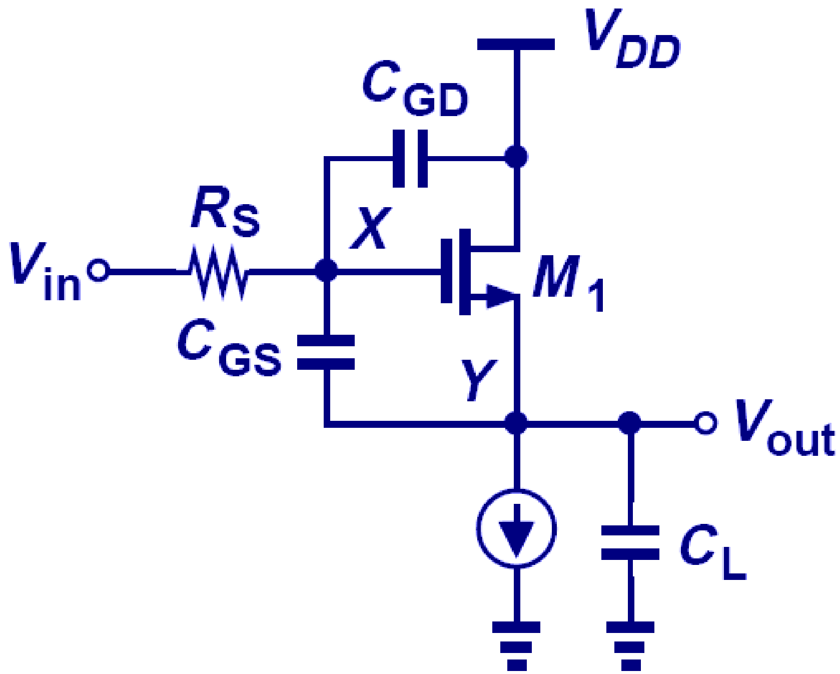
$$\boxed{\frac{V_{out}}{V_{in}}=\frac{1+\dfrac{C_\pi}{g_m}s}{as^2+bs+1}}$$

with $r_\pi \gg g_m^{-1}$

$$\boxed{\begin{aligned}\text{where } a&=\frac{R_S}{g_m}\left(C_\mu C_\pi+C_\mu C_L+C_\pi C_L\right)\\ b&=R_S C_\mu+\frac{C_\pi}{g_m}+\left(1+\frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}\end{aligned}}$$

$$\left|\omega_z\right|=\frac{g_m}{C_\pi}\approx f_T$$

Direct Analysis of Source Follower Stage

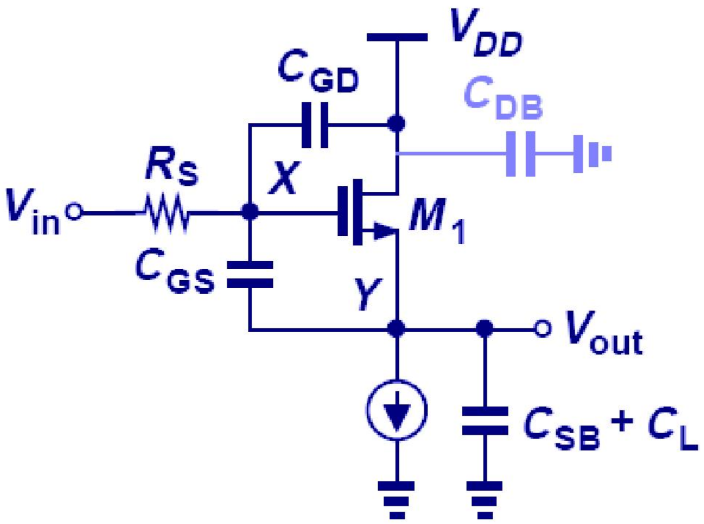


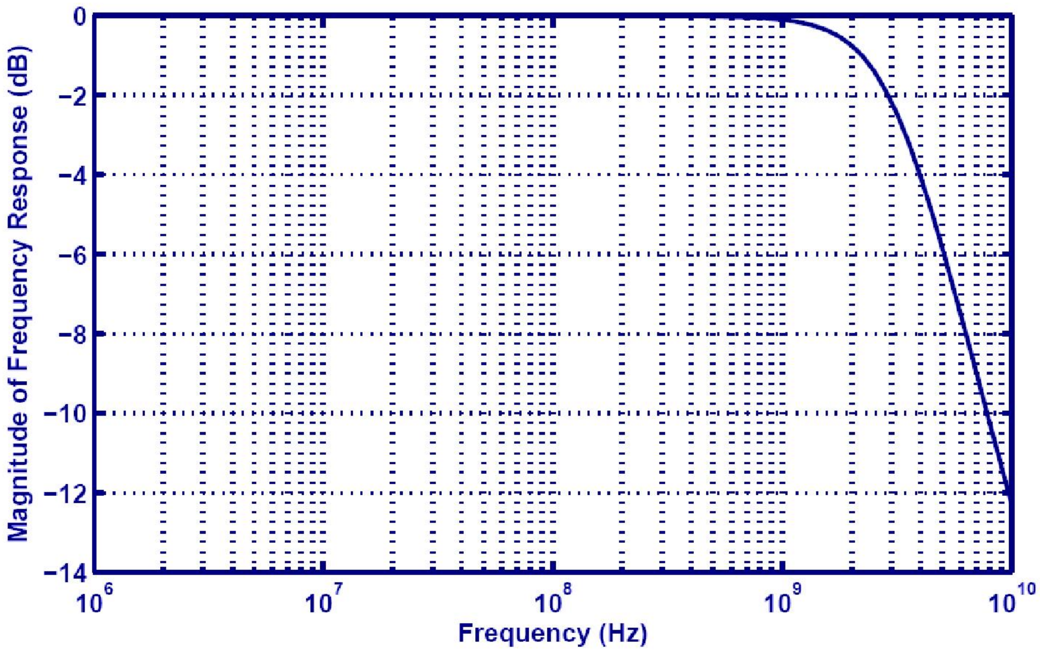
$$\frac{V_{out}}{V_{in}} = \frac{1 + \dfrac{C_{GS}}{g_m}s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_m}\left(C_{GD}C_{GS} + C_{GD}\left(C_{SB} + C_L\right) + C_{GS}\left(C_{SB} + C_L\right)\right)$$

$$b = R_S C_{GD} + \frac{C_{GD} + C_{SB} + C_L}{g_m}$$

Example: Frequency Response of Source Follower

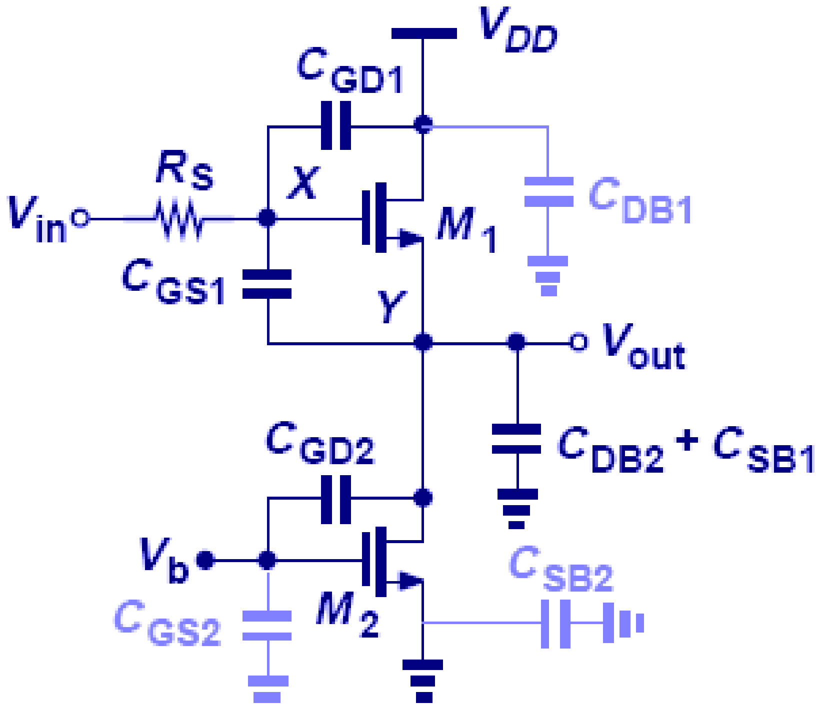




$$R_S = 200\ \Omega$$
$$C_L = 100\ \text{fF}$$
$$C_{GS} = 250\ \text{fF}$$
$$C_{GD} = 80\ \text{fF}$$
$$C_{DB} = 100\ \text{fF}$$
$$g_m = (150\ \Omega)^{-1}$$
$$\lambda = 0$$

$$a = 2.58 \times 10^{-21} s^{-2}$$
$$b = 5.8 \times 10^{-11} s$$
$$\omega_z = g_m / C_{GS} = 2\pi \times (4.24\ \text{GHz})$$
$$\omega_{p1} = 2\pi \left[-1.79\ \text{GHz} + j(2.57\ \text{GHz}) \right]$$
$$\omega_{p2} = 2\pi \left[-1.79\ \text{GHz} - j(2.57\ \text{GHz}) \right]$$

Example: Source Follower

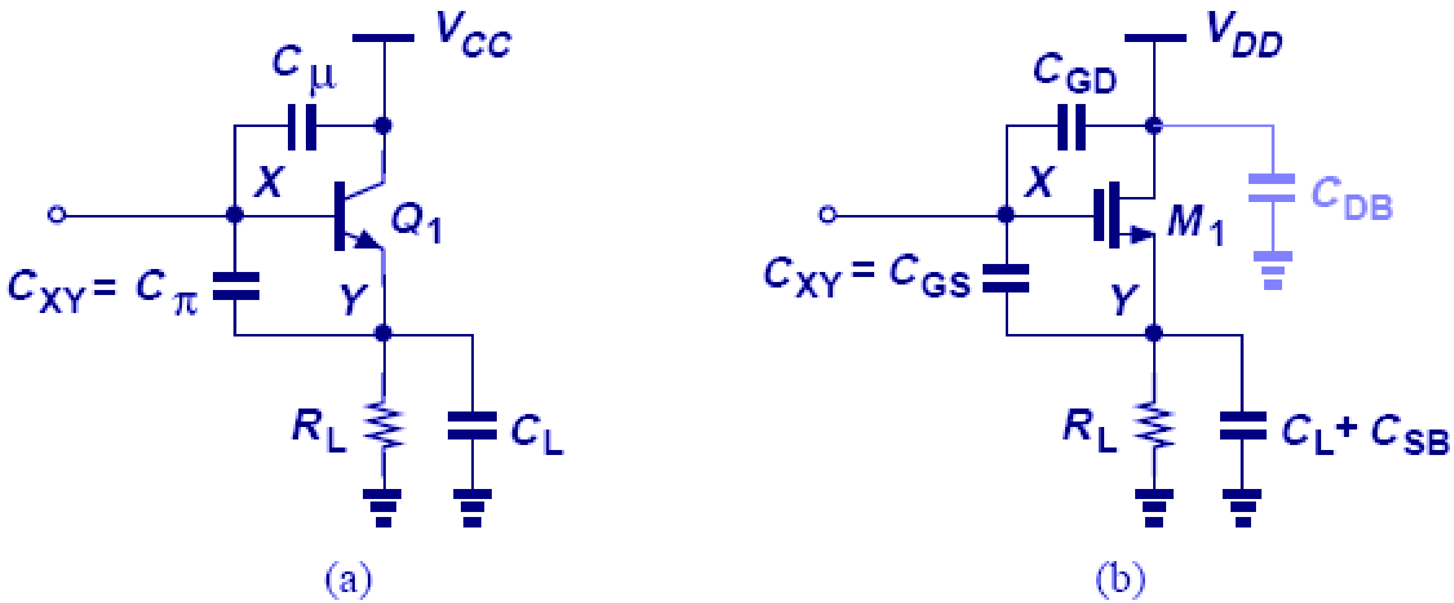


$$\frac{V_{out}}{V_{in}} = \frac{1+\dfrac{C_{GS}}{g_m}s}{as^2+bs+1}$$

$$a = \frac{R_S}{g_{m1}}\left[C_{GD1}C_{GS1} + (C_{GD1}+C_{GS1})(C_{SB1}+C_{GD2}+C_{DB2})\right]$$

$$b = R_S C_{GD1} + \frac{C_{GD1}+C_{SB1}+C_{GD2}+C_{DB2}}{g_{m1}}$$

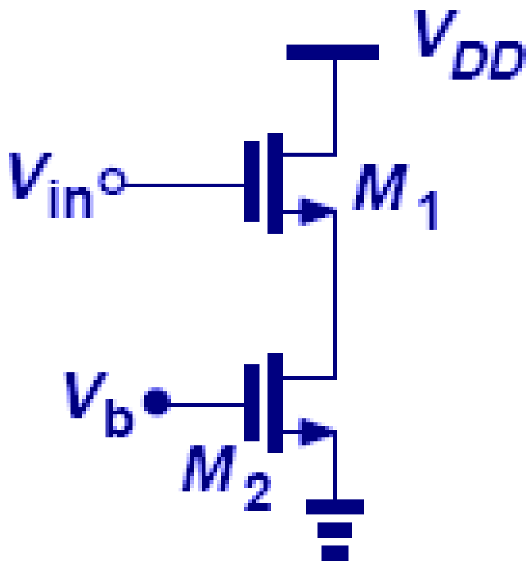
Input Capacitance of Emitter/Source Follower



$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} \Rightarrow C_X = (1 - A_v) C_{XY} = \frac{1}{1 + g_m R_L} C_{XY}$$

$$\therefore \boxed{C_{in} = (C_\mu \ or \ C_{GD}) + \frac{C_{XY}}{1 + g_m R_L}}$$

Example: Source Follower Input Capacitance

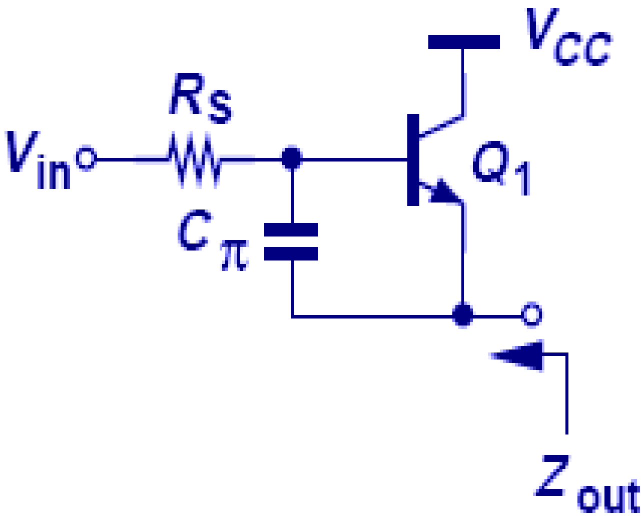


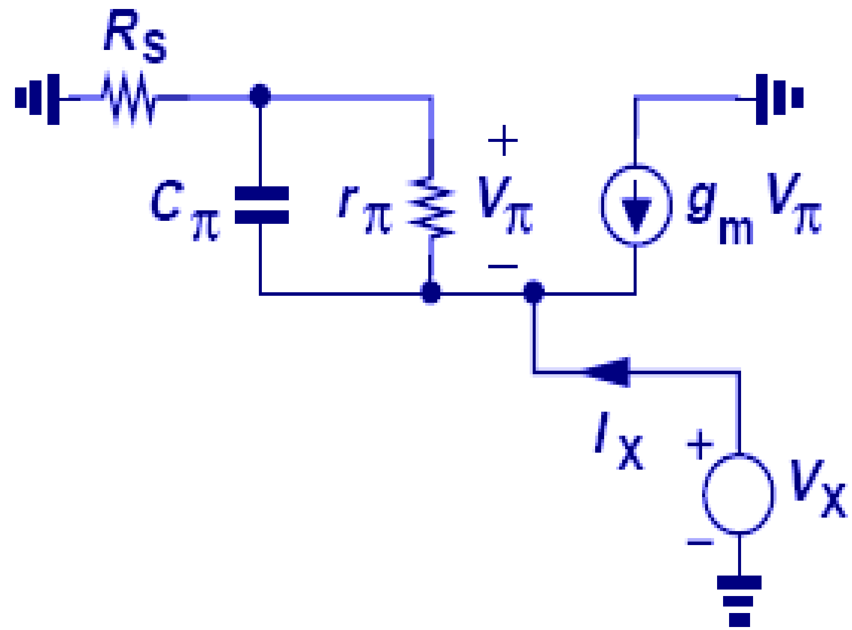
$$A_v = \frac{r_{O1} \| r_{O2}}{r_{O1} \| r_{O2} + \dfrac{1}{g_{m1}}}$$

$$\Rightarrow C_{in} = C_{GD1} + \left(1 - A_v\right) C_{GS1}$$

$$= \boxed{C_{GD1} + \frac{1}{1 + g_{m1}\left(r_{O1} \| r_{O2}\right)} C_{GS1}}$$

Output Impedance of Emitter Follower





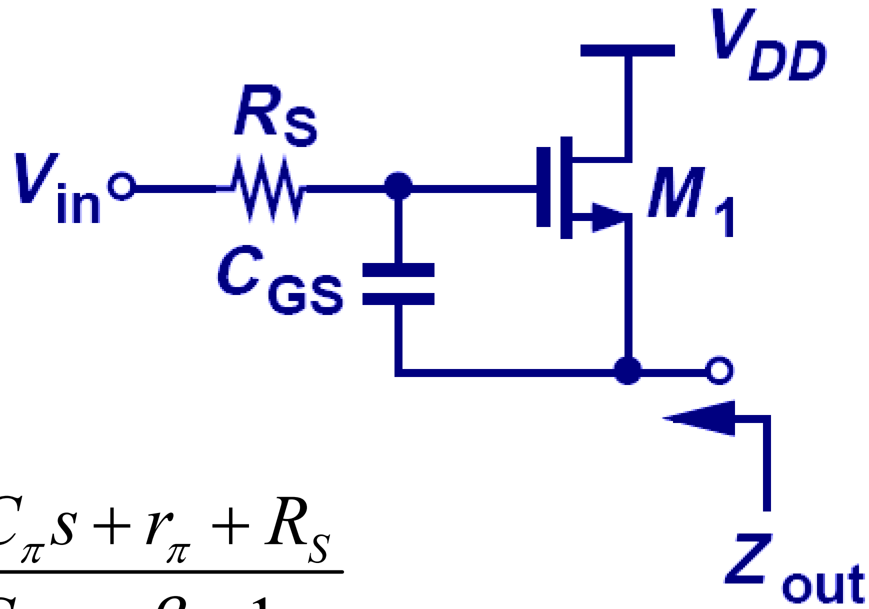
$$\left(I_X + g_m V_\pi\right)\left(r_\pi \left\| \frac{1}{C_\pi s}\right.\right) = -V_\pi$$

$$\Rightarrow V_\pi = -I_X \frac{r_\pi}{r_\pi C_\pi s + \beta + 1}$$

$$\left(I_X + g_m V_\pi\right) R_S - V_\pi = V_X$$

$$\Rightarrow \frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

Output Impedance of Source Follower

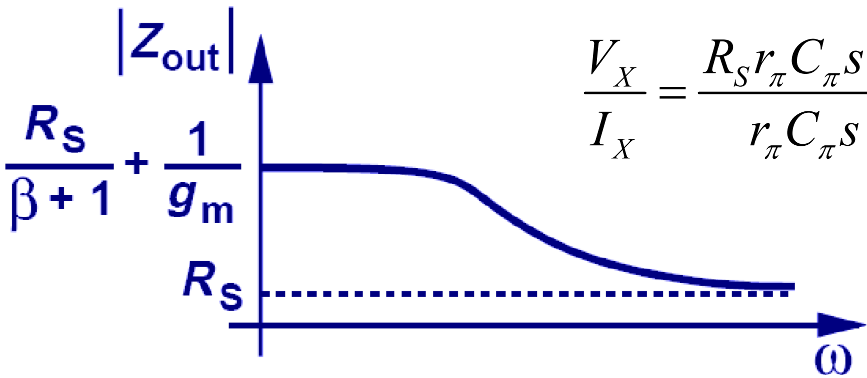


$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

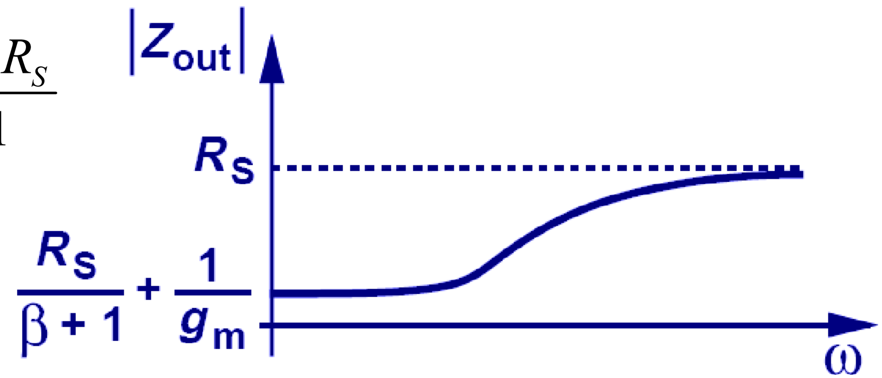
$$\text{with } g_m \cdot r_\pi = \beta \to \infty$$

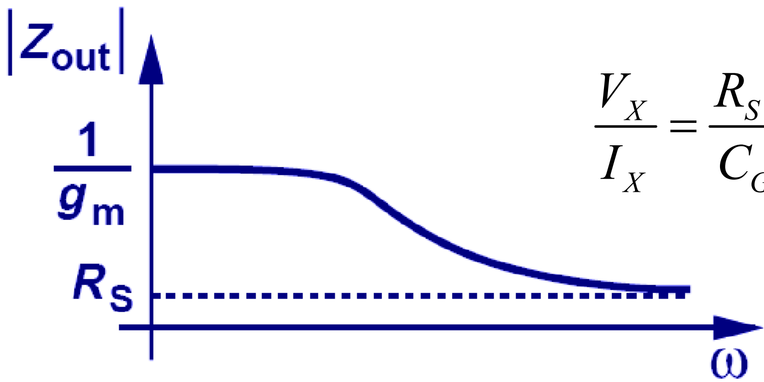
$$\boxed{\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}}$$

Active Inductor

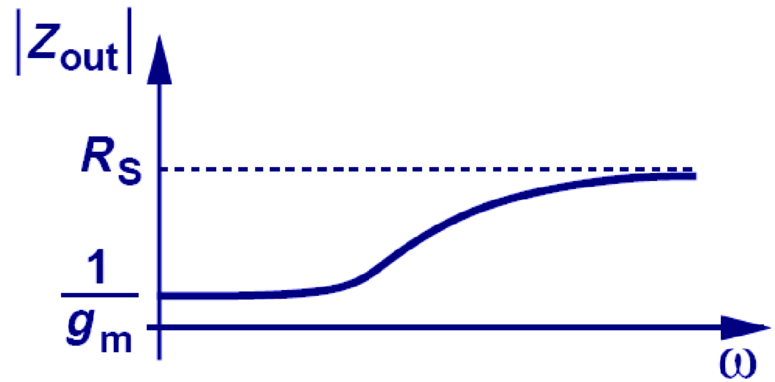


$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$



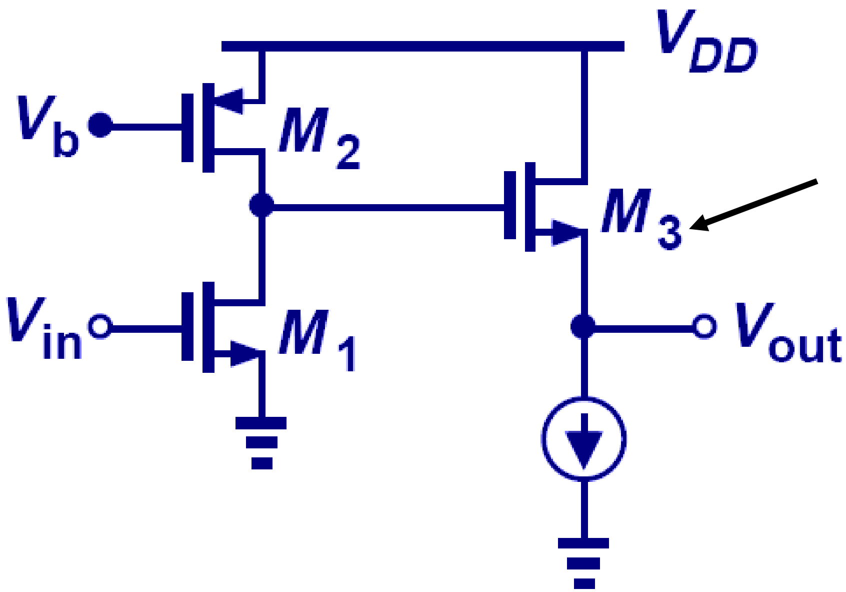


$$\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

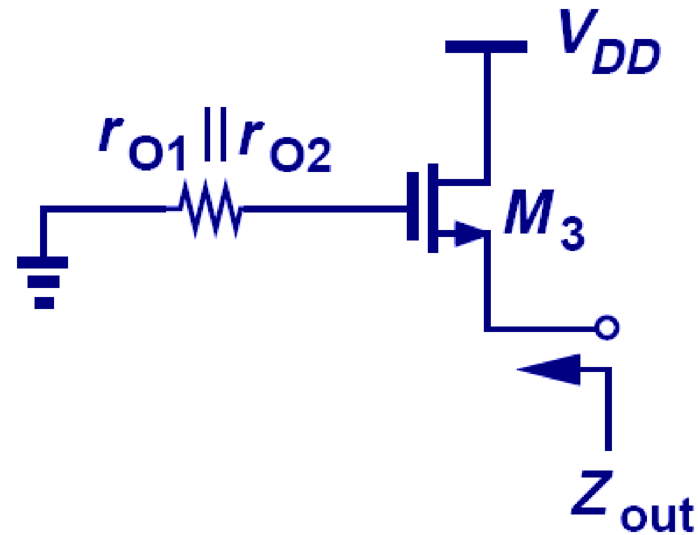


- The plot above shows the output impedance of emitter and source followers. Since a follower's primary duty is to lower the driving impedance ($R_S > 1/g_m$), the "active inductor" characteristic on the right is usually observed.

Example: Output Impedance

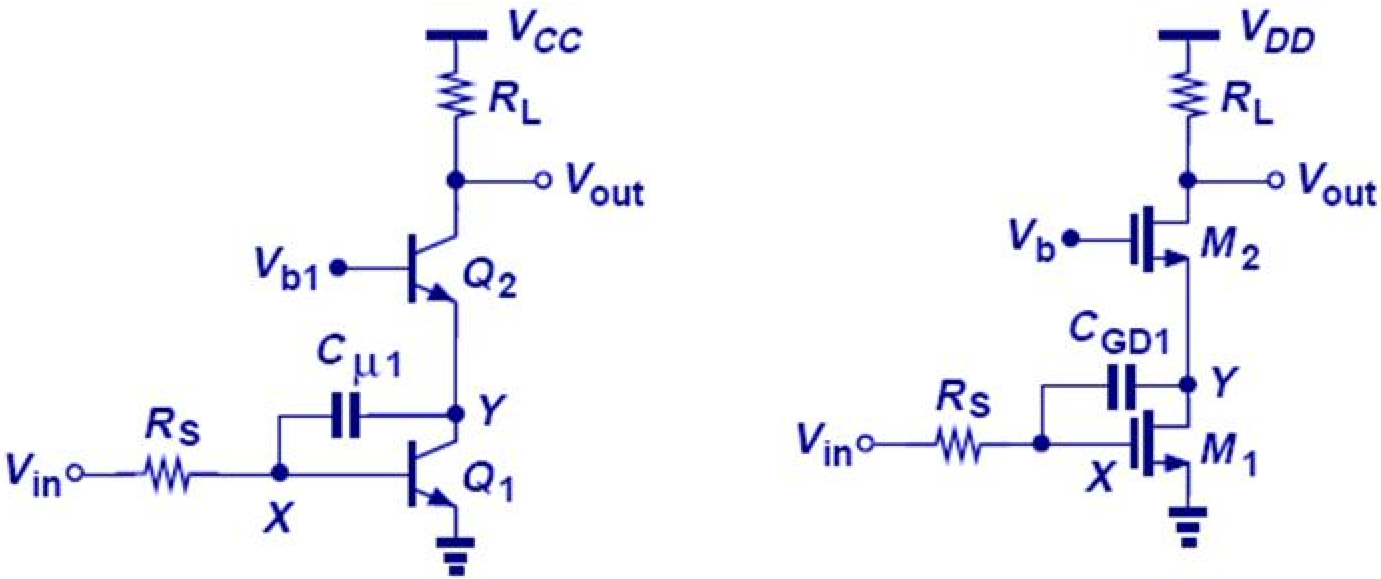


$r_{O3} = \infty$



$$\frac{V_X}{I_X} = \frac{(r_{O1} || r_{O2})C_{GS3}s + 1}{C_{GS3}s + g_{m3}}$$

Frequency Response of Cascode Stage



Assuming $r_o = \infty$ for all transistors,

$$A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1$$

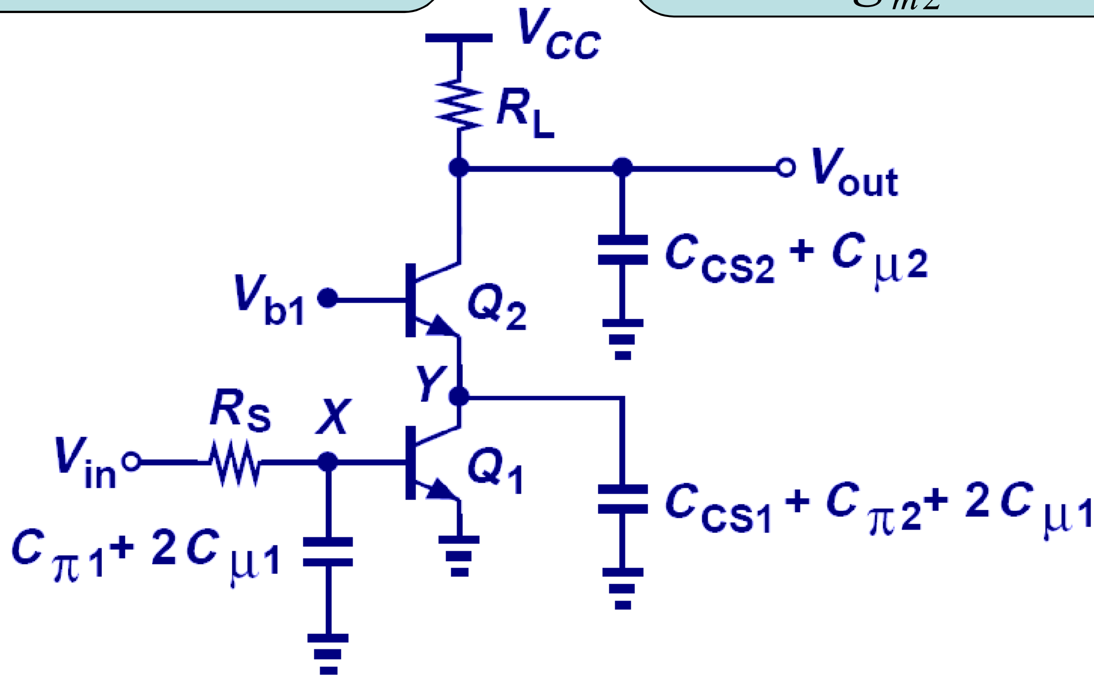
$$C_x = \left(1 - A_{v,XY}\right) C_{XY}$$
$$\approx 2 \cdot C_{XY}$$

- **For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.**

Poles of Bipolar Cascode

$$\omega_{p,X} = \frac{1}{(R_S \parallel r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}(C_{CS1} + C_{\pi 2} + 2C_{\mu 1})}$$

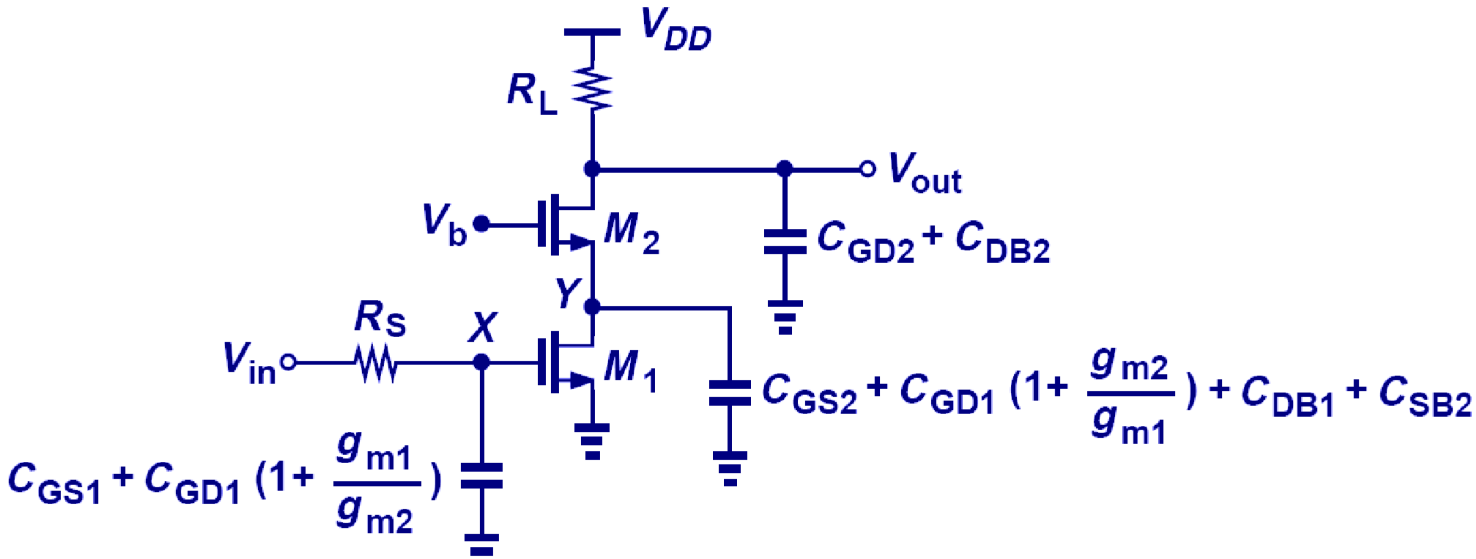


$$\omega_{p,out} = \frac{1}{R_L(C_{CS2} + C_{\mu 2})}$$

Poles of MOS Cascode

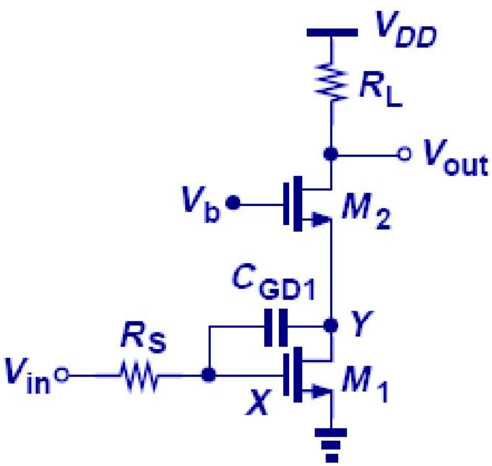
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

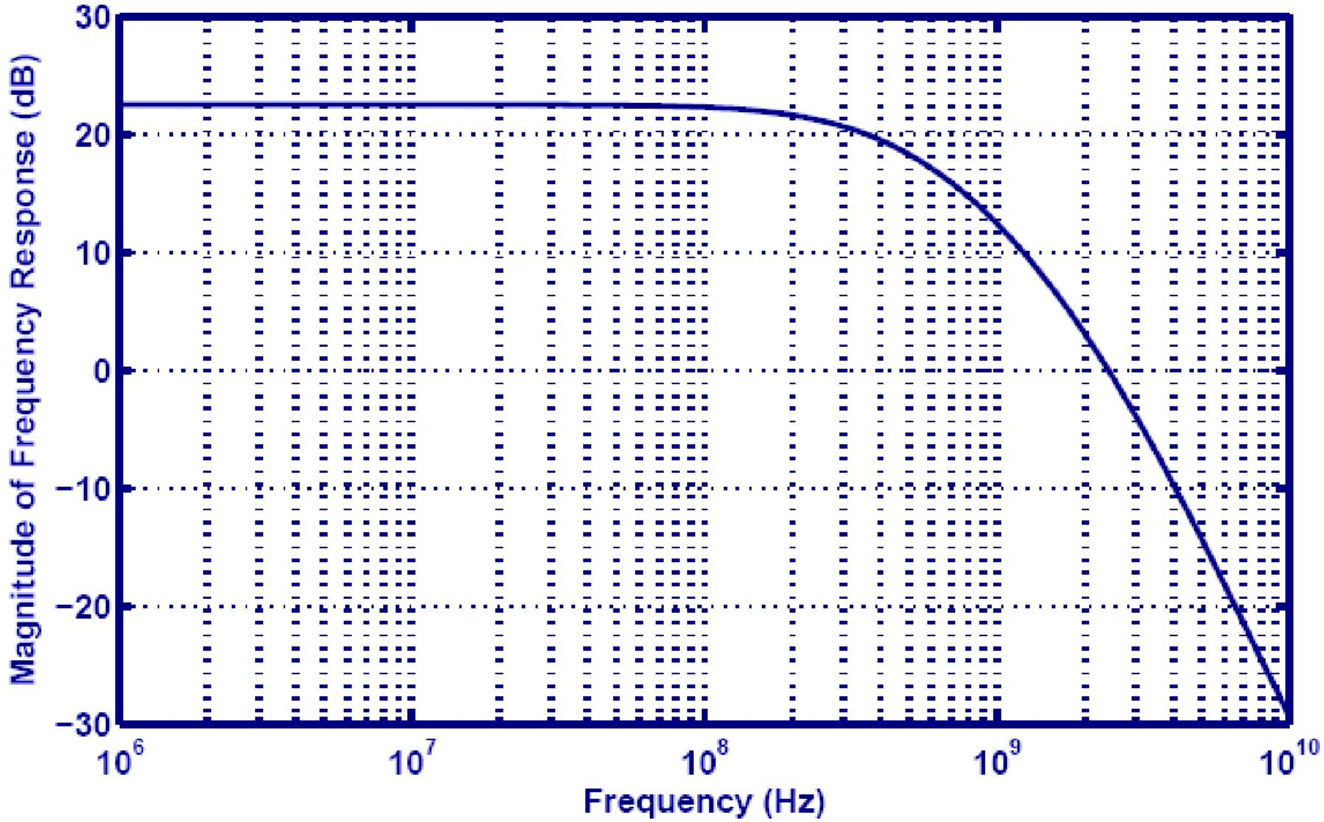
$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1}\right]}$$

Example: Frequency Response of Cascode





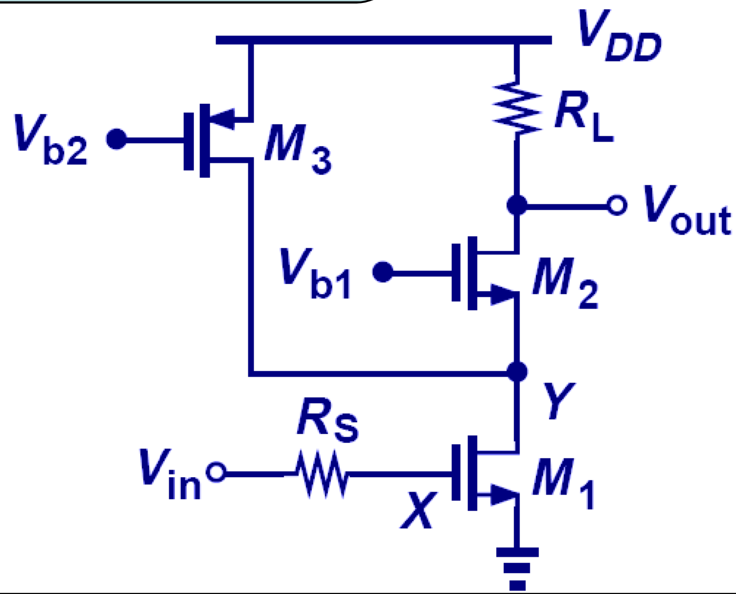
$$R_S = 200\ \Omega$$
$$C_{GS} = 250\ \text{fF}$$
$$C_{GD} = 80\ \text{fF}$$
$$C_{DB} = 100\ \text{fF}$$
$$g_m = (150\ \Omega)^{-1}$$
$$\lambda = 0$$
$$R_L = 2\ \text{k}\Omega$$

$$|\omega_{p,X}| = 2\pi \times (1.95\ \text{GHz})$$
$$|\omega_{p,Y}| = 2\pi \times (1.73\ \text{GHz})$$
$$|\omega_{p,out}| = 2\pi \times (442\ \text{MHz})$$

MOS Cascode Example

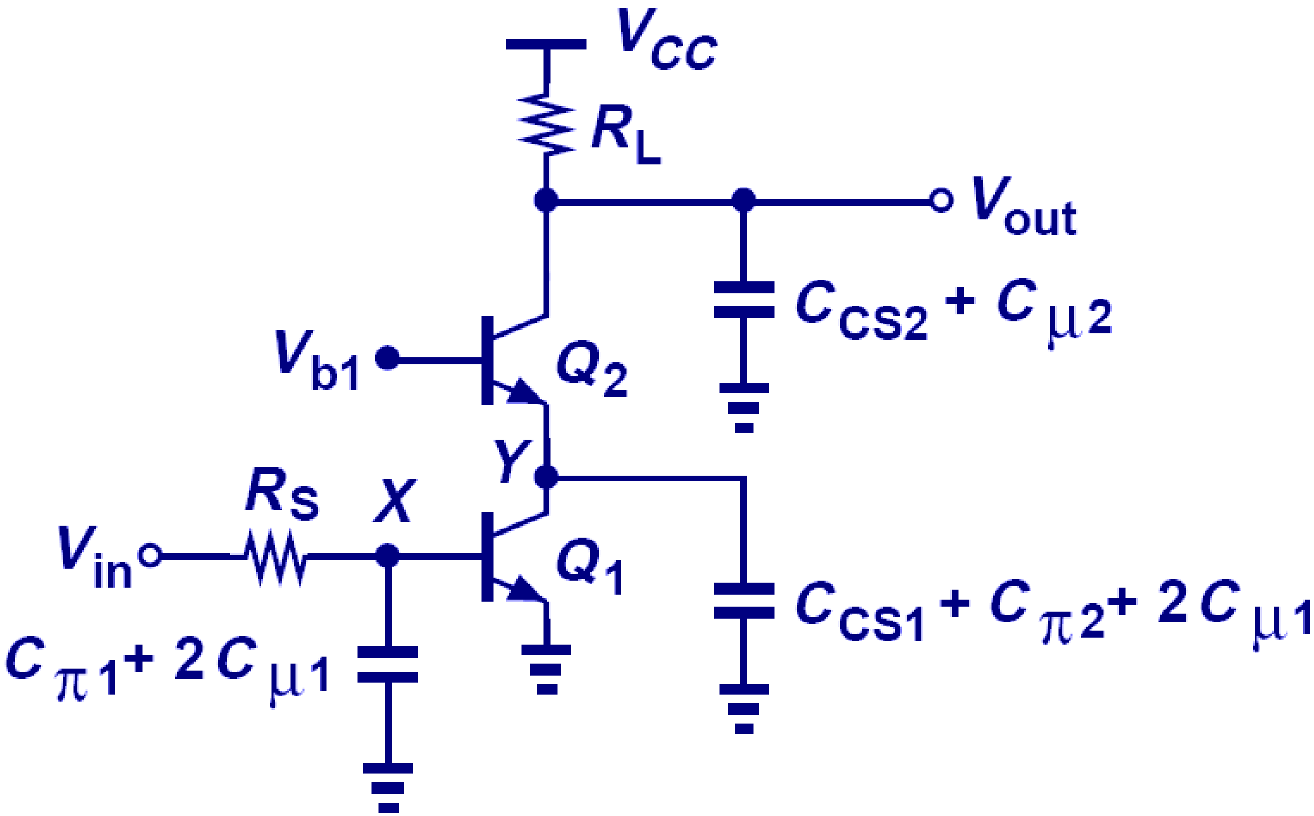
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{SB2} + C_{GD3} + C_{DB3}\right]}$$

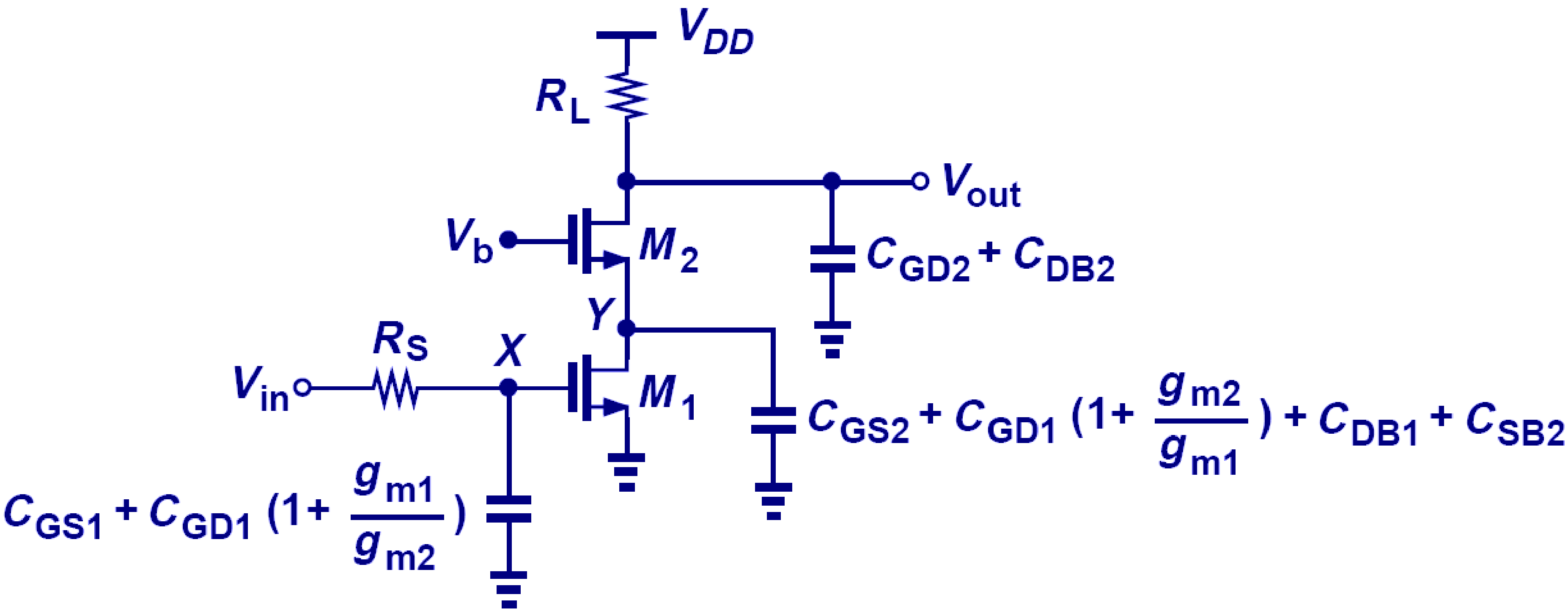
I/O Impedance of Bipolar Cascode



$$Z_{in} = r_{\pi 1} \parallel \frac{1}{(C_{\pi 1} + 2C_{\mu 1})s}$$

$$Z_{out} = R_L \parallel \frac{1}{(C_{\mu 2} + C_{CS2})s}$$

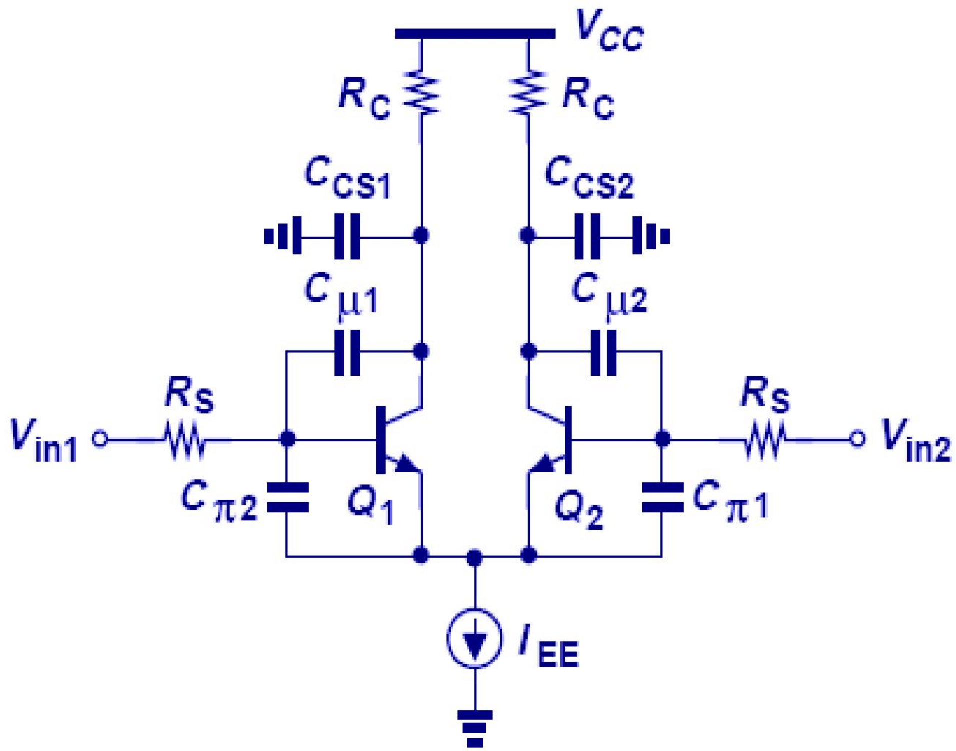
I/O Impedance of MOS Cascode

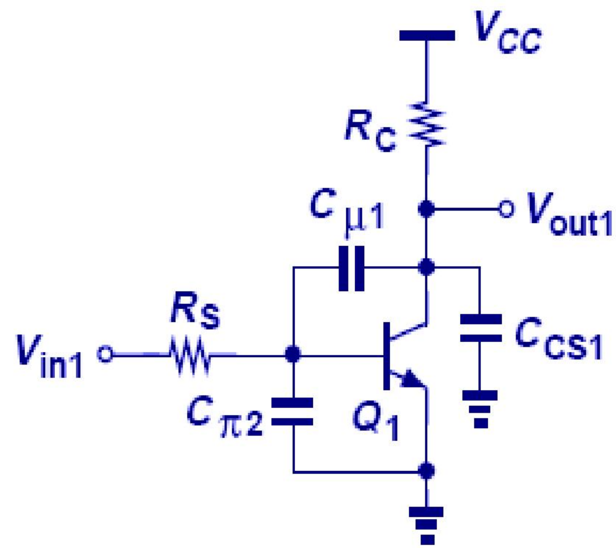


$$Z_{in} = \frac{1}{\left[C_{GS1} + \left(1+\dfrac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]s}$$

$$Z_{out} = R_L \,\|\, \frac{1}{\left(C_{GD2} + C_{DB2}\right)s}$$

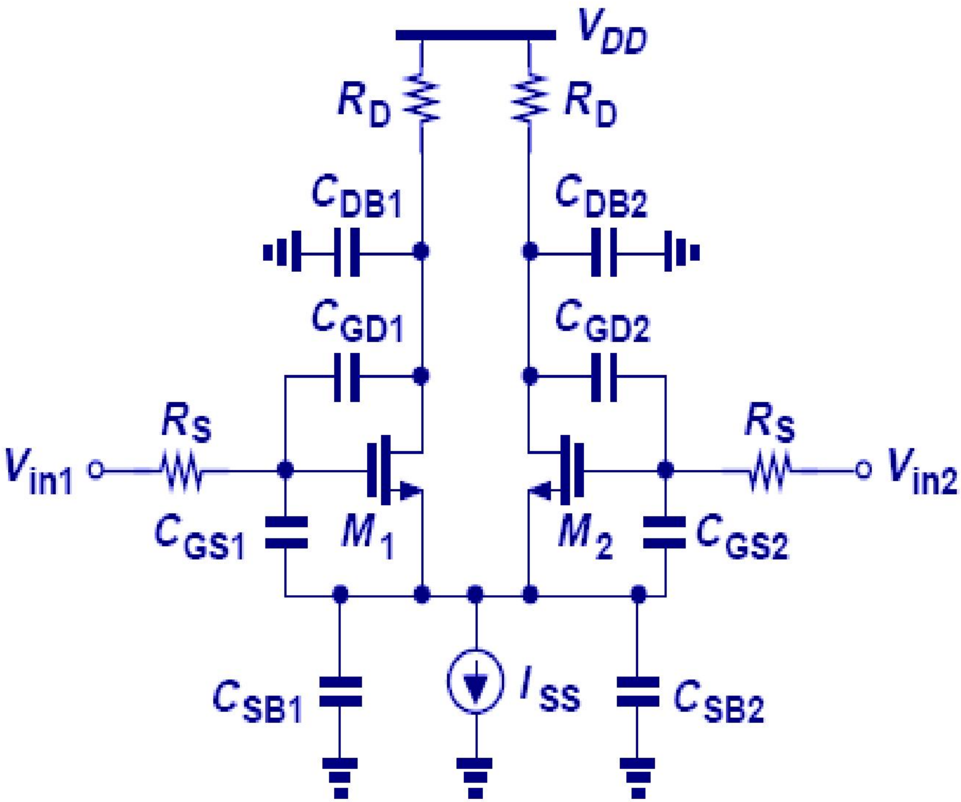
Bipolar Differential Pair Frequency Response

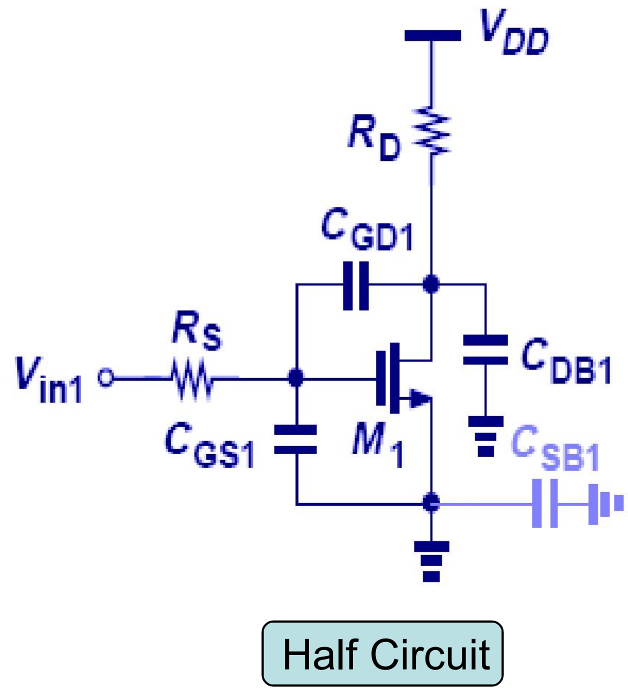




- Since bipolar differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

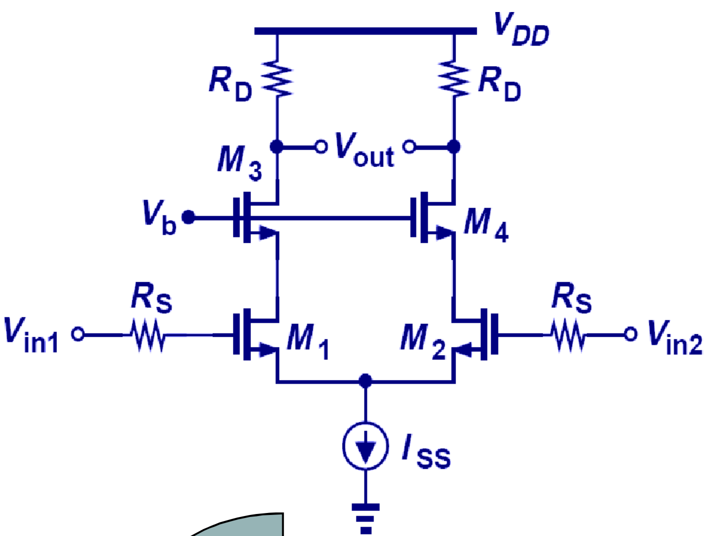
MOS Differential Pair Frequency Response





- Since MOS differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

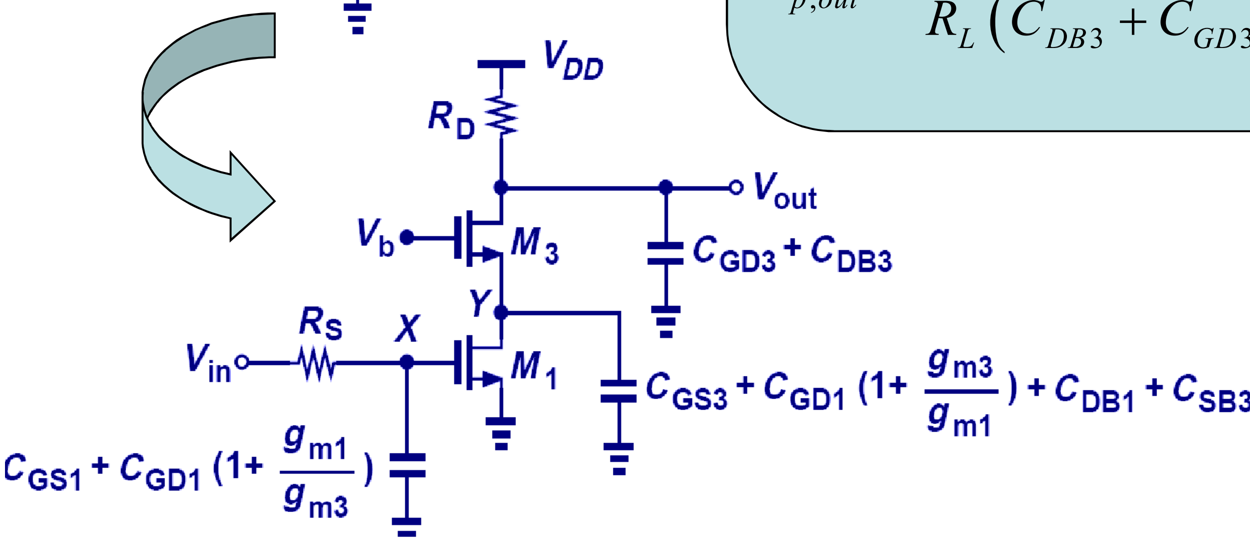
Example: MOS Differential Pair



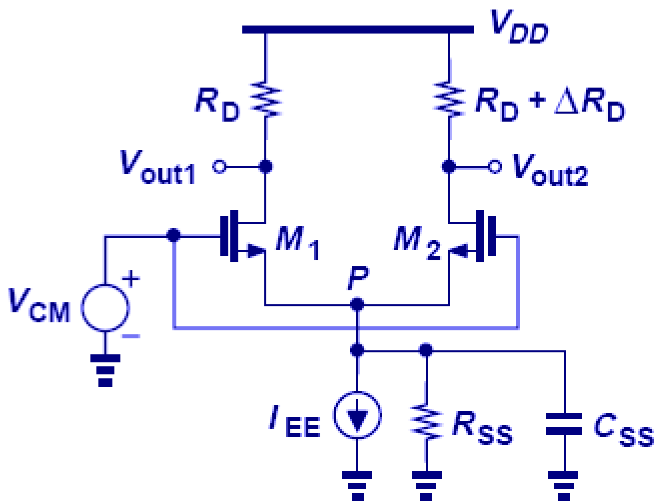
$$\omega_{p,X} = \frac{1}{R_S[C_{GS1} + (1 + g_{m1}/g_{m3})C_{GD1}]}$$

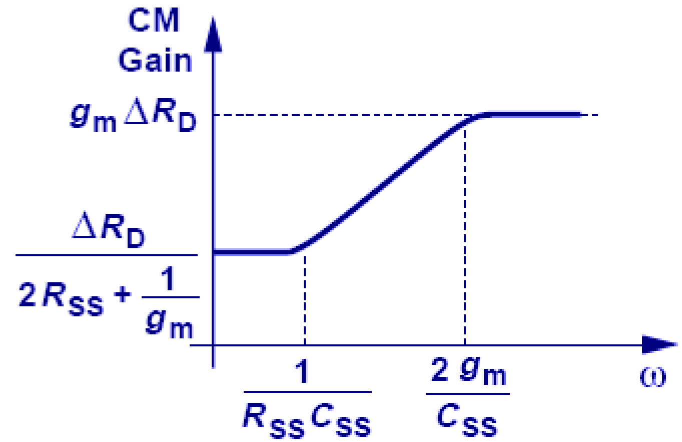
$$\omega_{p,Y} = \frac{1}{\dfrac{1}{g_{m3}}\left[C_{DB1} + C_{GS3} + C_{SB3} + \left(1 + \dfrac{g_{m3}}{g_{m1}}\right)C_{GD1}\right]}$$

$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB3} + C_{GD3}\right)}$$



Common Mode Frequency Response

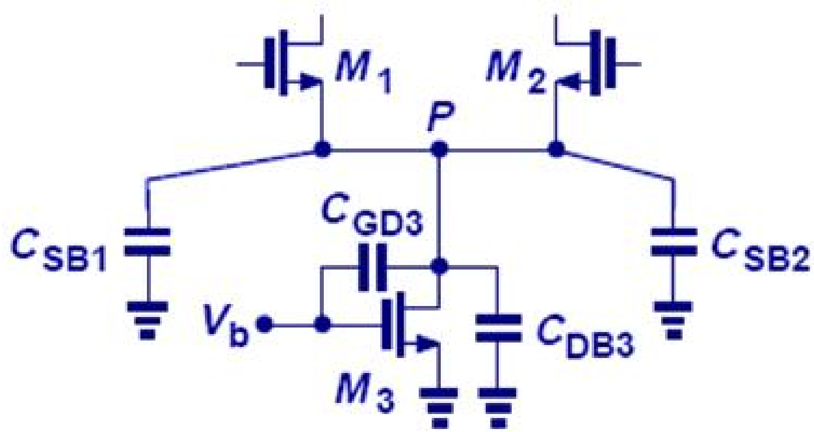




$$\left|\frac{\Delta V_{out}}{\Delta V_{CM}}\right| = \frac{\Delta R_D}{\dfrac{1}{g_m} + 2\left(R_{SS} \,\middle\|\, \dfrac{1}{C_{SS}s}\right)} = \frac{g_m \Delta R_D \left(R_{SS}C_{SS}s + 1\right)}{R_{SS}C_{SS}s + 2g_m R_{SS} + 1}$$

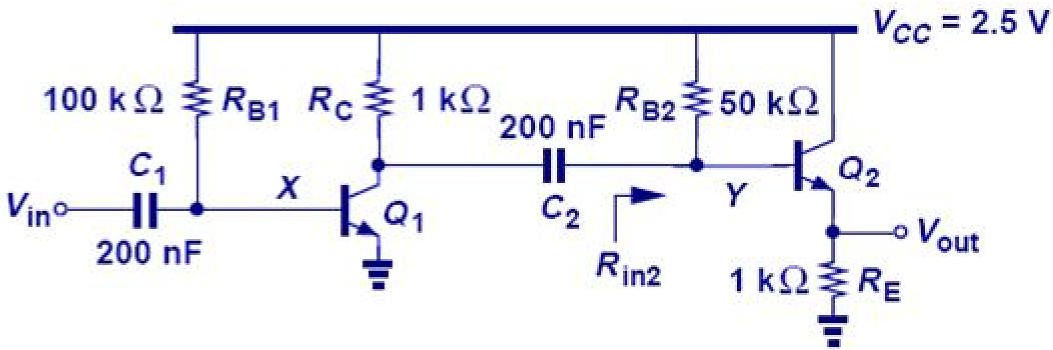
- C_{ss} will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.

Tail Node Capacitance Contribution



- Source-Body Capacitance of M_1, M_2
- Drain-Body Capacitance of M_3
- Gate-Drain Capacitance of M_3

Example: Capacitive Coupling



$$I_S = 5\times 10^{-16}\ \text{A}$$
$$\beta = 100$$
$$V_A = \infty$$

For Q_1, assuming $V_{BE1} = 800$ mV,

$$I_{C1} = \beta \frac{V_{CC} - V_{BE1}}{R_{B1}} = 1.7\ \text{mA}$$

$$\Rightarrow V_{BE1} = V_T \ln\left(I_{C1} / I_{S1}\right) = 748\ \text{mV}$$

$$\Rightarrow I_{C1} = 1.75\ \text{mA} \Rightarrow g_{m1} = \left(14.9\ \Omega\right)^{-1}$$

$$\Rightarrow r_{\pi 1} = 14.9\ \text{k}\Omega$$

For Q_2, assuming $V_{BE2} = 800$mV,

$$V_{CC} = I_{B2} R_{B2} + V_{BE2} + R_E I_{C2}$$

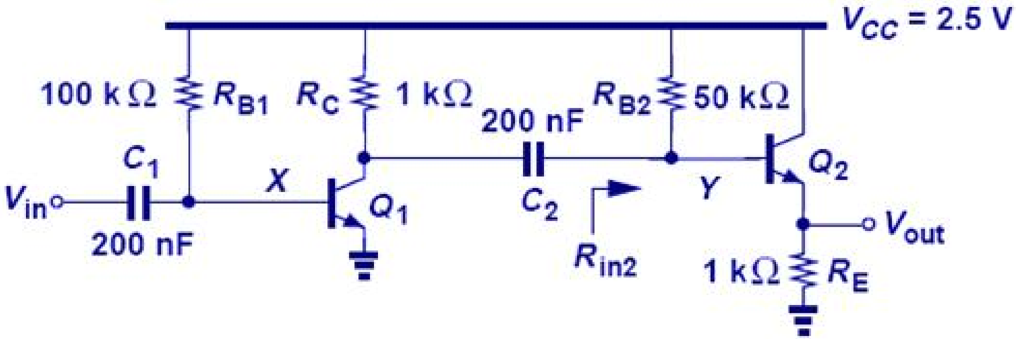
$$\Rightarrow I_{C2} = \frac{V_{CC} - V_{BE2}}{R_{B2} / \beta + R_E} = 1.13\ \text{mA}$$

Iteration yields

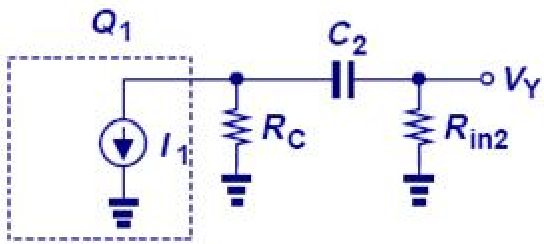
$$I_{C2} = 1.17\ \text{mA},\ g_{m2} = \left(22.2\ \Omega\right)^{-1}$$

$$\Rightarrow r_{\pi 2} = 2.22\ \text{k}\Omega$$

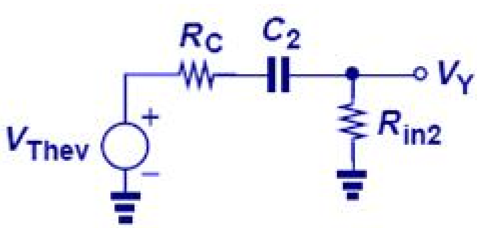
Example: Capacitive Coupling – cont'd



$$\omega_{L1} = \frac{1}{\left(r_{\pi 1} \parallel R_{B1}\right) C_1} = 2\pi \times \left(542\text{ Hz}\right)$$

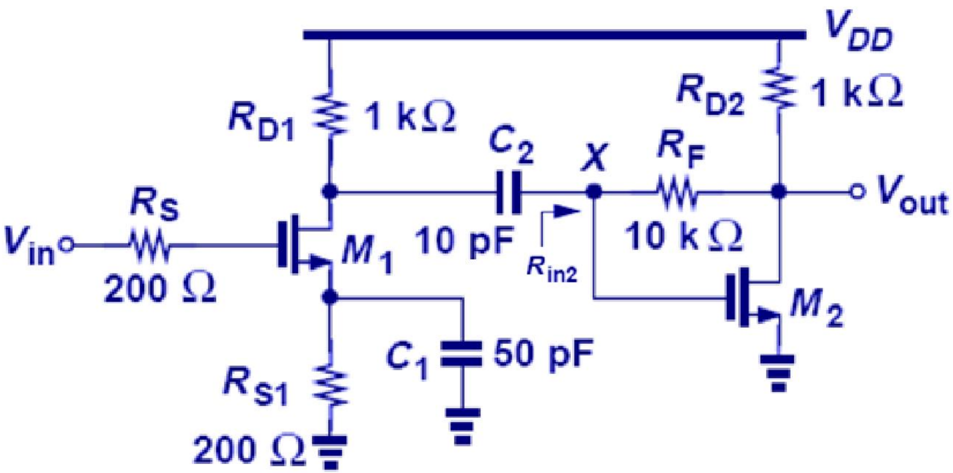


$$R_{in2} = R_{B2} \parallel \left[r_{\pi 2} + (\beta + 1) R_E\right]$$



$$\omega_{L2} = \frac{1}{\left(R_C + R_{in2}\right) C_2} = \pi \times \left(22.9\text{ Hz}\right)$$

Example: IC Amplifier – Low Frequency Behavior



$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$A_{v2} \approx -g_{m2} R_{D2} = -6.67$$

$$\Rightarrow R_{in2} = 1.30\ \mathrm{k}\Omega$$

$$\omega_{L1} = \frac{1}{\left(R_{S1} \parallel \frac{1}{g_{m1}}\right) C_1}$$

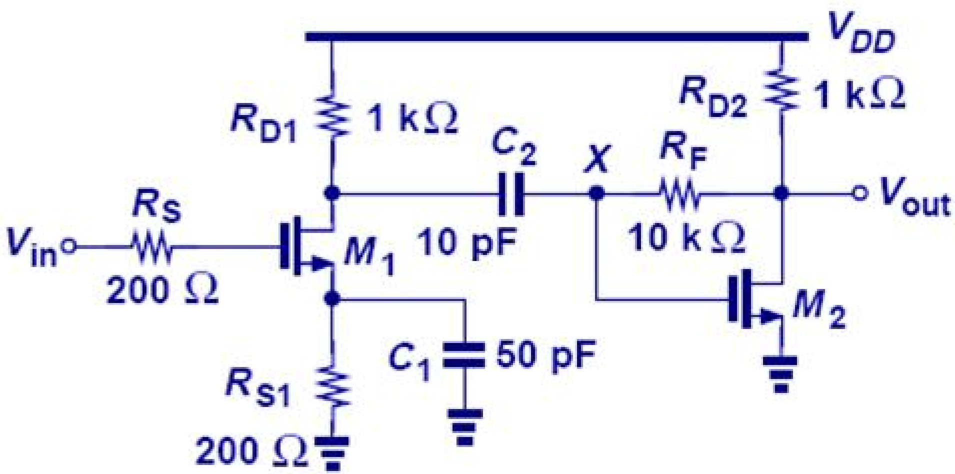
$$= \frac{g_{m1} R_{S1} + 1}{R_{S1} C_1}$$

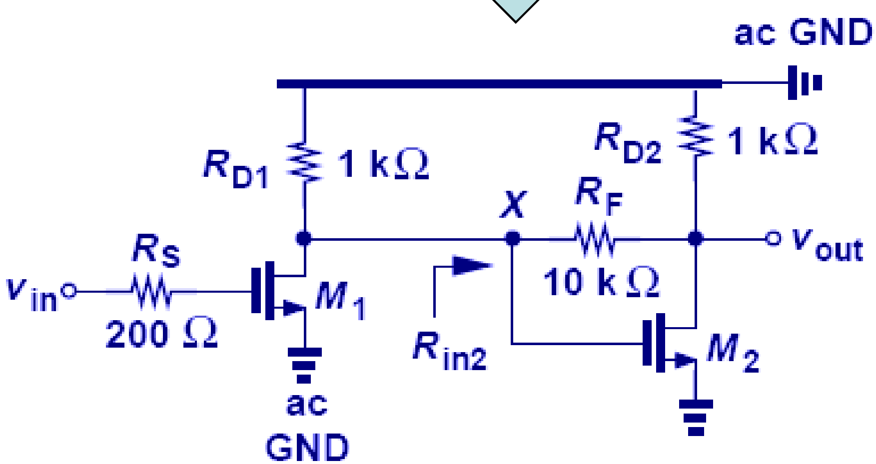
$$= 2\pi \times (42.4\ \mathrm{MHz})$$

$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2}) C_2}$$

$$= 2\pi \times (6.92\ \mathrm{MHz})$$

Example: IC Amplifier – Midband Behavior





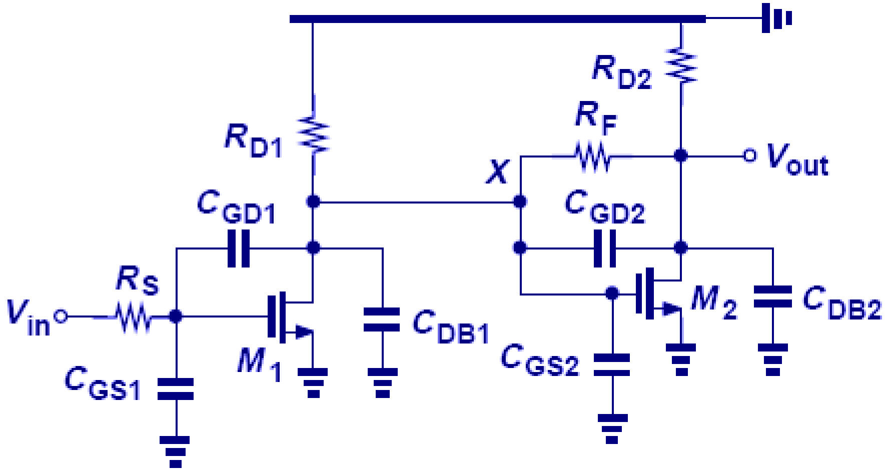
$$\frac{v_{out}}{v_{in}} = \left(\frac{v_X}{v_{in}}\right) \times \left(\frac{v_{out}}{v_X}\right)$$

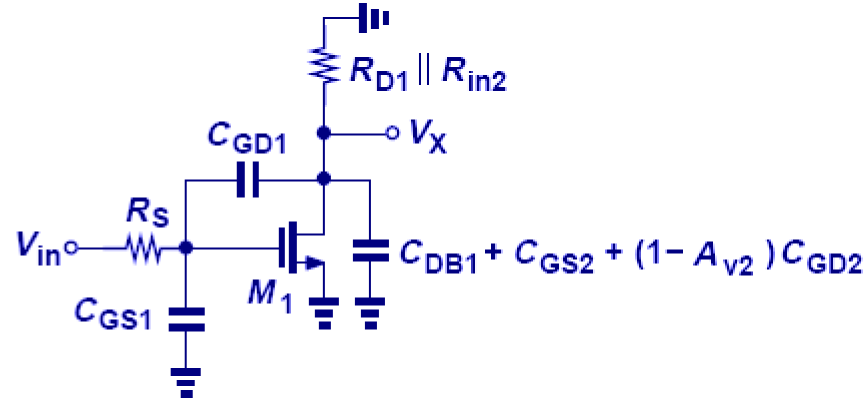
$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel R_{in2}\right) = -3.77$$

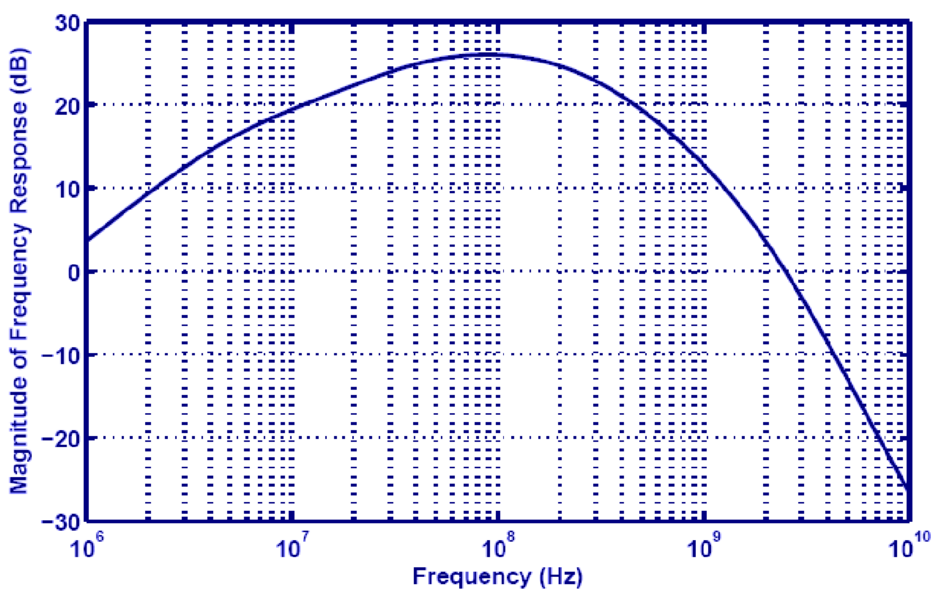
$$\frac{v_{out}}{v_X} \approx -g_{m2} R_{D2} = -6.67$$

$$\Rightarrow \frac{v_{out}}{v_{in}} \approx 25.1$$

Example: IC Amplifier – High Frequency Behavior







$$\left|\omega_{p1}\right| = 2\pi \times (308\ \text{MHz})$$

$$\left|\omega_{p2}\right| = 2\pi \times (2.15\ \text{GHz})$$

With Miller effect,

$$\left(1 - A_{v2}^{-1}\right)C_{GD2} \approx 1.15 \cdot C_{GD2}$$

$$\left|\omega_{p3}\right| = \frac{1}{R_{L2}(1.15 \cdot C_{GD2} + C_{DB2})} = 2\pi \times (1.21\ \text{GHz})$$