Chapter 14 Analog Filters

- 14.1 General Considerations
- 14.2 First-Order Filters
- 14.3 Second-Order Filters
- 14.4 Active Filters
- 14.5 Approximation of Filter Response

Outline of the Chapter

General Considerations

- Filter Characteristics
- Classification of Filters
- Transfer Function of Filters
- Problem of Sensitivity

→

Second-Order Filters

- Special Cases
- RLC Realizations

→

Active Filters

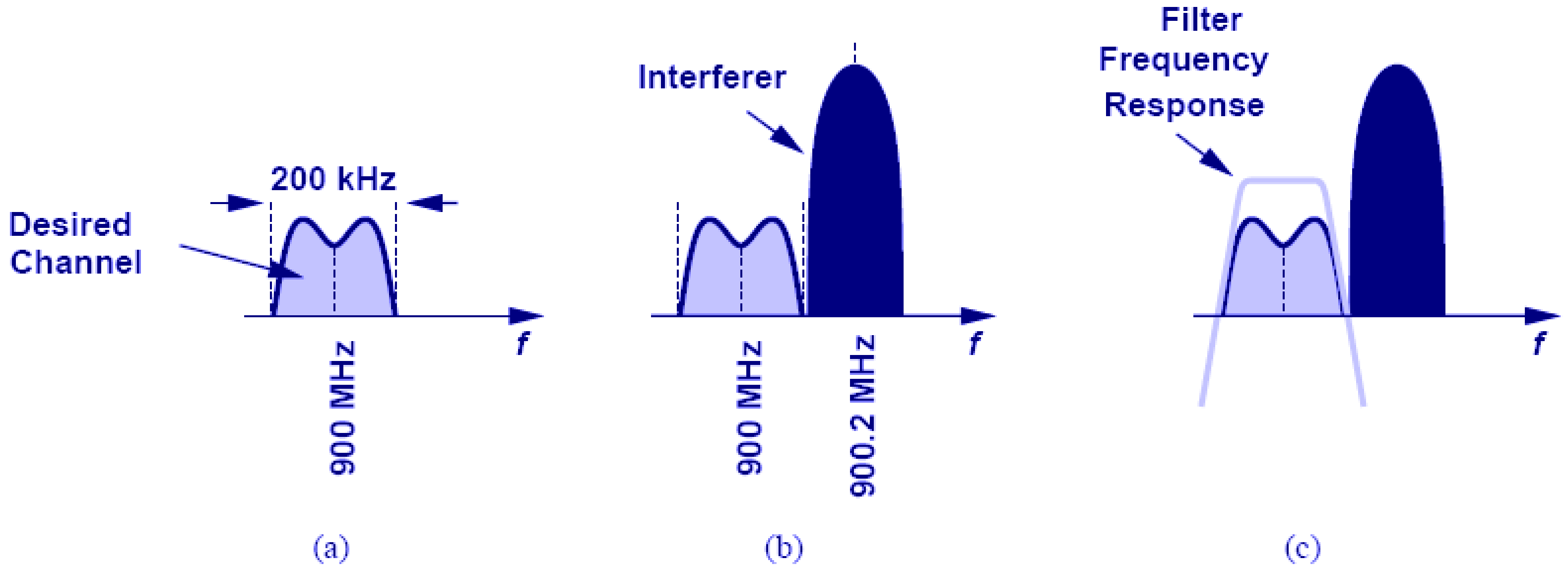
- Sallen and Key Filter
- KHN Biquad
- Tow-Thomas Biquad
- Biquads Based on Simulated Inductors

→

Approximation of Filter Response

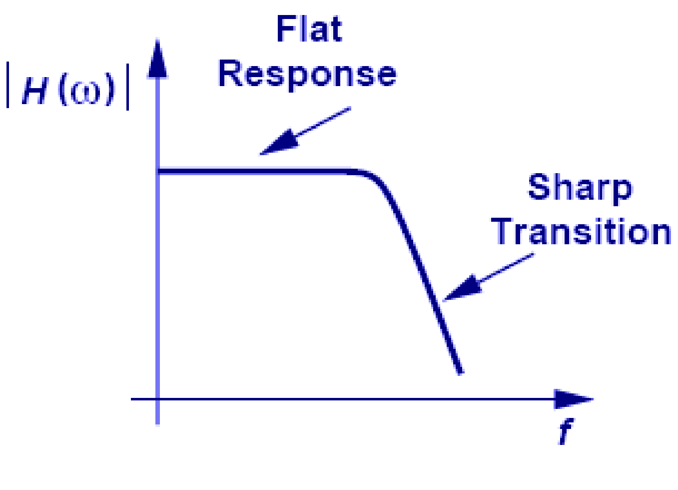
- Butterworth Response
- Chebyshev Response

Why We Need Filters

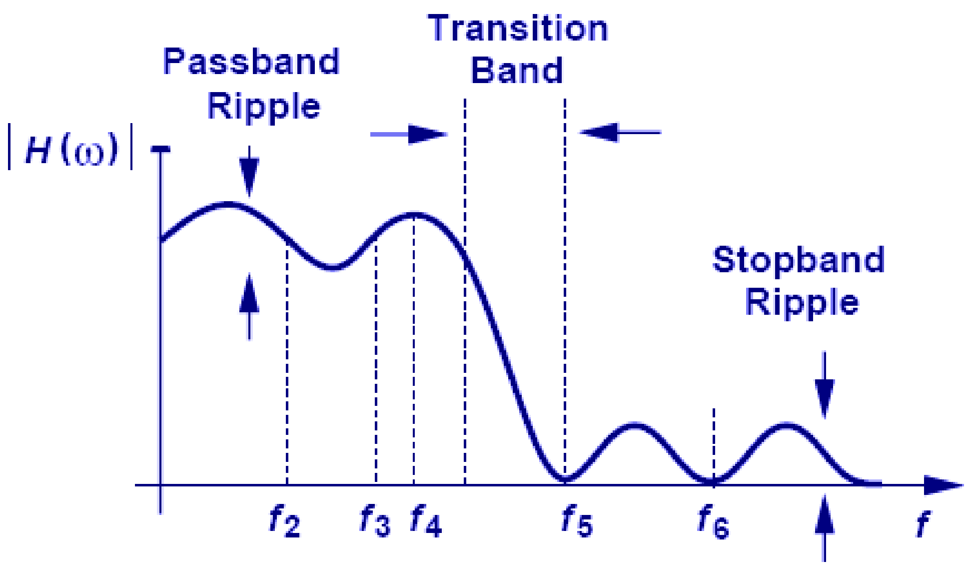


- In order to eliminate the unwanted interference that accompanies a signal, a filter is needed.

Filter Characteristics



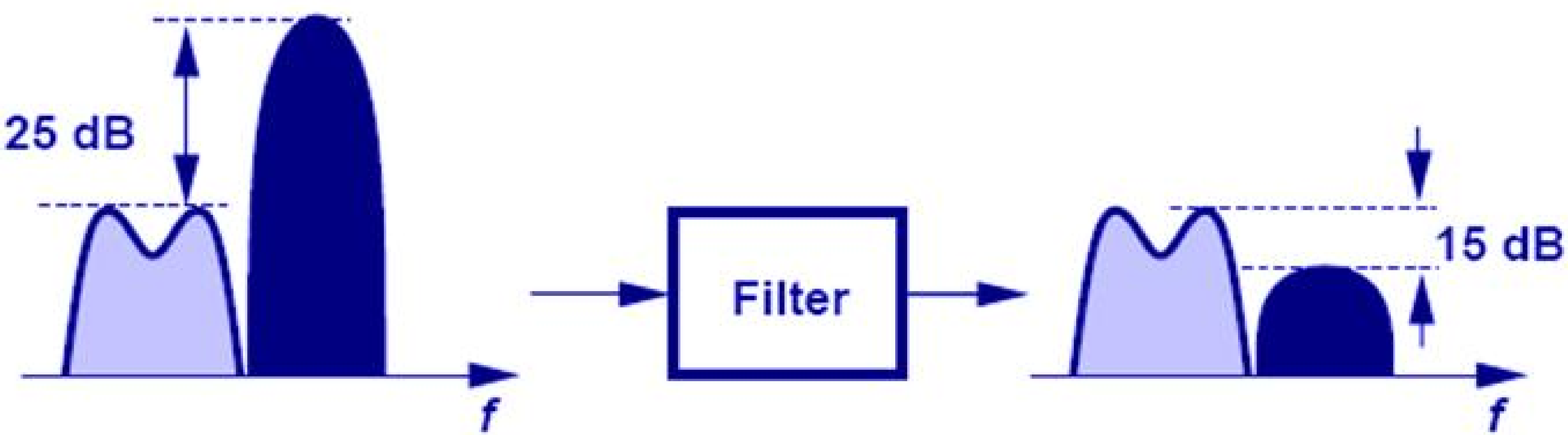
(a)



(b)

- **Ideally, a filter needs to have a flat pass band and a sharp roll-off in its transition band.**
- **Realistically, it has a rippling pass/stop band and a transition band.**

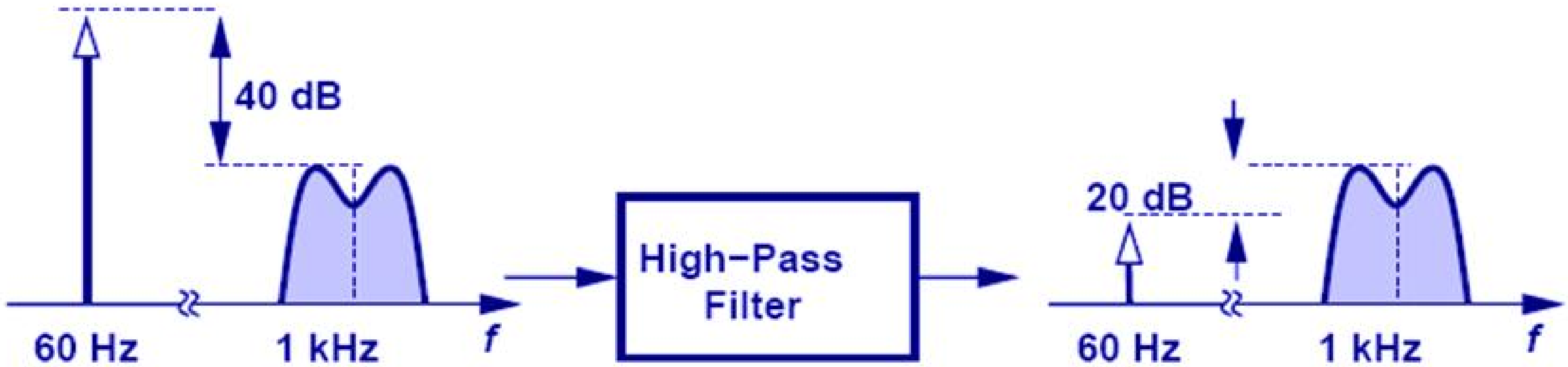
Example 14.1: Filter I



Problem : Adjacent channel interference is 25 dB above the signal. Determine the required stopband attenuation if Signal to Interference ratio must exceed 15 dB.

Solution: A filter with stopband attenuation of 40 dB

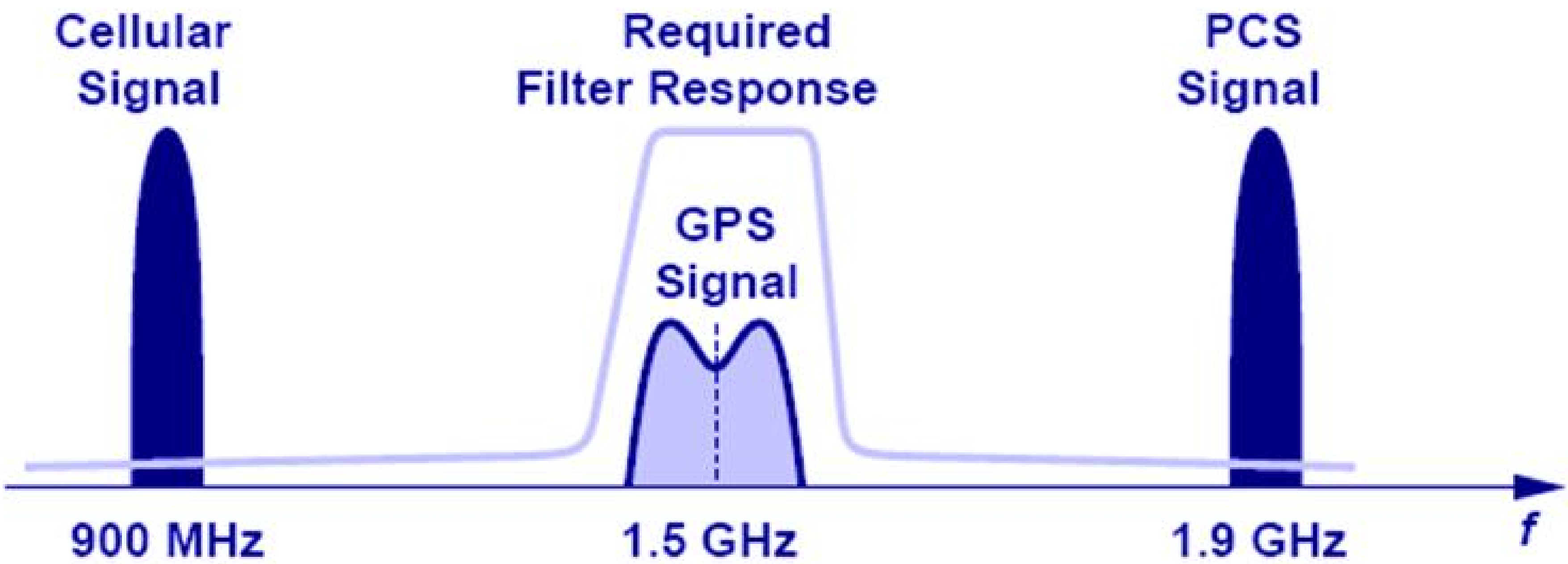
Example 14.2: Filter II



Problem: Adjacent 60-Hz channel interference is 40 dB above the signal. Determine the required stopband attenuation To ensure that the signal level remains 20dB above the interferer level.

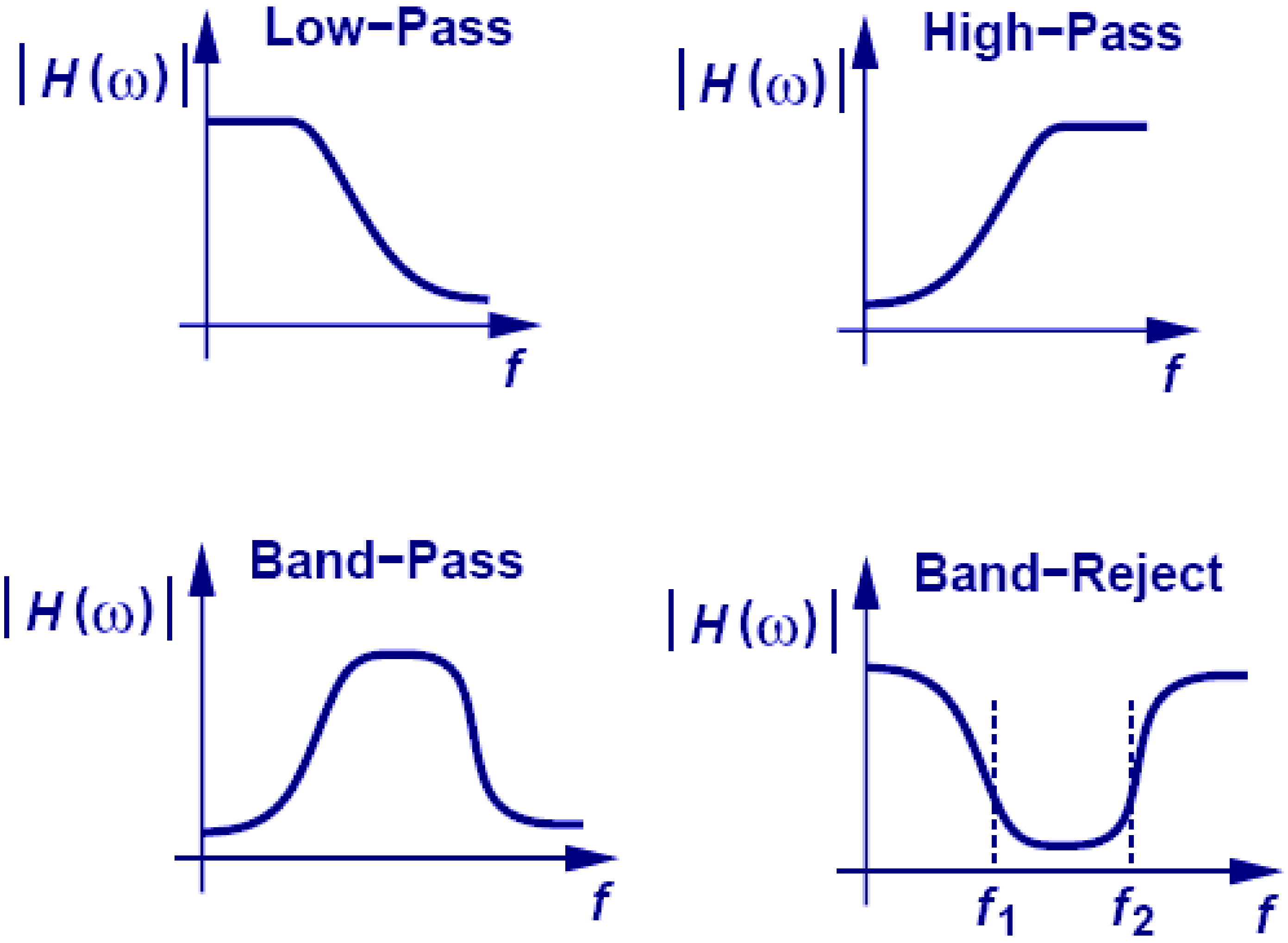
Solution: A high-pass filter with stopband attenuation of 60 dB at 60Hz.

Example 14.3: Filter III

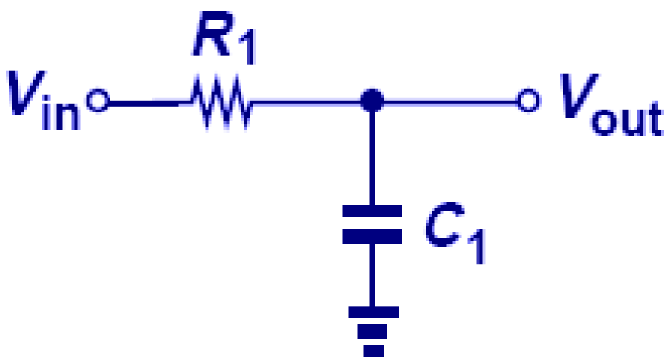


- A bandpass filter around 1.5 GHz is required to reject the adjacent Cellular and PCS signals.

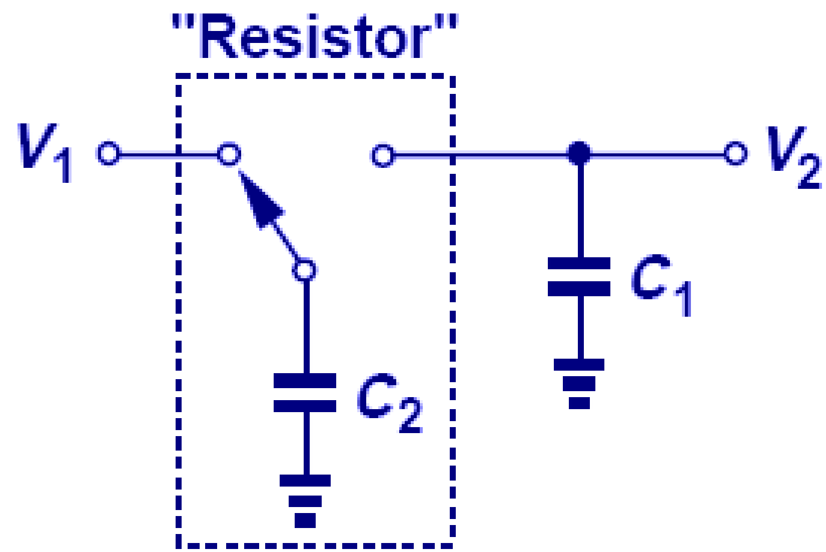
Classification of Filters I



Classification of Filters II



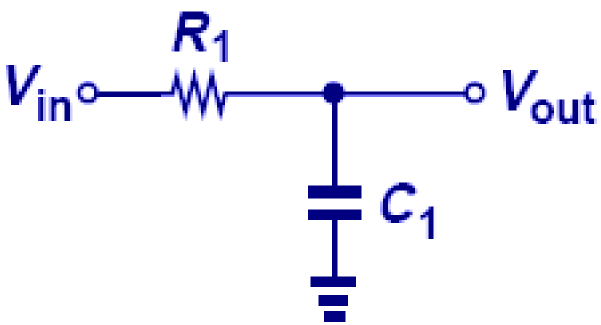
$$R_{eff} = \frac{1}{C_2 f}$$

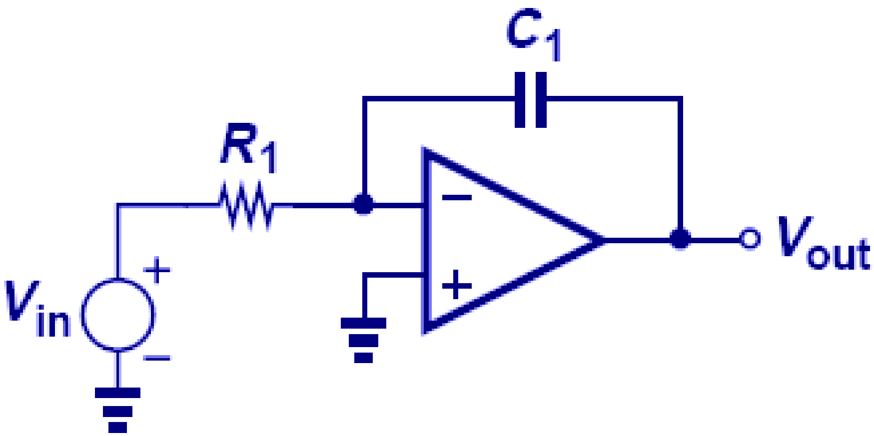


Continuous-time

Discrete-time

Classification of Filters III

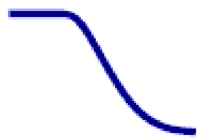
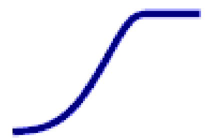
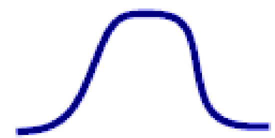
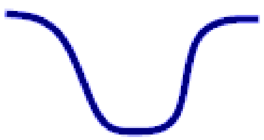
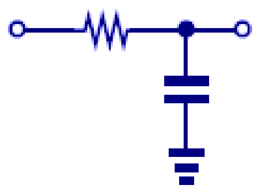
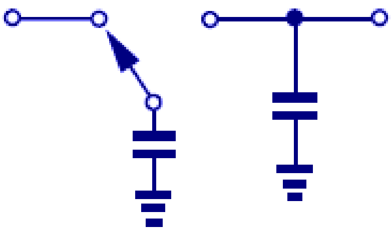
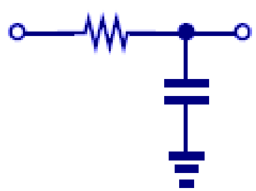
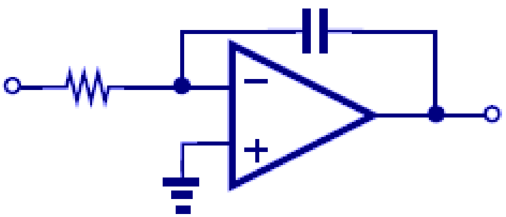




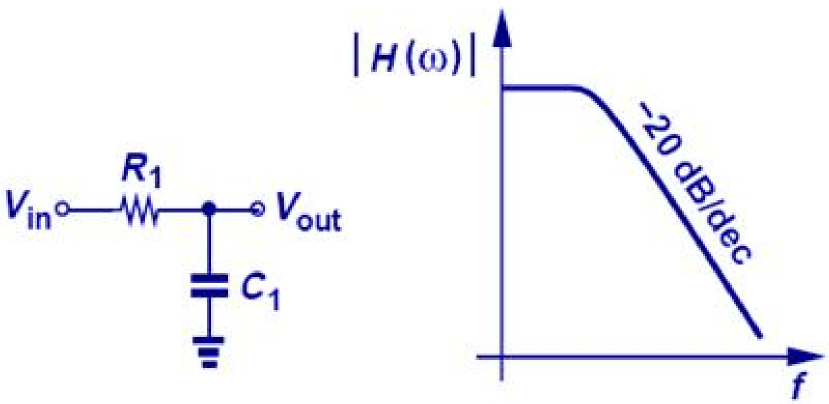
Passive

Active

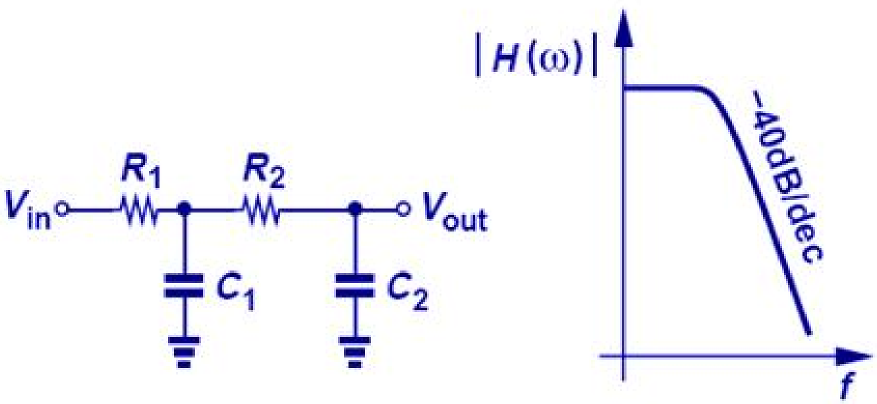
Summary of Filter Classifications

	Low-Pass	High-Pass	Band-Pass	Band-Reject
Frequency Response				
Continuous-Time and Discrete-Time				
Passive and Active				

Filter Transfer Function



(a)



(b)

- **Filter (a) has a transfer function with -20dB/dec roll-off.**
- **Filter (b) has a transfer function with -40dB/dec roll-off and provides a higher selectivity.**

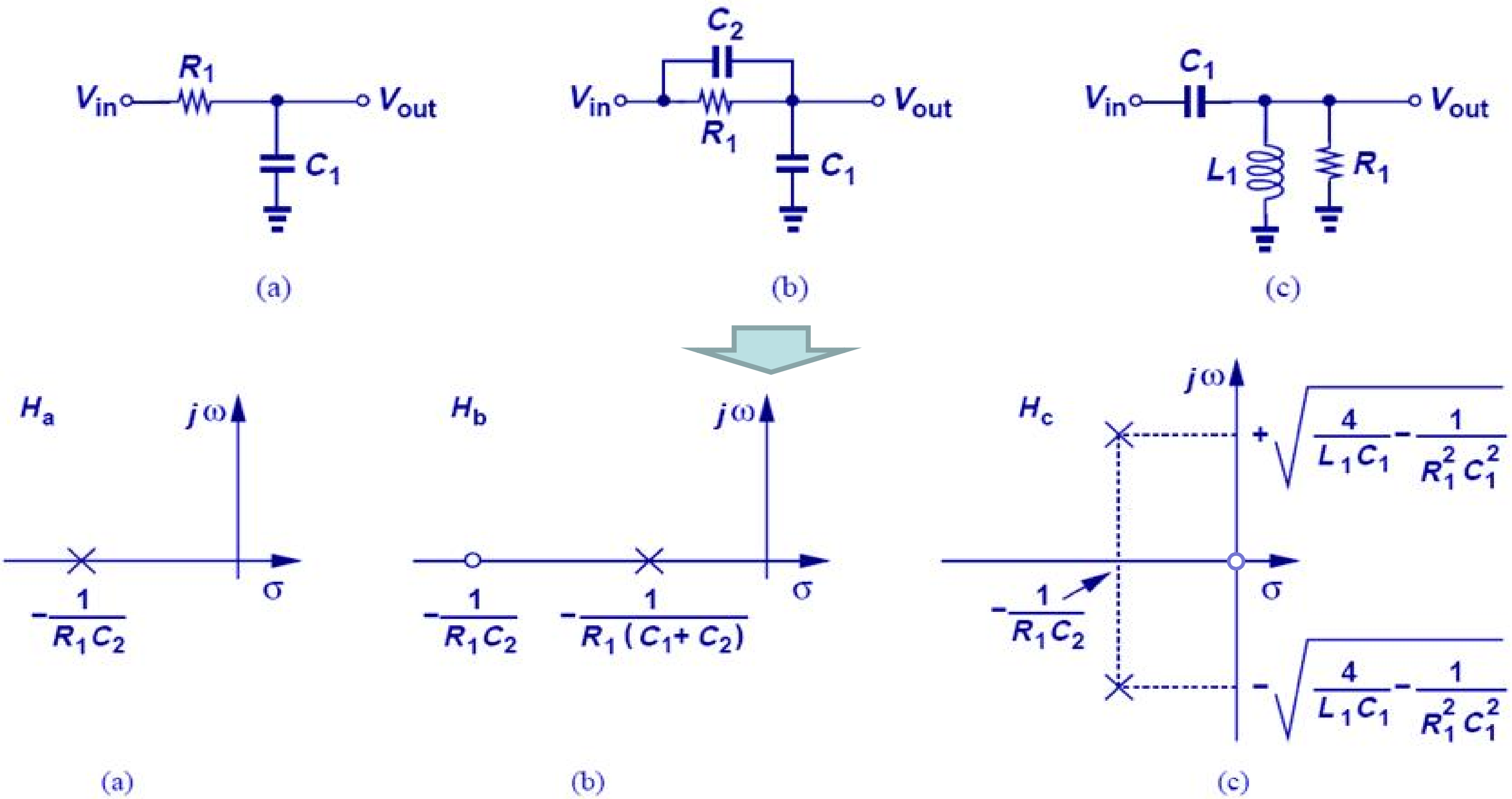
General Transfer Function

$$H(s) = \alpha \frac{(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)}$$

z_k = zero frequencies

p_k = pole frequencies

Example 14.4 : Pole-Zero Diagram

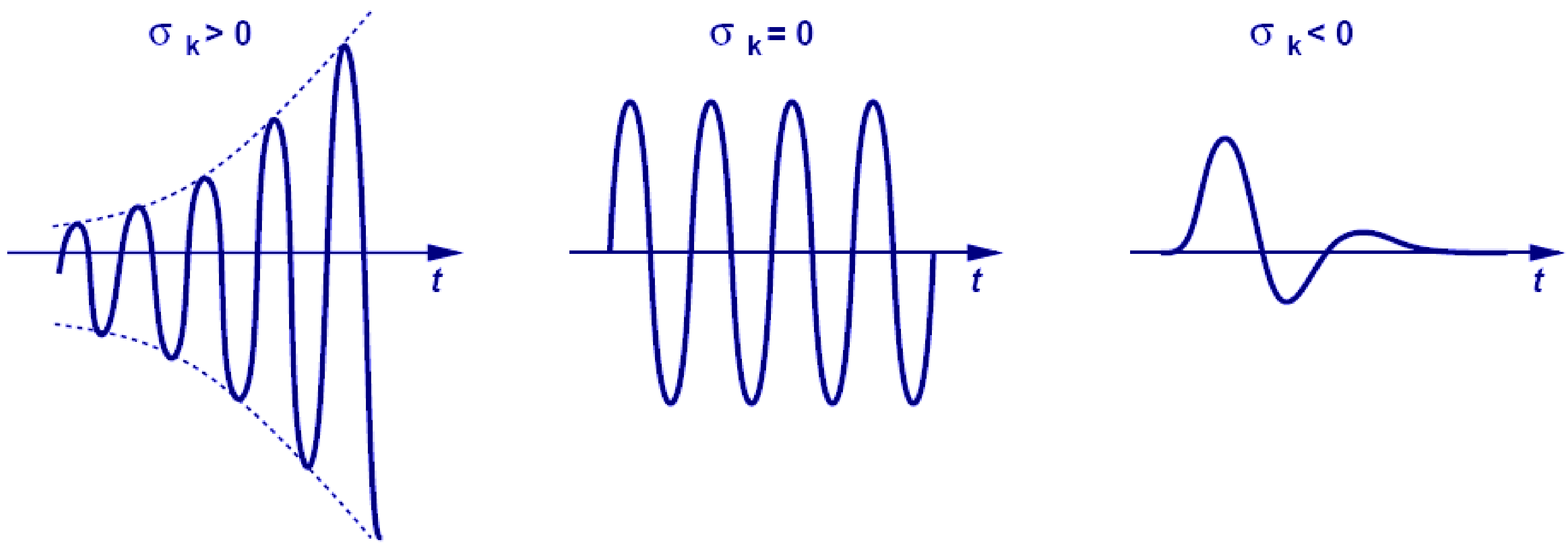


$$H_a(s) = \frac{1}{R_1C_1s+1} \qquad H_b(s) = \frac{R_1C_2s+1}{R_1(C_1+C_2)s+1} \qquad H_c(s) = \frac{C_1s}{R_1L_1C_1s^2+L_1s+R_1}$$

Example 14.5: Position of the poles

Impulse response contains

$$\exp(p_k t) = \exp(\sigma_k t)\exp(j\omega_k t)$$



Poles on the RHP
Unstable
(no good)

Poles on the jω axis
Oscillatory
(no good)

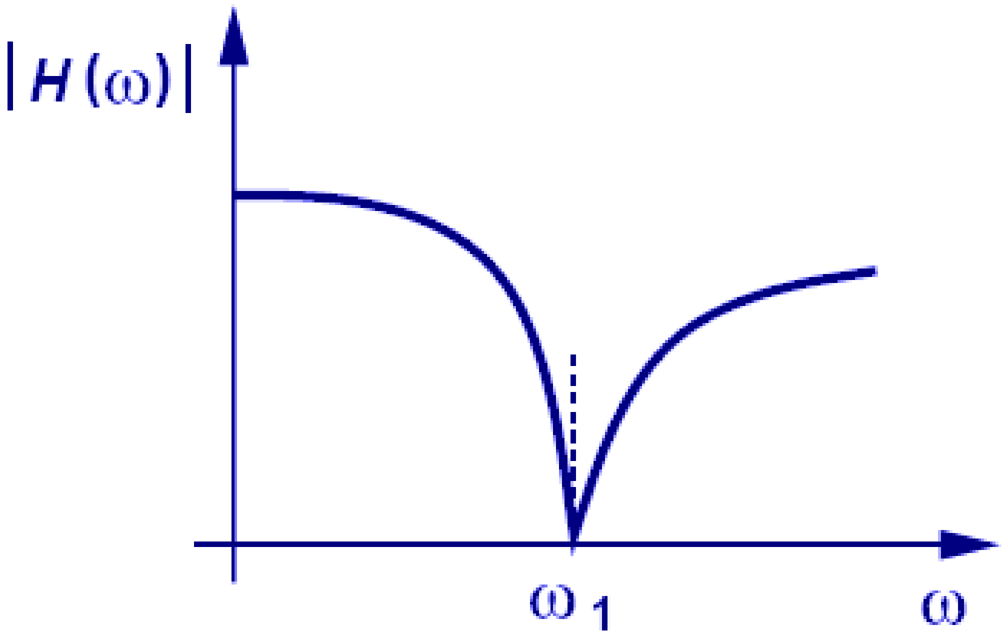
Poles on the LHP
Decaying
(good)

Transfer Function

$$H(s) = \alpha \frac{(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)}$$

- **The order of the numerator *m* ≤ The order of the denominator *n* Otherwise, H(s)→∞ as s→∞.**
- **For a physically-realizable transfer function, complex zeros or poles occur in conjugate pairs.**
- **If a zero is located on the jω axis, $z_{1,2} = \pm j\omega_1$, H(s) drops to zero at ω_1.**

Imaginary Zeros



- Imaginary zero is used to create a null at certain frequency. For this reason, imaginary zeros are placed only in the stop band.

Sensitivity

$$S_C^P = \frac{dP}{P} \Big/ \frac{dC}{C}$$

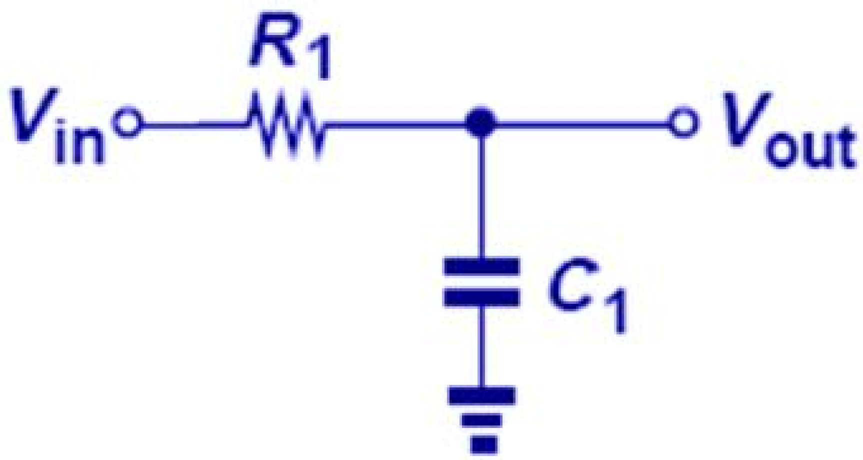
P=Filter Parameter

C=Component Value

- **Sensitivity indicates the variation of a filter parameter due to variation of a component value.**

Example 14.6: Sensitivity

Problem: Determine the sensitivity of ω_0 with respect to R_1.



$$\omega_0 = 1/(R_1 C_1)$$

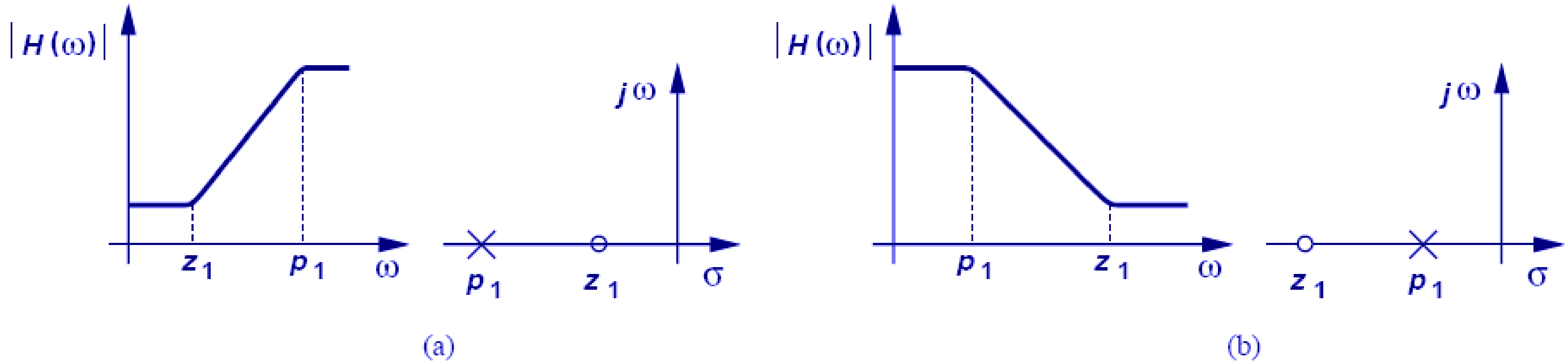
$$\frac{d\omega_0}{dR_1} = \frac{-1}{R_1^2 C_1}$$

$$\frac{d\omega_0}{\omega_0} = -\frac{dR_1}{R_1}$$

$$S_{R_1}^{\omega_0} = -1$$

- **For example, a +5% change in R_1 translates to a -5% error in ω_0.**

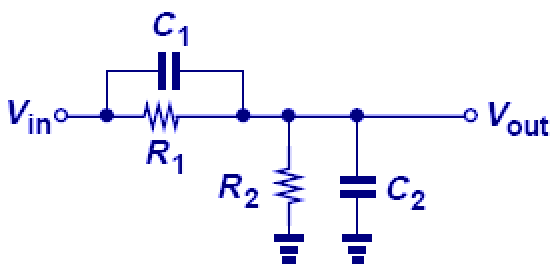
First-Order Filters



$$H(s) = \alpha \frac{s + z_1}{s + p_1}$$

- **First-order filters are represented by the transfer function shown above.**
- **Low/high pass filters can be realized by changing the relative positions of poles and zeros.**

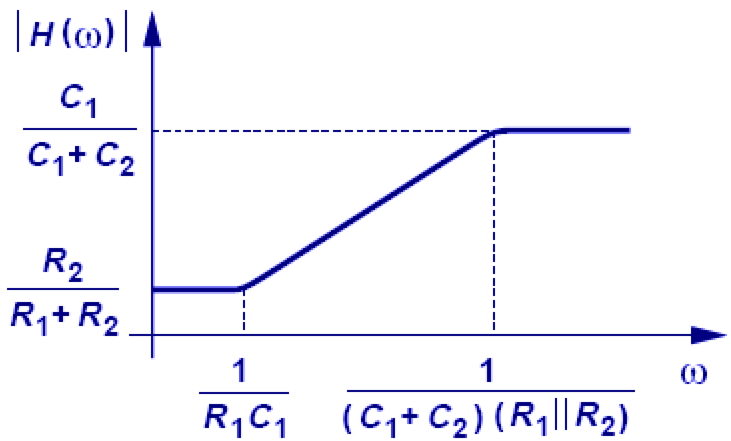
Example 14.8: First-Order Filter I



(a)

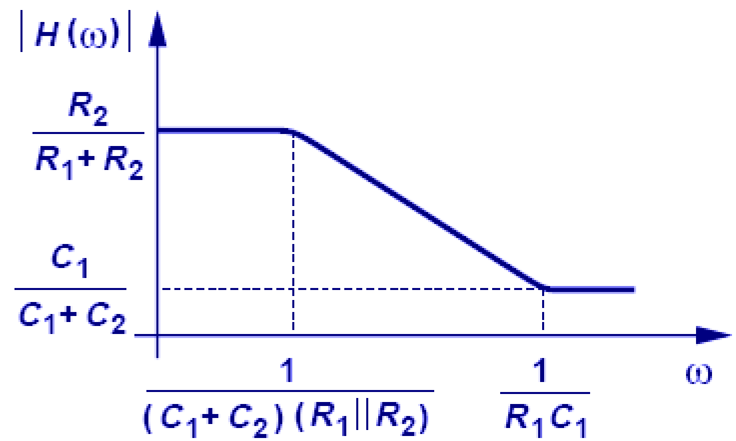
$$\frac{V_{out}}{V_{in}}(s) = \frac{R_2(R_1C_1s+1)}{R_1R_2(C_1+C_2)s+R_1+R_2}$$

$$z_1 = -1/(R_1C_1),\ p_1 = -[(C_1+C_2)R_1 \,||\, R_2]^{-1}$$



(b)

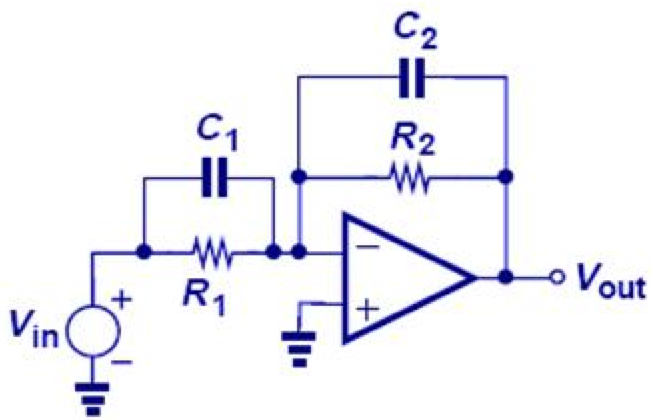
$R_2C_2 < R_1C_1$



(c)

$R_2C_2 > R_1C_1$

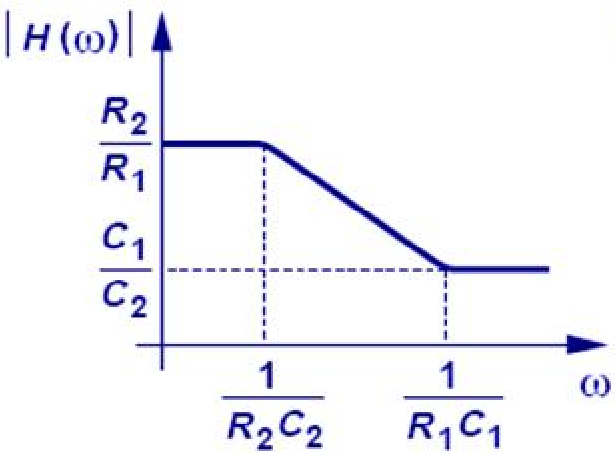
Example 14.9: First-Order Filter II



(a)

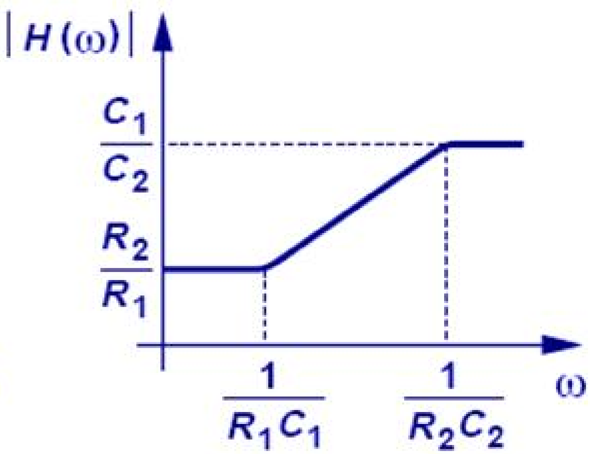
$$\frac{V_{out}}{V_{in}}(s) = \frac{-(R_2 \,||\, \frac{1}{C_2 s})}{R_1 \,||\, \frac{1}{C_1 s}}$$

$$= -\frac{R_2}{R_1} \cdot \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$$



(b)

$R_2C_2 < R_1C_1$



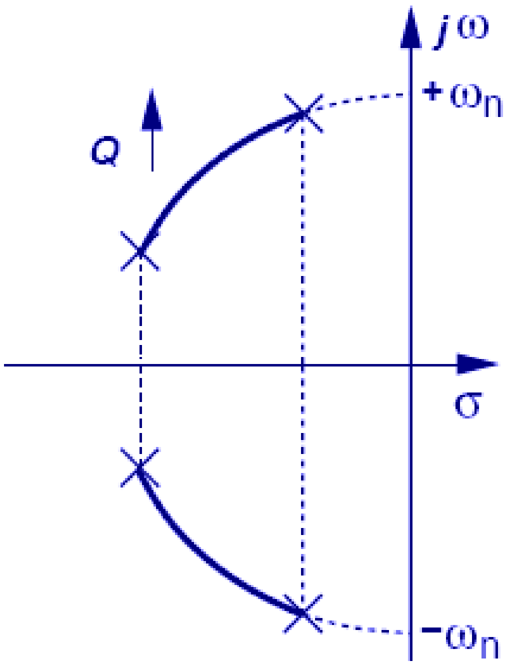
(c)

$R_2C_2 > R_1C_1$

Second-Order Filters

$$H(s) = \frac{\alpha s^2 + \beta s + \gamma}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}$$

$$p_{1,2} = -\frac{\omega_n}{2Q} \pm j\omega_n \sqrt{1 - \frac{1}{4Q^2}}$$

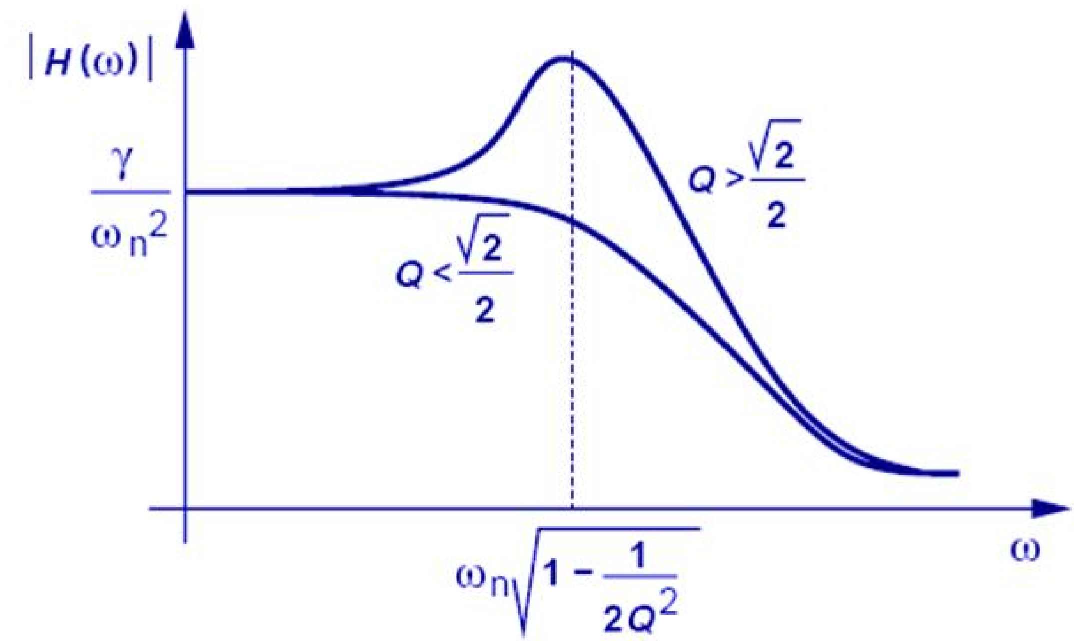


- **Second-order filters are characterized by the "biquadratic" equation with two complex poles shown above.**
- **When Q increases, the real part decreases while the imaginary part approaches $\pm\,\omega_n$.**
  => the poles look very imaginary thereby bringing the circuit closer to instability.

Second-Order Low-Pass Filter

$\alpha = \beta = 0$

$$|H(j\omega)|^2 = \frac{\gamma^2}{\left(\omega_n^2 - \omega^2\right)^2 + \left(\frac{\omega_n}{Q}\omega\right)^2}$$

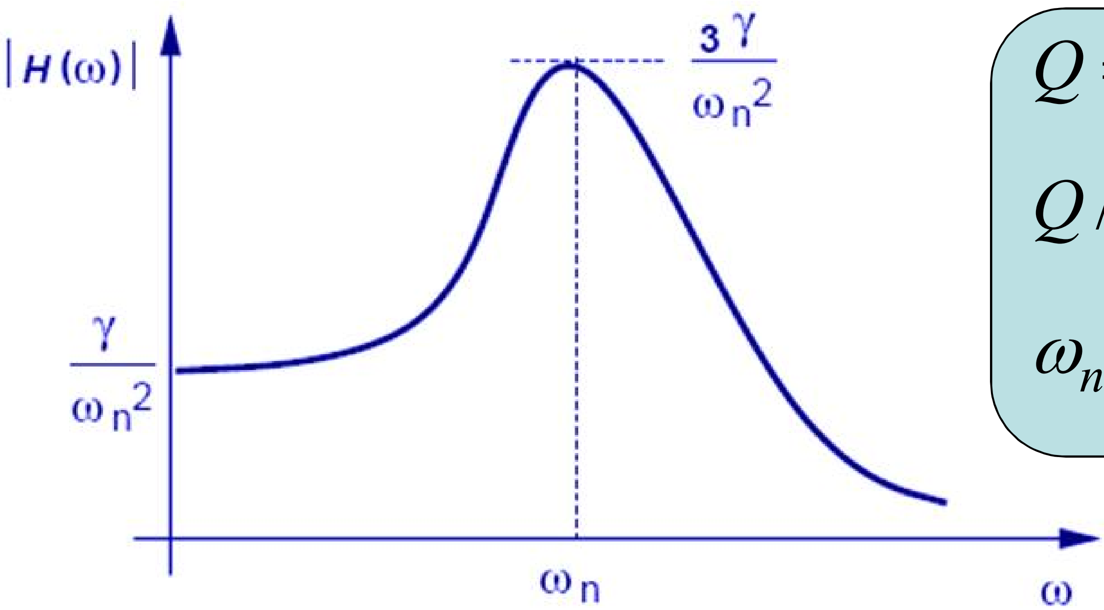


Peak magnitude normalized to the passband magnitude: $Q/\sqrt{1-(4Q^2)^{-1}}$

Example 14.10: Second-Order LPF

Problem: Q of a second-order LPF = 3.

Estimate the magnitude and frequency of the peak in the frequency response.



$$Q = 3$$

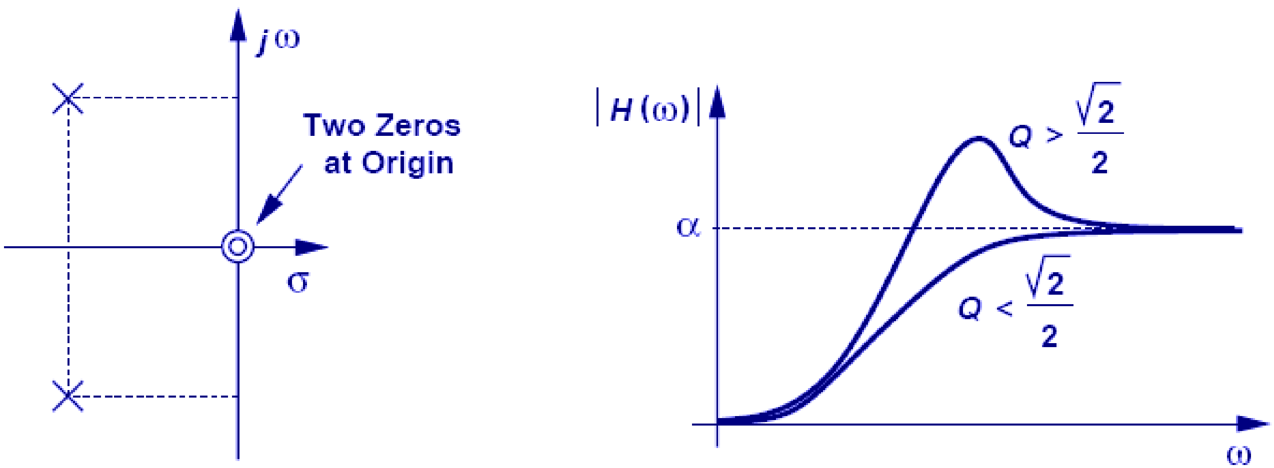
$$Q / \sqrt{1 - 1/(4Q^2)} \approx 3$$

$$\omega_n \sqrt{1 - 1/(2Q^2)} \approx \omega_n$$

Second-Order High-Pass Filter

$$H(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}$$

$\beta = \gamma = 0$



(a) (b)

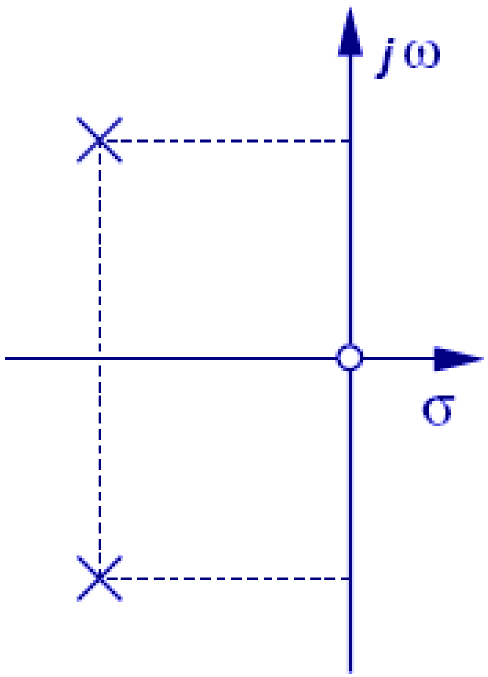
Frequency of the peak: $\omega_n / \sqrt{1 - 1/(2Q^2)}$

Peak magnitude normalized to the passband magnitude: $Q / \sqrt{1 - (4Q^2)^{-1}}$

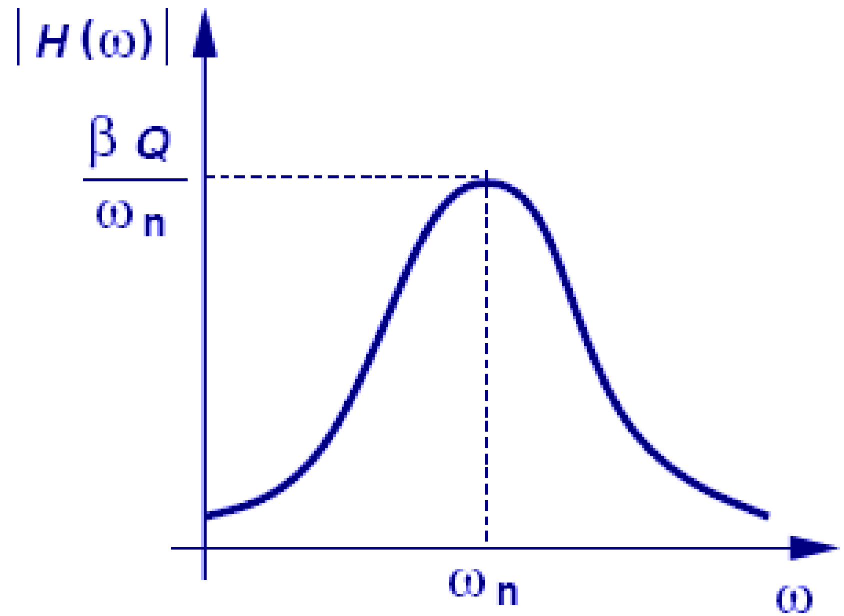
Second-Order Band-Pass Filter

$$H(s) = \frac{\beta s}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}$$

$\alpha = \gamma = 0$



(a)

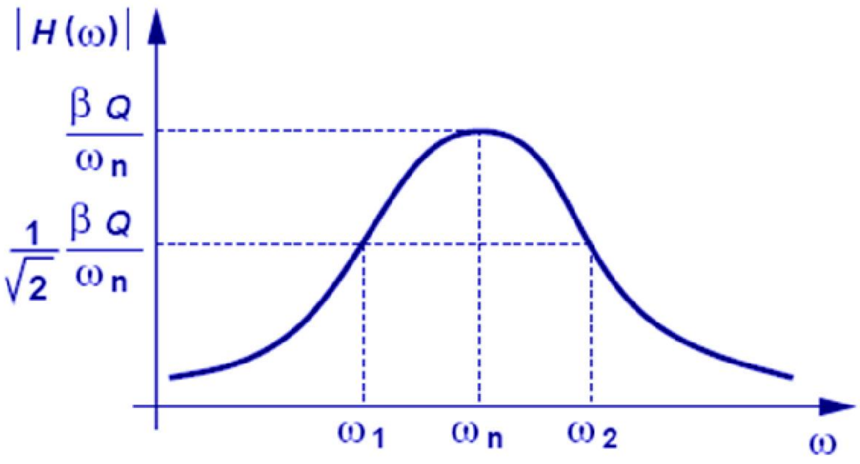


(b)

Example 14.2: -3-dB Bandwidth

Problem: Determine the -3dB bandwidth of a band-pass response.

$$H(s) = \frac{\beta s}{(s^2 + \frac{\omega_n}{Q} s + \omega_n^2)}$$

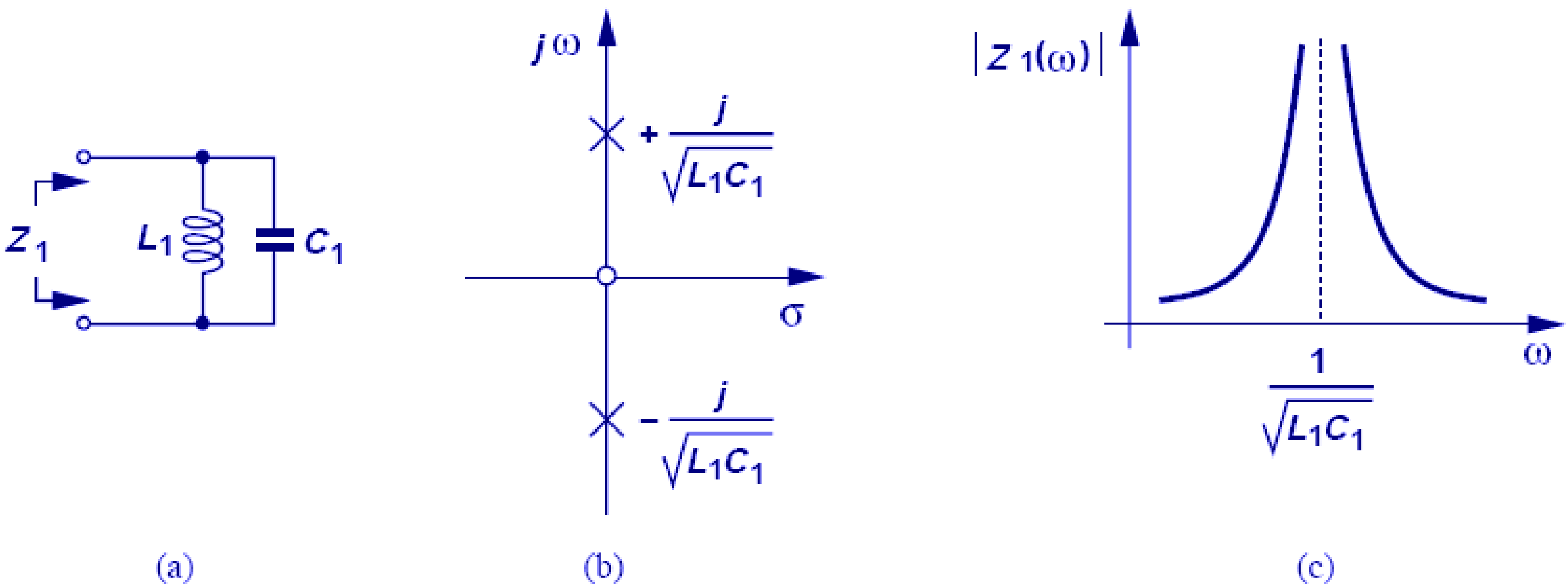


$$\frac{\beta^2 \omega^2}{(\omega_n{}^2 - \omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2} = \frac{\beta^2 Q^2}{2\omega_n{}^2}$$

$$\omega_{1,2} = \omega_0 \left[\sqrt{1 + \frac{1}{4Q^2}} \pm \frac{1}{2Q} \right]$$

$$BW = \frac{\omega_0}{Q}$$

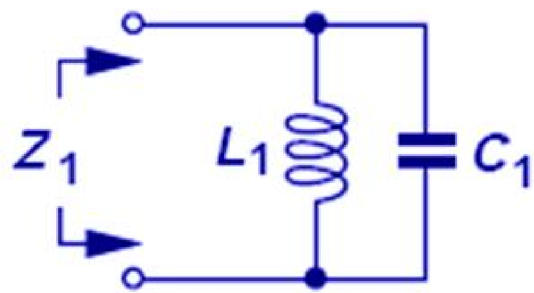
LC Realization of Second-Order Filters

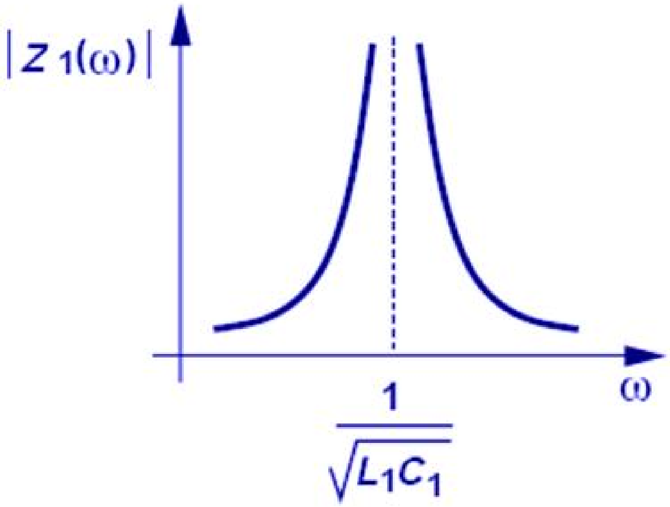


$$Z_1 = (L_1 s) \,||\, \frac{1}{C_1 s} = \frac{L_1 s}{L_1 C_1 s^2 + 1}$$

- **An LC tank realizes a second-order band-pass filter with two imaginary poles at $\pm j/(L_1C_1)^{1/2}$, which implies infinite impedance at $\omega=1/(L_1C_1)^{1/2}$.**

Example 14.13: LC Tank





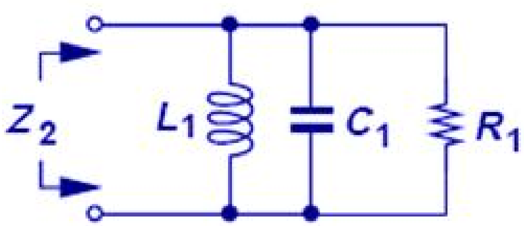
$$Z_1 = (L_1 s) \,||\, \frac{1}{C_1 s} = \frac{L_1 s}{L_1 C_1 s^2 + 1}$$

- **At ω=0, the inductor acts as a short.**
- **At ω=∞, the capacitor acts as a short.**

RLC Realization of Second-Order Filters

$$Z_2 = R_1 \parallel \frac{L_1 s}{L_1 C_1 s^2 + 1} = \frac{R_1 L_1 s}{R_1 L_1 C_1 s^2 + L_1 s + R_1}$$

$$= \frac{R_1 L_1 s}{R_1 L_1 C_1 (s^2 + \frac{1}{R_1 C_1} s + \frac{1}{L_1 C_1})} = \frac{R_1 L_1 s}{R_1 L_1 C_1 (s^2 + \frac{\omega_n}{Q} s + \omega_n{}^2)}$$



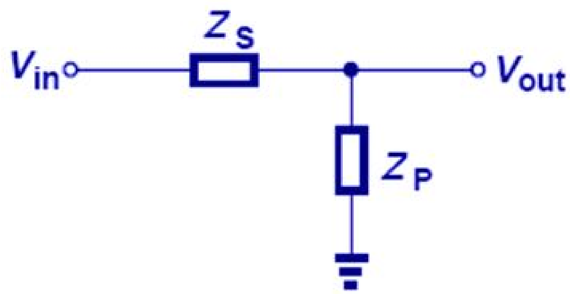
$$\omega_n = \frac{1}{\sqrt{L_1 C_1}}, \quad Q = R_1 \sqrt{\frac{C_1}{L_1}}$$

$$p_{1,2} = -\frac{\omega_n}{2Q} \pm j\omega_n \sqrt{1 - \frac{1}{4Q^2}}$$

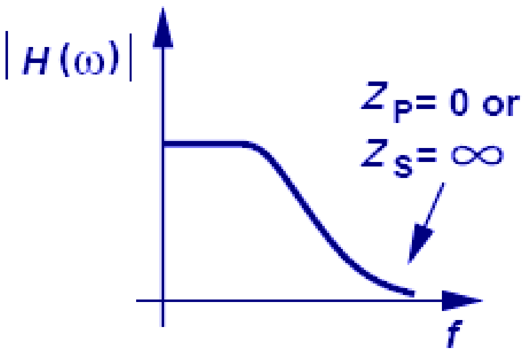
$$= -\frac{1}{2R_1 C_1} \pm j \frac{1}{\sqrt{L_1 C_1}} \sqrt{1 - \frac{L_1}{4R_1^2 C_1}}$$

- **With a resistor, the poles are no longer pure imaginary which implies there will be no infinite impedance at any ω.**

Voltage Divider Using General Impedances

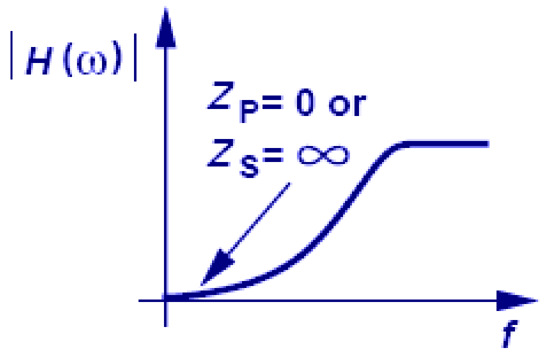


$$\frac{V_{out}}{V_{in}}(s) = \frac{Z_P}{Z_S + Z_P}$$



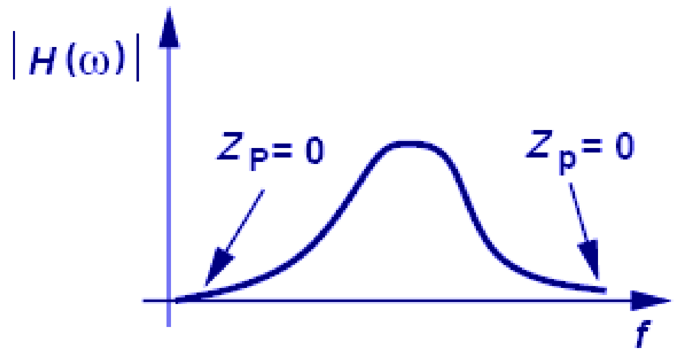
(a)

Low-pass



(b)

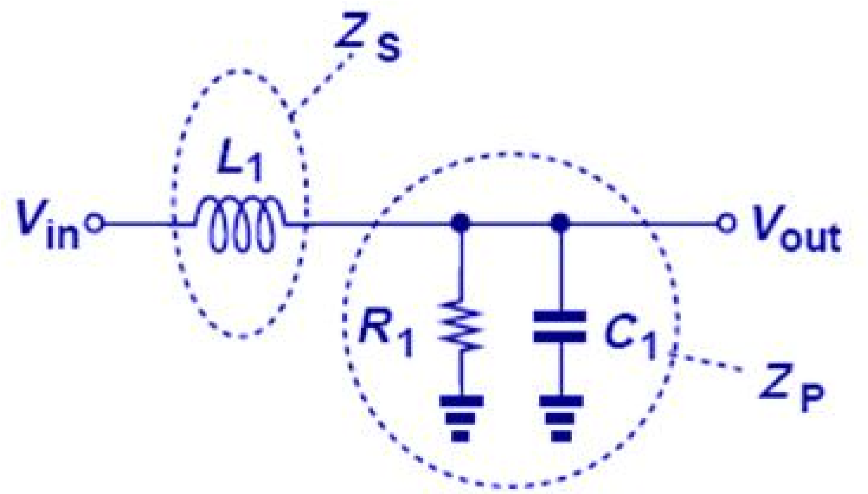
High-pass



(c)

Band-pass

Low-pass Filter Implementation with Voltage Divider

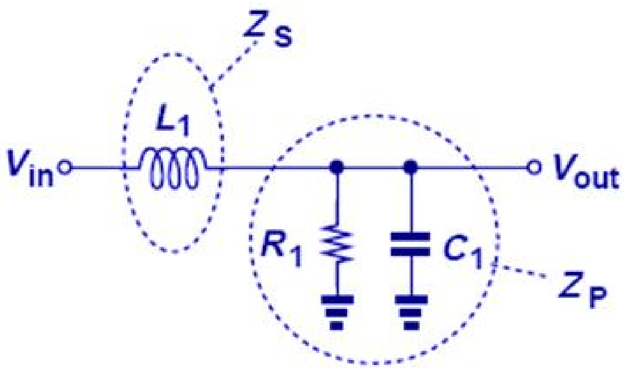


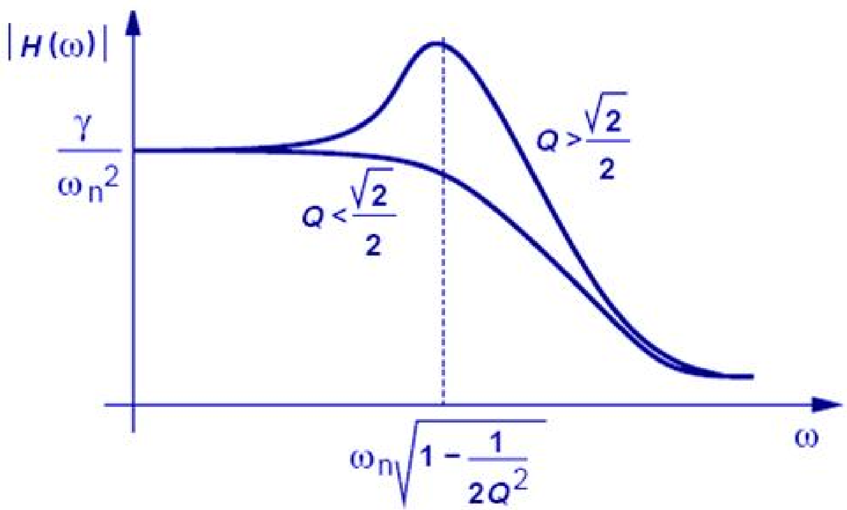
$$Z_S = L_1 s \to \infty \text{ as } s \to \infty$$

$$Z_P = \frac{1}{C_1 s} \parallel R_1 \to 0 \text{ as } s \to \infty$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{R_1}{R_1 C_1 L_1 s^2 + L_1 s + R_1}$$

Example 14.14: Frequency Peaking





$$\frac{V_{out}}{V_{in}}(s) = \frac{R_1}{R_1 C_1 L_1 s^2 + L_1 s + R_1}$$

$$\text{Let } |D|^2 = \left(R_1 - R_1 C_1 L_1 \omega^2\right)^2 + L_1^2 \omega^2$$

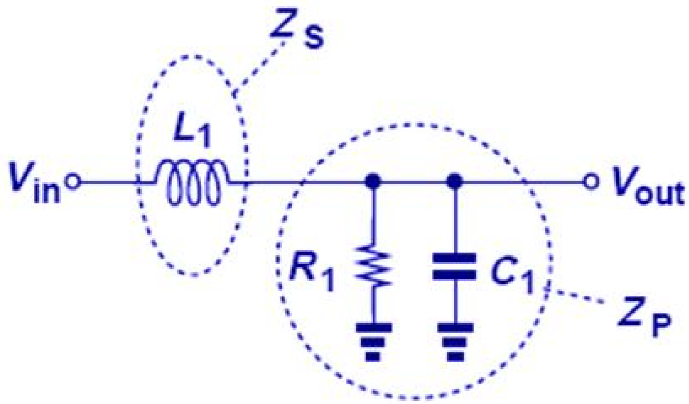
Voltage gain greater than unity (peaking) occurs when a solution exists for

$$\frac{d|D|^2}{d(\omega^2)} = 2(-R_1 C_1 L_1)(R_1 - R_1 C_1 L_1 \omega^2) + L_1^2$$
$$= 0$$

Thus, when $Q = R_1 \cdot \sqrt{\frac{C_1}{L_1}} > \frac{1}{\sqrt{2}}$,

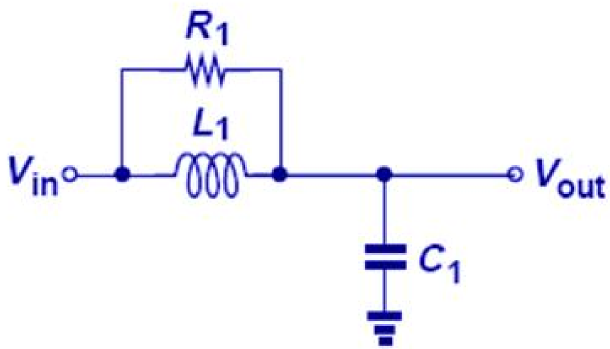
peaking occurs.

Example 14.15: Low-pass Circuit Comparison



(a)

Good

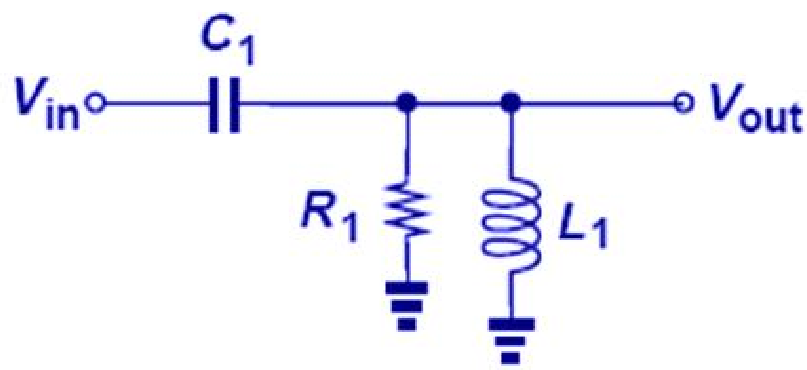


(b)

Bad

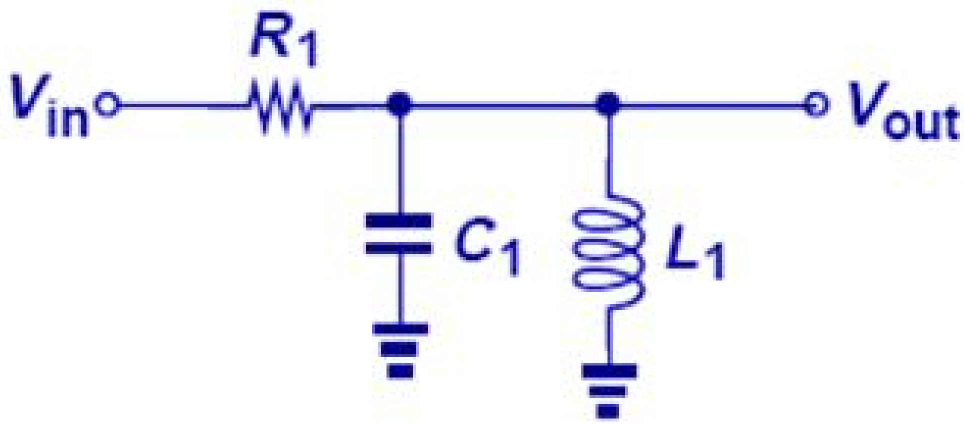
- **The circuit (a) has a -40dB/dec roll-off at high frequency.**
- **However, the circuit (b) exhibits only a -20dB/dec roll-off since the parallel combination of L_1 and R_1 is dominated by R_1 because $L_1\omega \rightarrow \infty$, thereby reduces the circuit to R_1 and C_1.**

High-pass Filter Implementation with Voltage Divider



$$\frac{V_{out}}{V_{in}}(s) = \frac{(L_1 s)\,||\,R_1}{(L_1 s)\,||\,R_1 + \frac{1}{C_1 s}} = \frac{L_1 C_1 R_1 s^2}{R_1 C_1 L_1 s^2 + L_1 s + R_1}$$

Band-pass Filter Implementation with Voltage Divider

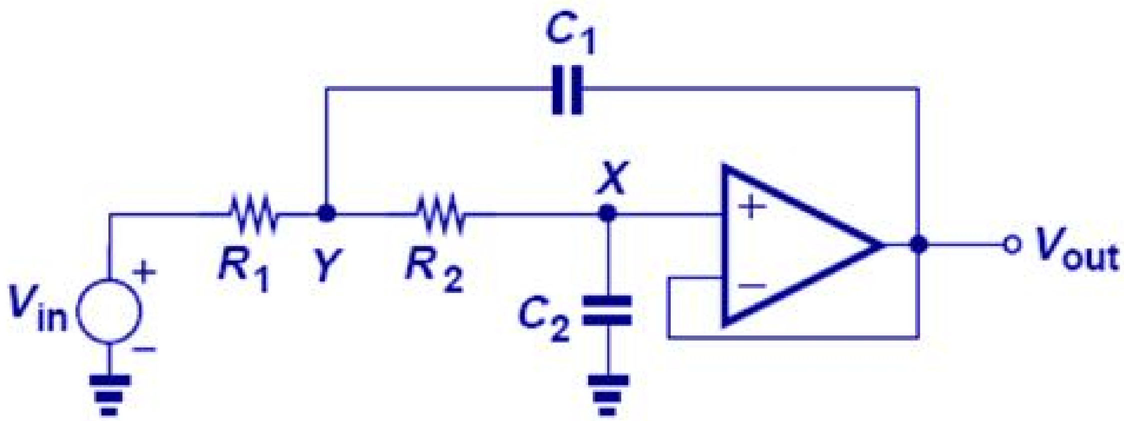


$$\frac{V_{out}}{V_{in}}(s) = \frac{(L_1 s) \,||\, \frac{1}{C_1 s}}{(L_1 s) \,||\, \frac{1}{C_1 s} + R_1} = \frac{L_1 s}{R_1 C_1 L_1 s^2 + L_1 s + R_1}$$

Why Active Filter?

- Passive filters constrain the type of transfer function.
- They may require bulky inductors.

Sallen and Key (SK) Filter: Low-Pass



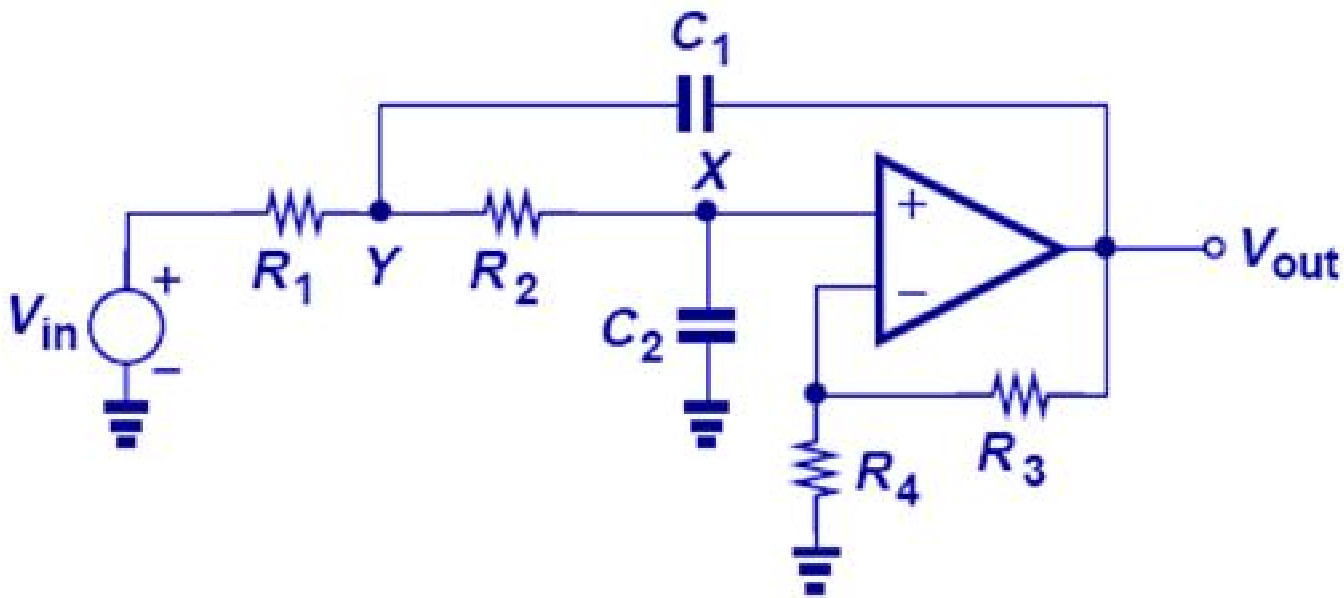
$$\frac{V_{out}}{V_{in}}(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1 + R_2)C_2s + 1}$$

$$Q = \frac{1}{R_1 + R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$$

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

- **Sallen and Key filters are examples of active filters. This particular filter implements a low-pass, second-order transfer function.**

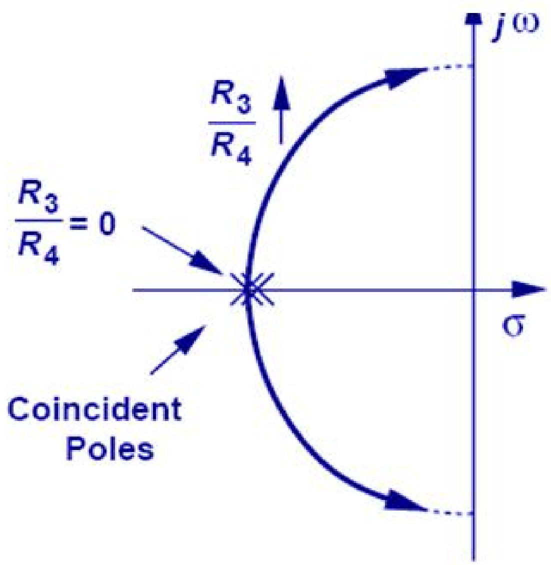
Example 14.16: SK Filter with Voltage Gain



$$\frac{V_{out}}{V_{in}}(s) = \frac{1+\dfrac{R_3}{R_4}}{R_1R_2C_1C_2s^2+\left(R_1C_2+R_2C_2-R_1\dfrac{R_3}{R_4}C_1\right)s+1}$$

Example 14.17: SK Filter Poles

Problem: Assuming $R_1=R_2$, $C_1=C_2$, Does such a filter contain complex poles?



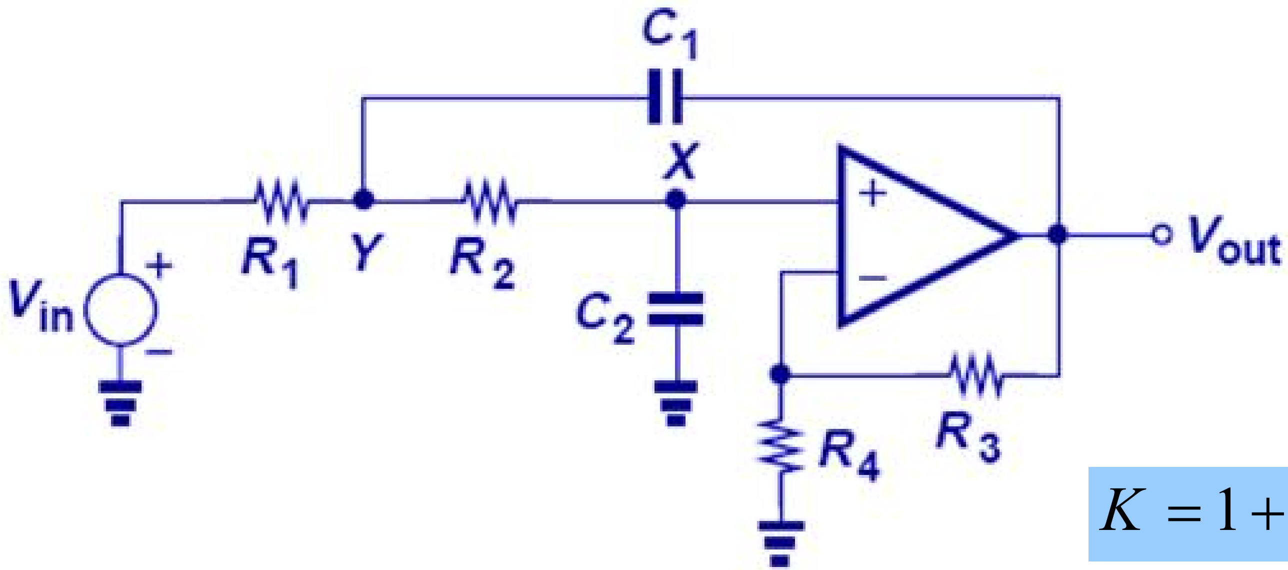
$$\frac{V_{out}}{V_{in}}(s) = \frac{1+\frac{R_3}{R_4}}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1)s + 1}$$

$$\frac{1}{Q} = \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}} - \sqrt{\frac{R_1C_1}{R_2C_2}}\frac{R_3}{R_4}$$

$$Q = \frac{1}{2-\frac{R_3}{R_4}}$$

- **The poles begin with real, equal values for $R_3/R_4 = 0$ and become complex for $R_3/R_4 > 0$.**

Sensitivity in Low-Pass SK Filter



$$K = 1 + R_3 / R_4$$

$$S_{R_1}^{\omega_n} = S_{R_2}^{\omega_n} = S_{C_1}^{\omega_n} = S_{C_2}^{\omega_n} = -\frac{1}{2}$$

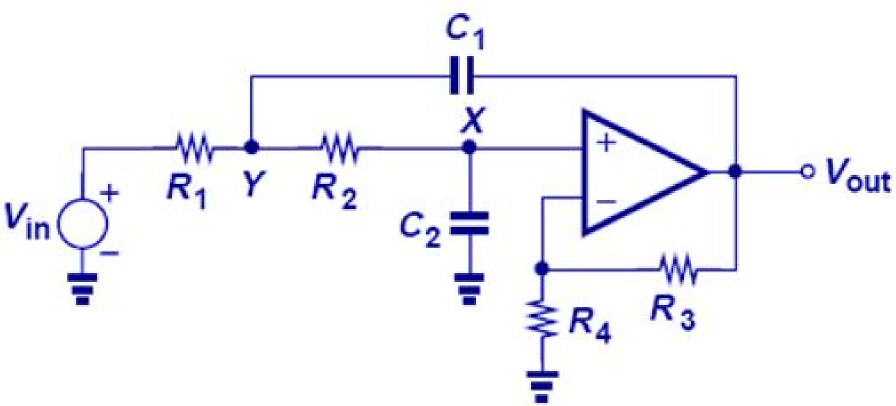
$$S_{C_1}^{Q} = -S_{C_2}^{Q} = -\frac{1}{2} + Q\left(\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}}\right)$$

$$S_{R_1}^{Q} = -S_{R_2}^{Q} = -\frac{1}{2} + Q\sqrt{\frac{R_2 C_2}{R_1 C_1}}$$

$$S_{K}^{Q} = QK\sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

Example 14.18: SK Filter Sensitivity I

Problem: Determine the Q sensitivities of the SK filter for the common choice $R_1=R_2=R$, $C_1=C_2=C$.



$$S_{R_1}^{Q} = -S_{R_2}^{Q} = -\frac{1}{2} + \frac{1}{3-K}$$

$$S_{C_1}^{Q} = -S_{C_2}^{Q} = -\frac{1}{2} + \frac{2}{3-K}$$

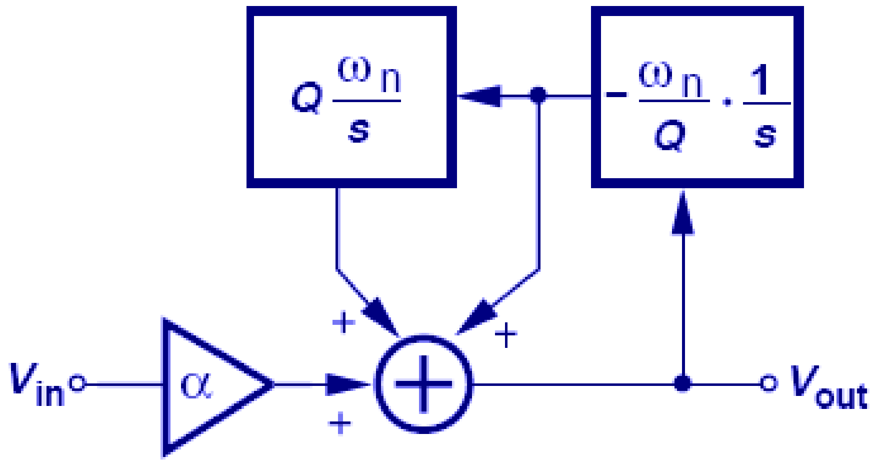
$$S_{K}^{Q} = \frac{K}{3-K}$$

With $K=1$,

$$\left|S_{R_1}^{Q}\right| = \left|S_{R_2}^{Q}\right| = 0$$

$$\left|S_{C_1}^{Q}\right| = \left|S_{C_2}^{Q}\right| = \left|S_{K}^{Q}\right| = \frac{1}{2}$$

Integrator-Based Biquads

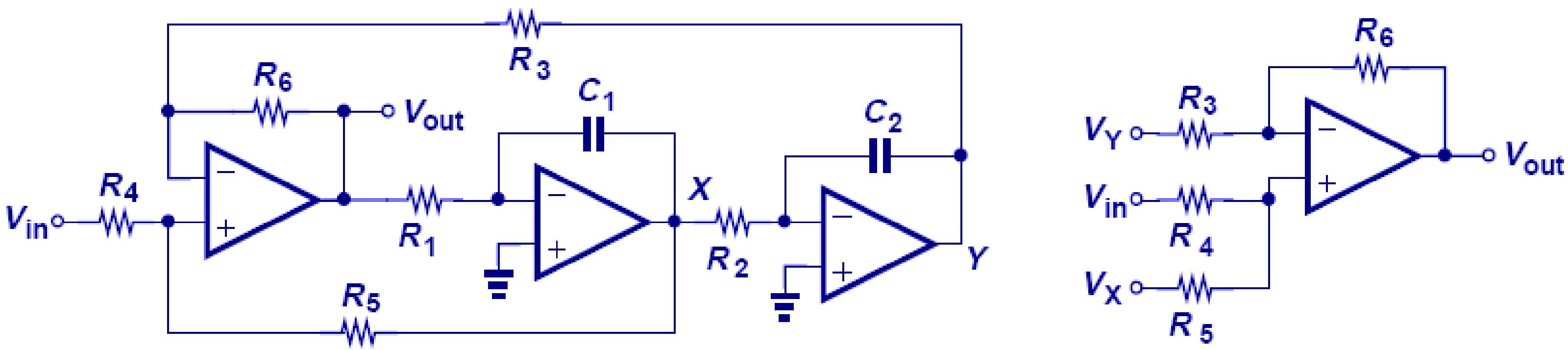


$$\frac{V_{out}}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$V_{out}(s) = \alpha V_{in}(s) - \frac{\omega_n}{Q}\cdot\frac{1}{s}V_{out}(s) - \frac{\omega_n^2}{s^2}V_{out}(s)$$

- **It is possible to use integrators to implement biquadratic transfer functions.**

KHN (Kerwin, Huelsman, and Newcomb) Biquads



$$V_X = -\frac{1}{R_1 C_1 s} V_{out}, \quad V_Y = -\frac{1}{R_2 C_2 s} V_X = \frac{1}{R_1 R_2 C_1 C_2 s^2} V_{out}$$

$$V_{out} = \frac{V_{in} R_5 + V_X R_4}{R_4 + R_5}\left(1 + \frac{R_6}{R_3}\right) - V_Y \frac{R_6}{R_3}$$

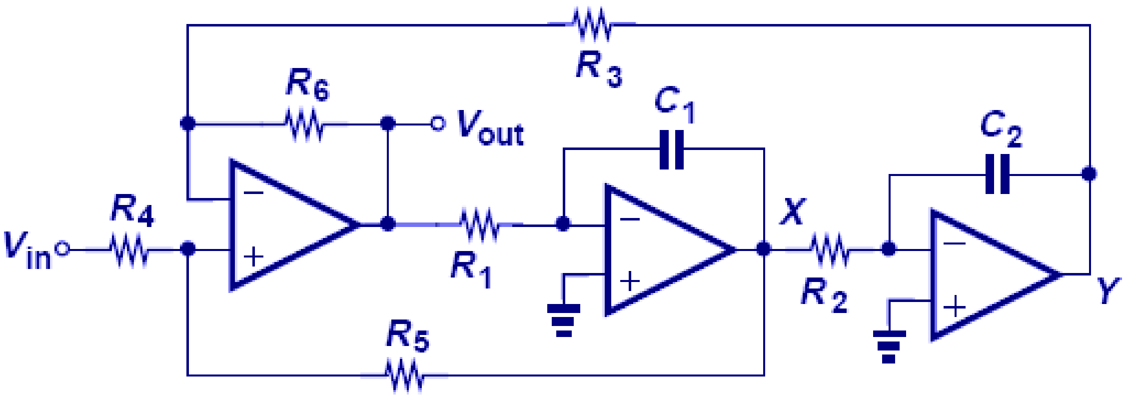
Comparing with $V_{out}(s) = \alpha V_{in}(s) - \frac{\omega_n}{Q} \cdot \frac{1}{s} V_{out}(s) - \frac{\omega_n^2}{s^2} V_{out}(s)$

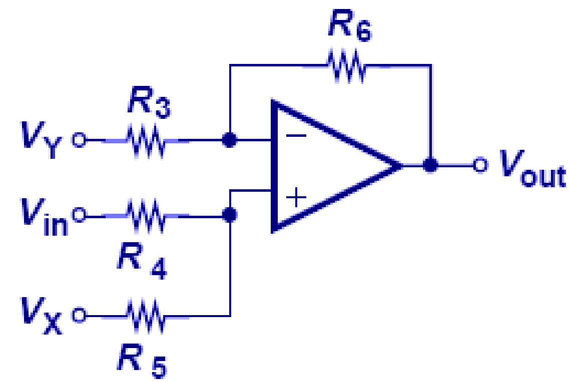
$$\alpha = \frac{R_5}{R_4 + R_5}\left(1 + \frac{R_6}{R_3}\right)$$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5} \cdot \frac{1}{R_1 C_1} \cdot \left(1 + \frac{R_6}{R_3}\right)$$

$$\omega_n^2 = \frac{R_6}{R_3} \cdot \frac{1}{R_1 R_2 C_1 C_2}$$

Versatility of KHN Biquads



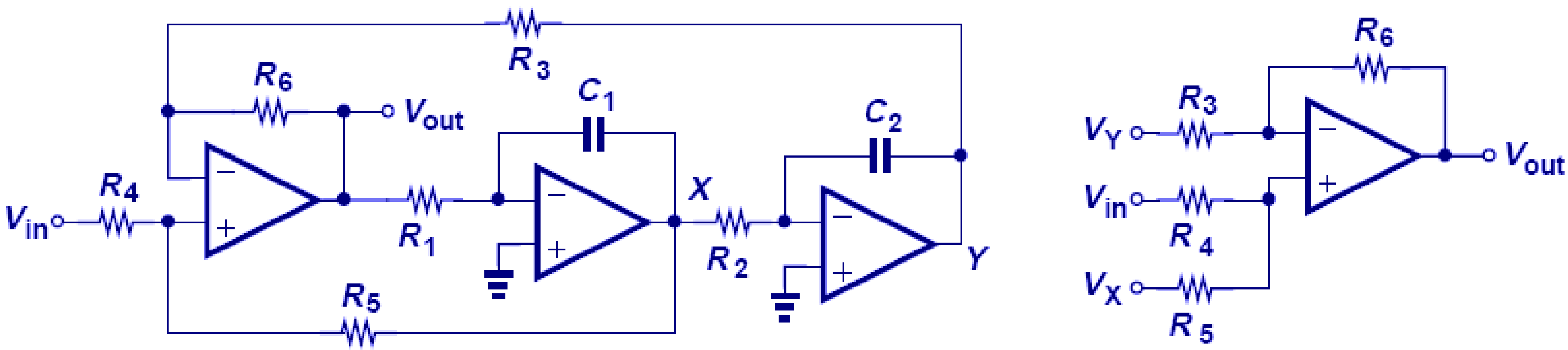


$$\text{High-pass: } \frac{V_{out}}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$\text{Band-pass: } \frac{V_X}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \cdot \frac{-1}{R_1 C_1 s}$$

$$\text{Low-pass: } \frac{V_Y}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \cdot \frac{1}{R_1 R_2 C_1 C_2 s^2}$$

Sensitivity in KHN Biquads

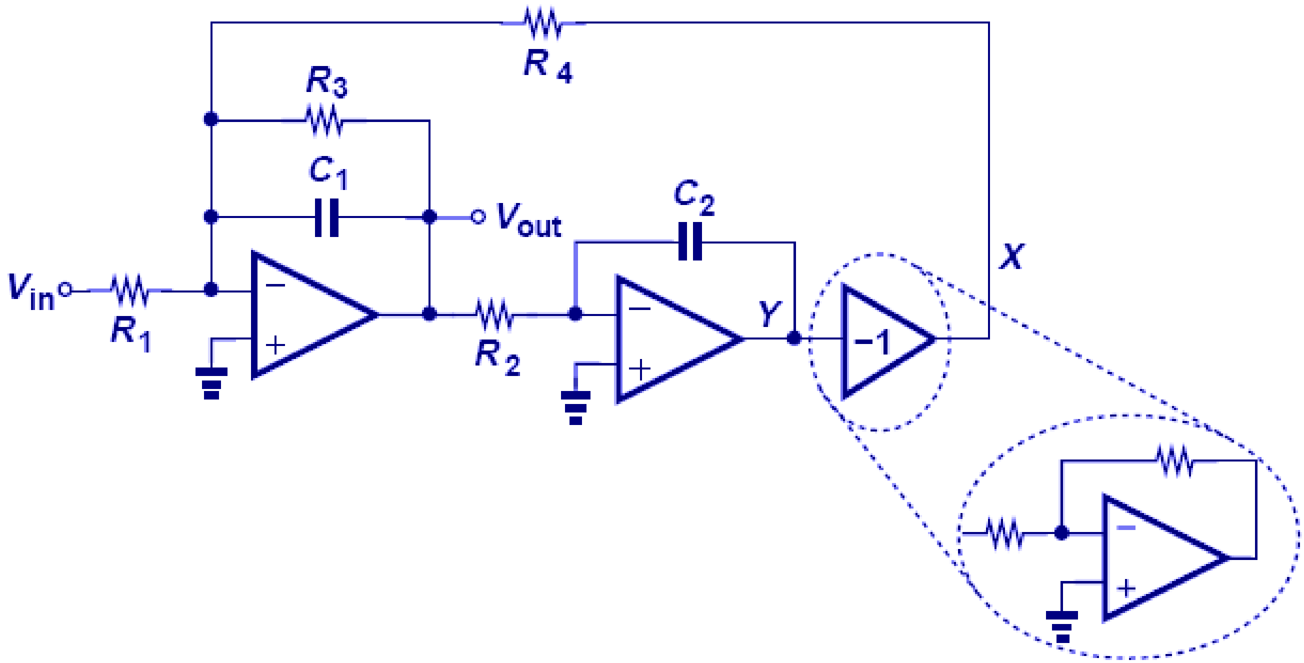


$$\left| S^{\omega_n}_{R_1, R_2, C_1, C_2, R_4, R_5, R_3, R_6} \right| = 0.5$$

$$\left| S^{Q}_{R_1, R_2, C_1, C_2} \right| = 0.5, \quad \left| S^{Q}_{R_4, R_5} \right| = \frac{R_5}{R_4 + R_5} < 1,$$

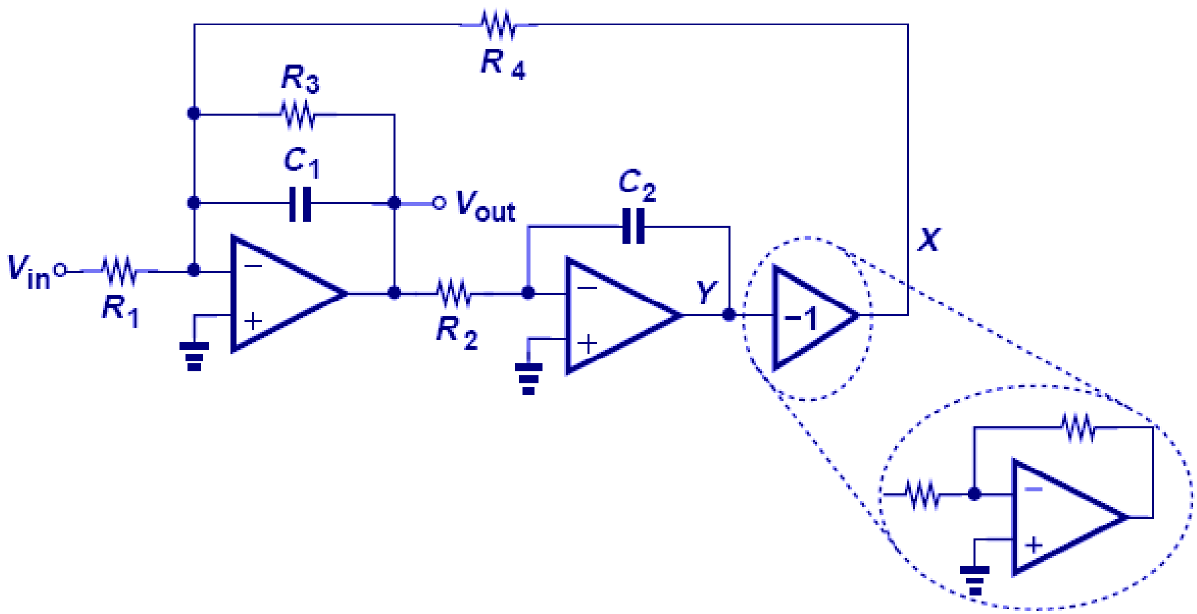
$$\left| S^{Q}_{R_3, R_6} \right| = \frac{Q}{2} \frac{\left| R_3 - R_6 \right|}{1 + \frac{R_5}{R_4}} \sqrt{\frac{R_2 C_2}{R_3 R_6 R_1 C_1}}$$

Tow-Thomas Biquad



$$\left(\frac{V_{out}}{R_2C_2s}\cdot\frac{1}{R_4}+\frac{V_{in}}{R_1}\right)\left(R_3 \,\|\, \frac{1}{sC_1}\right)=-V_{out}$$

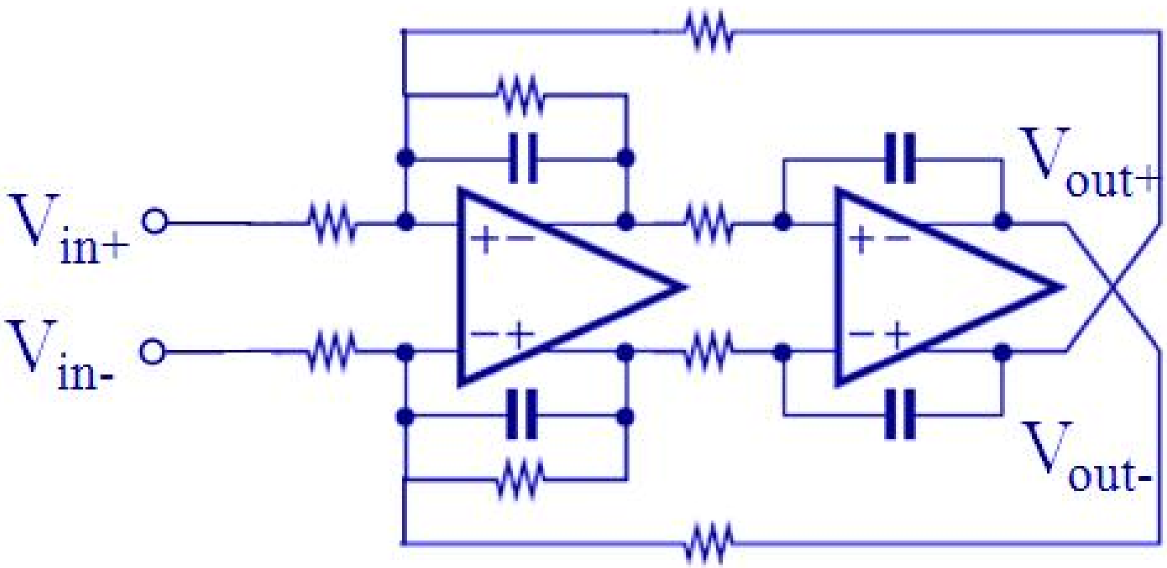
Tow-Thomas Biquad



Band-pass: $$\frac{V_{out}}{V_{in}} = -\frac{R_2R_3R_4}{R_1} \cdot \frac{C_2 s}{R_2R_3R_4C_1C_2s^2 + R_2R_4C_2s + R_3}$$

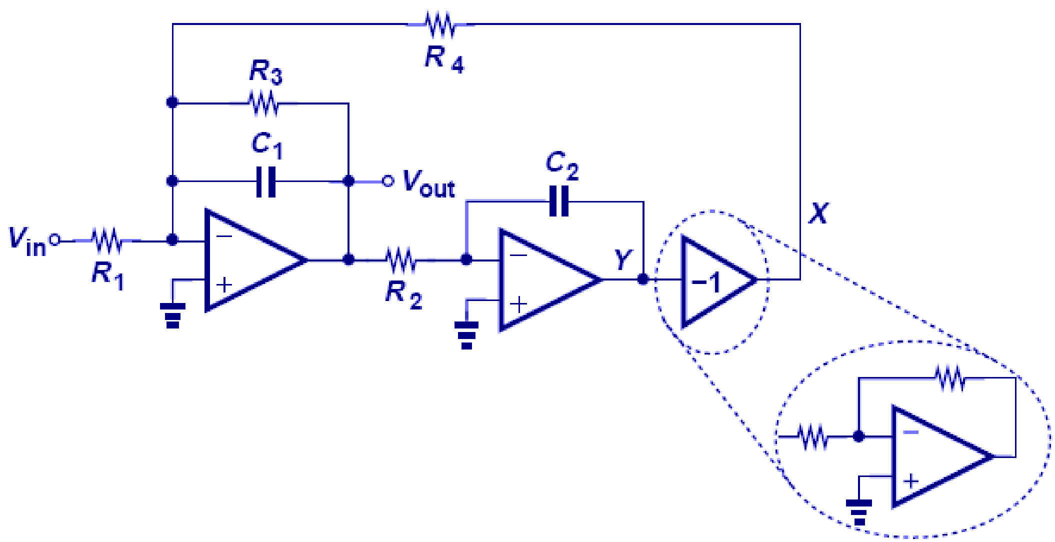
Low-pass: $$\frac{V_Y}{V_{in}} = \frac{R_3R_4}{R_1} \cdot \frac{1}{R_2R_3R_4C_1C_2s^2 + R_2R_4C_2s + R_3}$$

Differential Tow-Thomas Biquads



- An important advantage of this topology over the KHN biquad is accrued in integrated circuit design, where differential integrators obviate the need for the inverting stage in the loop.

Example 14.20: Tow-Thomas Biquad



Note that ω_n and Q of the Tow-Thomas filter can be adjusted (tuned) independently.

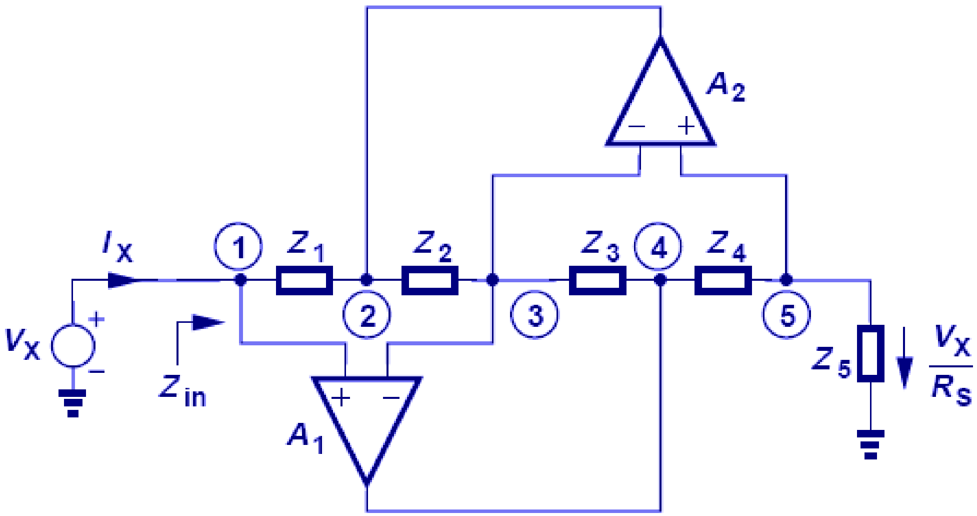
$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

Adjusted by R_2 or R_4

$$Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

Adjusted by R_3

Antoniou General Impedance Converter



$$Z_{in} = \frac{Z_1 Z_3}{Z_2 Z_4} Z_5$$

$$V_1 = V_3 = V_5 = V_X$$

$$V_4 = \frac{V_X}{Z_5} Z_4 + V_X$$

$$I_{Z3} = \frac{V_4 - V_3}{Z_3}$$

$$= \frac{V_X}{Z_5} \cdot \frac{Z_4}{Z_3}$$

$$V_2 = V_3 - Z_2 I_{Z3}$$

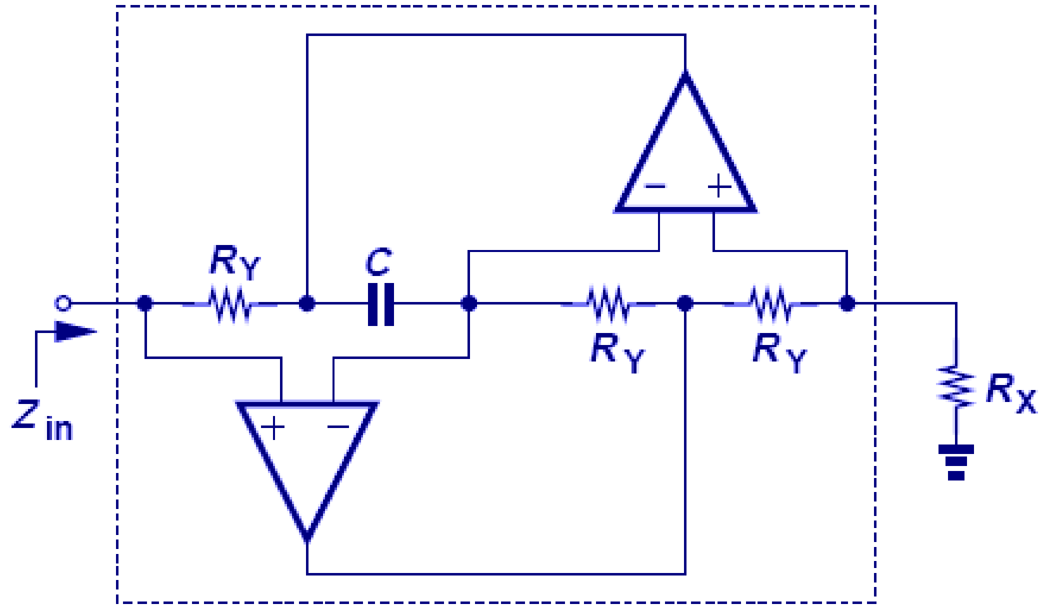
$$= V_X - Z_2 \cdot \frac{V_X}{Z_5} \cdot \frac{Z_4}{Z_3}$$

$$I_X = \frac{V_X - V_2}{Z_1}$$

$$= V_X \frac{Z_2 Z_4}{Z_1 Z_3 Z_5}$$

- It is possible to simulate the behavior of an inductor by using active circuits in feedback with properly chosen passive elements.

Simulated Inductor

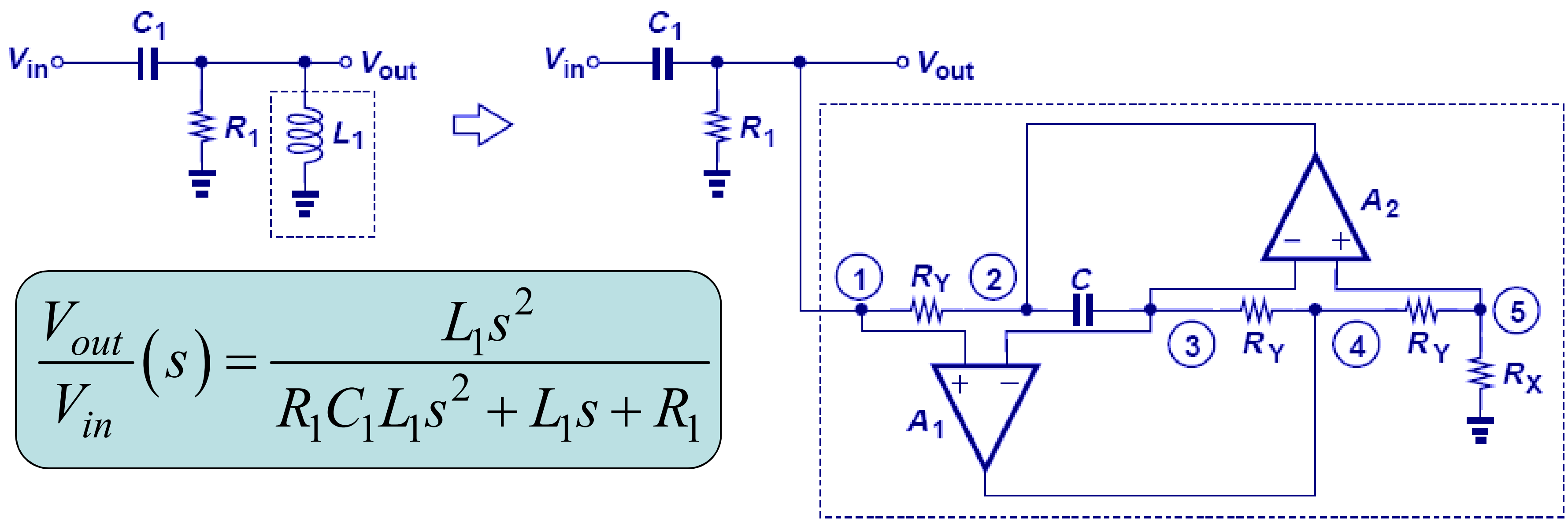


$$Z_{in} = R_X R_Y C s$$

$$\text{Thus, } L_{eq} = R_X R_Y C$$

- By proper choices of Z_1-Z_4, Z_{in} has become an impedance that increases with frequency, simulating inductive effect.

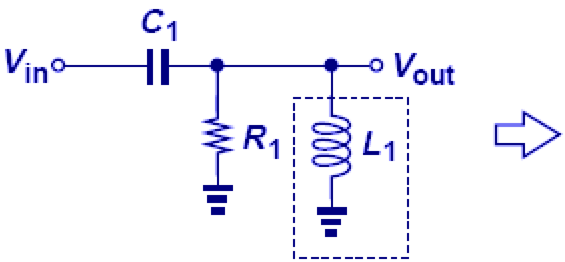
High-Pass Filter with SI

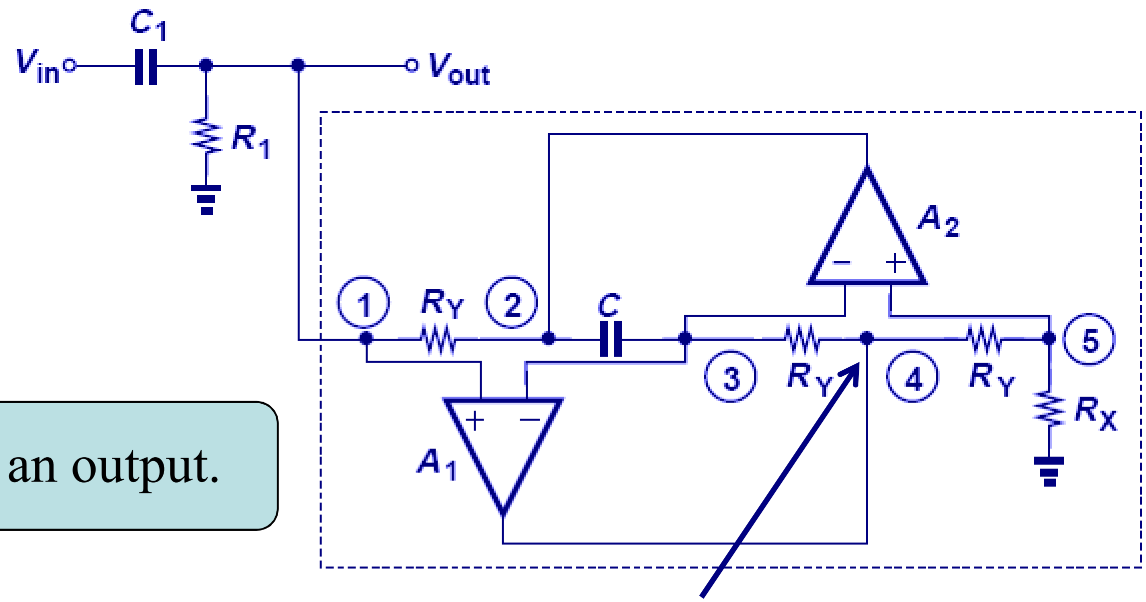


$$\frac{V_{out}}{V_{in}}(s) = \frac{L_1 s^2}{R_1 C_1 L_1 s^2 + L_1 s + R_1}$$

- With the inductor simulated at the output, the transfer function resembles a second-order high-pass filter.

Example 14.22: High-Pass Filter with SI





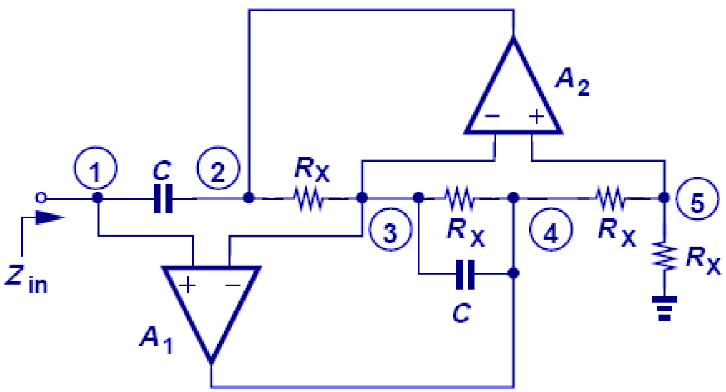
Node 4 can also serve as an output.

$$V_4 = V_{out}\left(1+\frac{R_Y}{R_X}\right)$$

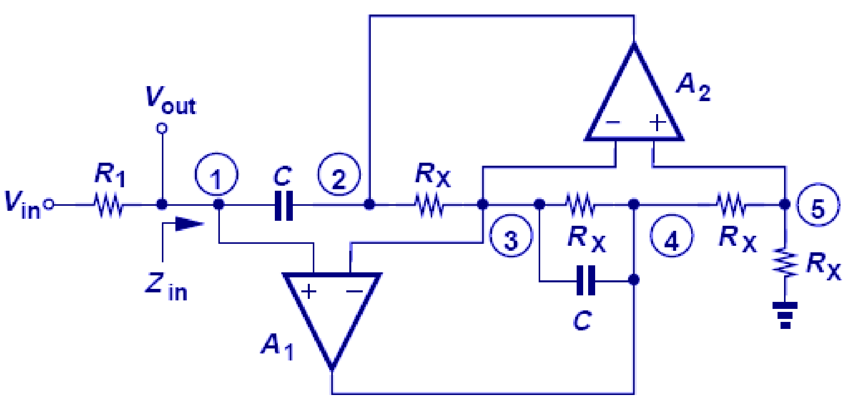
- V_4 **is better than** V_{out} **since the output impedance is lower.**

Low-Pass Filter with Super Capacitor

- How to build a floating inductor to derive a low-pass filter? Not possible. So use a super capacitor.



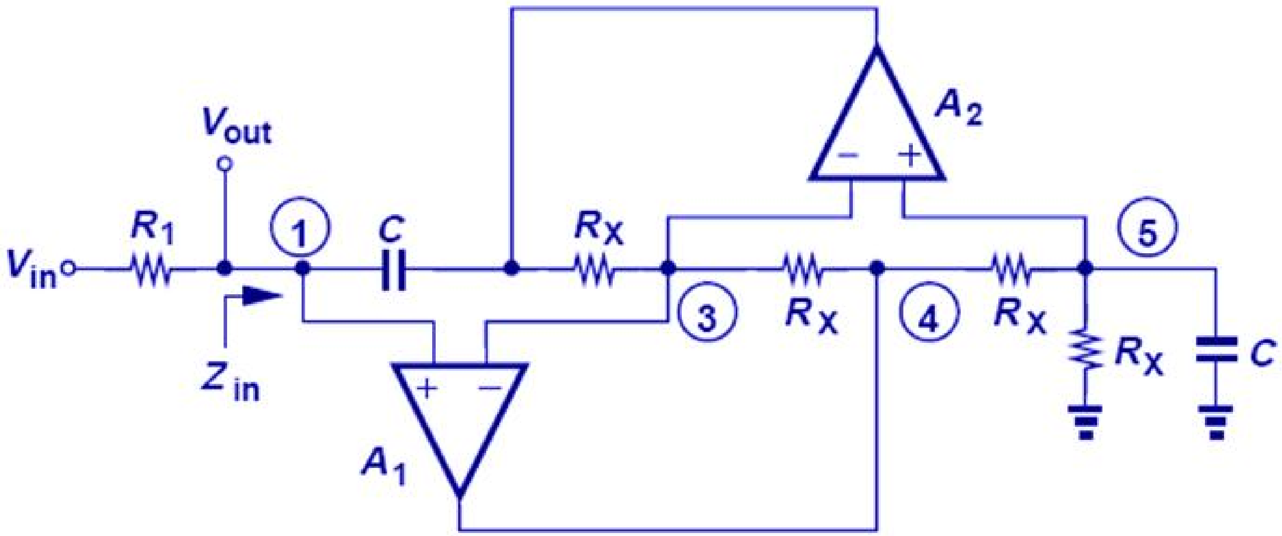
$$Z_{in} = \frac{1}{Cs(R_X Cs + 1)}$$



$$\frac{V_{out}}{V_{in}} = \frac{Z_{in}}{Z_{in} + R_1}$$

$$= \frac{1}{R_1 R_X C^2 s^2 + R_1 Cs + 1}$$

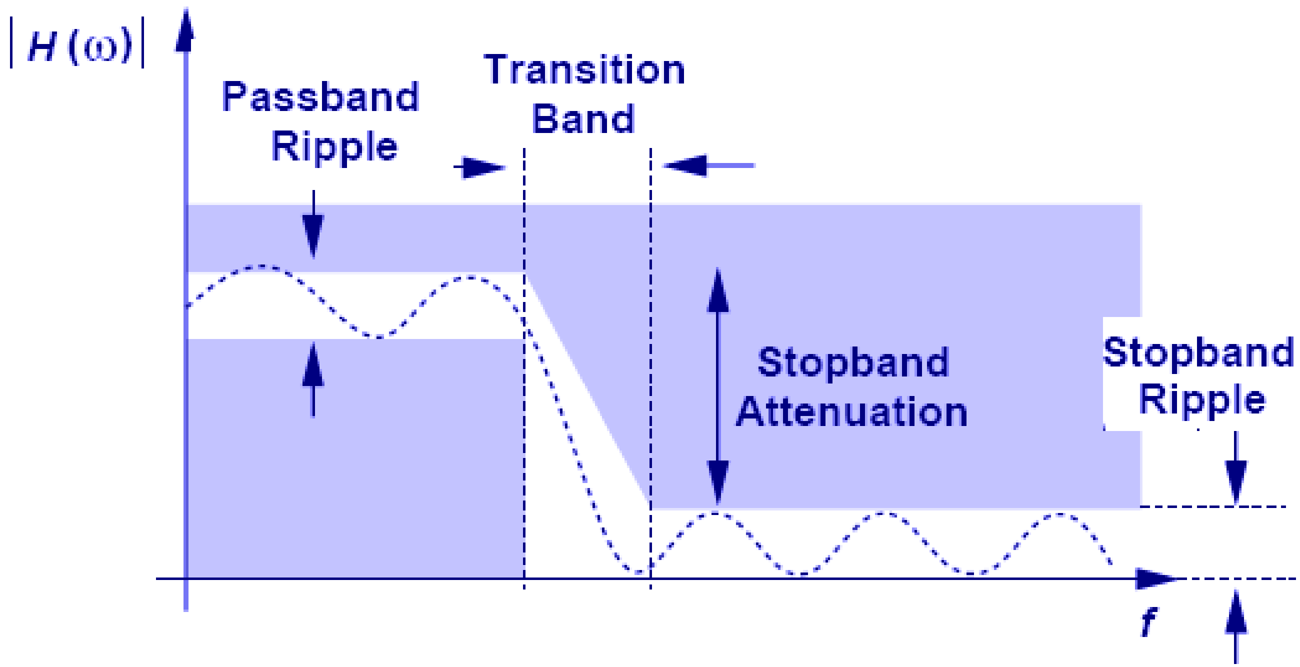
Example 14.24: Poor Low-Pass Filter



$$V_4 = \left[V_{out} \left(\frac{1}{R_X} + Cs \right) \right] R_X + V_{out} = V_{out} \left(2 + R_X C s \right)$$

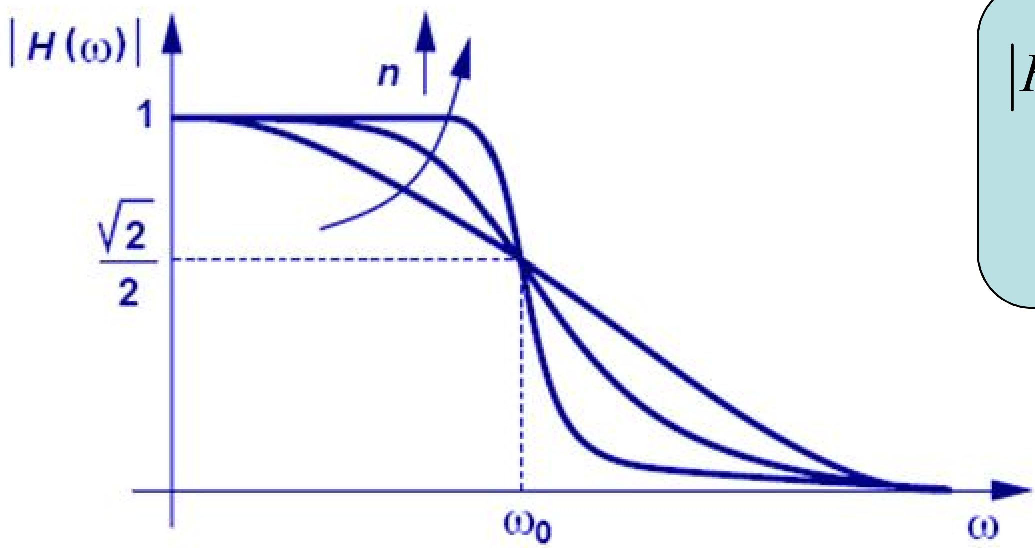
- Node 4 is no longer a scaled version of the V_{out}. Therefore the output can only be sensed at node 1, suffering from a high impedance.

Frequency Response Template



- With all the specifications on pass/stop band ripples and transition band slope, one can create a filter template that will lend itself to transfer function approximation.

Butterworth Response



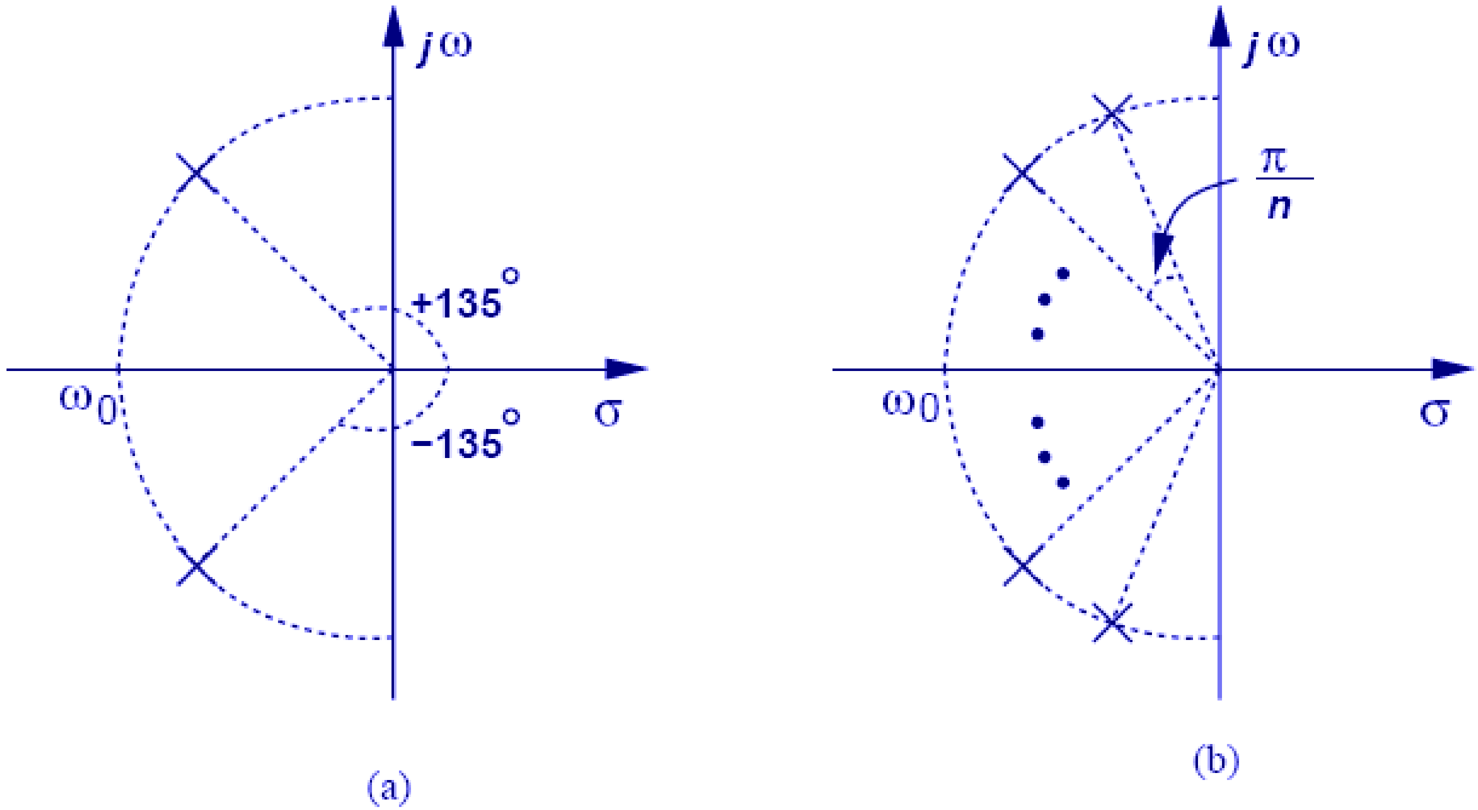
$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}}$$

$\omega_{-3dB}=\omega_{0,}$ **for all n.**

- **The Butterworth response completely avoids ripples in the pass/stop bands at the expense of the transition band slope.**

Poles of the Butterworth Response

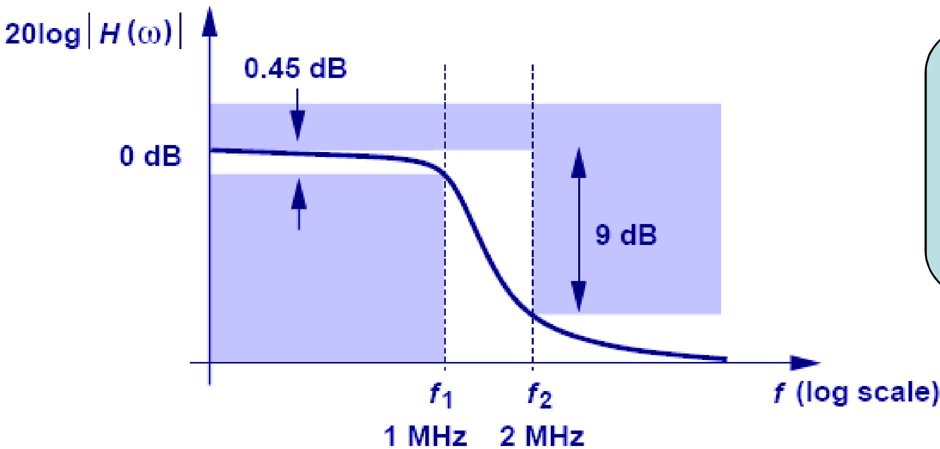
$$p_k = \omega_0 \exp\frac{j\pi}{2}\exp\left(j\frac{2k-1}{2n}\pi\right), k = 1, 2, \cdots, n$$



2nd-Order

nth-Order

Example 14.24: Order of Butterworth Filter



Specification: passband flatness of 0.45 dB for f < f_1=1 MHz, stopband attenuation of 9 dB at f_2=2 MHz.

$$|H(f_1 = 1\text{MHz})| = 0.95$$

$$\frac{1}{1+\left(\frac{2\pi f_1}{\omega_0}\right)^{2n}} = 0.95^2$$

$$|H(f_2 = 2\text{MHz})| = 0.355$$

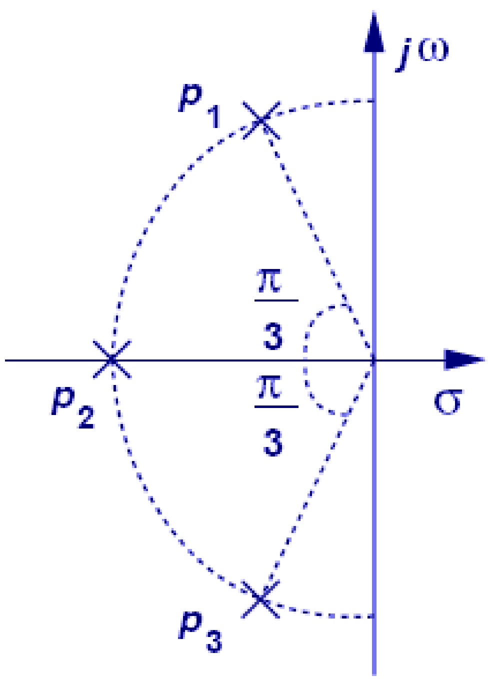
$$\frac{1}{1+\left(\frac{2\pi f_2}{\omega_0}\right)^{2n}} = 0.355^2$$

$$\left(\frac{f_2}{f_1}\right)^{2n} = 64.2 \Rightarrow f_2 = 2f_1 \Rightarrow n = 3, \quad \omega_0 = 2\pi \times (1.45\text{MHz})$$

- **The minimum order of the Butterworth filter is three.**

Example 14.25: Butterworth Response

Using a Sallen and Key topology, design a Butterworth filter for the response derived in Example 14.24.



$$p_1 = 2\pi * (1.45MHz) * \left(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} \right)$$

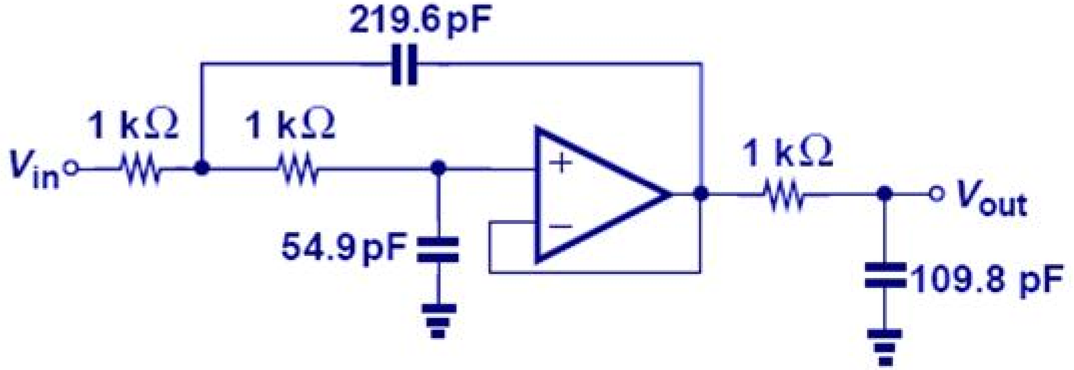
$$p_3 = 2\pi * (1.45MHz) * \left(\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} \right)$$

$$p_2 = 2\pi * (1.45MHz)$$

2nd-order SK (p_1, p_3)

RC section (p_2)

Example 14.25: Butterworth Response (cont'd)



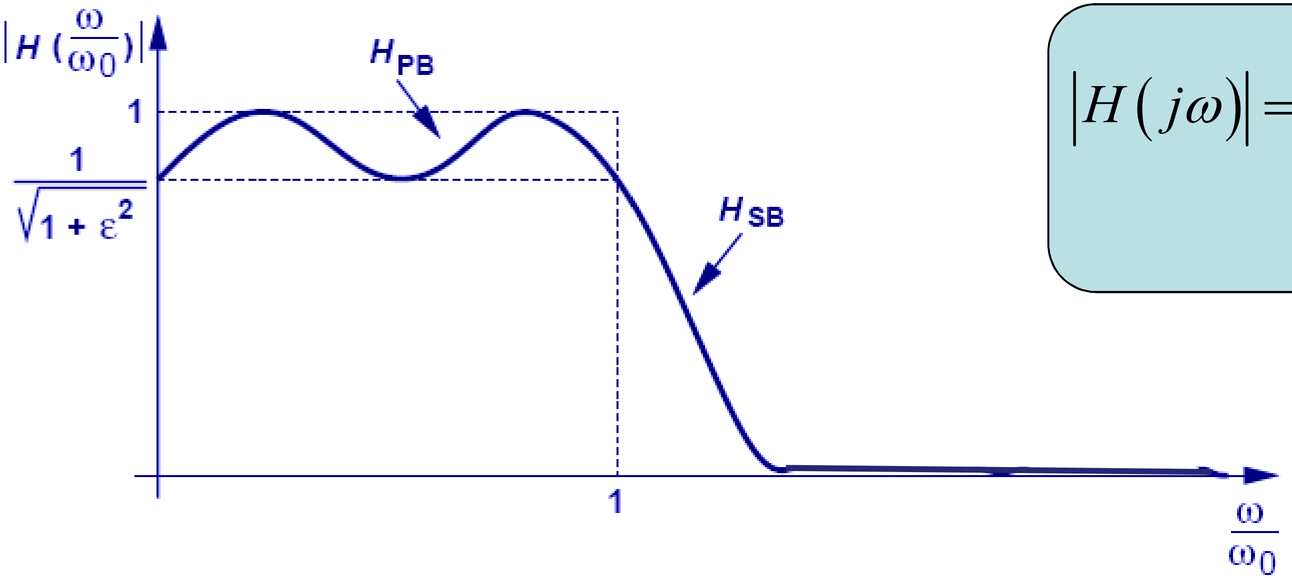
$$H_{SK}(s) = \frac{(-p_1)(-p_3)}{(s-p_1)(s-p_3)} = \frac{[2\pi \times (1.45\text{MHz})]^2}{s^2 - \left[4\pi \times (1.45\text{MHz})\cos(2\pi/3)\right]s + [2\pi \times (1.45\text{MHz})]^2}$$

$$\omega_n = 2\pi \times (1.45\text{MHz}) \text{ and } Q = 1/2\cos\frac{2\pi}{3} = 1 \rightarrow$$

$$R_1 = R_2 = 1\text{k}\Omega,\ C_2 = 54.9\text{pF},\ \text{and } C_1 = 4C_2$$

$$\frac{1}{R_3 C_3} = 2\pi \times (1.45\text{MHz}) \rightarrow R_3 = 1\text{k}\Omega \text{ and } C_3 = 109.8\text{pF}$$

Chebyshev Response

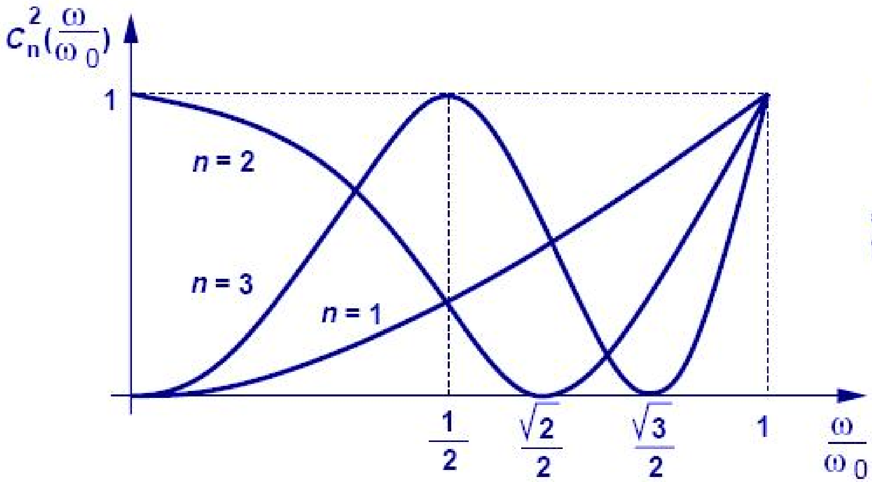


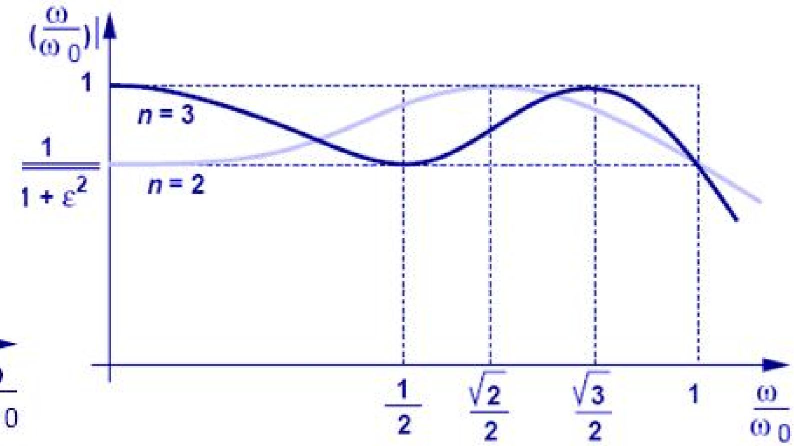
$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2\left(\frac{\omega}{\omega_0}\right)}}$$

Chebyshev Polynomial

- **The Chebyshev response provides an "equiripple" pass/stop band response.**

Chebyshev Polynomial





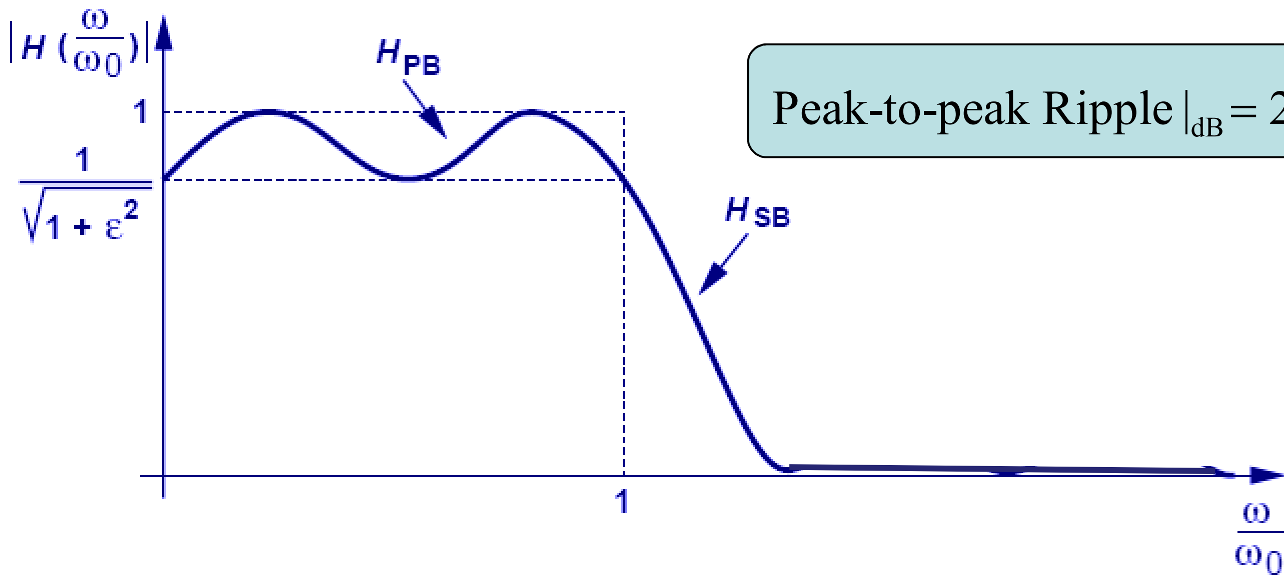
Chebyshev polynomial for n=1,2,3

Resulting transfer function for n=2,3

$$C_n\left(\frac{\omega}{\omega_0}\right) = \cos\left(n\cos^{-1}\frac{\omega}{\omega_0}\right), \ \omega < \omega_0$$

$$= \cosh\left(n\cosh^{-1}\frac{\omega}{\omega_0}\right), \ \omega > \omega_0$$

Chebyshev Response

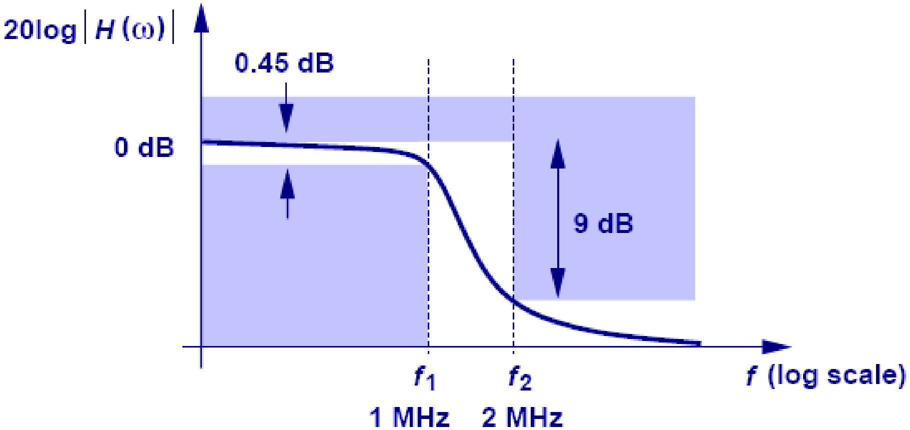


$$\text{Peak-to-peak Ripple}\,\big|_{\text{dB}} = 20\log\sqrt{1+\varepsilon^2}$$

$$|H_{PB}(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\cos^2\left(n\cos^{-1}\dfrac{\omega}{\omega_0}\right)}}$$

$$|H_{SB}(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\cosh^2\left(n\cosh^{-1}\dfrac{\omega}{\omega_0}\right)}}$$

Example 14.26: Chebyshev Response



Suppose the filter required in Example 14.24 is realized with third-order Chebyshev response. Determine the attenuation at 2MHz.

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.95 \rightarrow \varepsilon = 0.329$$

$$\omega_0 = 2\pi\ (1\text{MHz})$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2\left[4\left(\frac{\omega}{\omega_0}\right)^3 - 3\frac{\omega}{\omega_0}\right]^2}}$$

$$\left|H(j2\pi(2\text{MHz}))\right| = 0.116 = -18.7\text{dB}$$

- **A third-order Chebyshev response provides an attenuation of -18.7 dB a 2MHz.**

Example 14.27: Order of Chebyshev Filter

Specification:

Passband ripple: 1 dB

Bandwidth: 5 MHz

Attenuation at 10 MHz: 30 dB

What's the order?

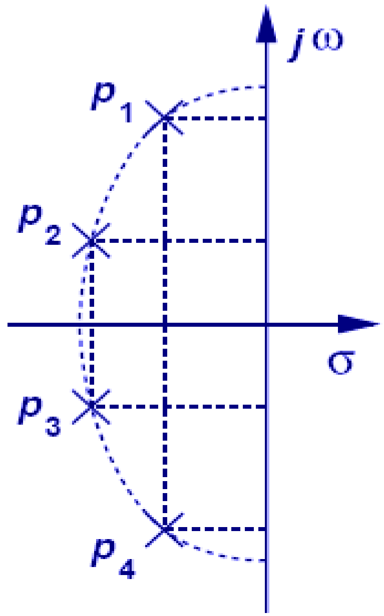
$$1\text{ dB} = 20\log\sqrt{1+\varepsilon^2} \rightarrow \varepsilon = 0.509$$

Attenuation at $\omega = 2\omega_0 = 10$ MHz: 30 dB

$$\frac{1}{\sqrt{1+0.509^2\cosh^2(n\cosh^{-1}2)}} = 0.0316$$

$$\cosh^2(1.317n) = 3862 \rightarrow n > 3.66 \rightarrow n = 4$$

Example 14.28: Chebyshev Filter Design



Using two SK stages, design a filter that satisfies the requirements in Example 14.27.

$$p_k = -\omega_0 \sin\frac{(2k-1)\pi}{2n} \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_0 \cos\frac{(2k-1)\pi}{2n} \cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right)$$

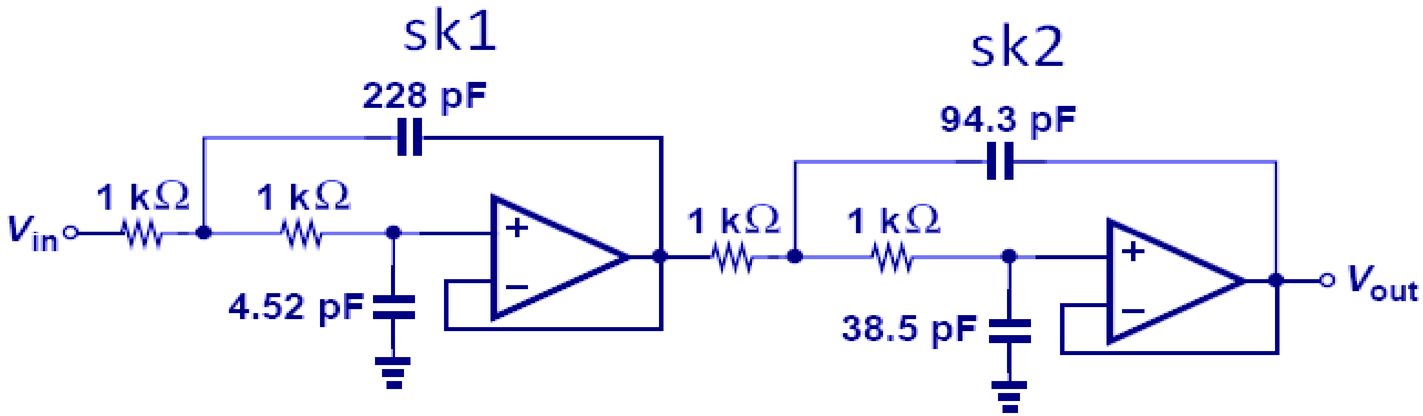
$$p_{1,4} = -0.140\omega_0 \pm 0.983 j\omega_0$$

SK1

$$p_{2,3} = -0.337\omega_0 \pm 0.407 j\omega_0$$

SK2

Example 14.28: Chebyshev Filter Design (cont'd)



$$H_{SK1}(s) = \frac{(-p_1)(-p_4)}{(s-p_1)(s-p_4)} = \frac{0.986\omega_0^2}{s^2 + 0.28\omega_0 s + 0.986\omega_0^2}$$

$$\omega_{n1} = 0.993\omega_0 = 2\pi \times (4.965\text{MHz})$$

$$Q_1 = 3.55$$

$$R_1 = R_2 = 1\text{ k}\Omega,\ C_1 = 50.4C_2$$

$$\frac{1}{\sqrt{50.4R_1C_2}} = 2\pi \times (4.965\text{MHz})$$

$$\to C_2 = 4.52\text{ pF},\ C_1 = 227.8\text{ pF}$$

$$H_{SK2}(s) = \frac{(-p_2)(-p_3)}{(s-p_2)(s-p_3)} = \frac{0.279\omega_0^2}{s^2 + 0.674\omega_0 s + 0.279\omega_0^2}$$

$$\omega_{n2} = 0.528\omega_0 = 2\pi \times (2.64\text{MHz})$$

$$Q_2 = 0.783.$$

$$R_1 = R_2 = 1\text{ k}\Omega,\ C_1 = 2.45C_2$$

$$\frac{1}{\sqrt{2.45R_1C_2}} = 2\pi \times (2.64\text{ MHz})$$

$$\to C_2 = 38.5\text{ pF},\ C_1 = 94.3\text{ pF}$$