Chapter 5
Circuit Theorems

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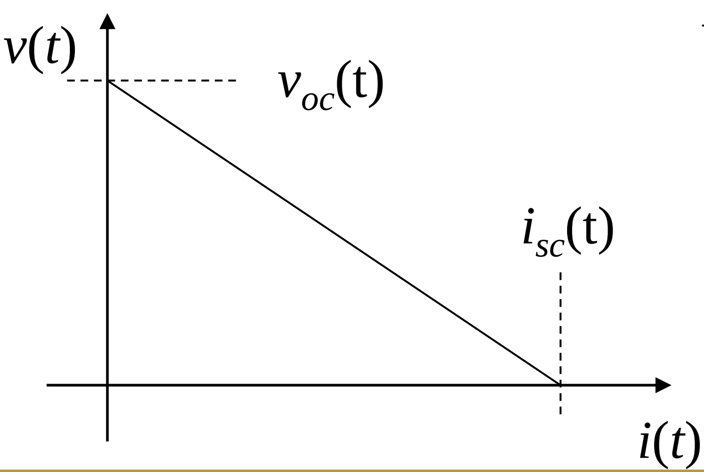
Non ideal sources

- Ideal sources: voltage source and current source:

  There is no source resistances and there is no loading effect.
- Non ideal voltage source with loading effect:

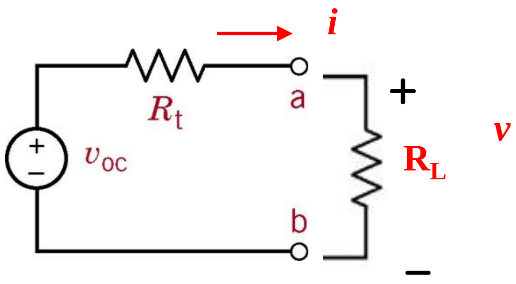
  v decreases with i.

  (As R_L decreases, i increases and v decreases linearly (to i)).
- Intercepts are $v_{oc}(t)$ and $i_{sc}(t)$, Slope is R_{Th}



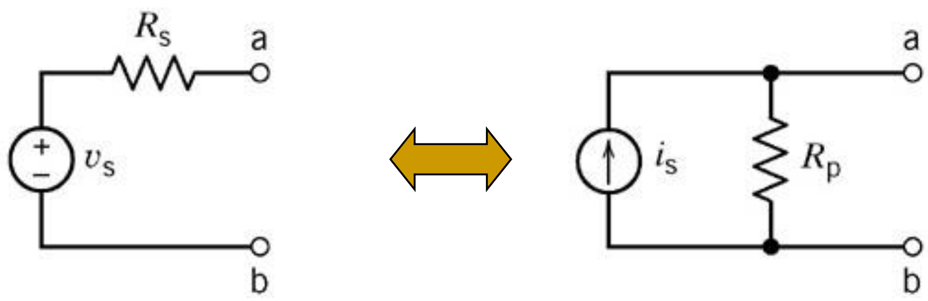
$$v = v_{oc} - R_{th} i$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}}$$

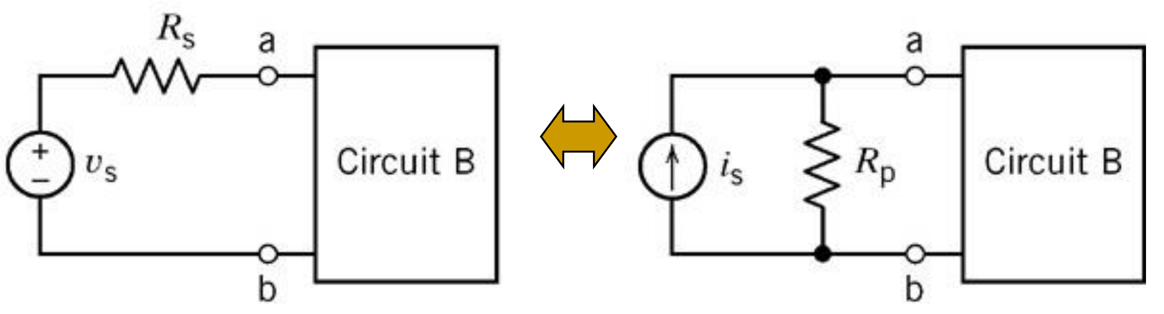


Source Transformations

- Source transformation
  - The conversion of a nonideal voltage source to a nonideal current source. Or vice versa.



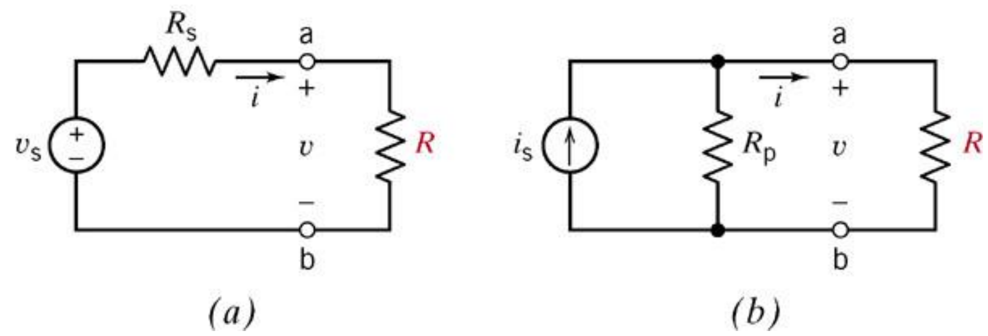
  - Replacing the nonideal voltage (or current) source by the the other nonideal current (or voltage) source should not change the voltage or current of any element in circuit B (Then the two sources are equivalent.)



Source Transformations (cont'd)

- Source transformation

  : For two extreme cases,



- For short circuit condition (R=0)

$$(a)\ i = \frac{v_s}{R_s} \quad (b)\ i = i_s \quad \Rightarrow \quad i_s = \frac{v_s}{R_s}$$

- For open circuit condition (R=∞)

$$(a)\ v = v_s \quad (b)\ v = i_s R_p \quad \Rightarrow \quad v_s = i_s R_p$$

$$v_s = i_s R_p = \left(\frac{v_s}{R_s}\right) R_p$$

$$\therefore R_p = R_s$$

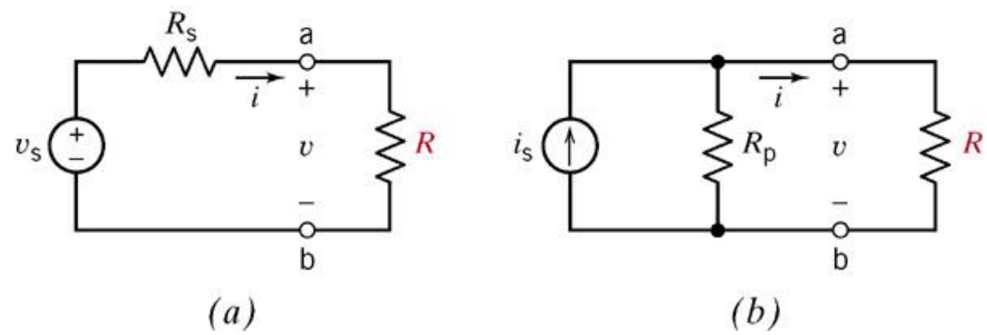
- For both circuits to be equivalent

$$v_s = i_s R_s, \quad R_p = R_s$$

Source Transformations (cont'd)

- Source transformation

  : In general,



- For the circuit of figure 5.2-2a we use KVL to obtain

$$v_s = iR_s + v$$

- Dividing by R_s gives

$$\frac{v_s}{R_s} = i + \frac{v}{R_s}$$

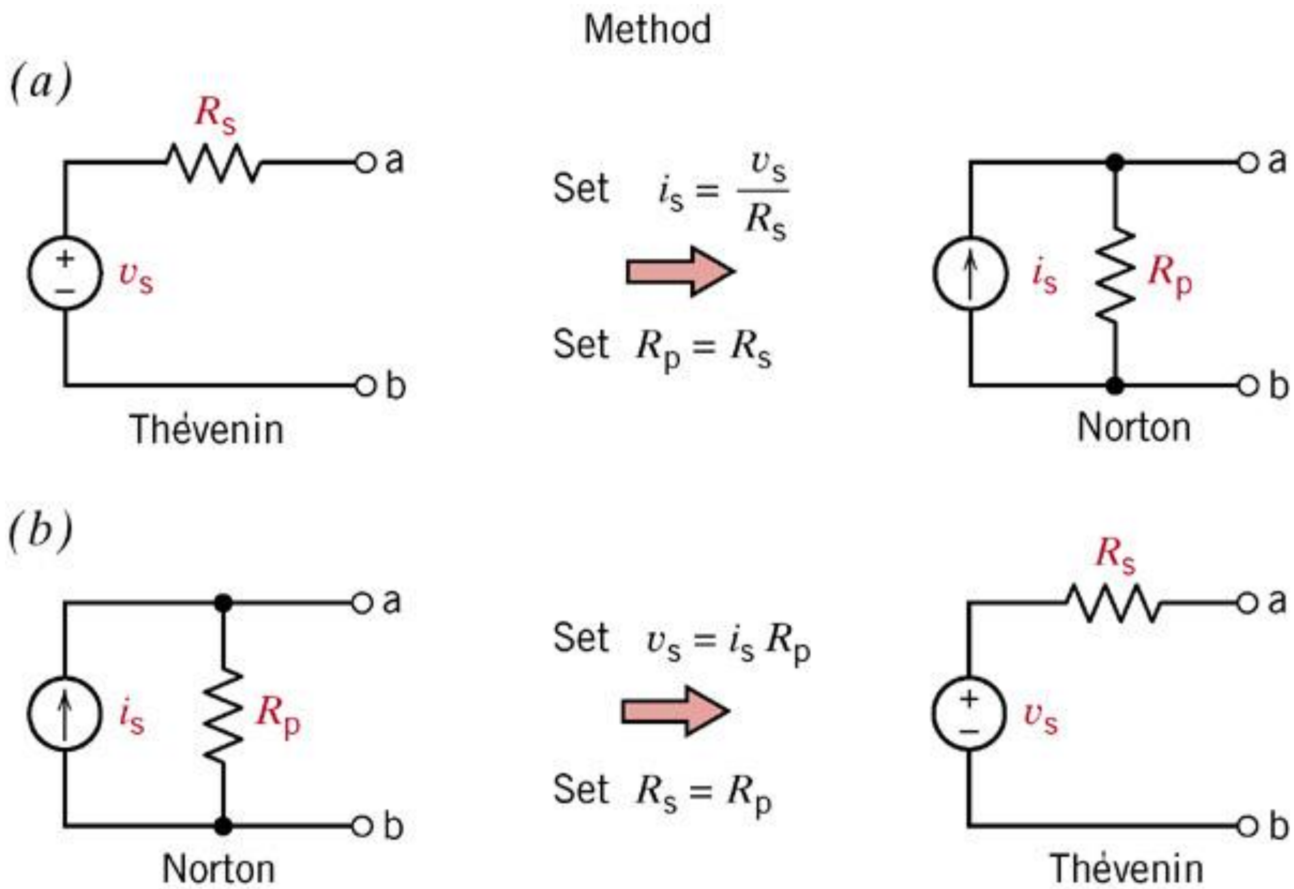
- If we use KCL for the circuit of figure 5.2-2b, we have

$$i_s = i + \frac{v}{R_p}$$

- Thus, the two circuits are equivalent when $i_s=v_s/R_s$ and $R_s=R_p$

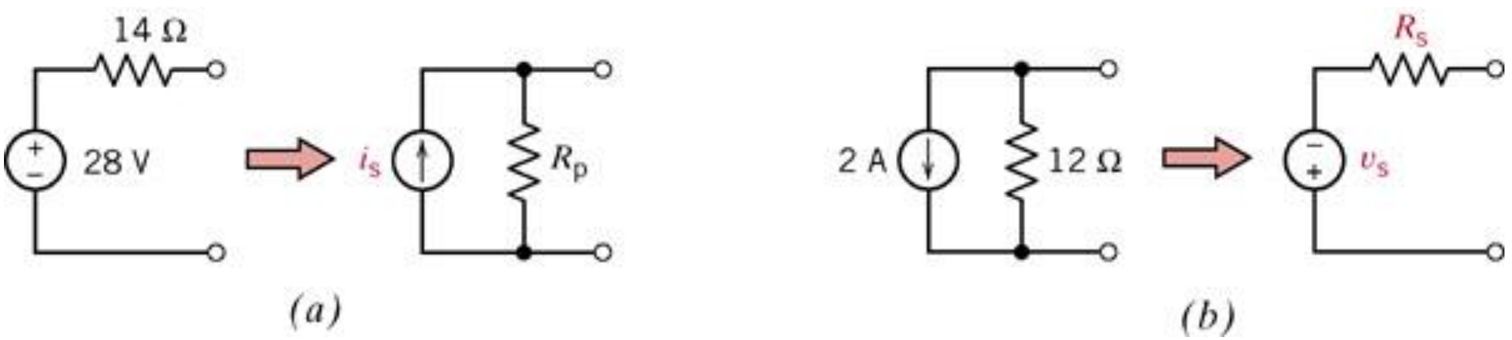
Source Transformations

- Method for Source transformations

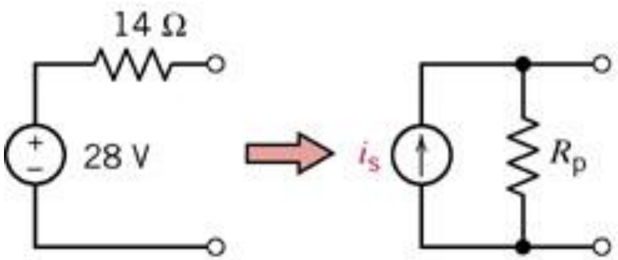


Example) Source Transformations

- Find the source transformation for the circuits shown in figure a, b.



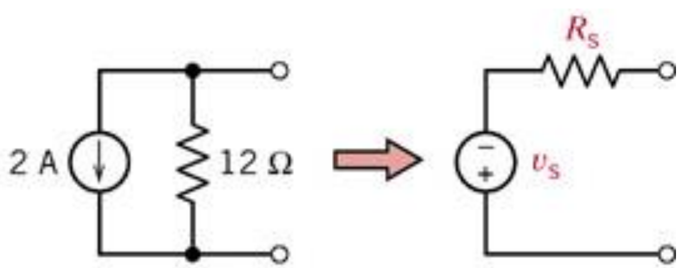
Solution



(a)

Current source

$$i_s = \frac{v_s}{R_s} = \frac{28}{14} = 2\text{A}$$



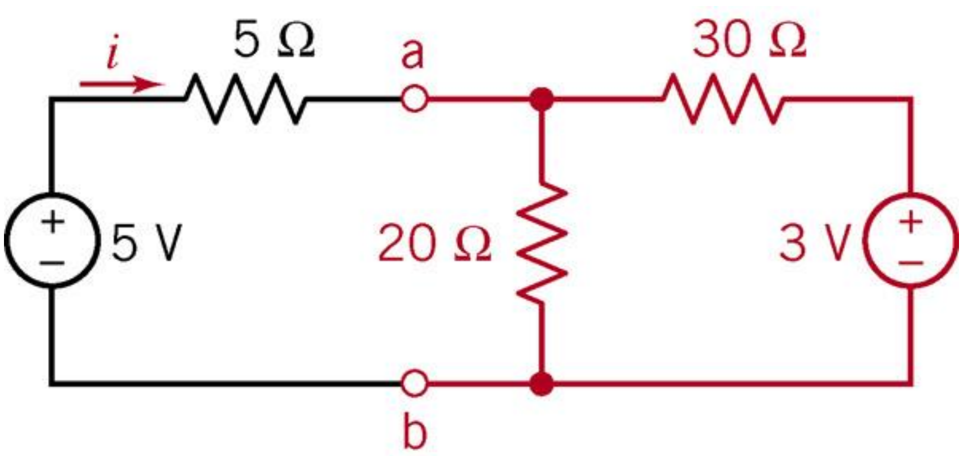
(b)

Voltage source

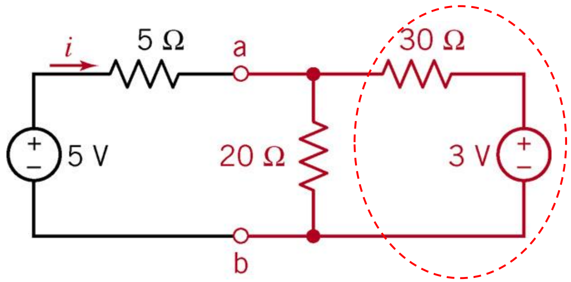
$$v_s = i_s R_p = 2(12) = 24\text{V}$$

Example) Source Transformations

- A circuit is shown in figure. Find the current i by reducing the circuit to the right of terminals a-b to its simplest form using source transformations.

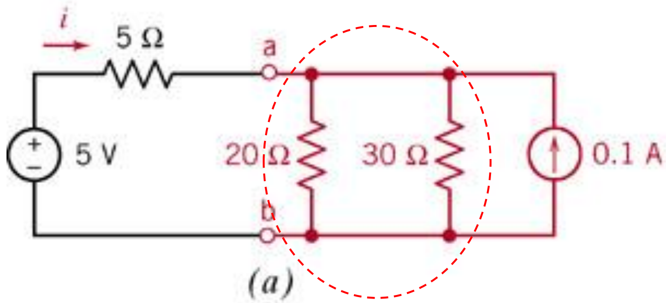


Solution



Current source

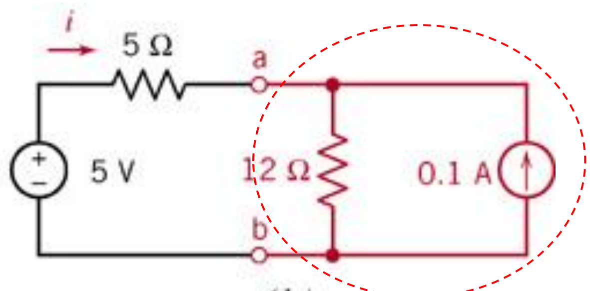
$$i_s = \frac{v_s}{R_p} = \frac{3}{30} = 0.1\text{A}$$



(a)

Combining two parallel resistances

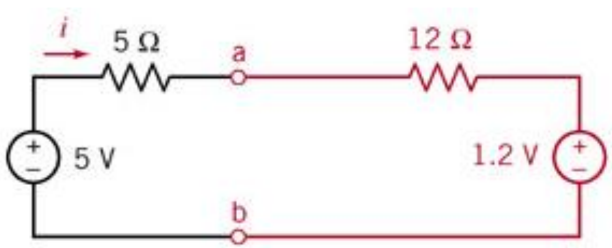
$$R_{p2} = 20\Omega // 30\Omega = 12\Omega$$



(b)

Voltage source

$$v_s = i_s R_{s2} = 0.1(12) = 1.2\text{V}$$



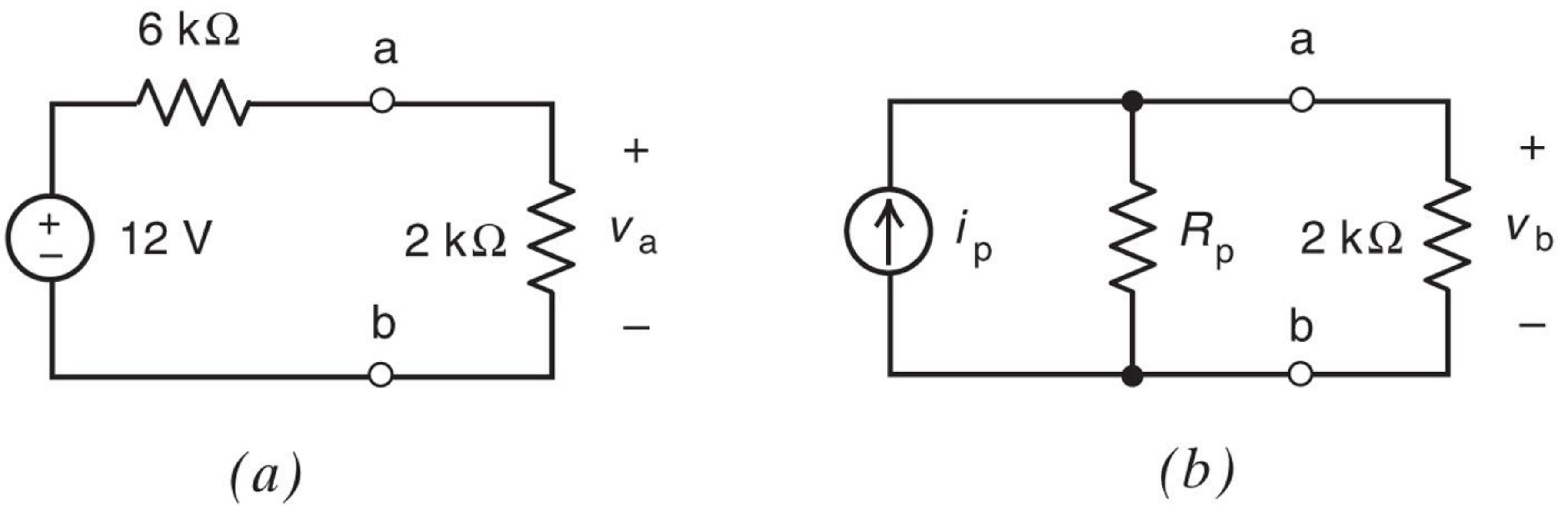
(c)

Current *i*

$$i = \frac{(5-1.2)\text{V}}{(5+12)\Omega} = 0.224\text{A}$$

Example 5.2-1 Source Transformations

- First, determine the values of i_p and R_p that cause the part of the circuit connected to the 2 kohm resistor in Figure a. Next, determine the values of v_a and v_b.



Solution

- Source transformation

$$i_p = \frac{12}{6000} = 0.002A = 2mA \qquad R_p = 6k\Omega$$

- Voltage division

$$v_a = \frac{2000}{2000+6000}(12) = 3V$$

- Voltage across the parallel resistors

$$v_b = \frac{2000+R_p}{2000+R_p} i_p = \frac{2000(6000)}{2000+6000}(0.002) = 1500(0.002) = 3V$$

→ As expected, the source transformation did not change the value of the voltage across the 2kohm resistor

Superposition

- A Linear element satisfies superposition when it satisfies fallowing response and excitation relationship.

$$i_1 \longrightarrow v_1$$

$$i_2 \longrightarrow v_2$$

$$i_1 + i_2 \longrightarrow v_1 + v_2$$

where, the arrows imply the excitation and the resulting response.

Superposition (cont'd)

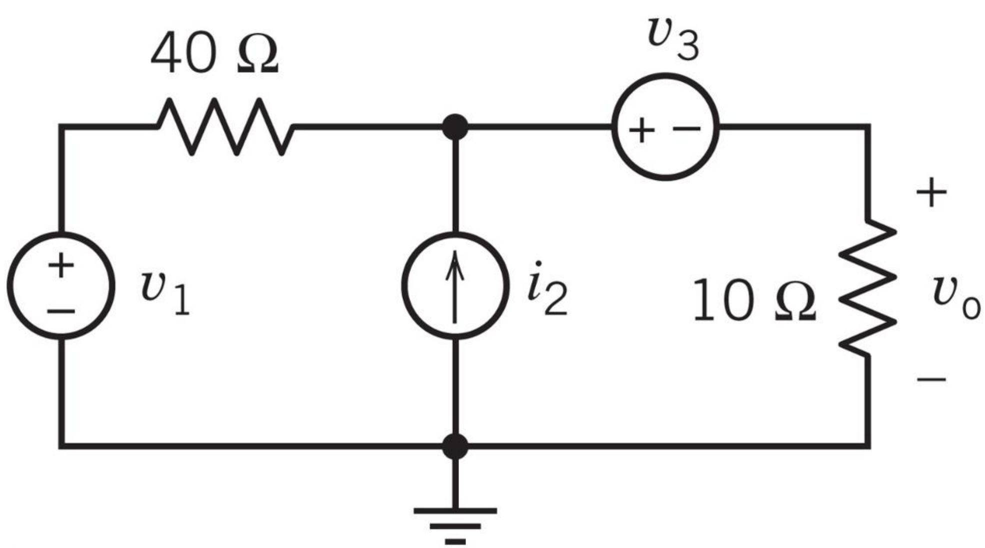
- Superposition principle: For a linear circuit consisting of linear elements and independent sources, we can determine the total response by finding the response to each independent source with all other independent source set to zero and then summing these individual responses

- Source deactivation
  - Independent voltage source: short circuit
  - Independent current source: open circuit
  - Dependent source must remain active during the superposition process

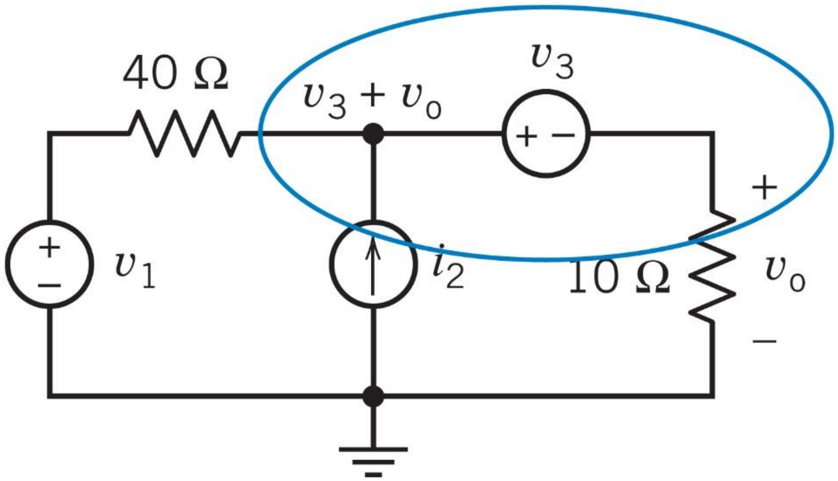
- Power and Superposition
  - For dc circuit analysis, the principle of superposition does not apply to power calculation.

Example 5.3-1 Superposition

- The circuit shown in Figure a has one output, v_o, and three inputs, v_1, i_2, and v_3. (As expected, the inputs are voltages of independent voltage sources and the currents of independent current sources.) Express the output as a linear combination of the inputs.



Solution



- Apply KCL to the supernode to get

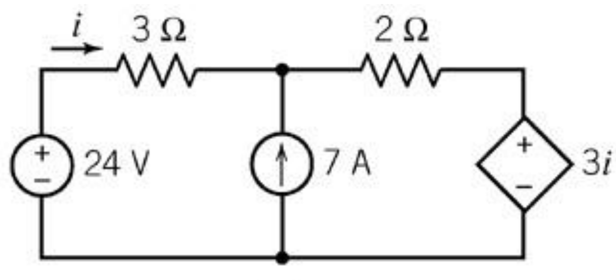
$$\frac{v_1-(v_3+v_o)}{40}+i_2=\frac{v_o}{10}$$

$$v_1-(v_3+v_o)+40i_2=4v_o \Rightarrow v_1+40i_2-v_3=5v_o$$

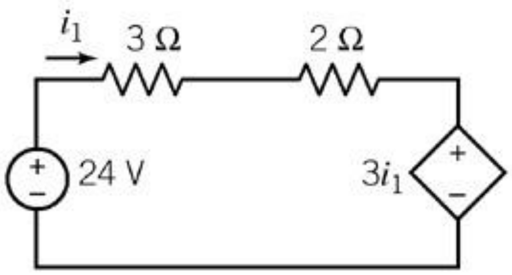
$$v_o=\frac{1}{5}(v_1+40i_2-v_3)$$

Example 5.3-2 Superposition

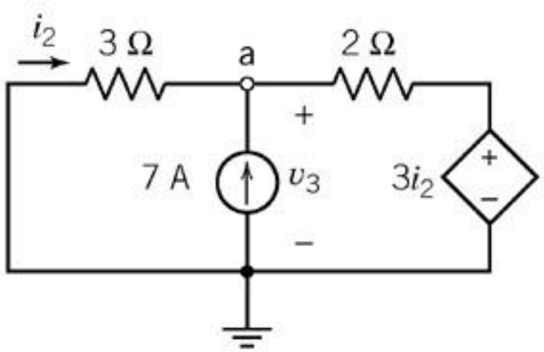
- Find the current i for the circuit of Figure 5.3-6a.



(a)



(b)



(c)

Solution

- Independent voltage source acting alone (Fig. 5.3-2b)
  Apply KVL to the loop

$$-24+(3+2)i_1+3i_1=0 \quad \Rightarrow \quad i_1=3\text{A}$$

- Independent current source acting alone (Fig. 5.3-2c)
  (a) Controlling current of the dependent source

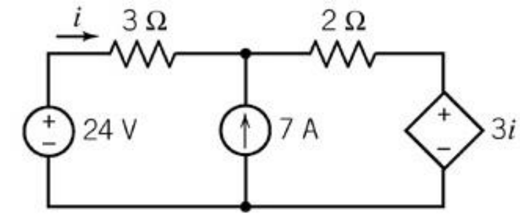
$$i_2=-\frac{v_a}{3} \quad \Rightarrow \quad v_a=-3i_2$$

  (b) Apply KCL at node a

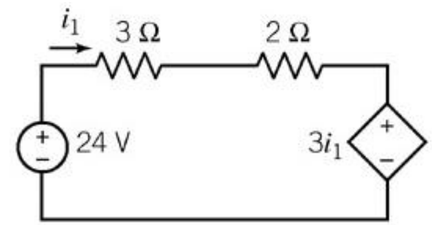
$$i_2+7=\frac{v_a-3i_2}{2} \quad \Rightarrow \quad i_2+7=\frac{-3i_2-3i_2}{2} \quad \Rightarrow \quad i_2=-\frac{7}{4}\text{A}$$

- The current, i, is equal to the sum of the currents, i1, i2

$$i=i_1+i_2=3-\frac{7}{4}=\frac{5}{4}\text{A}$$



(a)



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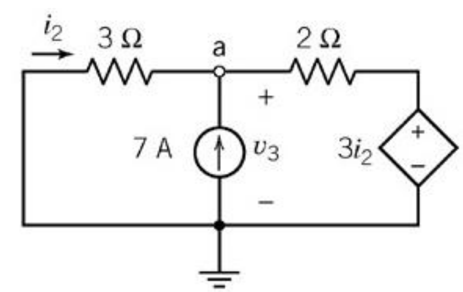
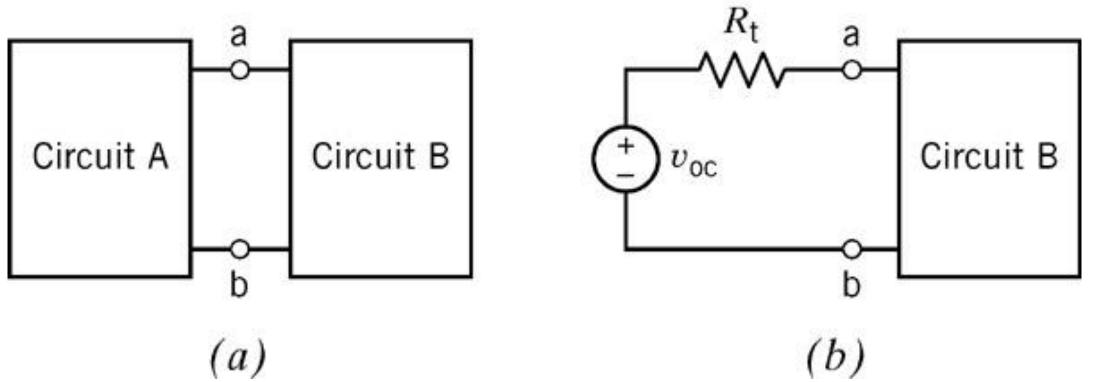


(c)

Thévenin's theorem

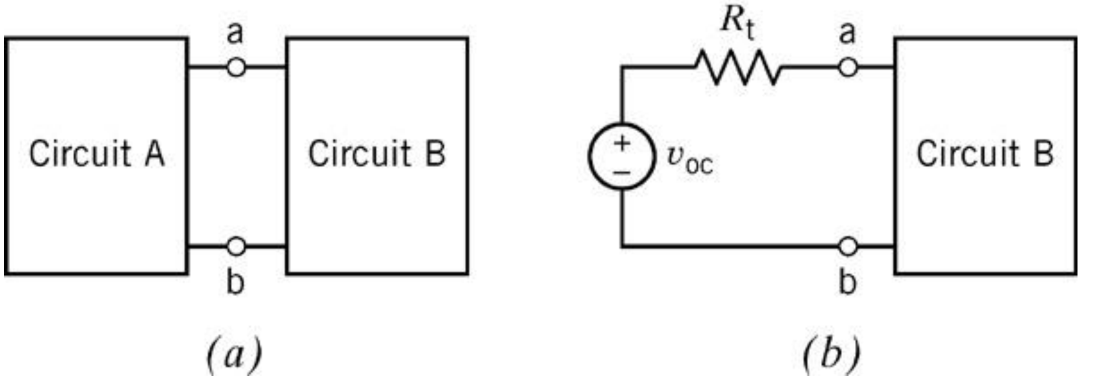
- Any circuit with sources (dependent and/or independent) and resistors can be replaced by an equivalent circuit containing a single source and a single resistor.
- Thevenin's theorem implies that we can replace arbitrarily complicated networks with simple networks for purposes of analysis.



(a) A circuit partitioned with two parts: circuit A and circuit B

(b) Replacing circuit A by its Thevenin equivalent circuit

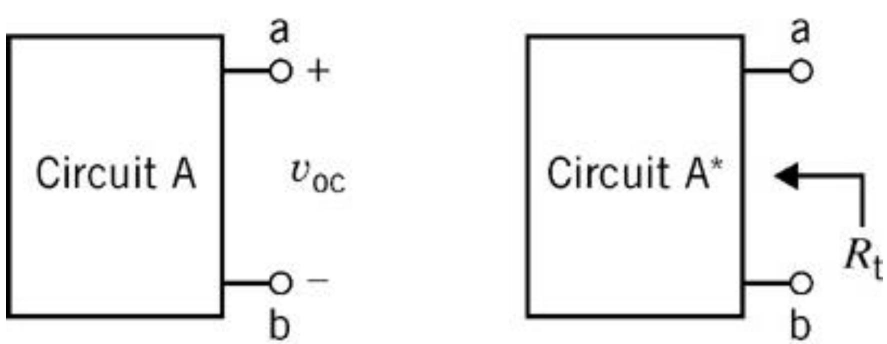
Thévenin's theorem(1)



"Given any linear circuit, rearrange it in the form of two networks A and B connected by two wires. Define a voltage voc as the open-circuit voltage which appears across to the terminals of A when B is disconnected. Then all currents and voltages in B will remain unchanged if all independent voltage and current sources in A are "killed" or "zeroed out" and an independent voltage source voc is connected , with proper polarity, in series with the dead (inactive) A network."

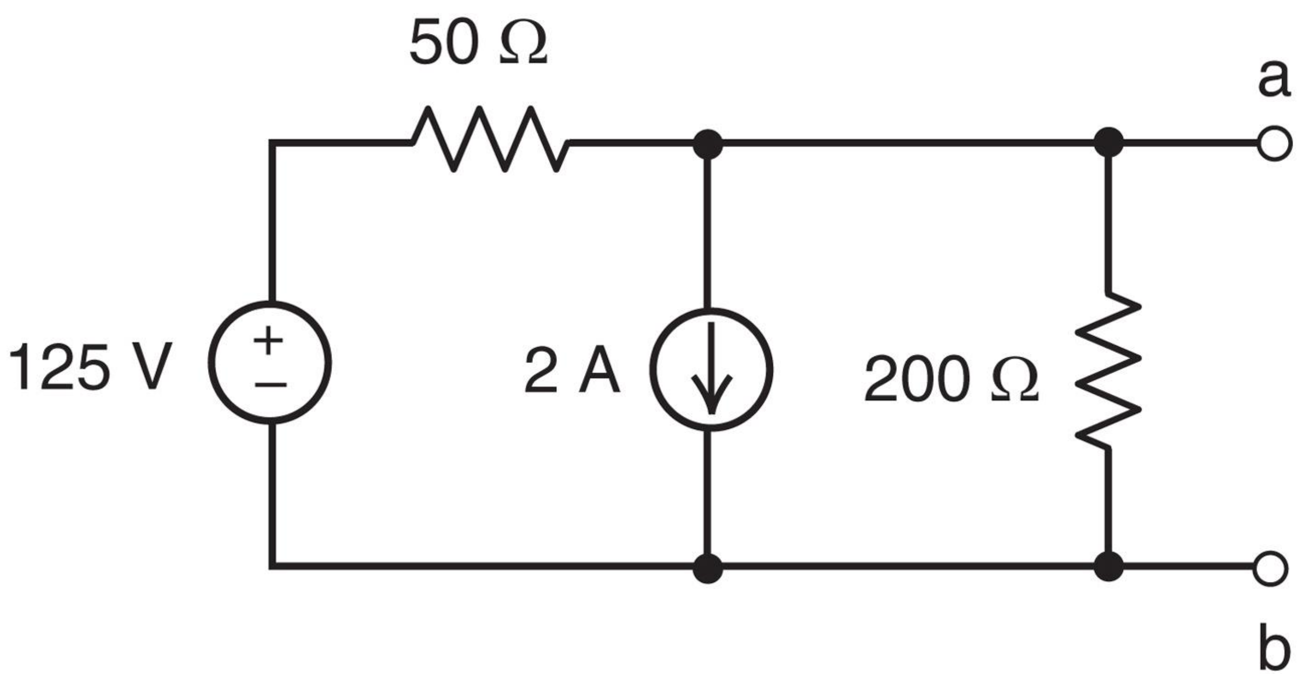
Find the Thévenin's Equivalent Circuit
: Independent Source

- **If the circuit contains resistors and independent sources,**
  - V_{oc} **: Open circuit으로 구함**
  - R_t **: independent voltage source → short**
    independent current source → open으로 놓고 (즉, 전원을 deactivate 시키고, 두 단자 사이의 등가 저항을 구한다.



Example 5.4-1 Thévenin Equivalent Circuit

- Using Thévenin's theorem, find the current i through the resistor R in the circuit of Figure 5.4-5.



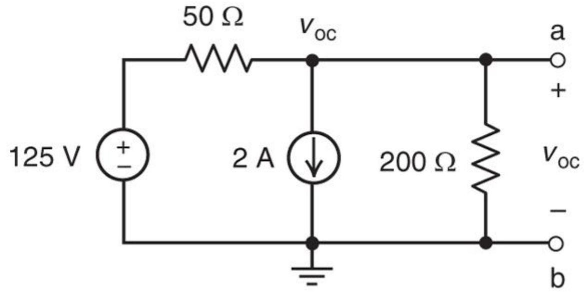
Solution

$$\frac{125 - v_{oc}}{50} = 2 + \frac{v_{oc}}{200}$$

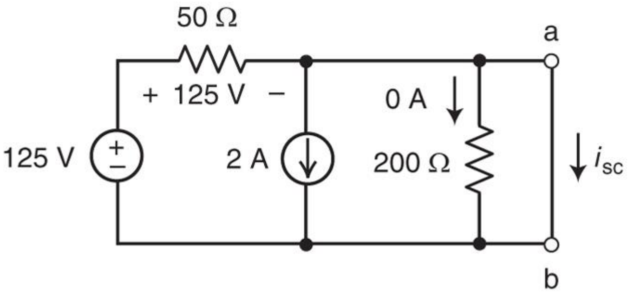
$$v_{oc} = 20V$$

$$\frac{125}{50} = 2 + 0 + i_{sc}$$

$$i_{sc} = 0.5A$$



(a)



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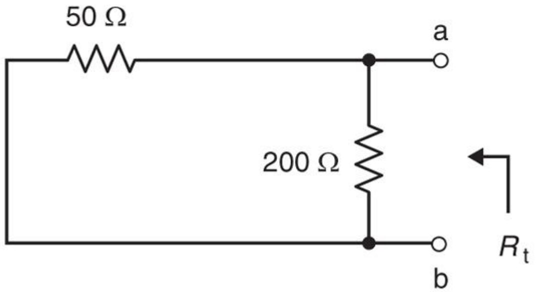

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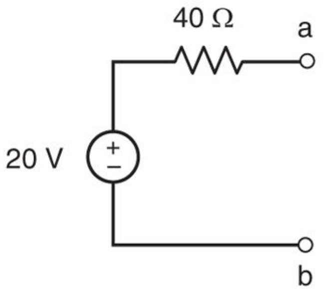
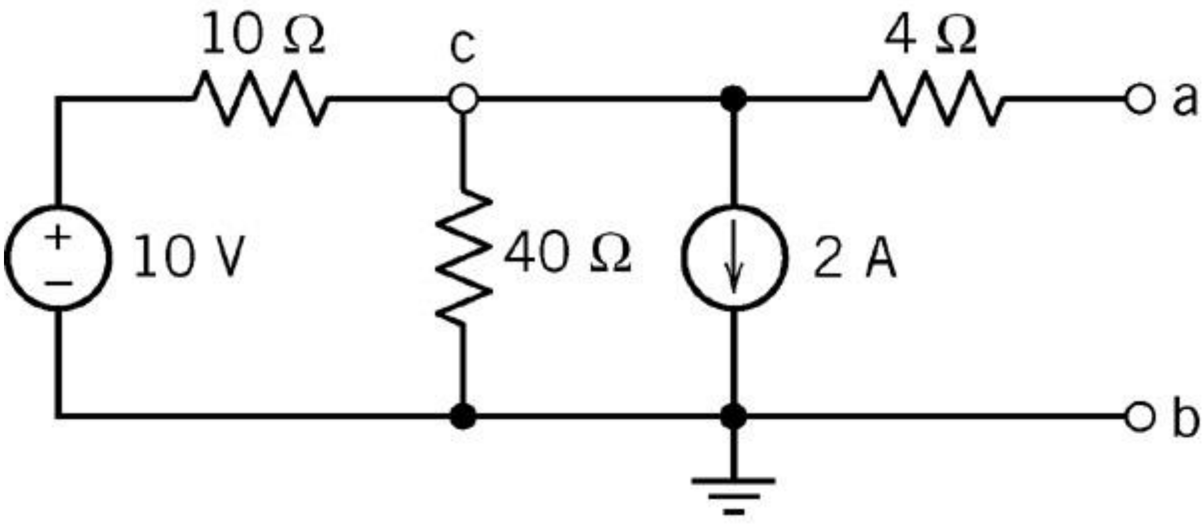


(d)

Example) Thévenin Equivalent Circuit

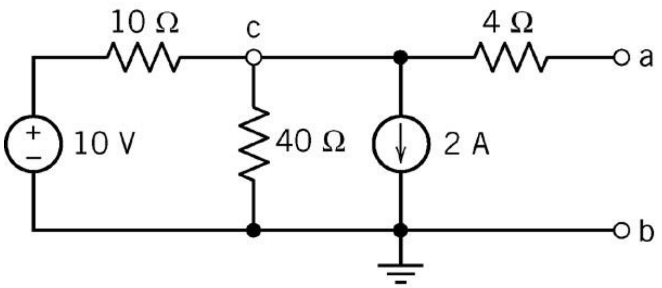
- Find the Thévenin equivalent circuit for the circuit shown in Figure 5.4-7



Solution

1. All the source deactivated (Fig. 5.4-8)

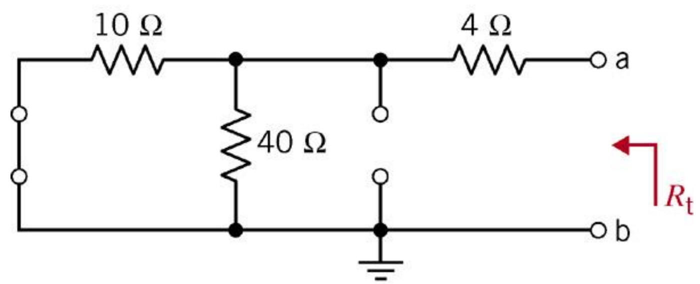
$$R_t = 12\Omega$$



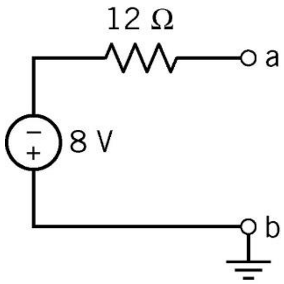
2. Open-circuit voltage at terminals a-b. (Fig 5.4-7)

$$\frac{v_c - 10}{10} + \frac{v_c}{40} + 2 = 0$$

$$v_c = -8\text{V}$$

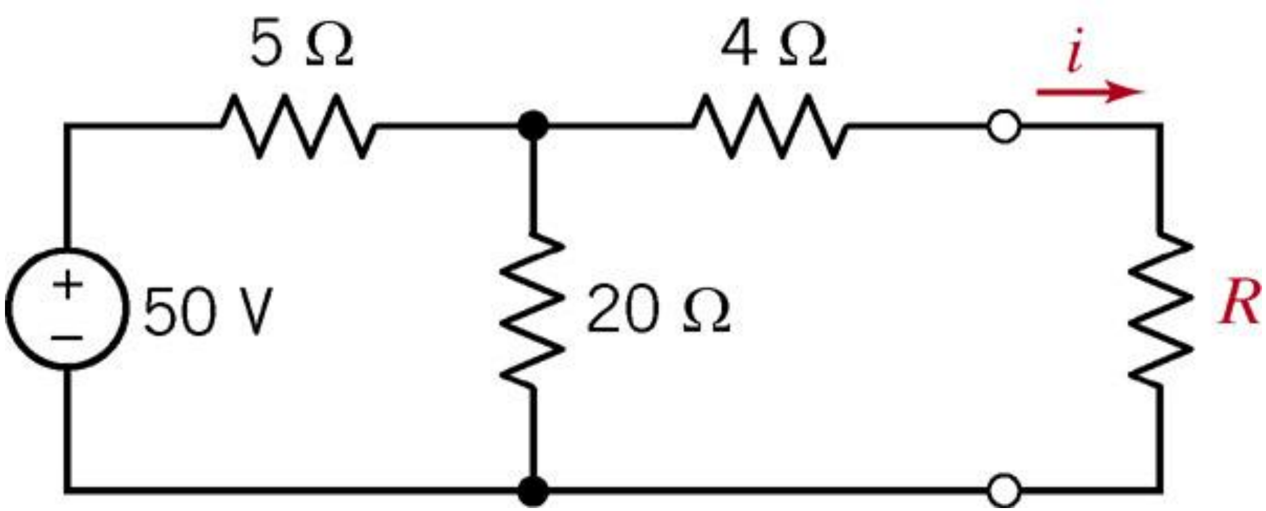


3. Therefore, the Thevenin equivalent circuit is as shown in Figure 5.4-9

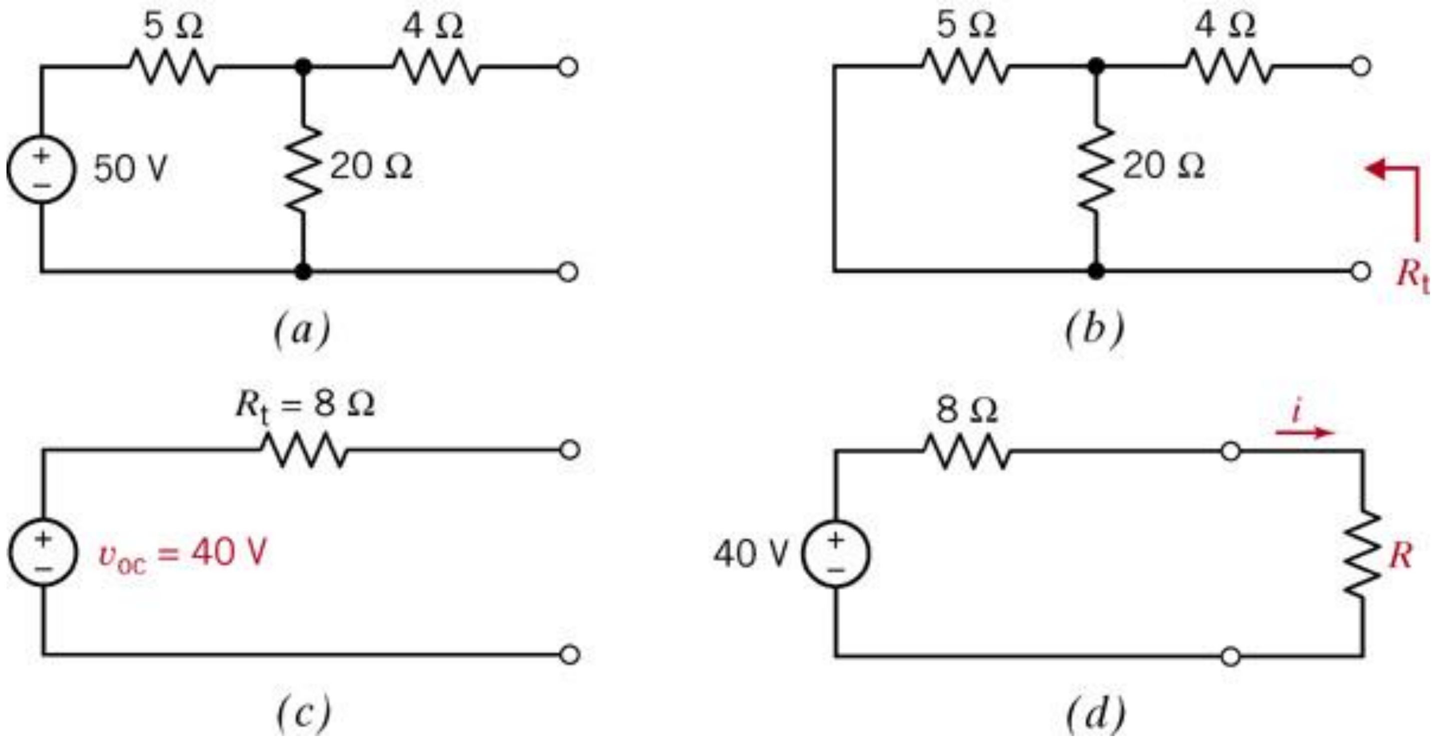


Example 5.4-3 Thévenin Equivalent Circuit

- Using Thévenin's theorem shown in Figure 5.4-13. Determine the current i as a function of R.



Solution



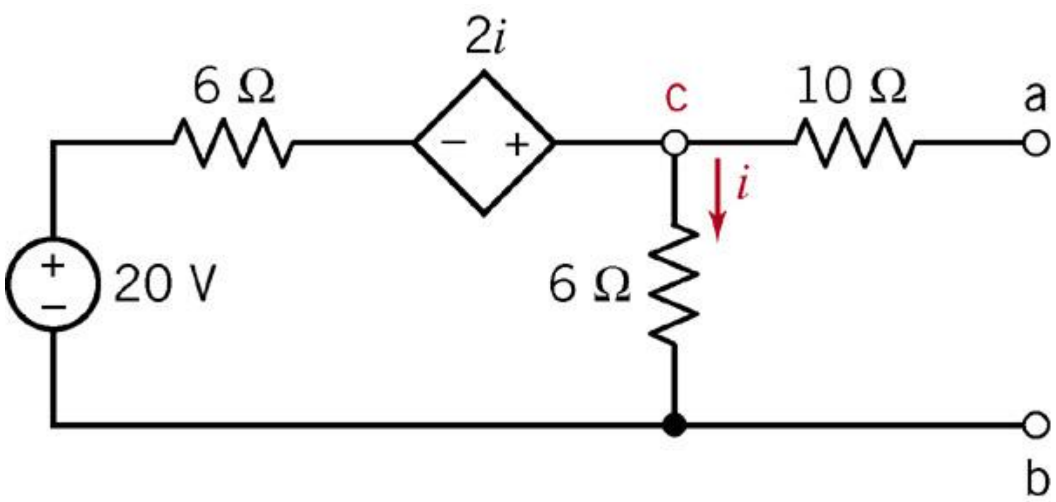
$$i = \frac{40}{R+8}\,\text{A}$$

Find the Thévenin's Equivalent Circuit : independent and dependent sources

- **If the circuit contains resistors and two kinds of sources,**
- **Find Voc and Isc, then Rt= Voc/Isc**
  - **Connect an open circuit between terminals a and b . Find Voc=Vab, the voltage across the open circuit.**
  - **Connect a short circuit between terminals a and b Find Isc, the current directed from a to b in the short circuit.**

Example) Thévenin Equivalent Circuits and Dependent Source

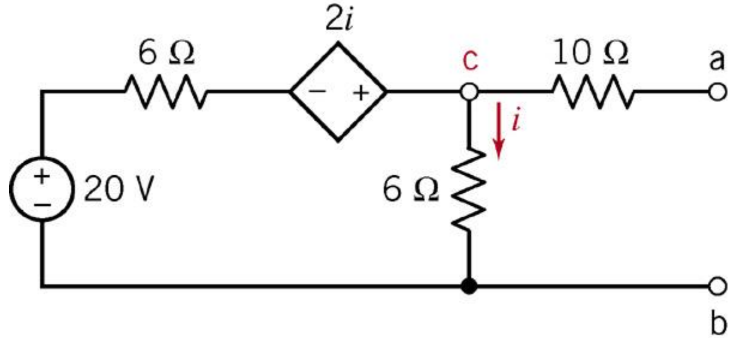
- Find the Thévenin equivalent circuit for the circuit shown in Figure 5.4-8.



Solution

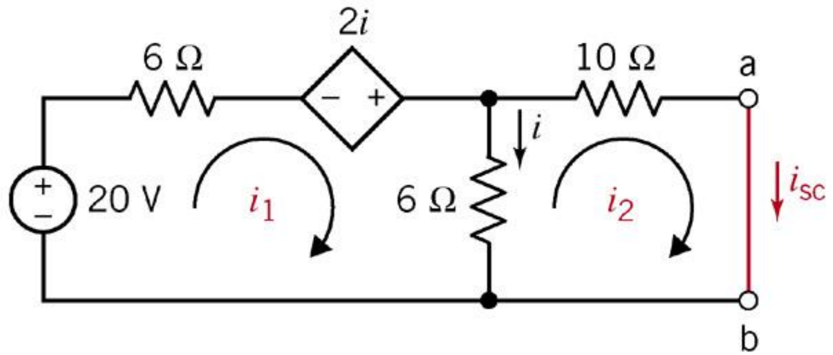
1. Open circuit voltage $v_{oc}=v_{ab}$.
   KVL around the mesh of Fig 5.4-11

$$-20+6i-2i+6i=0$$
$$i=2\text{A}$$
$$v_{oc}=6i=12\text{V}$$



2. Short circuit current
   for the circuit of Fig 5.4-12
   Using two mesh currents

$$-20+6i_1-2i+(6i_1-i_2)=0$$
$$6(i_1-i_2)+10i_2=0$$

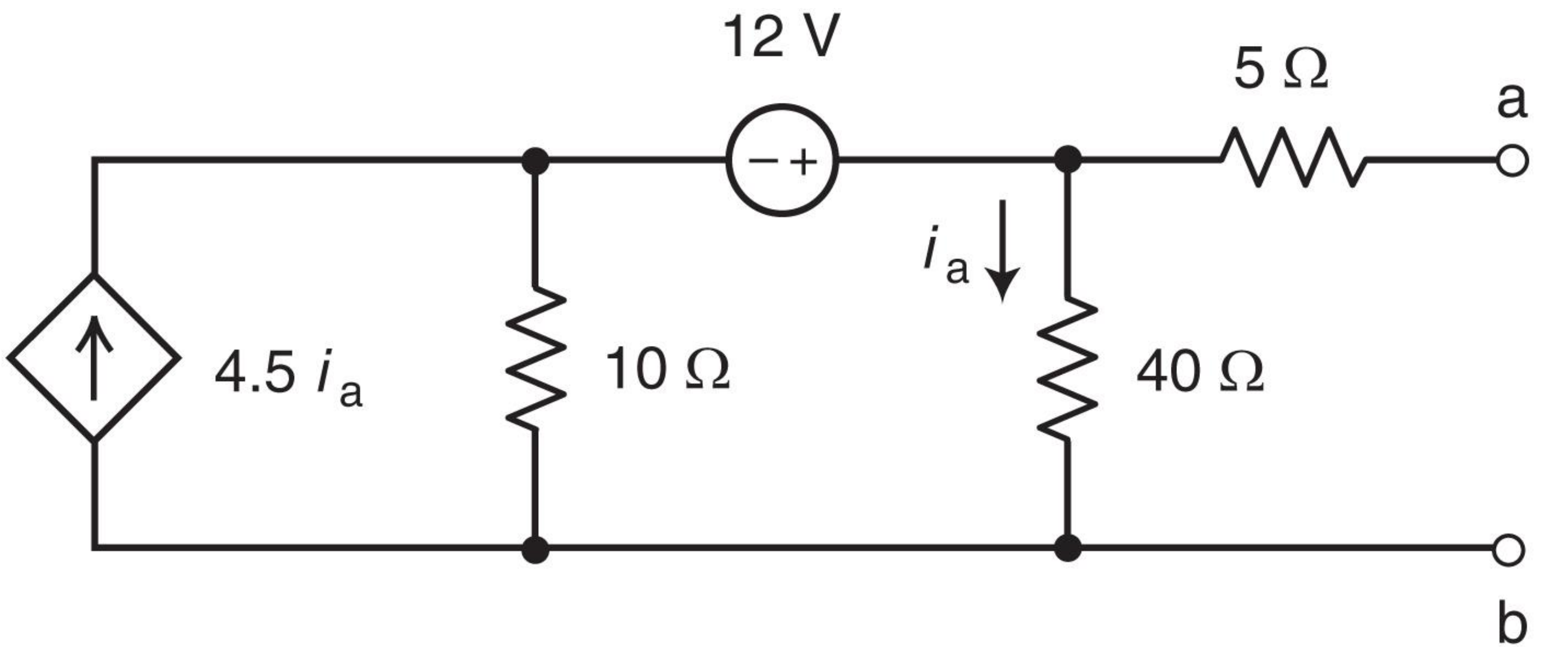


Therefore

$$i_2=i_{sc}=120/136\text{A}$$
$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{12}{120/136}=13.6\Omega$$

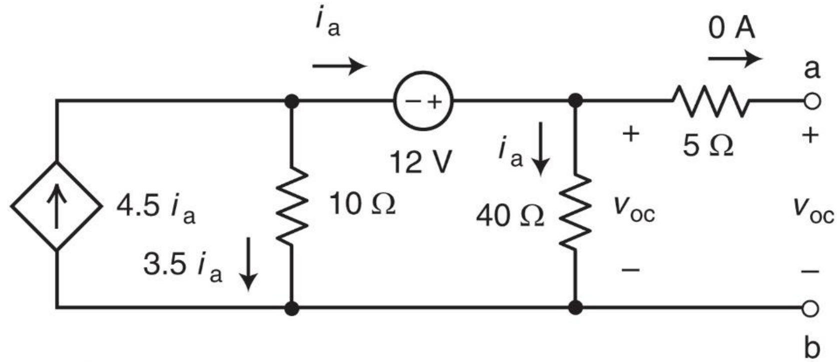
Example 5.4-2 Thévenin Equivalent Circuits and Dependent Source

- Determine the Thévenin equivalent circuit for the circuit shown below



Solution

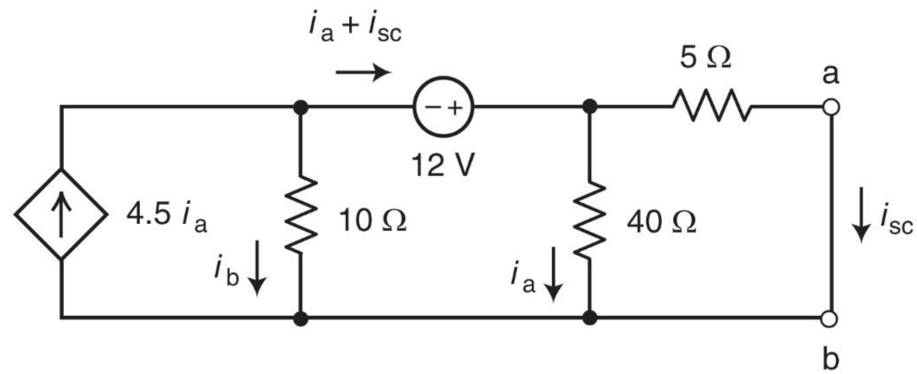
1. Open circuit voltage



$$i_a = \frac{v_{oc}}{40} \qquad 0 = -12 + v_{oc} - 10(3.5i_a)$$

$$v_{oc} = 96V$$

2. Short circuit current

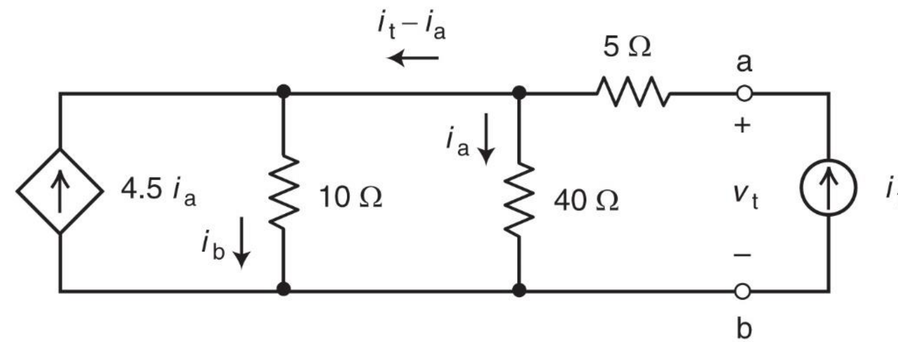


$$5i_{sc} - 40i_a = 0 \quad \Rightarrow i_a = \frac{i_{sc}}{8}$$

$$4.5i_a = i_b + (i_a + i_{sc}) \quad \Rightarrow i_b = 3.5i_{sc} - i_{sc} = -\frac{9}{16}i_{sc}$$

$$i_{sc} = 1.1294A$$

3. Thévenin resistance



$$4.5i_a + (i_t - i_a) = i_b \Rightarrow i_b = 3.5i_a + i_t$$

$$40i_a = 10i_b = 10(3.5i_a + i_t) \Rightarrow i_a = 2i_t$$

$$v_t = 5i_t + 10i_b \quad R_t = \frac{v_t}{i_t} = 85\Omega$$

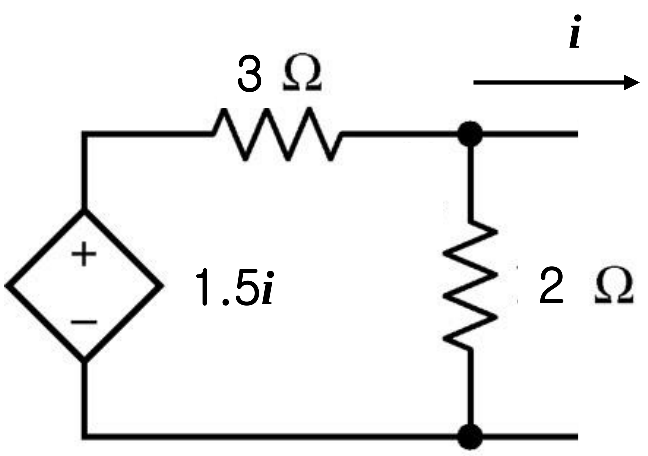
Find the Thévenin's Equivalent Circuit
: Dependent Source (As in the transistor circuit)

- **If the circuit contains resistors and only dependent sources,**
- **One Ampere Method:**
  - ***Determine Voc (it can be zero)***
  - ***Find Rt by connecting a one ampere source at load.***
  - ***Rt=Vab/1.***

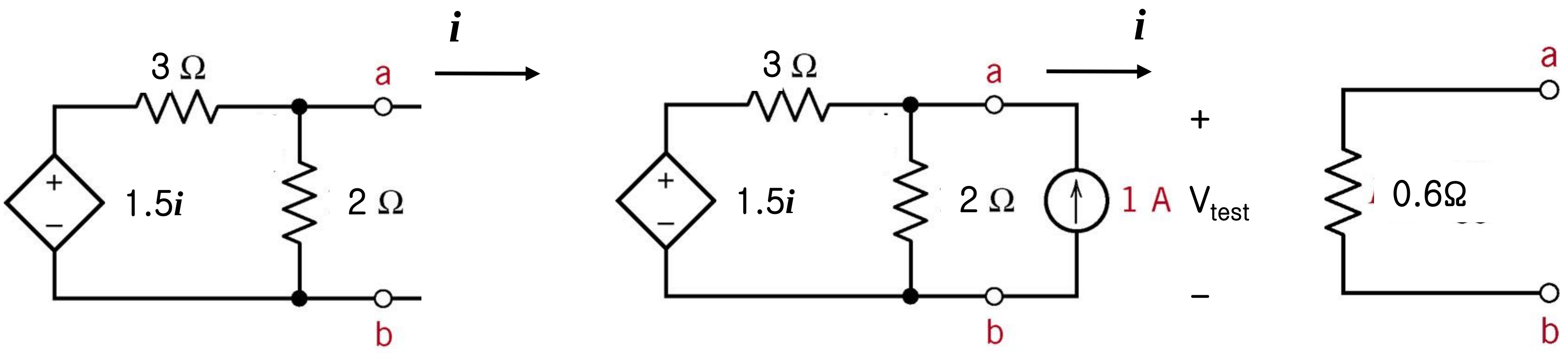
Example) Thévenin Equivalent Circuit

"1Amp method"

- Find the Thevenin equivalent of the circuit shown in Figure



Solution



- Since the rightmost terminals are already open-circuited, i=0. Consequently, the dependent source is dead, so v_{oc}=0
- We apply a 1-A source externally, measure the voltage v_{test} that results, and then set $R_{TH}=v_{test}/1$. Apply node analysis

$$\frac{v_{test}-1.5i}{3\Omega}+\frac{v_{test}}{2\Omega}=1 \quad \Leftarrow \quad i=-1$$

$$\therefore\ v_{test}=0.6\text{V}$$

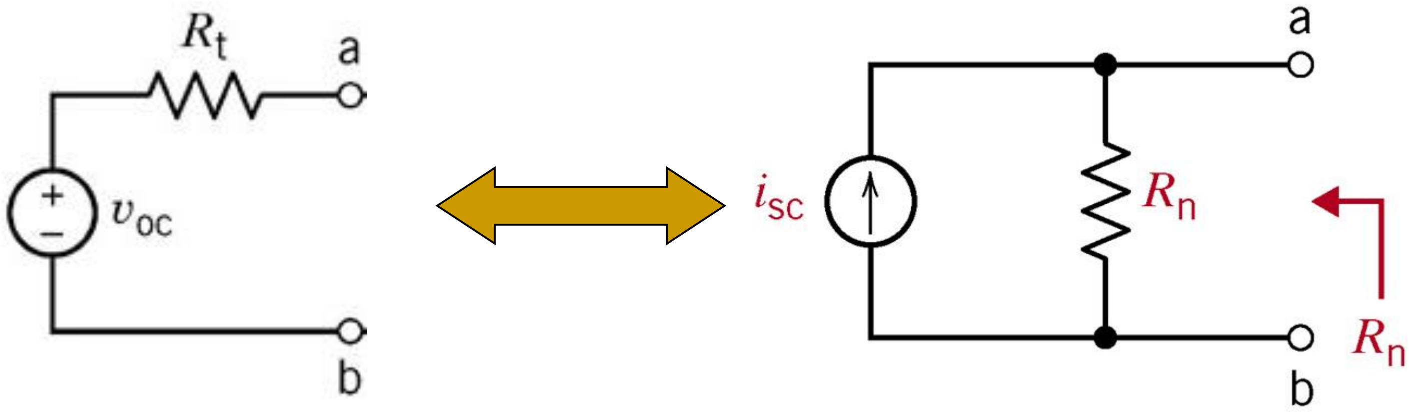
$$\therefore R_{th}=v_{test}=0.6\Omega$$

Thévenin's theorem (cont'd)

Method of finding a Thevenin Equivalent Circuit		
Number of Method	If the circuit contains:	Thévenin equivalent circuit
1	Resistors and independent sources	1.(a) Connect an open circuit between terminals a and b. Find $v_{oc}=v_{ab}$, the voltage across the open circuit (b) Deactivate the independent sources. Find R_t by circuit resistance reduction. 또는 2.(a) Connect an open circuit between terminals a and b. Find $v_{oc}=v_{ab}$, the voltage across the open circuit (b) Connect a short circuit between terminals a and b. Find i_{sc}, the current directed from a to b in the short circuit $R_t=v_{oc}/i_{sc}$ 또는 3. Set all independent sources to zero, then connect a 1-A current source from terminal b to terminal a. Determine v_{ab}. Then $R_t=v_{ab}/1$
2	Resistors and independent and dependent sources	2.또는 3.(위와 마찬가지)
3	Resistors and dependent source (no independent sources)	3과 같이(a) find v_{oc}(this can be zero) (b) Connect a 1-A current source from terminal b to terminal a. Determine v_{ab}. Then $R_t=v_{ab}/1$

Norton's theorem

- Norton equivalent circuit
  - The source transformation of the Thévenin equivalent circuit



Thévenin equivalent circuit

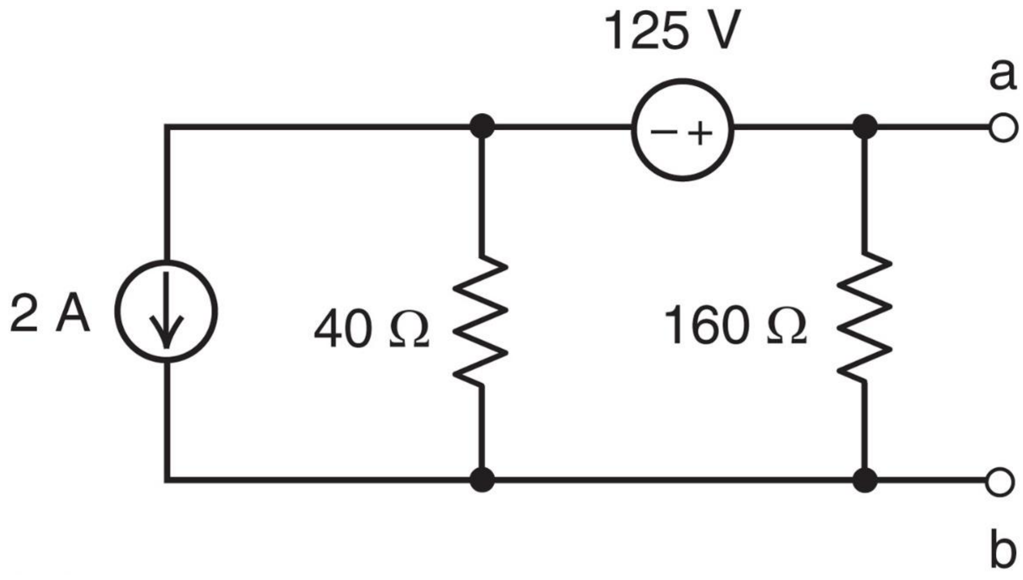
Norton equivalent circuit

Norton's theorem (cont'd)

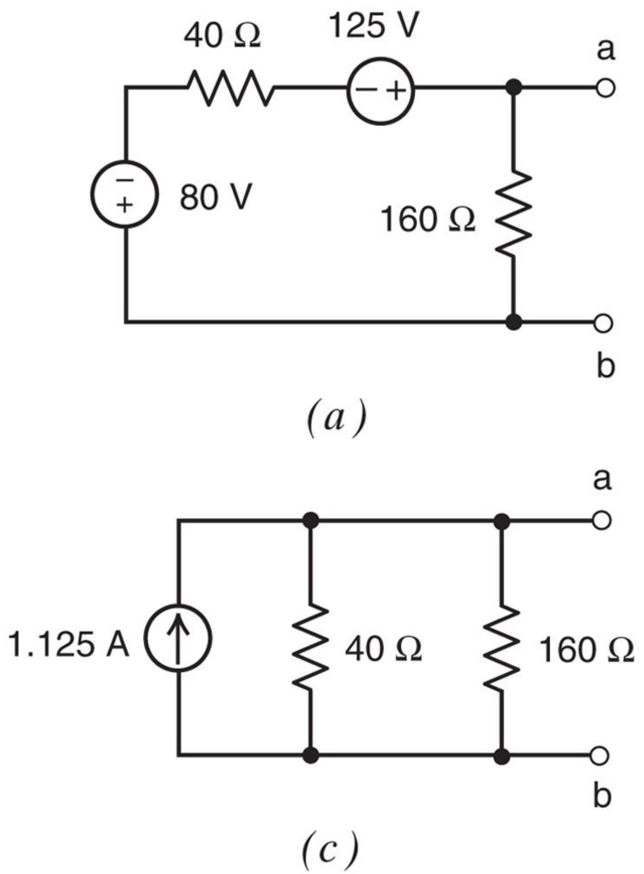
Methods of finding a Norton Equivalent Circuit		
Number of Method	If the circuit contains:	Norton equivalent circuit
1	Resistors and independent sources	1. (a) Connect an short circuit between terminals a and b. Find i_{sc}, the current directed from a to b in the short circuit (b) Deactivate the independent sources. Find $R_n = R_t$ by circuit resistance reduction. 또는 2. (a) Connect an open circuit between terminals a and b. Find $v_{oc}=v_{ab}$, the voltage across the open circuit (b) Connect an short circuit between terminals a and b. Find i_{sc}, the current directed from a to b in the short circuit, Then $R_n=R_t=v_{oc}/i_{sc}$ 또는 3. Set all independent sources to zero, then connect a 1-A current source from terminal b to terminal a. Determine v_{ab}. Then $R_n=R_t=v_{ab}/1$
2	Resistors and independent and dependent sources	2 또는 3(상동)
3	Resistors and dependent source (no independent sources)	3과 같이 (a) Note that $i_{sc}=0$ (b) Connect a 1-A current source from terminal b to terminal a. Determine v_{ab}. Then $R_n=R_t=v_{ab}/1$

Example 5.5-1 Norton Equivalent Circuit

- Find the Norton equivalent circuit for the circuit of Figure 5.5-2



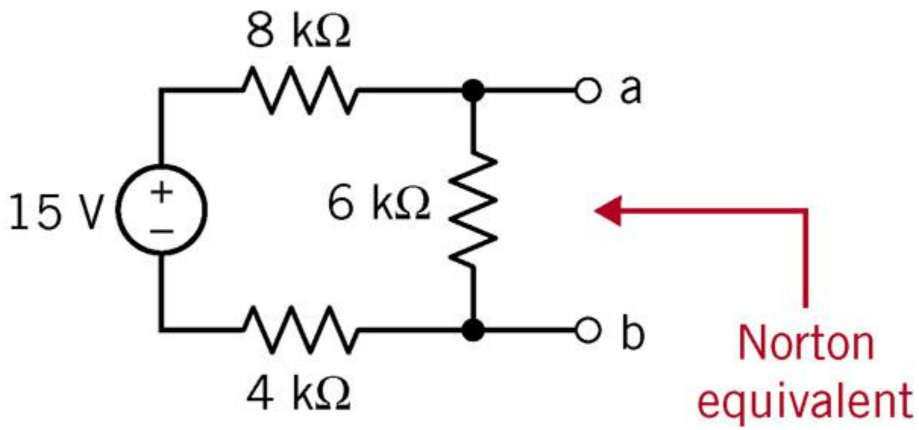
Solution



$$i_{sc} = 1.125A \qquad R_t = R_n = 32\Omega$$

Example) Norton Equivalent Circuit

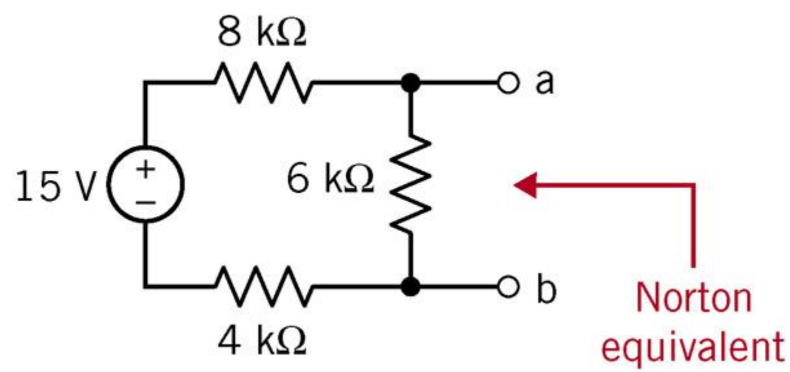
- Find the Norton equivalent circuit for the circuit of Figure 5.5-2



Solution

1. Deactivate the source and find R_n
   Replacing the voltage source by a short circuit

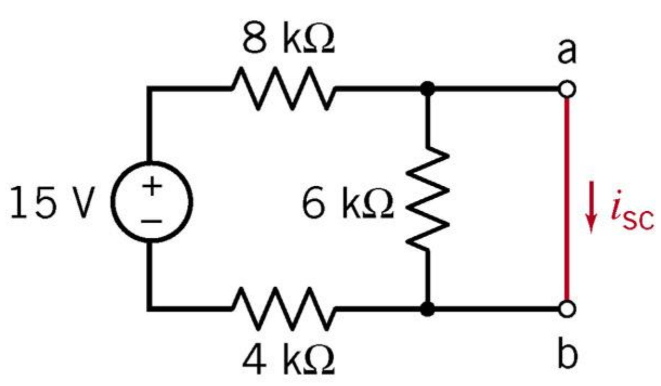
$$R_n = \frac{6 \times 12}{6+12} = 4k\Omega$$



2. Short circuit connected to output terminals
   KCL at node a in Figure 5.5-3

$$-\frac{15\text{V}}{12k\Omega} + i_{sc} = 0$$

$$i_{sc} = 1.25\text{mA}$$



3. Thus, Norton equivalent has

$$R_n = 4k\Omega, \quad i_{sc} = 1.25\text{mA}$$

Maximum Power Transfer

- Consider the circuit A shown in Figure 5.6-1. The Thévenin equivalent circuit is shown in Figure 5.6-2
  We wish to find the value of load resistance RL such that maximum power is delivered to it.
  First, we need to find the power from

$$p = i^2 R_L$$

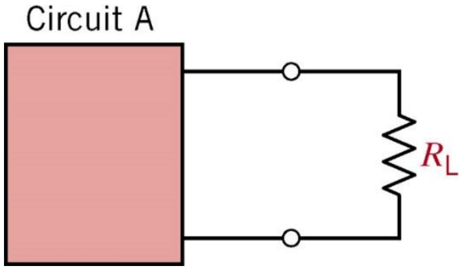


Fig 5.6-1

Since the current ***i*** is

$$i = \frac{v_s}{R_L + R_t}$$

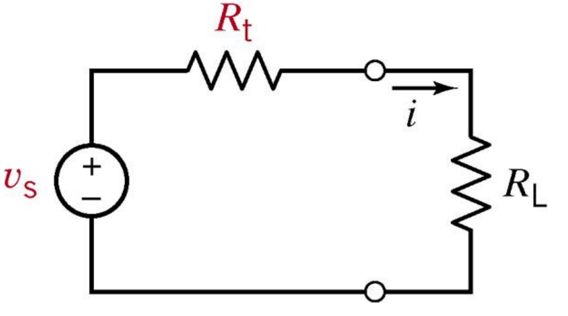


Fig 5.6-2

We find that the power is

$$p = \left(\frac{v_s}{R_L + R_t} \right)^2 R_L$$

Maximum Power Transfer (cont'd)

- Assuming that v_s and R_t are fixed for a given source, the maximum power is a function of RL.
  To find the value of R_L that maximizes the power, we use the differential calculus to find where the derivative dp/dR_L equals zero.
  Taking the derivative, we obtain

$$\frac{dp}{dR_L} = v_s^{\ 2} \frac{(R_t + R_L)^2 - 2(R_t + R_L)R_L}{(R_L + R_t)^4}$$

  The derivative is zero when

$$(R_t + R_L)^2 - 2(R_t + R_L)R_L = 0$$

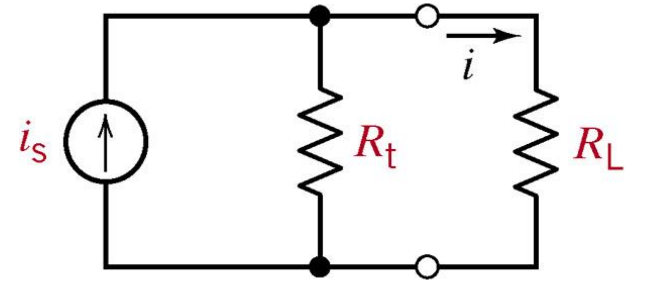
$$R_L = R_t$$

  We find that the maximum power is

$$p_{\max} = \left(\frac{v_s}{R_L + R_t} \right)^2 R_L = \frac{v_s^{\ 2} R_t}{(2R_t)^2} = \frac{v_s^{\ 2}}{4R_t}$$

Maximum Power Transfer (cont'd)

- We may also use Norton's equivalent circuit to represent the circuit A shown in Figure 5.6-1. The equivalent circuit is shown in Figure 5.6-4 The current i may be obtained to yield



$$i = \frac{R_t}{R_t + R_L} i_s$$

Therefore the power p is

$$p = i^2 R_L$$

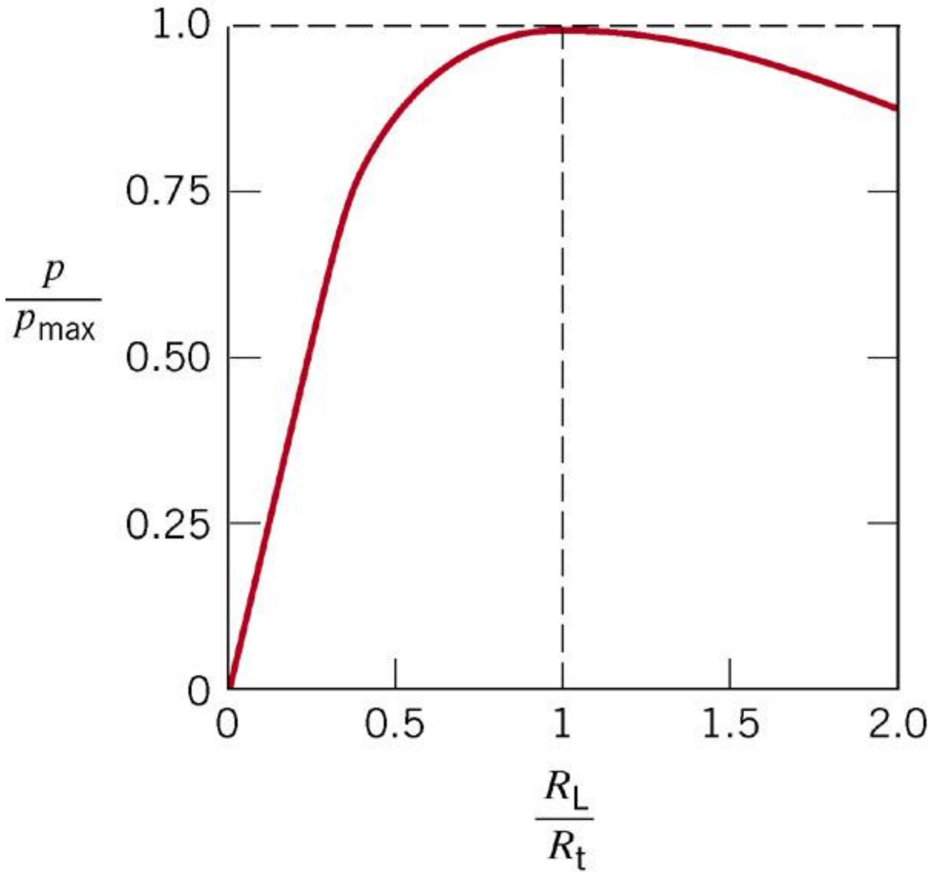
Using calculus, we can show that the maximum power occurs when

$$R_L = R_t$$

Then the maximum power delivered to the load is $p_{\max} = \dfrac{R_t {i_s}^2}{4}$

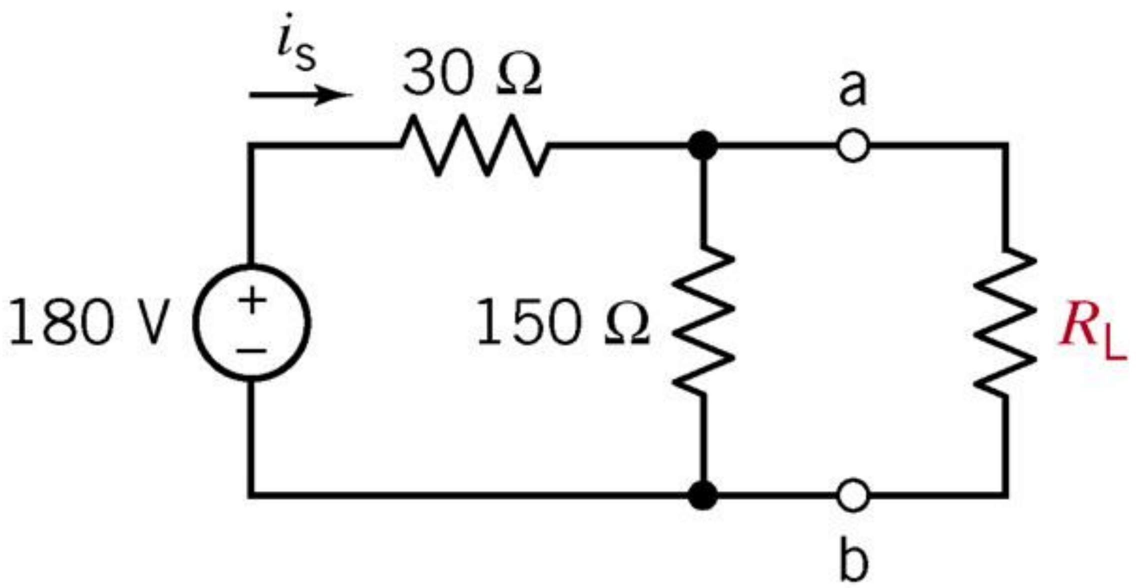
Maximum Power Transfer (cont'd)

- Power actually attained as R_L varies in relation to R_t.



Example 5.6-1 Maximum Power Transfer

- Find the load resistance R_L that will result in maximum power delivered to the load for the circuit of Figure 5.6-5. Also determine the maximum power delivered to the load resistor.



Solution

- Disconnect the load resistor. The Thévenin voltage source v_t is

$$v_t = \frac{150}{180} \times 180 = 150\text{V}$$

- The Thévenin resistance R_t is

$$R_t = \frac{30 \times 150}{30 + 150} = 25\Omega$$

- The Thévenin circuit connected to the load resistor is shown in Figure 5.6-6. Maximum power transfer is obtained when

$$R_L = R_t = 25\Omega$$

- Then the maximum power is

$$p_{\max} = \frac{v_s^{\ 2}}{4R_L} = \frac{(150)^2}{4 \times 25} = 225\text{W}$$

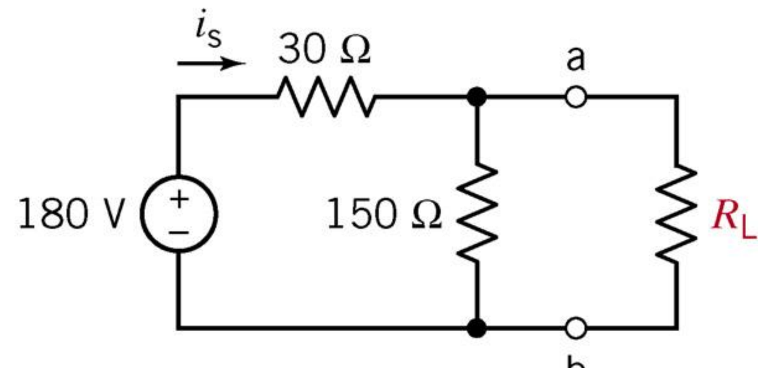


Fig. 5.6-5

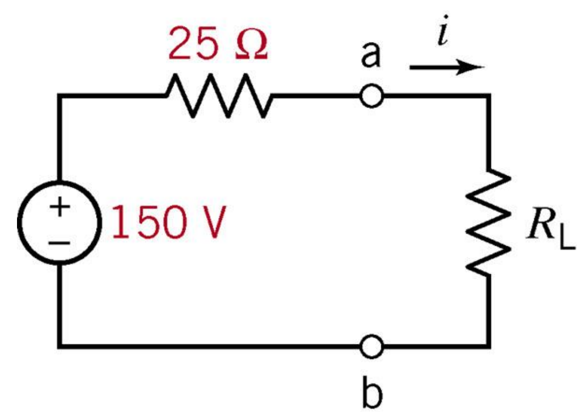
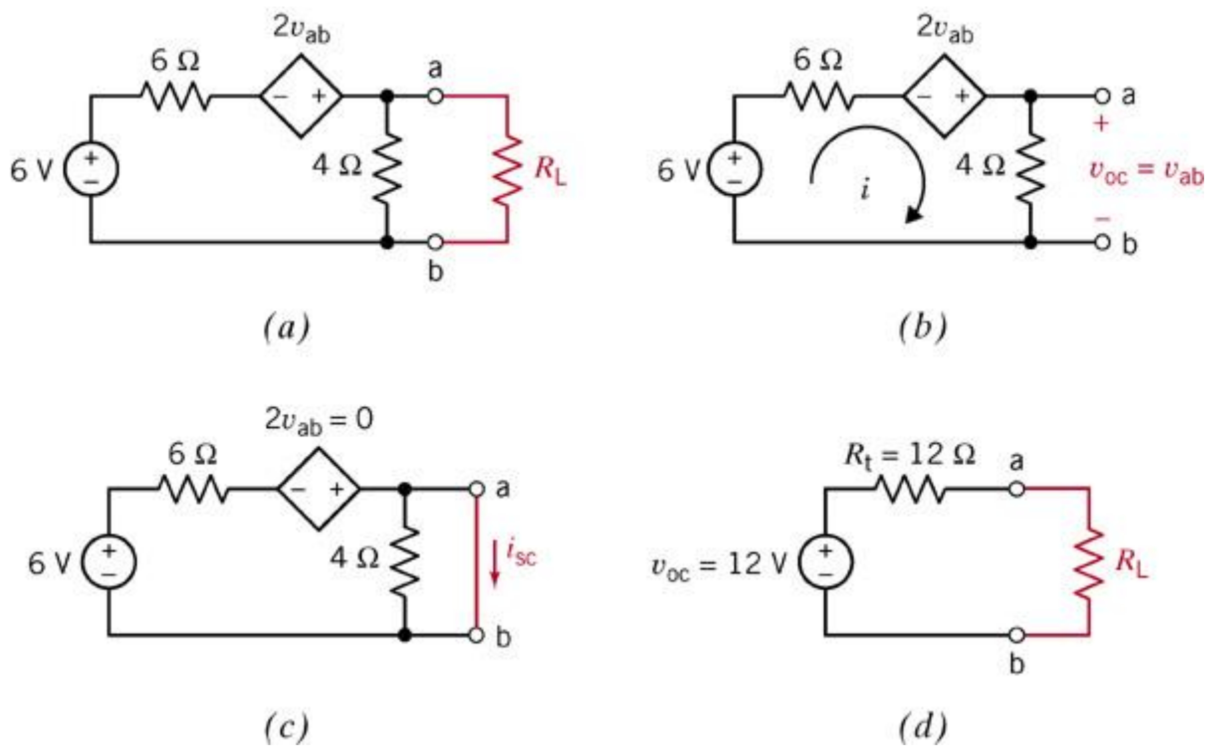


Fig. 5.6-6

Example 5.6-2 Maximum Power Transfer

- Find the load R_L that will remain in maximum power delivered to the load of the circuit of Figure 5.6-7a. Also determine $p_{max.}$ delivered.



Department of Electrical and Computer Engineering, SNU
Prof. SungJune Kim

Solution

1. In Figure 5.6-7b

$$-6+10i-2v_{ab}=0$$
$$v_{ab}=4i$$
$$i=3,$$
$$v_{ab}=12$$

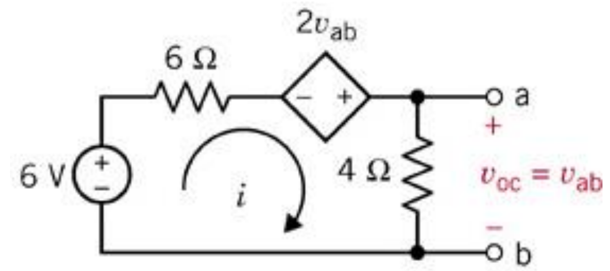


Fig. 5.6−7b

2. In Figure 5.6-7c

$$-6+6i_{sc}=0$$
$$i_{sc}=1\text{A}$$

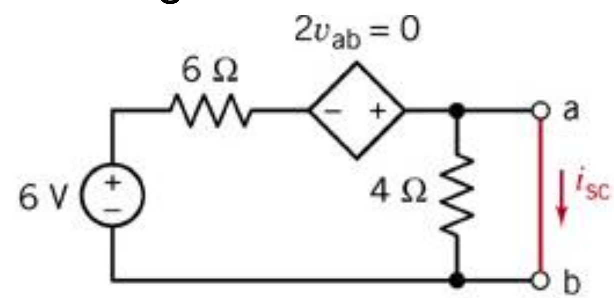


Fig. 5.6−7c

3. In Figure 5.6-7d
   Then the maximum power is

$$p_{\max}=\frac{v_{oc}{}^{2}}{4R_L}=\frac{(12)^2}{4\times 12}=3\text{W}$$

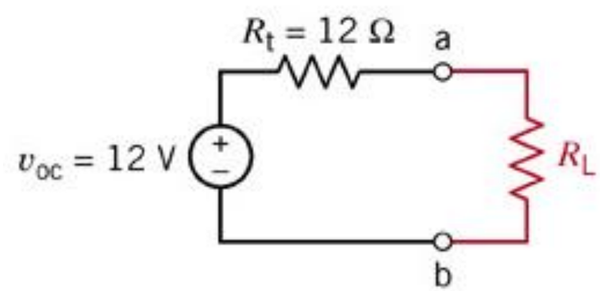


Fig. 5.6−7d