

2019 Fall

Introduction to Materials Science and Engineering

10. 22. 2019

Eun Soo Park

Office: 33-313

Telephone: 880-7221

Email: espark@snu.ac.kr

Office hours: by appointment

Chapter 7: Diffusion

I. Introduction of diffusion

: Movement of atoms to reduce its chemical potential μ .

 **driving force: Reduction of G**

Down-hill diffusion movement of atoms from a high C_B region to low C_B region.

Up-hill diffusion movement of atoms from a low C_B region to high C_B region.

II. Diffusion mechanisms

Vacancy diffusion vs. Interstitial diffusion

(a) Self-diffusion

(b) Interdiffusion

III. Steady-state diffusion

Concentration varies with position.

Fick's First law:

$$J = -D \frac{dC}{dx}$$

Estimation of Diffusion Depth:

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

IV. Non-steady-state diffusion

Concentration varies with time and position.

Fick's Second law:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

Diffusion coefficient:

$$D = D_0 \exp\left(\frac{Q_d}{RT}\right)$$

V. Factors that influences diffusion

Diffusion FASTER for...	Diffusion SLOWER for...
open crystal structures	close-packed structures
lower melting T materials	higher melting T materials
materials w/secondary bonding	materials w/covalent bonding
smaller diffusing atoms	larger diffusing atoms
cations	anions
lower density materials	higher density materials

H2 : Chapter 7 example

IH : Chapter 7 연습문제

중간고사

10월 25일 6시 부터 9시 33동 225호, 226호

시험범위 1장부터 - 7장까지
주교재 - 영문기준 207 page까지

Contents for previous class

Chapter 1: Introduction

Chapter 2-6:

Atomic structure and interatomic bonding

Fundamentals of **crystallography**

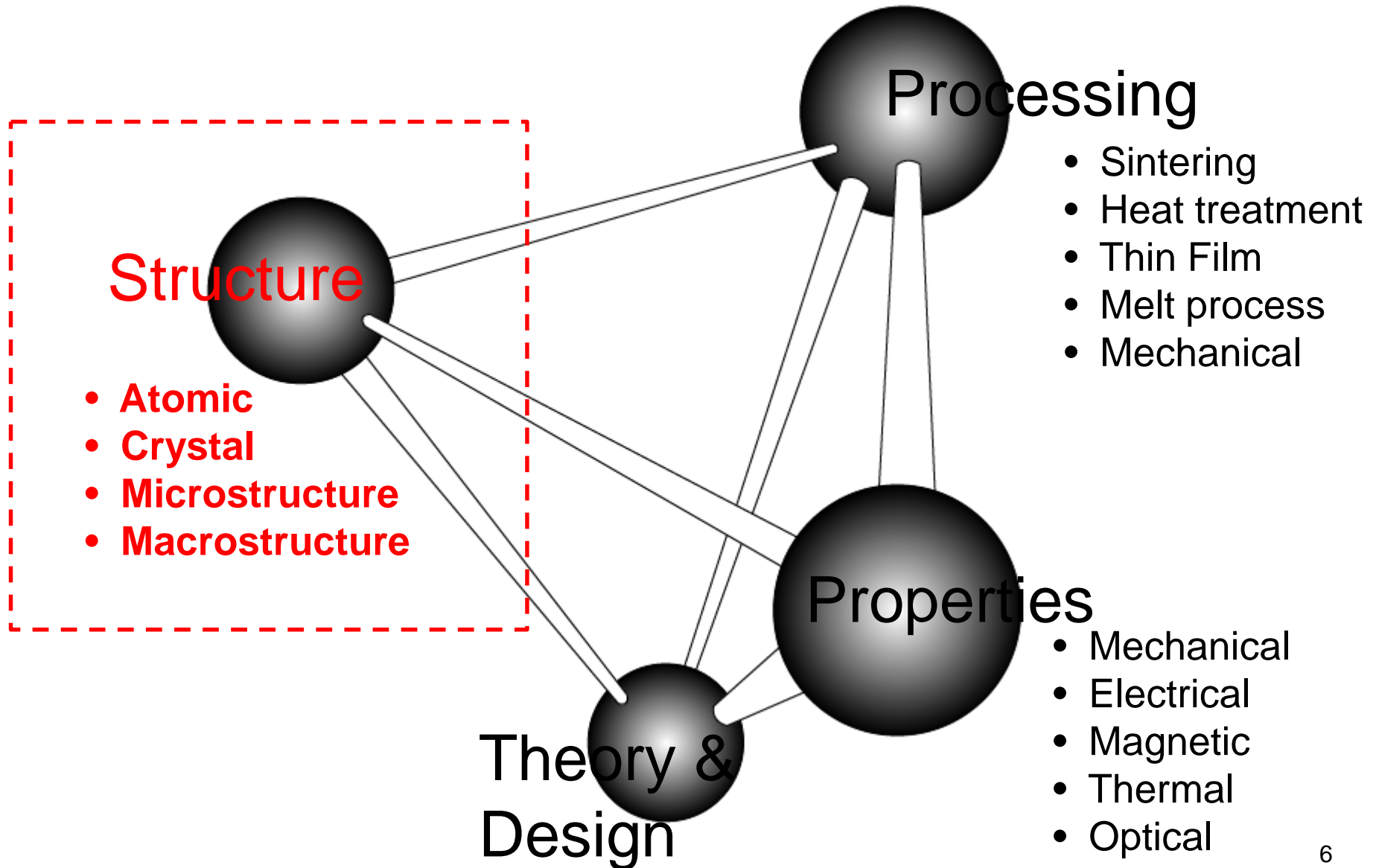
The structure of **crystalline solid**

Structure of Polymers

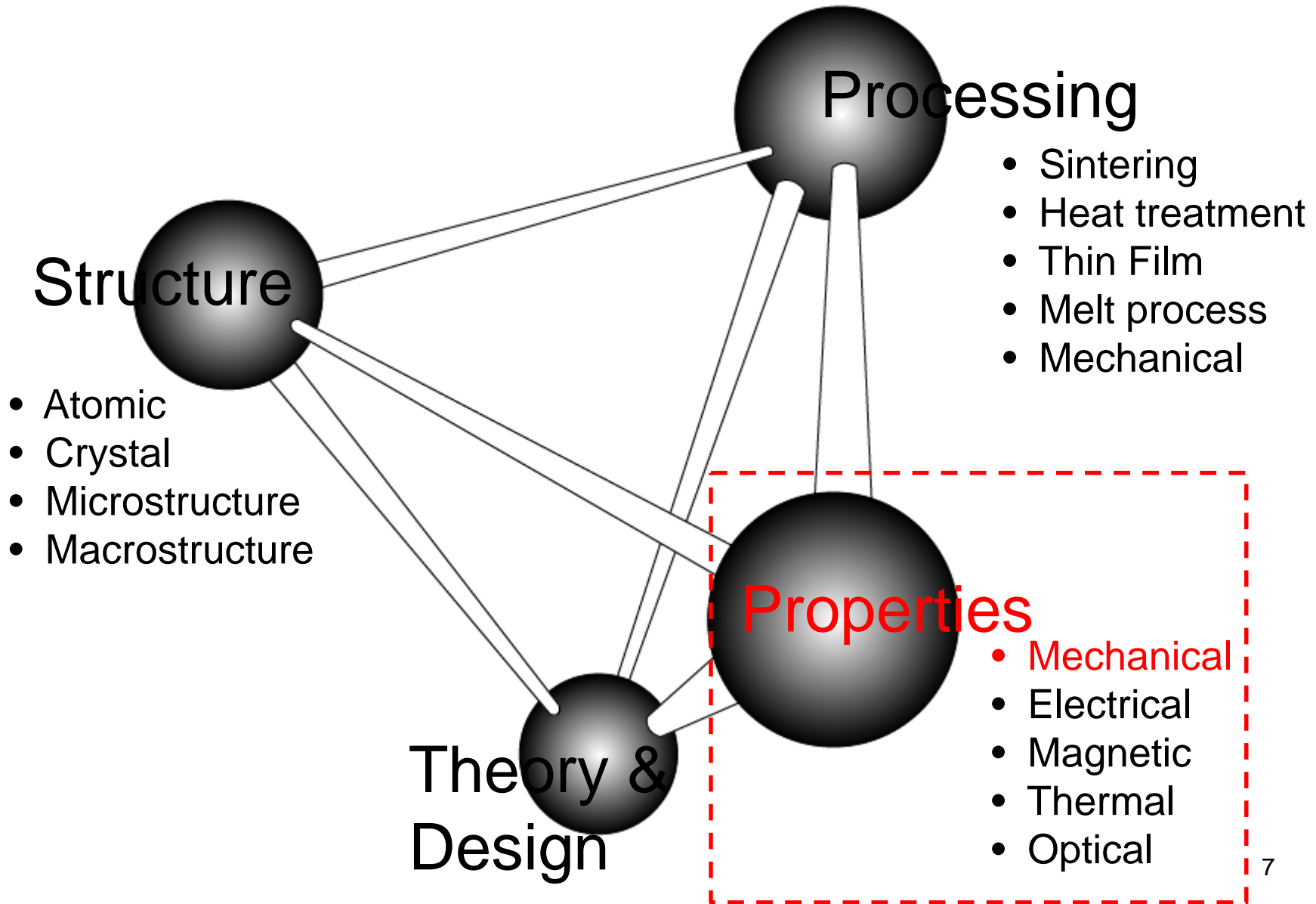
Imperfections in solids

Chapter 7: Diffusion

Materials Science and Engineering



Materials Science and Engineering



Chapter 8: Mechanical Properties of Metals

- **Stress and strain:**

What are they and why are they used instead of load and deformation?

- **Elastic behavior:**

When loads are small, how much deformation occurs?

What materials deform least?

- **Plastic behavior:**

At what point do dislocations cause permanent deformation?

What materials are most resistant to permanent deformation?

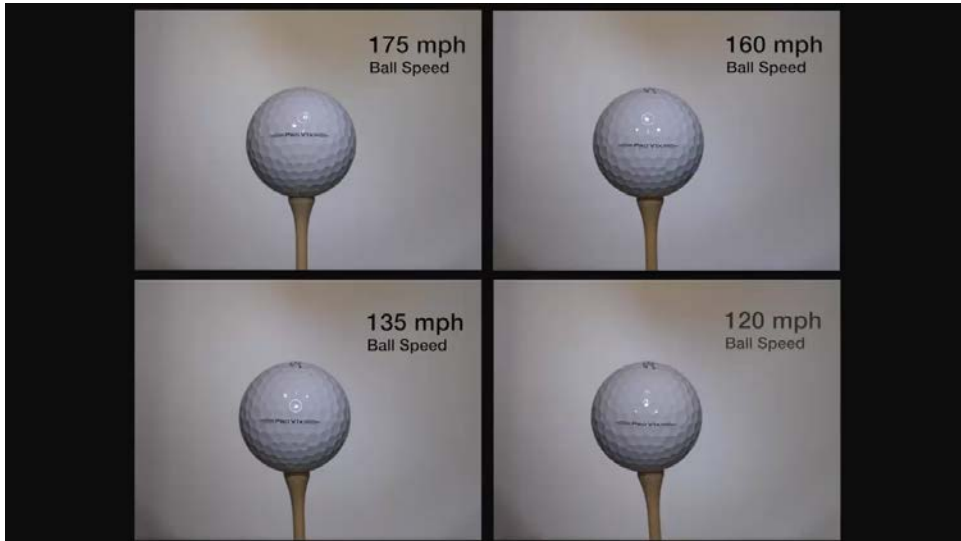
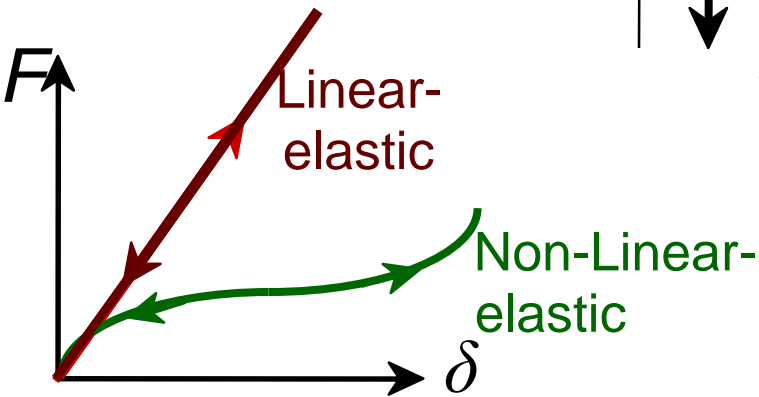
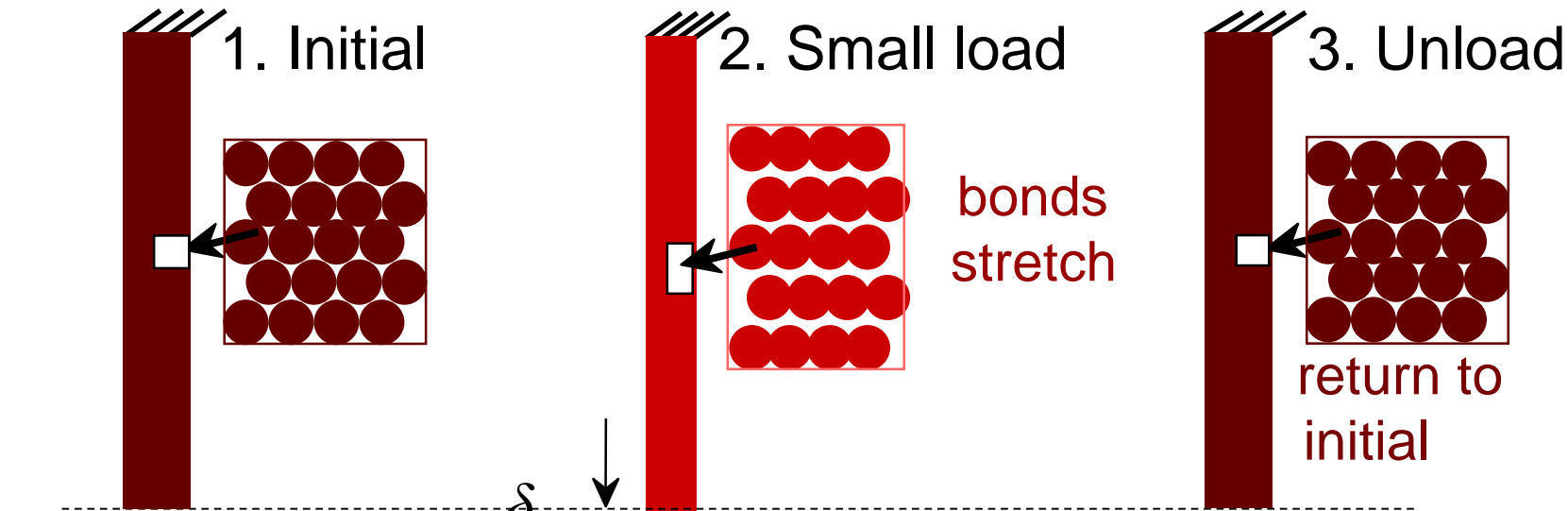
- **Toughness and ductility:**

What are they and how do we measure them?

- **Load** - The force applied to a material during testing
- **Stress** - Force or load per unit area of cross-section over which the force or load is acting
- **Strain** - Elongation change in dimension per unit length
- **Engineering stress** - The applied load, or force, divided by the original cross-sectional area of the material
- **Engineering strain** - The amount that a material deforms per unit length in a tensile test
- **True stress** The load divided by the actual cross-sectional area of the specimen at that load
- **True strain** The strain calculated using actual and not original dimensions, given by $\epsilon_t \ln(l/l_0)$
- **Young's modulus (E)** - The slope of the linear part of the stress-strain curve in the elastic region, same as modulus of elasticity
- **Shear modulus (G)** - The slope of the linear part of the shear stress-shear strain curve

I. Elastic deformation vs Plastic deformation

Elastic Deformation



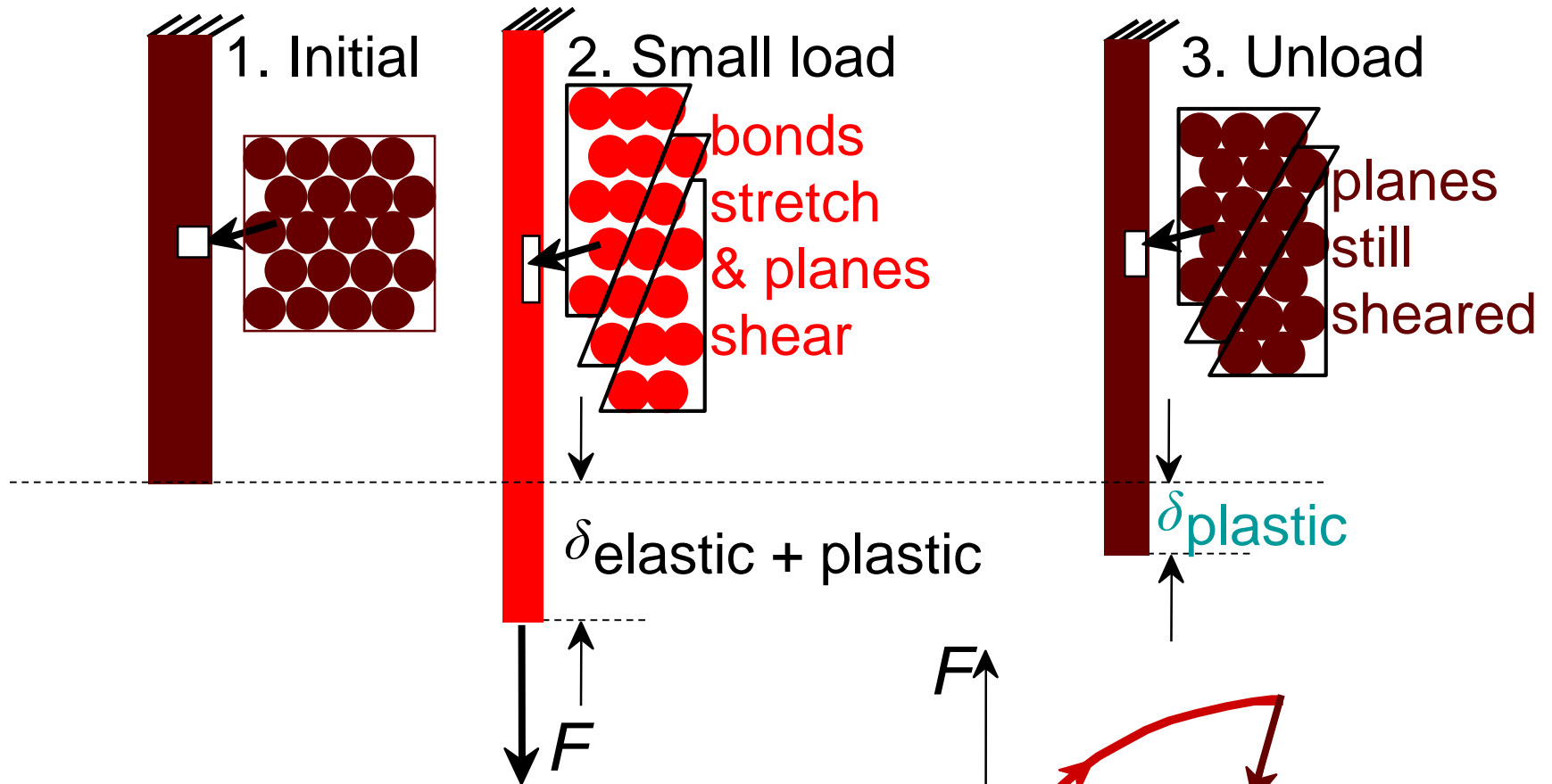
Elastic means **reversible!**



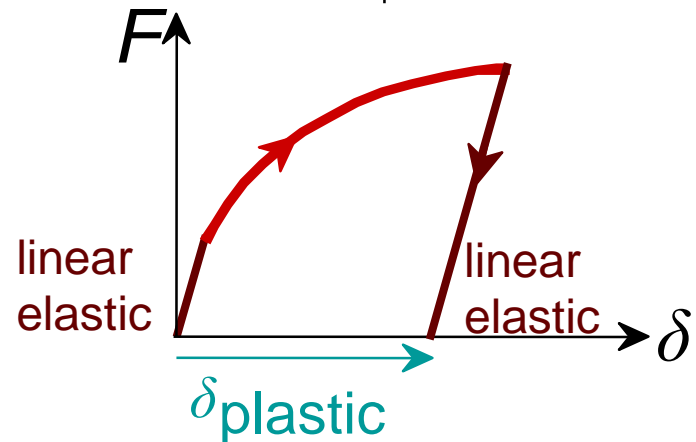
Steel vs Steel

<https://www.youtube.com/watch?v=SIFfY-MS3yA>

Plastic Deformation (Metals)

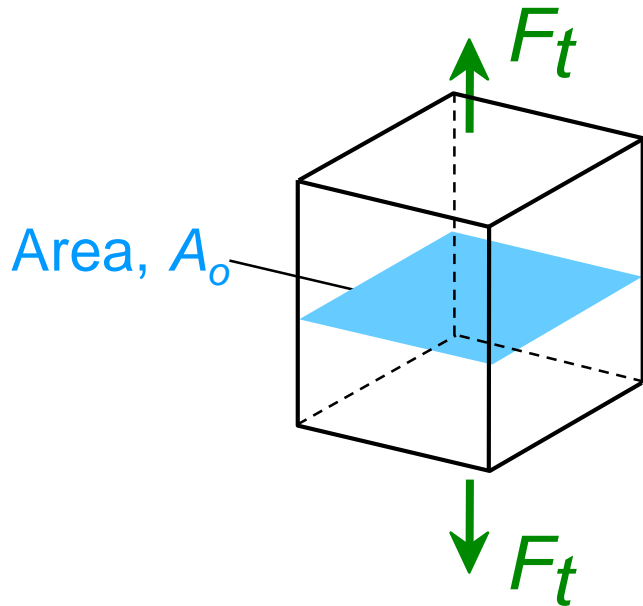


Plastic means permanent!



II. Engineering Stress

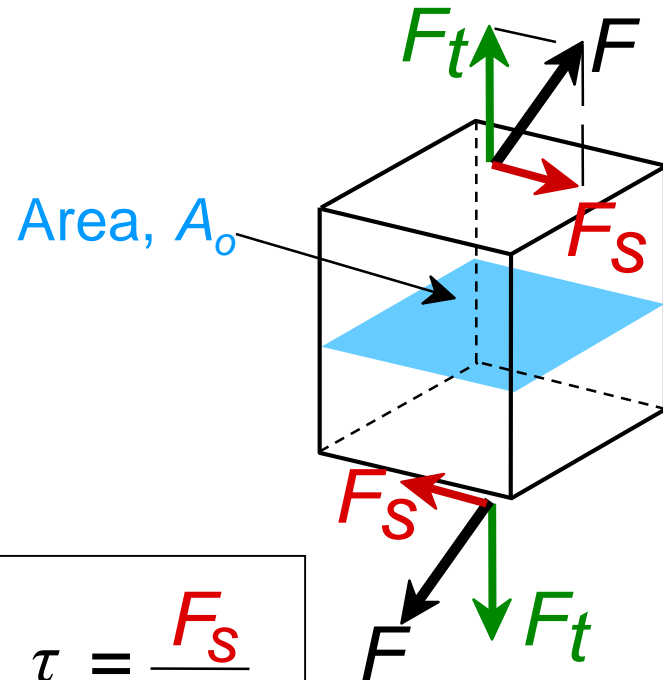
- Tensile stress, σ :



$$\sigma = \frac{F_t}{A_0} = \frac{\text{N}}{\text{m}^2}$$

original cross-sectional area before loading

- Shear stress, τ :

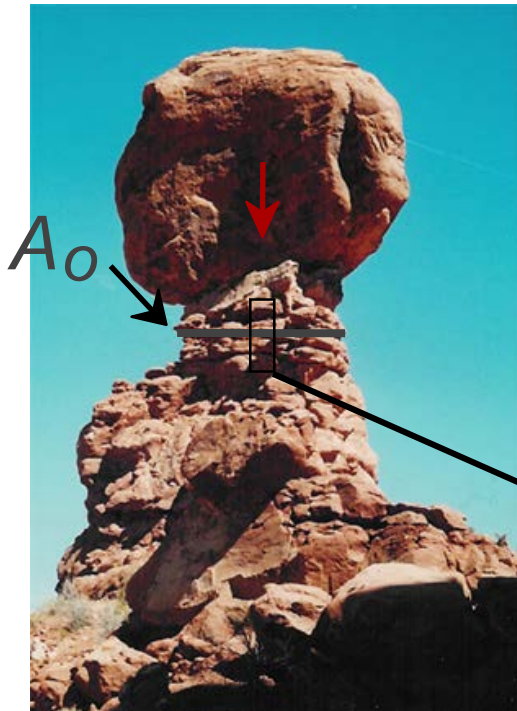


$$\tau = \frac{F_s}{A_0}$$

\therefore Stress has units:
N/m²

II. Common States of Stress

a. Simple **compression**:



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)



Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$



Note: compressive structure member ($\sigma < 0$ here).

II. Common States of Stress

b. Simple tension: cable



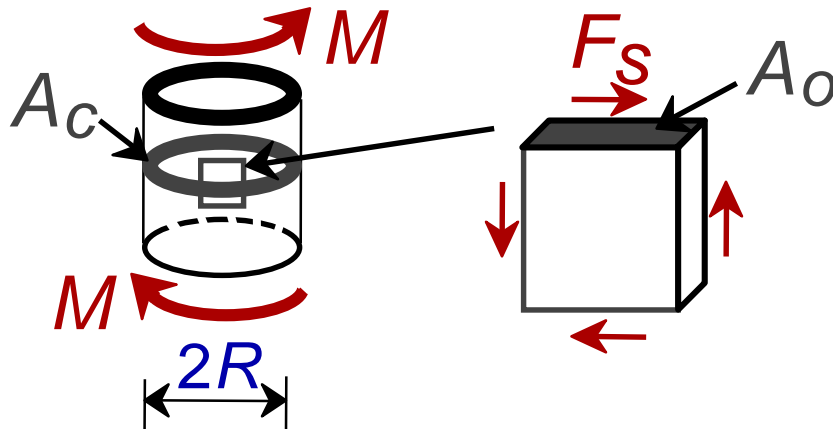
A_0 = cross-sectional area
(when unloaded)

$$\sigma = \frac{F}{A_0}$$

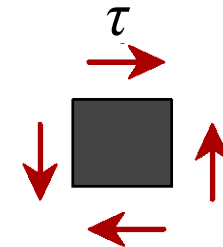


c. Torsion (a form of shear): drive shaft

Ski lift (photo courtesy
P.M. Anderson)



$$\tau = \frac{F_s}{A_0}$$



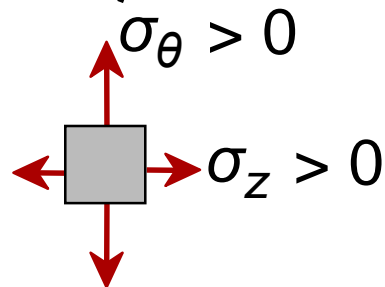
Note: $\tau = M/A_c R$ here.

II. Common States of Stress

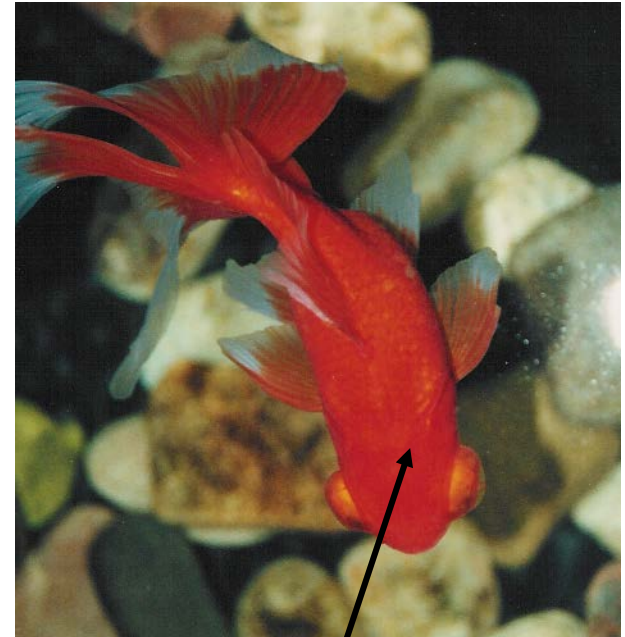
d. **Bi-axial tension:**



Pressurized tank
(photo courtesy
P.M. Anderson)

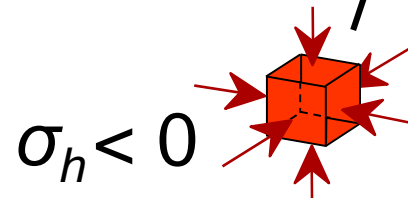


e. **Hydrostatic compression:**



Fish under water

(photo courtesy
P.M. Anderson)



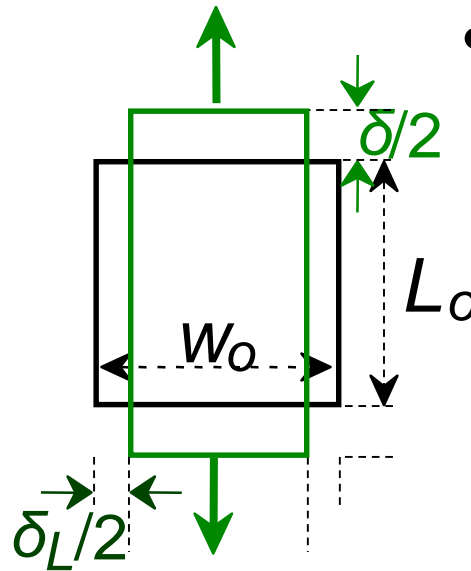
III. Engineering Strain

- **Tensile strain:**

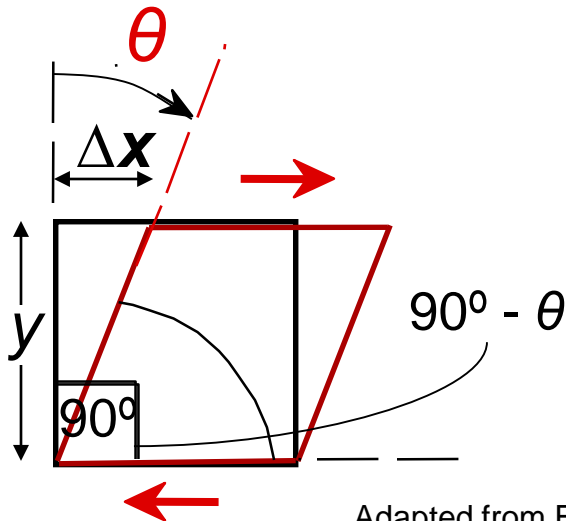
$$\varepsilon = \frac{\delta}{L_0}$$

- **Lateral strain:**

$$\varepsilon_L = -\frac{\delta_L}{W_0}$$



- **Shear strain:**



$$\gamma = \Delta x / y = \tan \theta$$

Strain is always dimensionless.

Adapted from Fig. 8.1 (a) and (c), *Callister & Rethwisch 9e*.

Stress-Strain Testing

- **Typical tensile test machine**

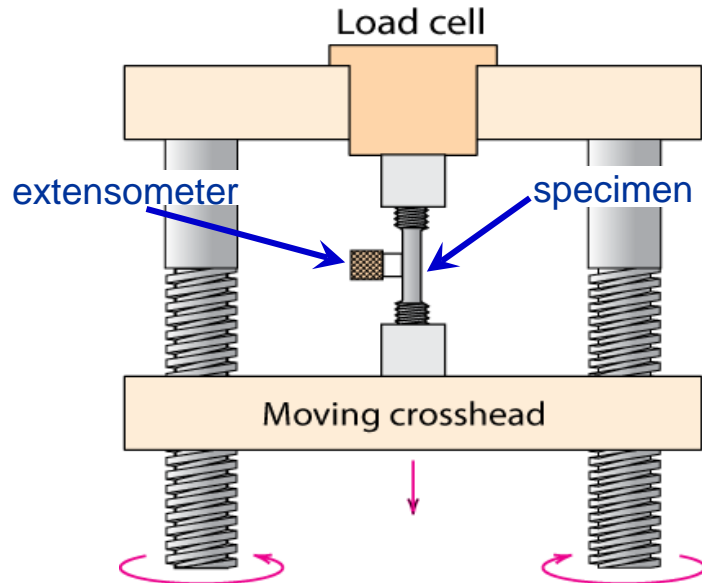
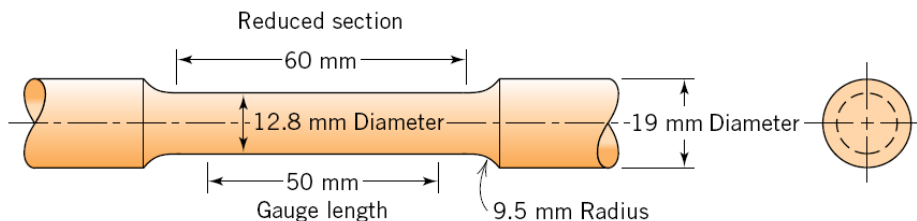


Fig. 8.3, *Callister & Rethwisch 9e*.

(Taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)



- **Typical tensile specimen**



➤ **Other types of tests:**

- **compression: brittle materials**
(e.g. concrete)

- **torsion: cylindrical tubes, shafts**

Tensile testing of a steel specimen



IV. Deformation

➤ Elastic vs. Plastic region

➤ Key points

1. Elastic modulus (=Young's modulus), E

2. 0.2% (0.002 strain) yield stress, σ_y

3. Ultimate yield stress, σ_{UTS}

4. Ductility, ϵ_p

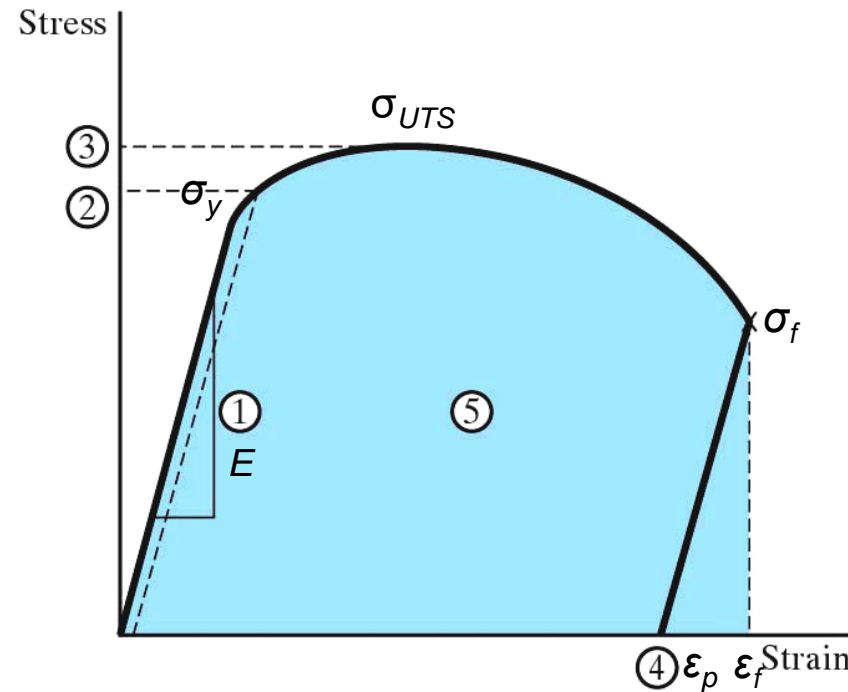
5. Toughness



$$\int \sigma d\epsilon$$

6. Fracture stress, σ_f

7. Fracture strain, ϵ_f

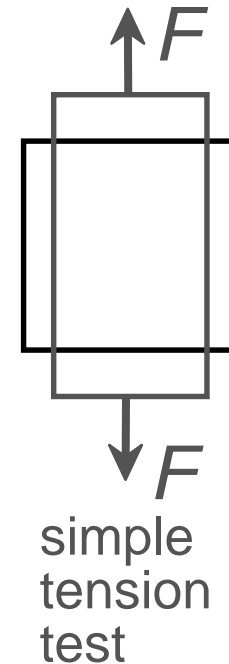
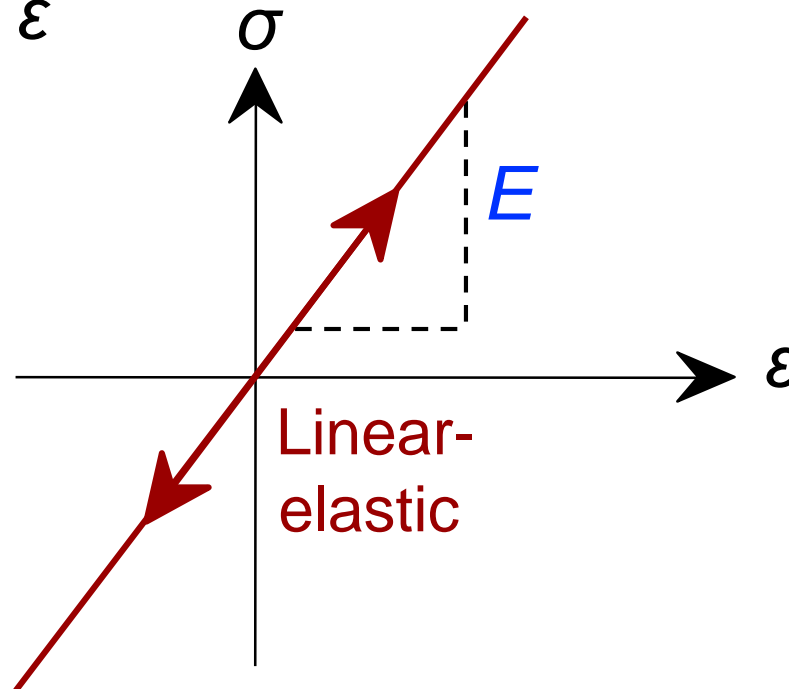


a. Elastic Properties

(1) **Modulus of Elasticity, E :**
(also known as Young's modulus)

• **Hooke's Law:**

$$\sigma = E \varepsilon$$



Properties from Bonding: E

- Slope of stress strain plot (which is proportional to the **elastic modulus, E**) depends on bond strength of metal

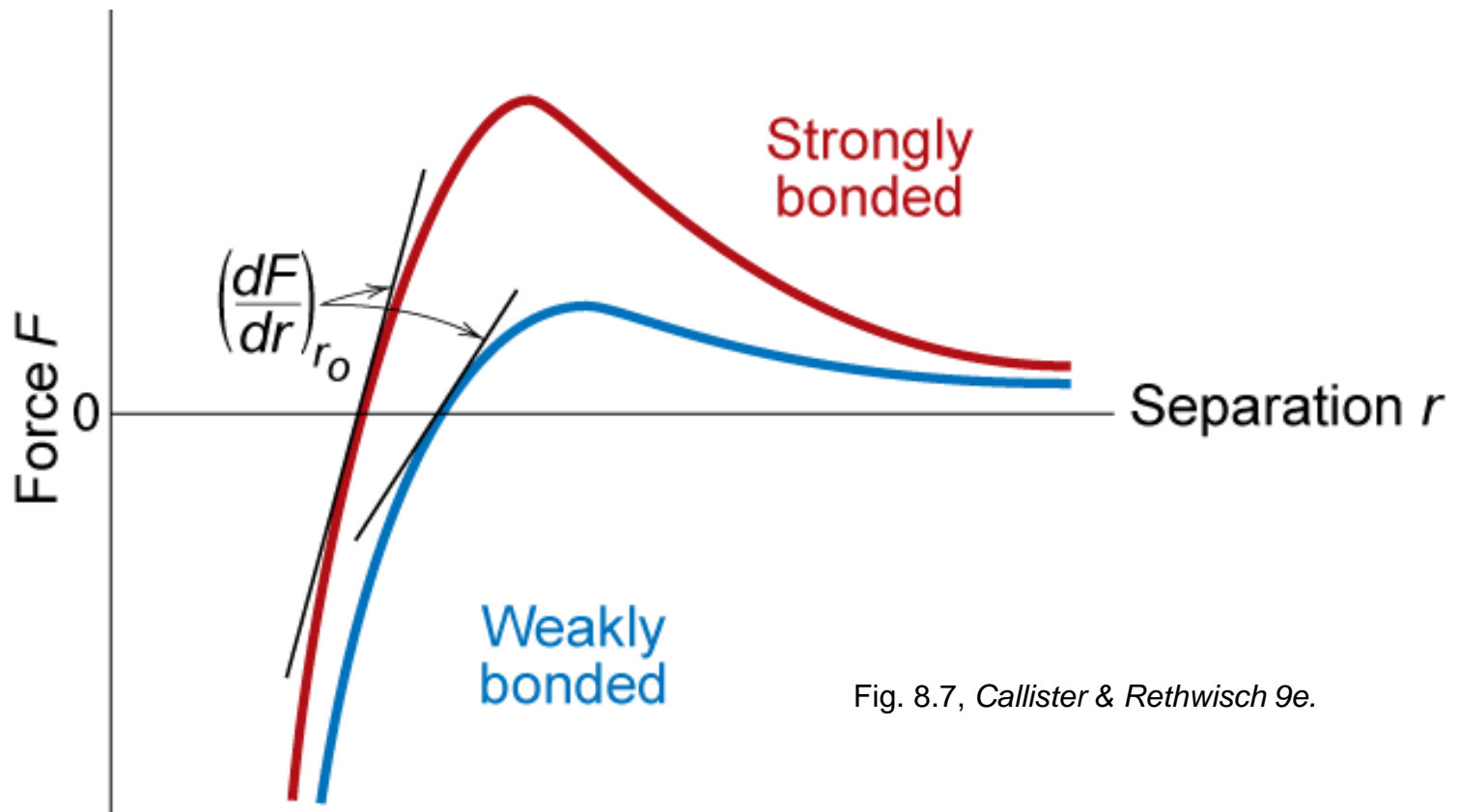
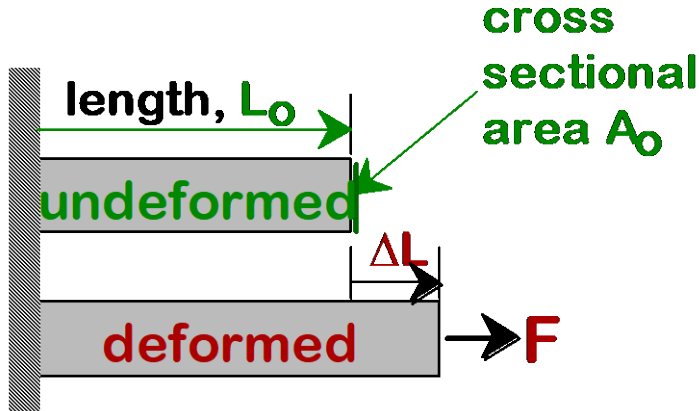


Fig. 8.7, Callister & Rethwisch 9e.

Properties from Bonding: E

- Elastic (Young's) modulus, E (Y)



Elastic modulus

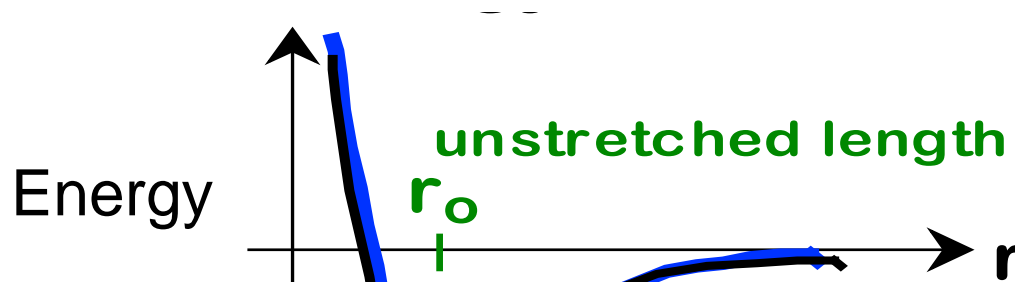
$$\frac{F}{A_0} = E \frac{\Delta L}{L_0}$$

$$\frac{F}{A_0} = E \frac{\Delta L}{L_0}$$

$$\sigma = Y \epsilon$$

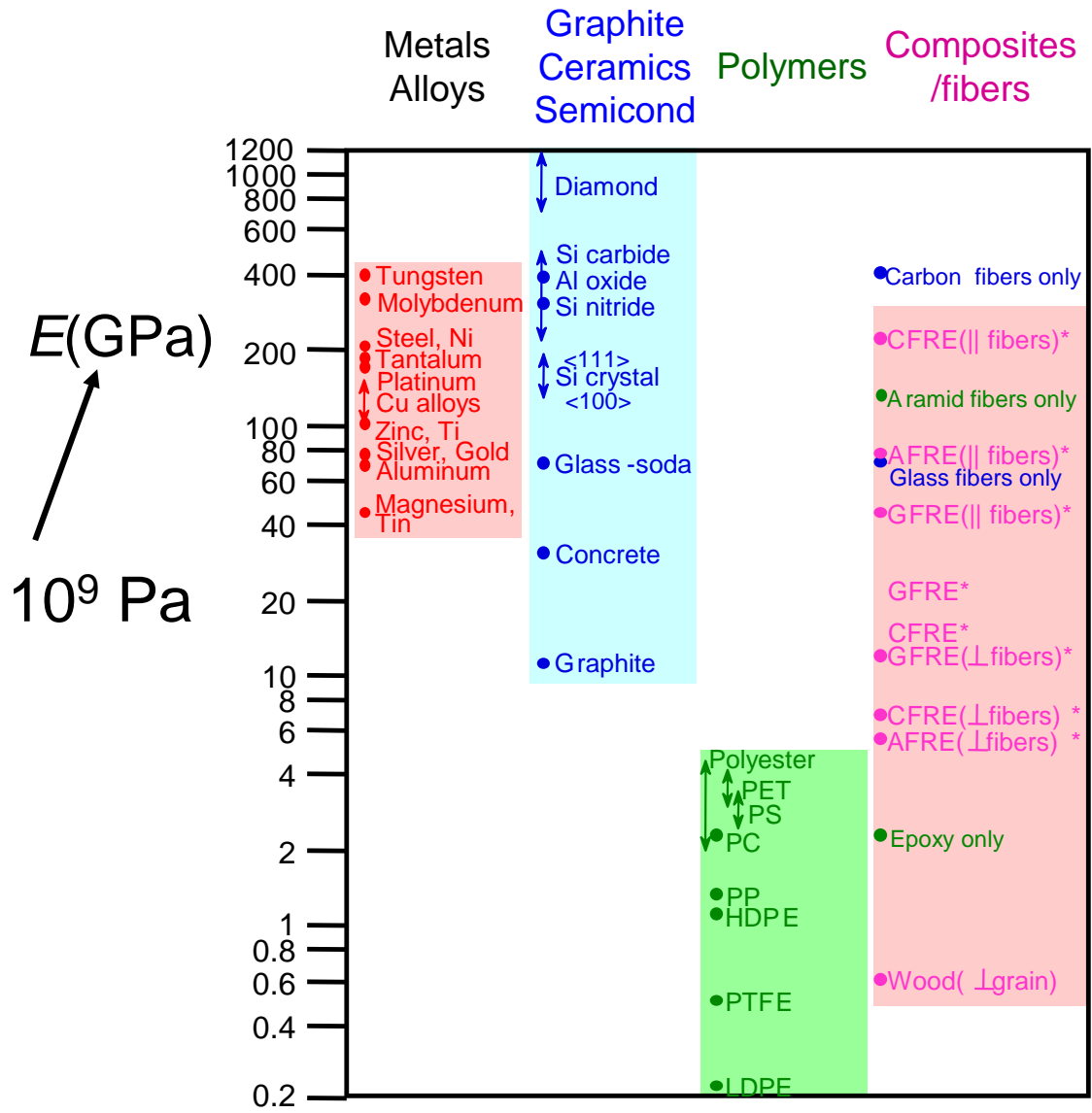
- $E \sim$ curvature at r_0 (the bottom of the well)

$$Y \sim \left(\frac{d^2 E}{dr^2} \right)_{r_0}$$



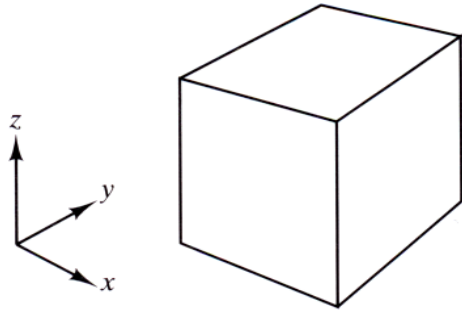
E is larger if E_0 is larger

Young's Moduli: Comparison

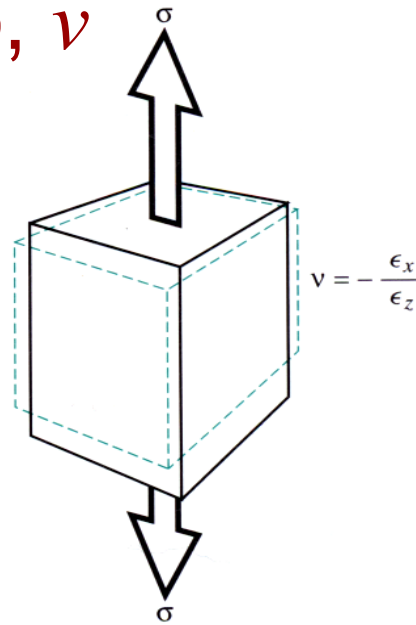


Based on data in Table B.2, *Callister & Rethwisch 9e*.
 Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

(2) Poisson's ratio, ν



(a) Unloaded



(b) Loaded

Poisson's ratio

$$\nu = -\frac{\epsilon_x}{\epsilon_z}$$

Estimation of ν

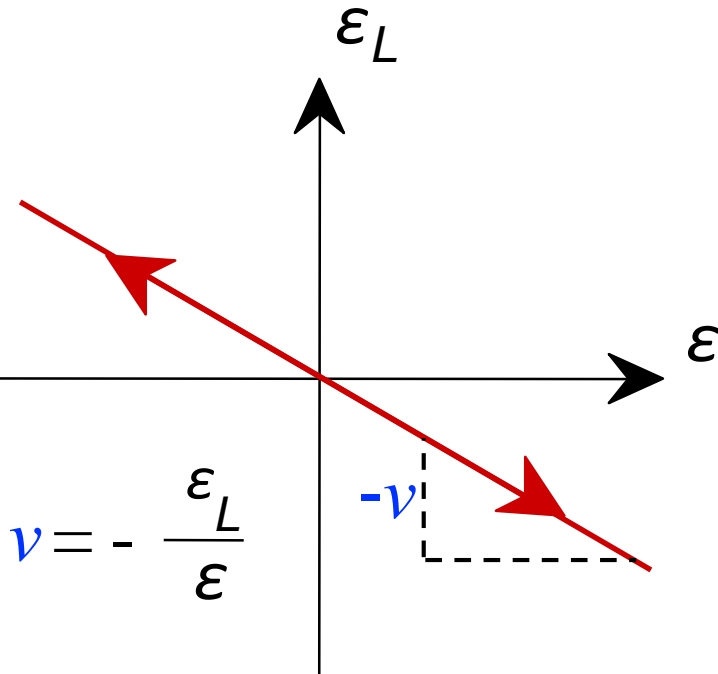
Units:

E : [GPa] or [psi]

ν : dimensionless

$\nu > 0.50$ density increases

$\nu < 0.50$ density decreases
(voids form)



Measured values of Poisson's ratio

metals: $\nu \sim 0.33$

ceramics: $\nu \sim 0.25$

polymers: $\nu \sim 0.40$

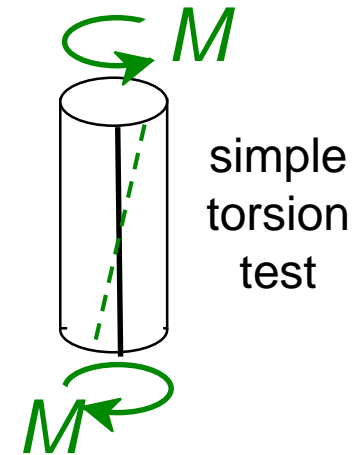
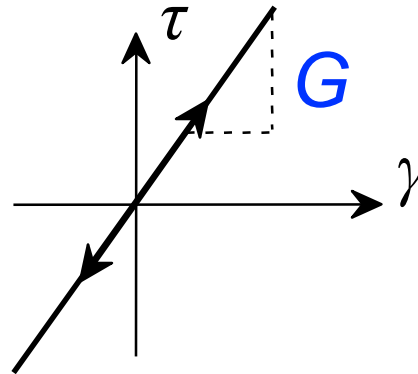
	ν
1. Al_2O_3	0.26
2. BeO	0.26
3. CeO_2	0.27–0.31
4. Cordierite ($2\text{MgO} \cdot 2\text{Al}_2\text{O}_3 \cdot 5\text{SiO}_2$)	0.31
5. Mullite ($3\text{Al}_2\text{O}_3 \cdot 2\text{SiO}_2$)	0.25
6. SiC	0.19
7. Si_3N_4	0.24
8. TaC	0.24
9. TiC	0.19
10. TiO_2	0.28
11. Partially stabilized ZrO_2	0.23
12. Fully stabilized ZrO_2	0.23–0.32
13. Glass-ceramic ($\text{MgO}-\text{Al}_2\text{O}_3-\text{SiO}_2$)	0.24
14. Borosilicate glass	0.2
15. Glass from cordierite	0.26

Source: Data from *Ceramic Source '86* and *Ceramic Source '87*, American Ceramic Society, Columbus, OH, 1985 and 1986.

(3) Other Elastic Properties

- **Elastic Shear modulus, G :**

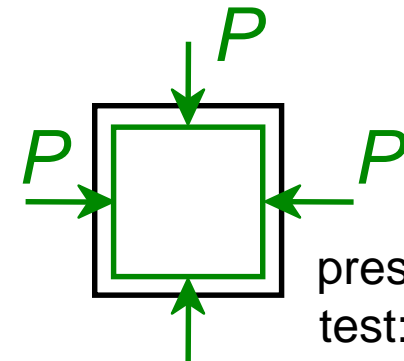
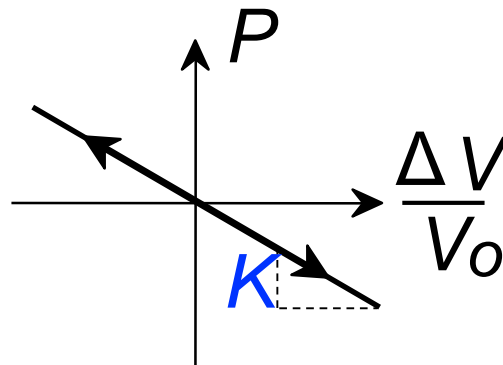
$$\tau = G \gamma$$



simple torsion test

- **Elastic Bulk modulus, K :**

$$P = -K \frac{\Delta V}{V_0}$$



pressure test: Init. vol = V_0 .
Vol chg. = ΔV

- **Special relations for isotropic materials:**

$$G = \frac{E}{2(1 + \nu)}$$

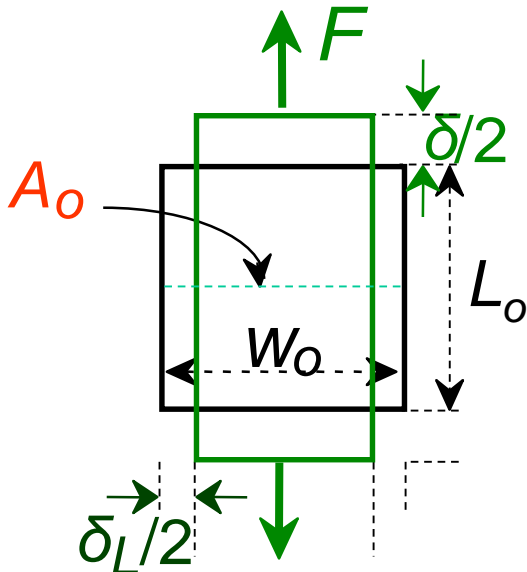
$$K = \frac{E}{3(1 - 2\nu)}$$

(4) Useful Linear Elastic Relationships

- Simple tension:

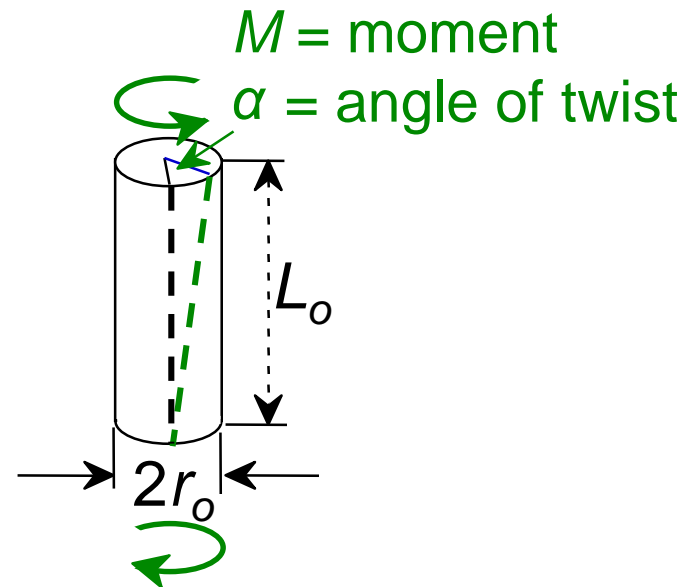
$$\delta = \frac{FL_o}{EA_o}$$

$$\delta_L = -\nu \frac{FW_o}{EA_o}$$



- Simple torsion:

$$\alpha = \frac{2ML_o}{\pi r_o^4 G}$$

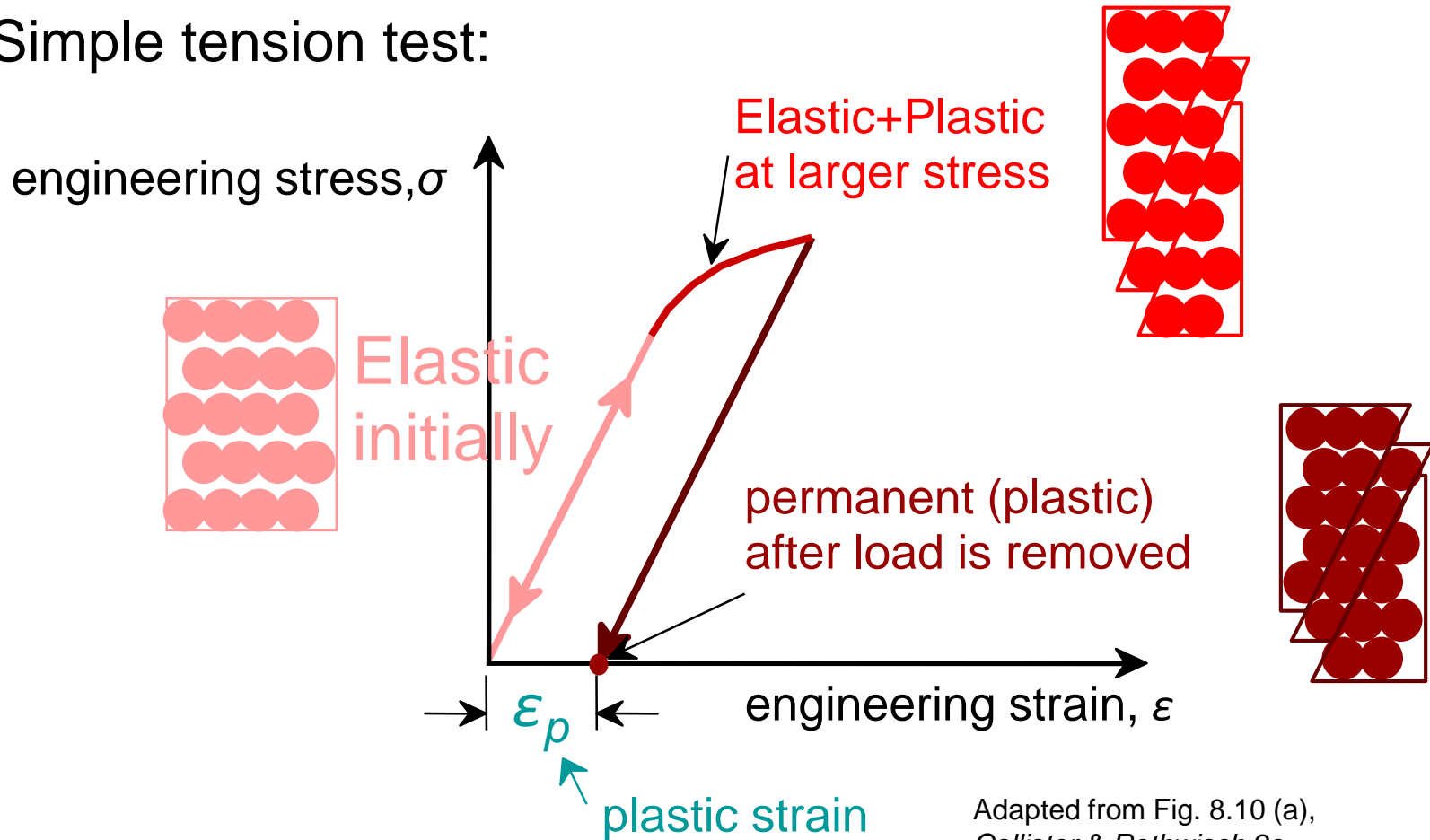


- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

- Simple tension test:

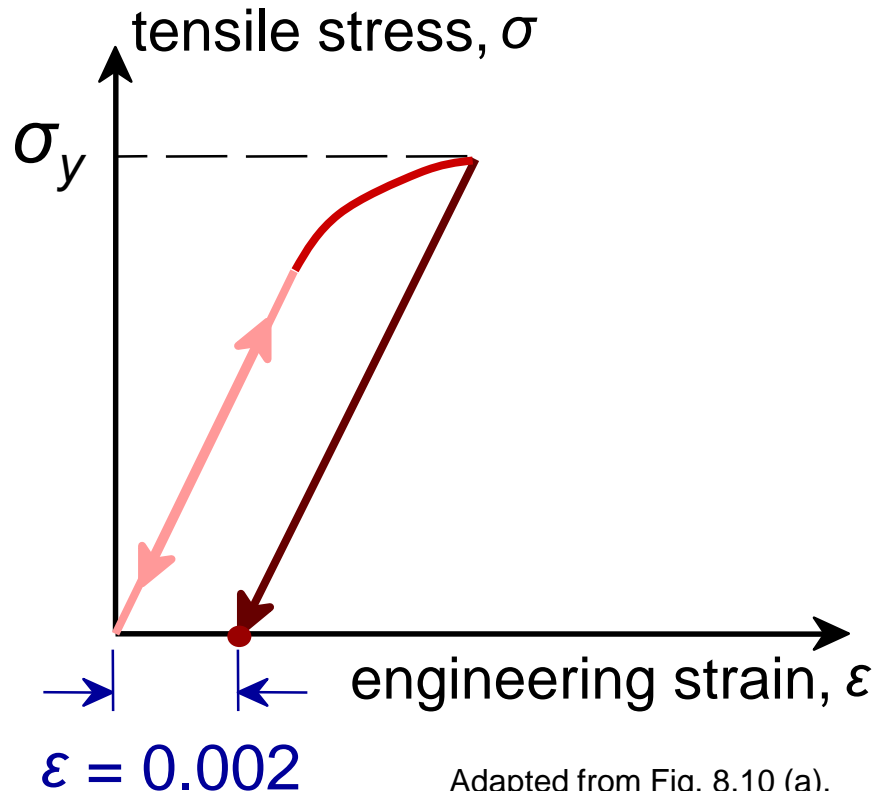


Adapted from Fig. 8.10 (a),
Callister & Rethwisch 9e.

b. Yield Strength, σ_y

- Stress at which *noticeable* plastic deformation has occurred.

when $\varepsilon = 0.002$



$\sigma_y =$ yield strength

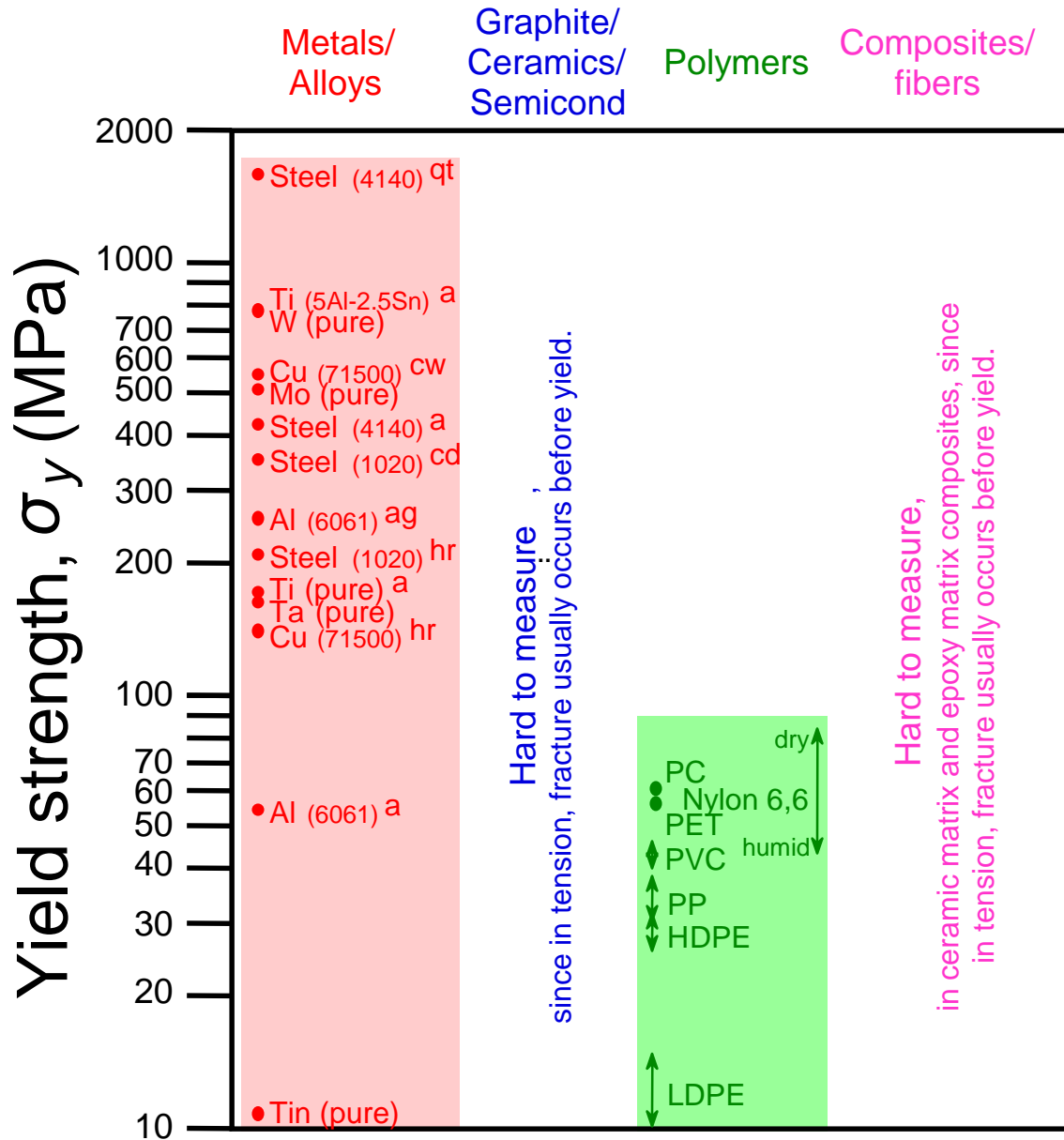
Note: for 2 inch sample

$$\varepsilon = 0.002 = \Delta z / z$$

$$\therefore \Delta z = 0.004 \text{ in}$$

Adapted from Fig. 8.10 (a),
Callister & Rethwisch 9e.

Yield Strength : Comparison



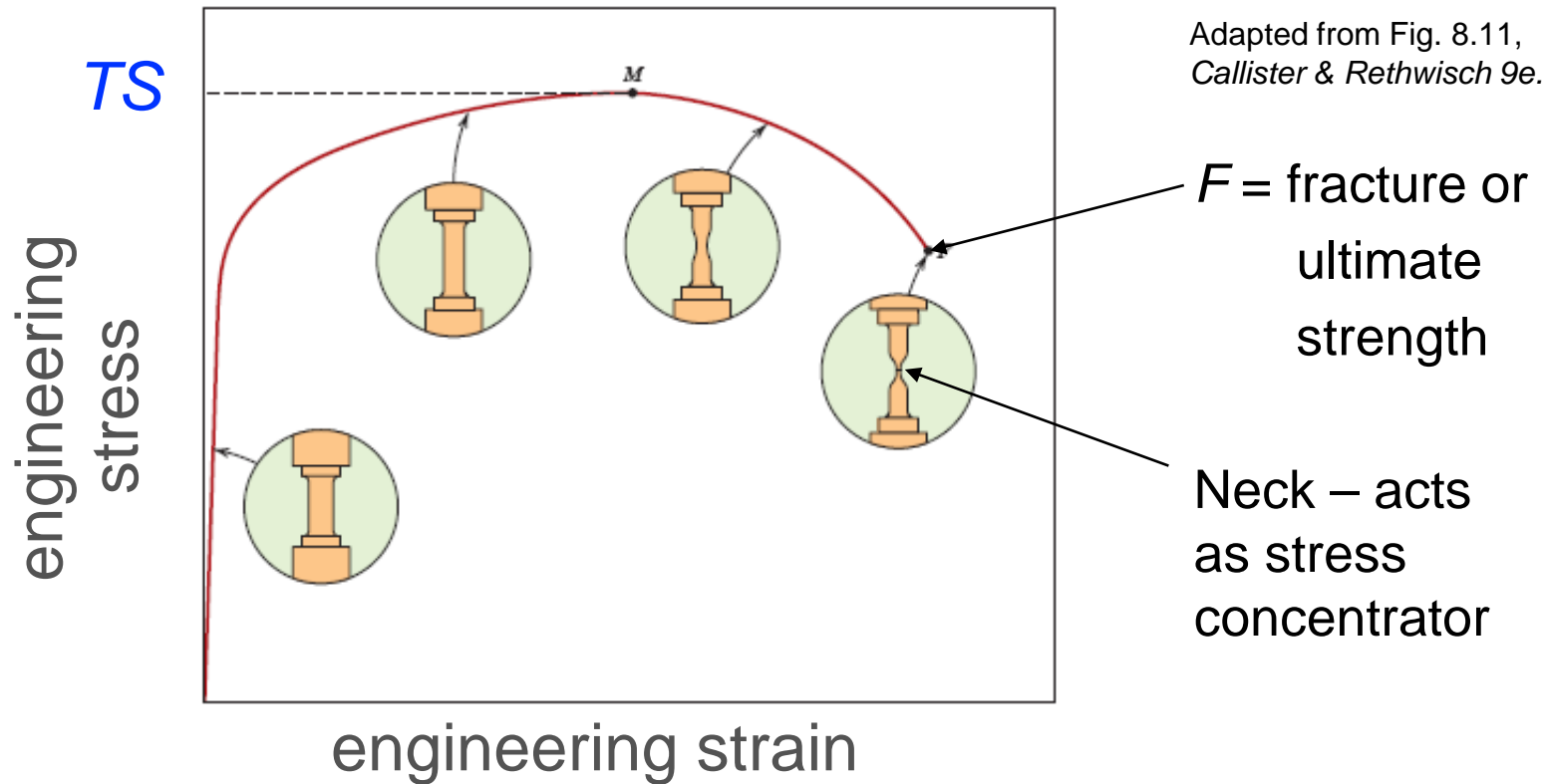
Room temperature values

Based on data in Table B.4,
Callister & Rethwisch 9e.

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered

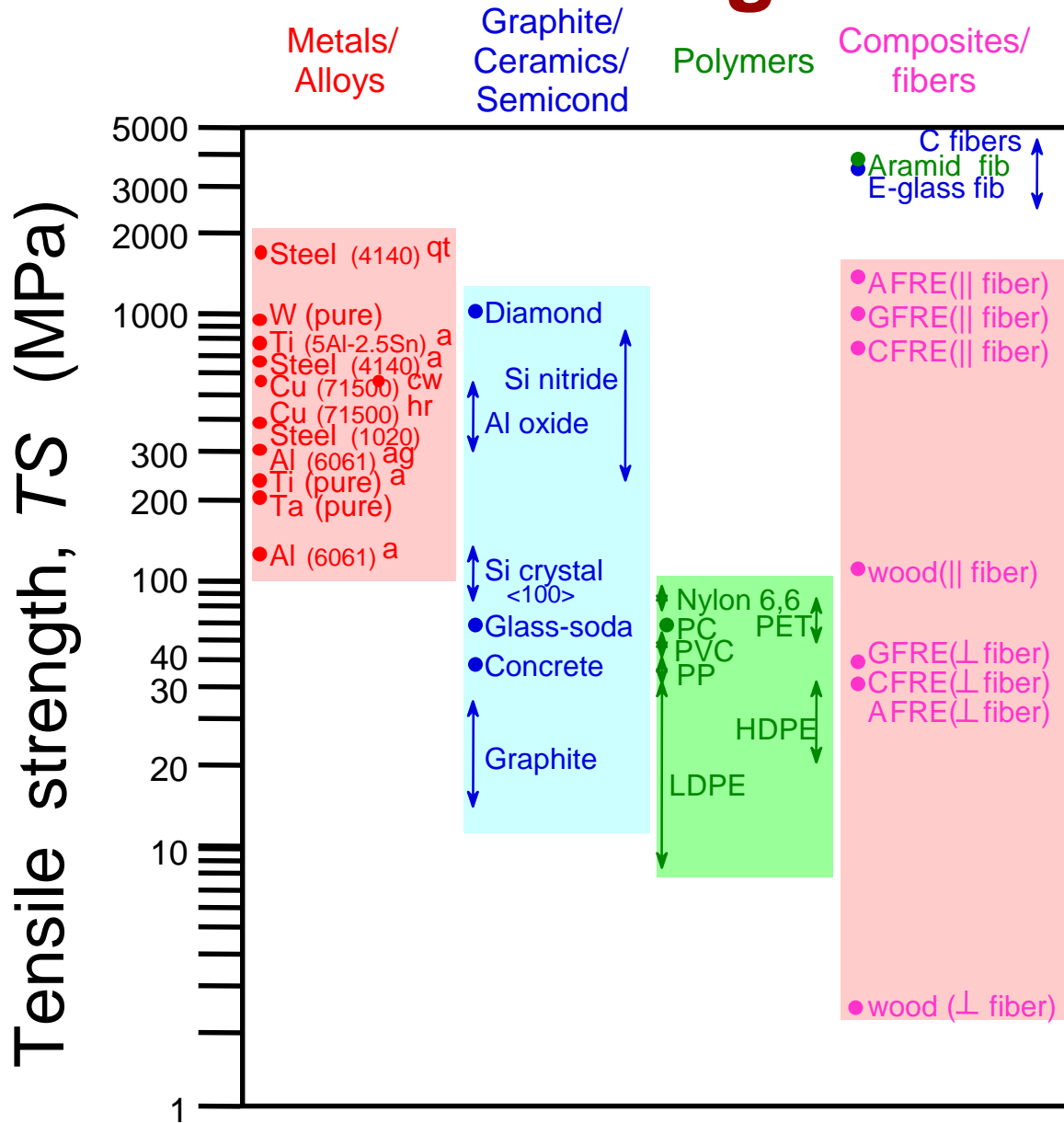
c. Tensile Strength, TS

- Maximum stress on engineering stress-strain curve.



- **Metals**: occurs when noticeable **necking** starts.
- **Polymers**: occurs when **polymer backbone chains** are aligned and about to break.

Tensile Strength: Comparison

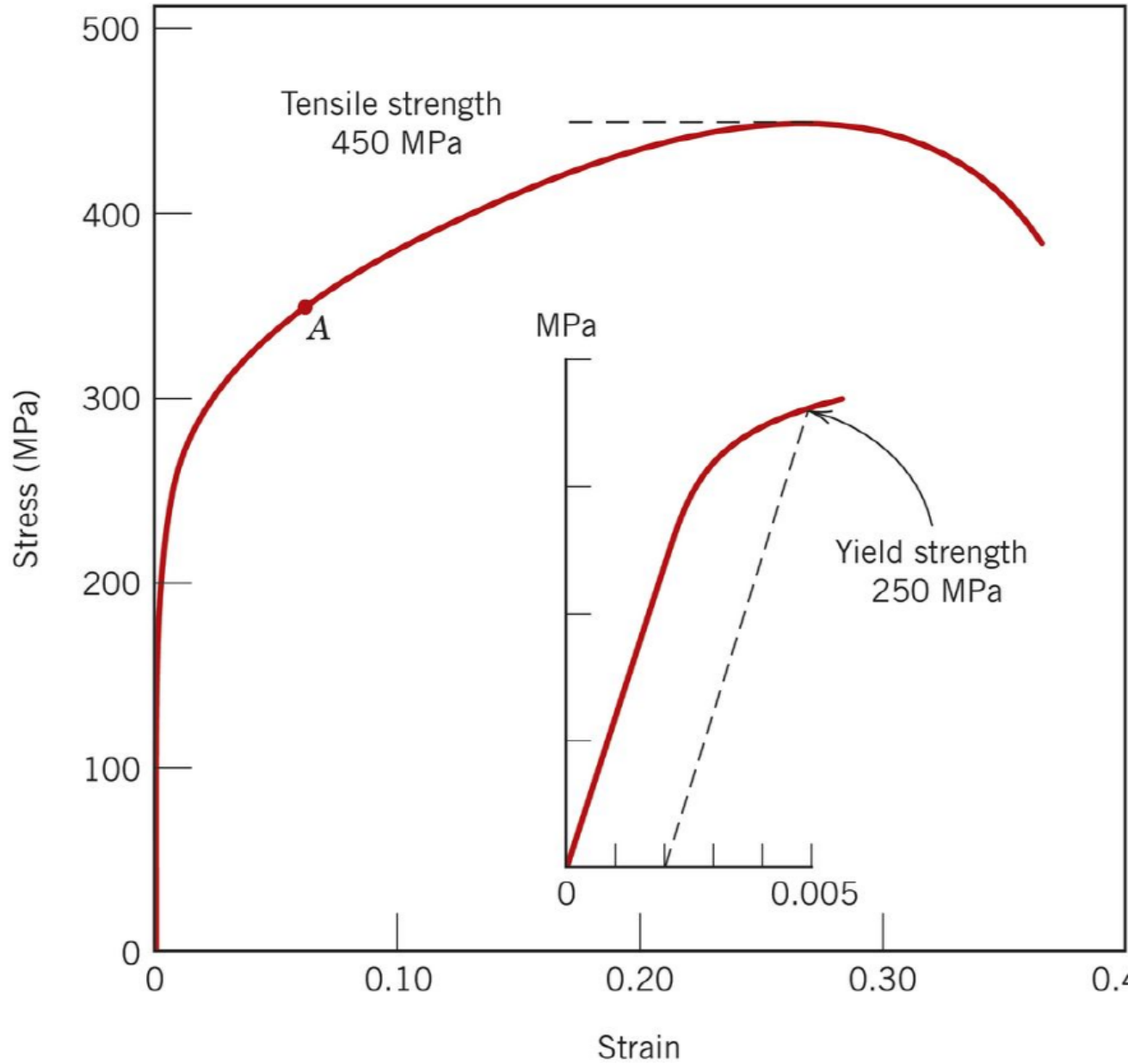


Room temperature values

Based on data in Table B4, *Callister & Rethwisch 9e*.

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered
- AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

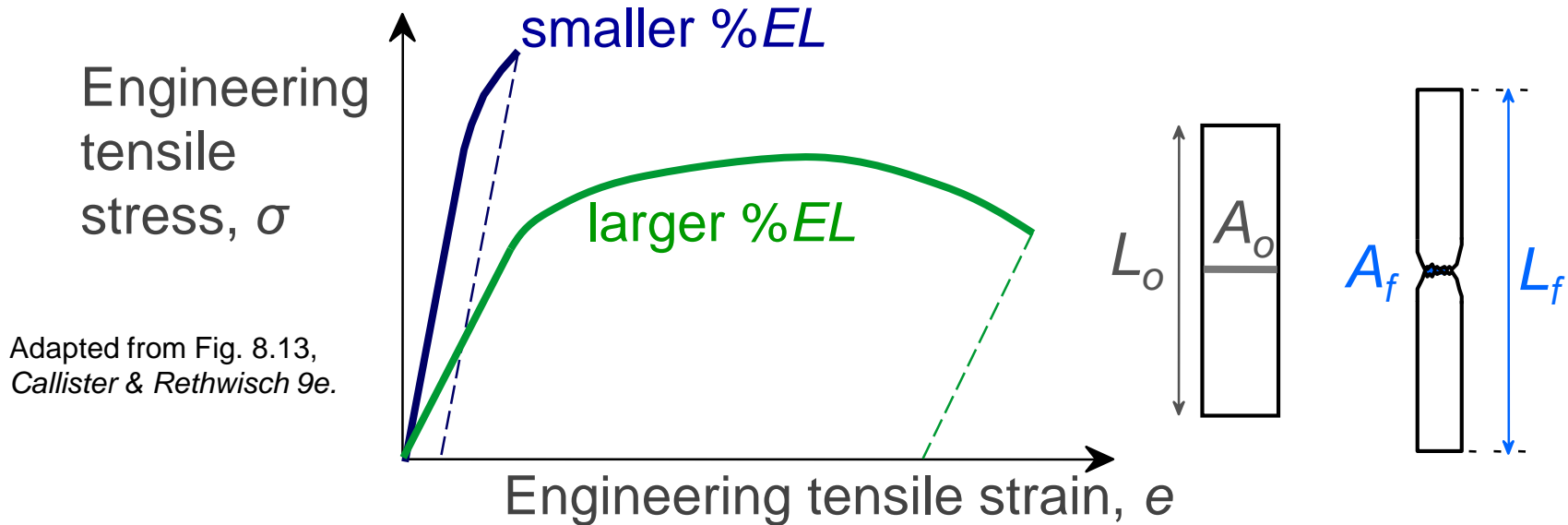
Figure 8.12 in the textbook



d. Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \times 100$$

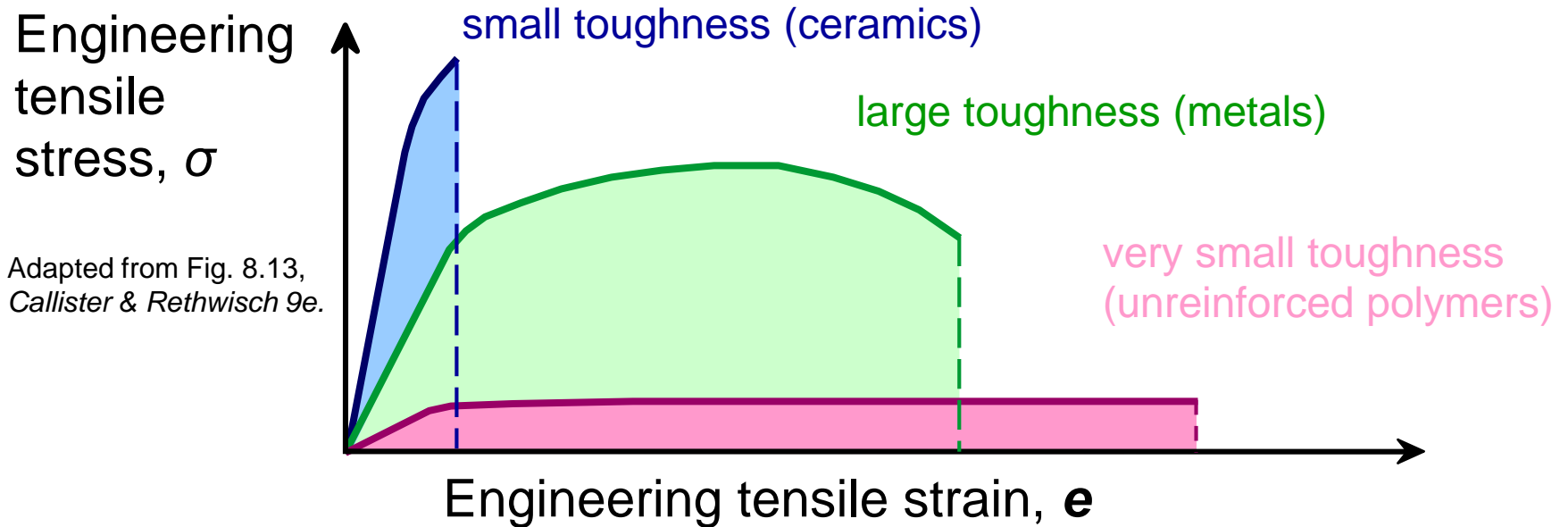


- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \times 100$$

e. Toughness

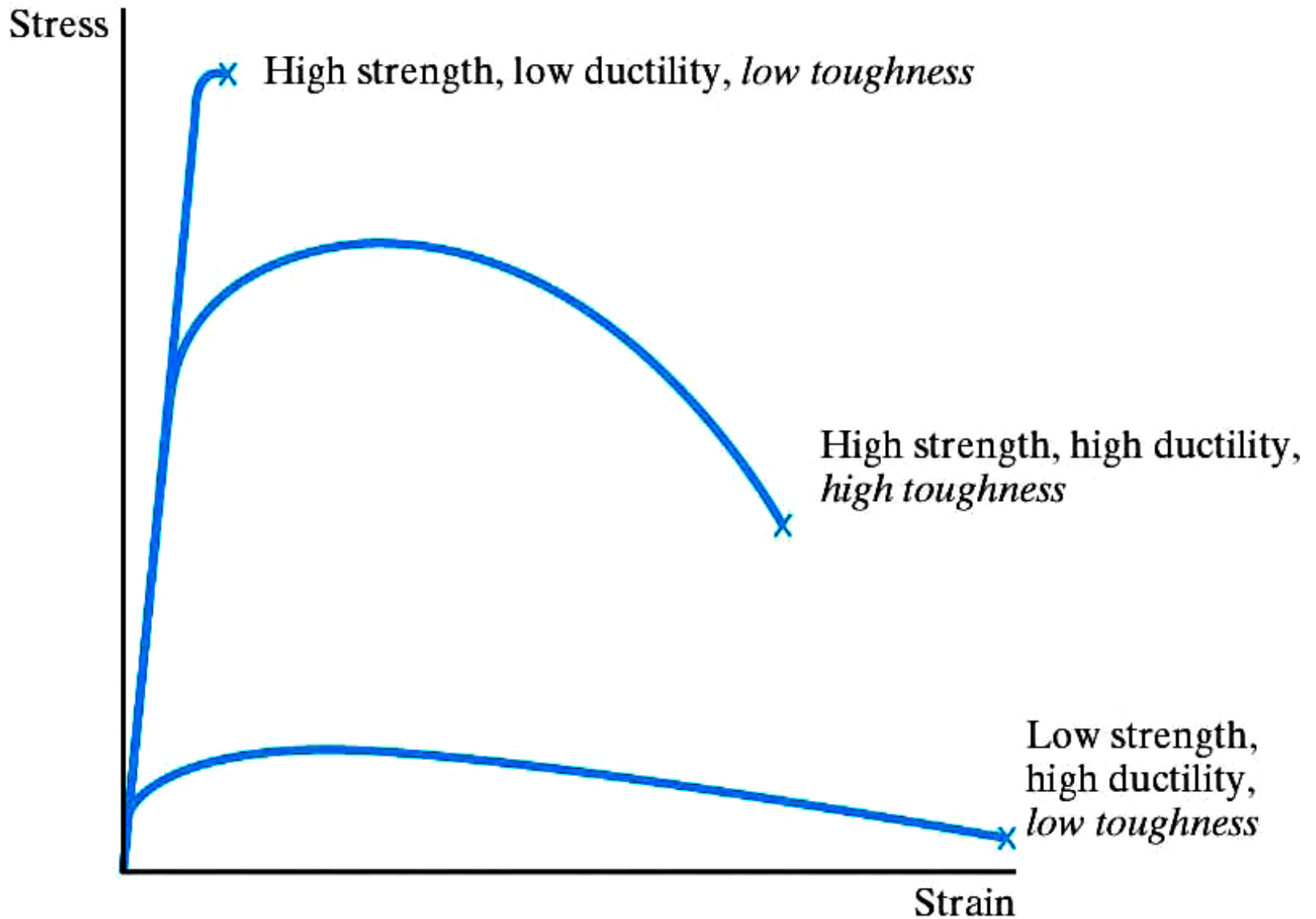
- **Energy to break a unit volume of material**
- Approximate by the area under the stress-strain curve.



Brittle fracture: elastic energy

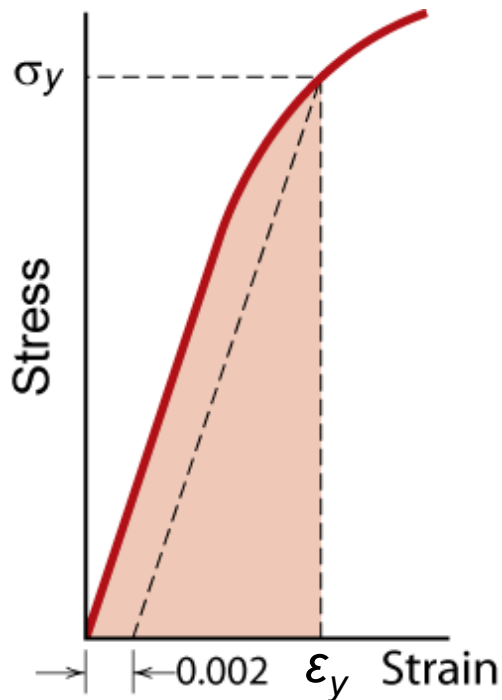
Ductile fracture: elastic + plastic energy

Tensile Test



f. Resilience, U_r

- Ability of a material to store energy
 - Energy stored best in elastic region



$$U_r = \int_0^{\epsilon_y} \sigma \, d\epsilon$$

If we assume a linear stress-strain curve this simplifies to

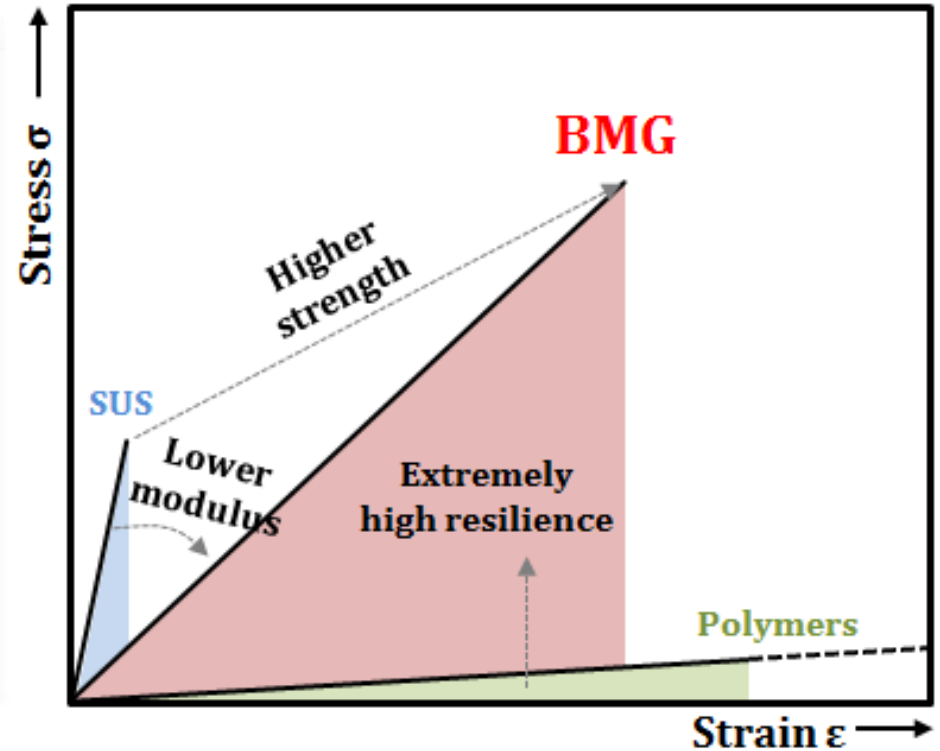
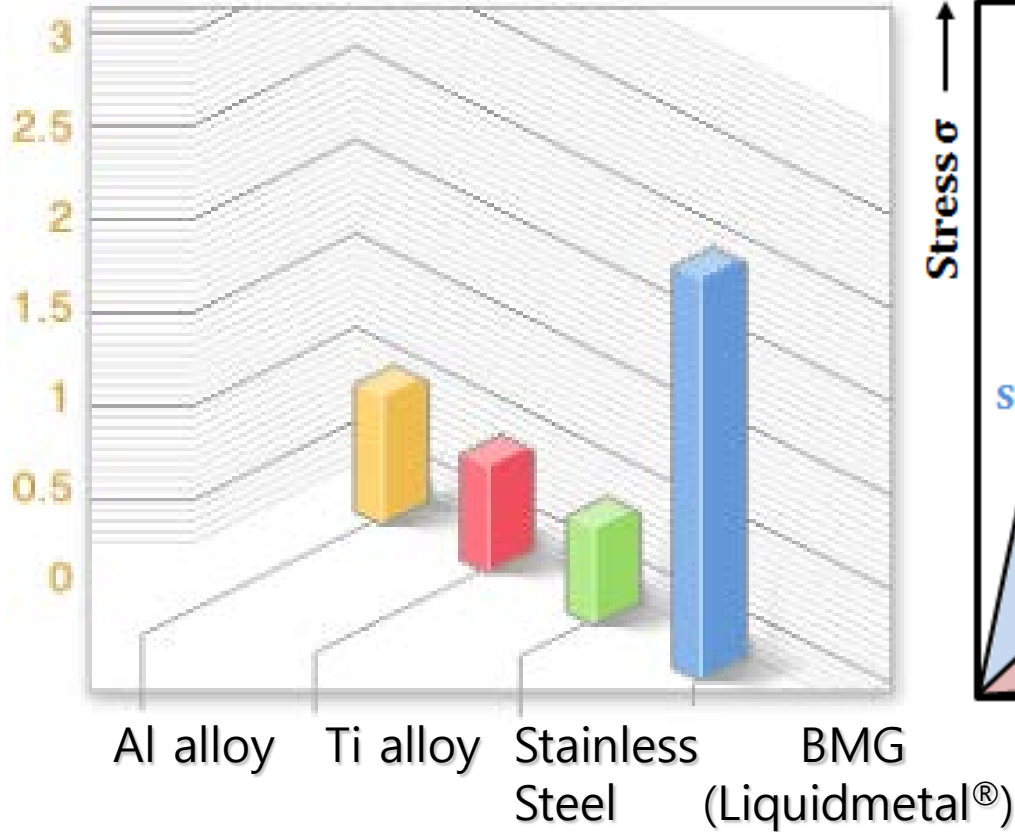
$$U_r \cong \frac{1}{2} \sigma_y \epsilon_y$$

Fig. 8.15, Callister & Rethwisch 9e.

Large elastic strain limit of BMGs

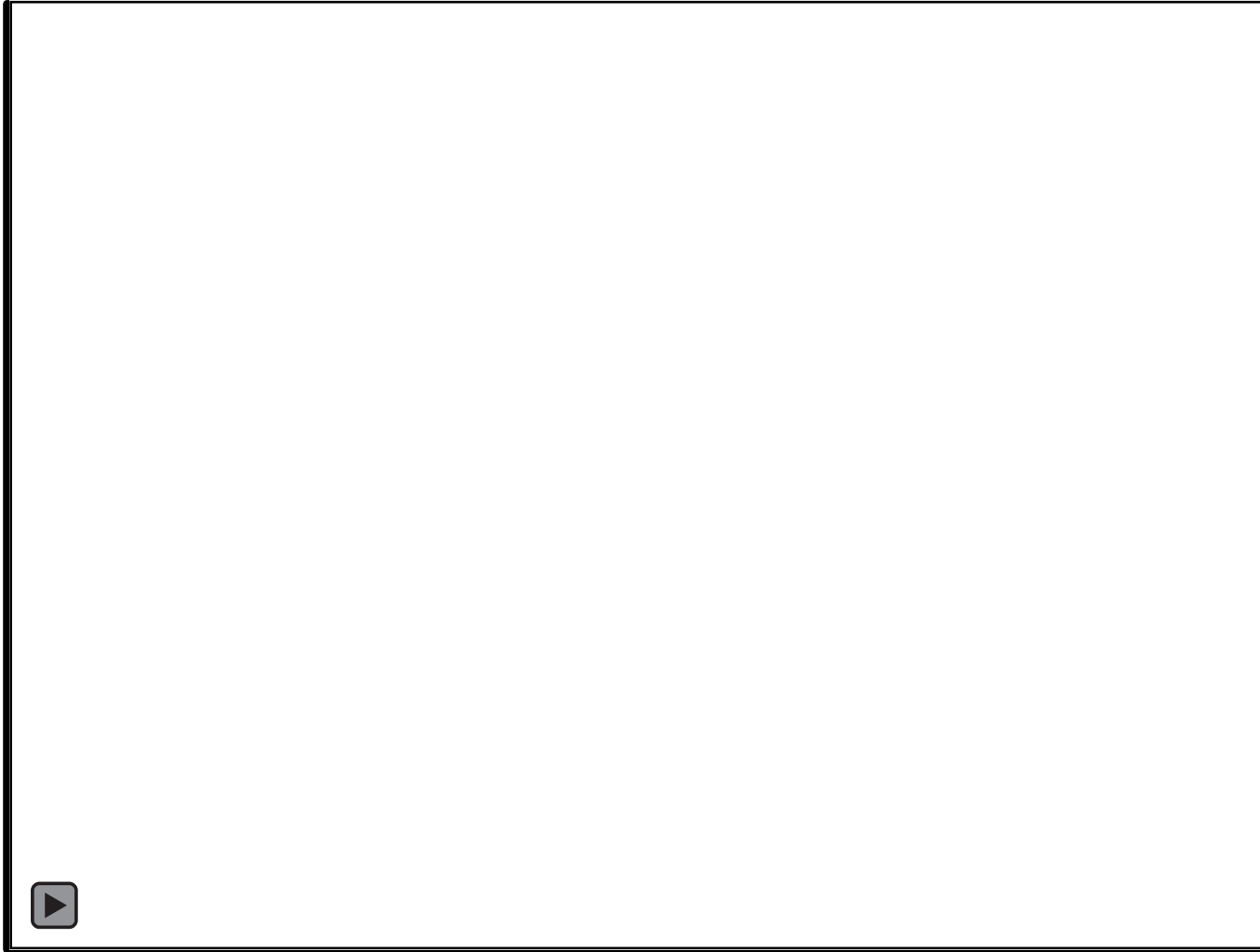
Elastic Strain Limit

[as % of Original Shape]



* Resilience: ability to return to the original form, position, etc. $\rightarrow U = \frac{\sigma_y^2}{2E}$

Large elastic strain limit of BMGs



VI. Elastic Strain Recovery

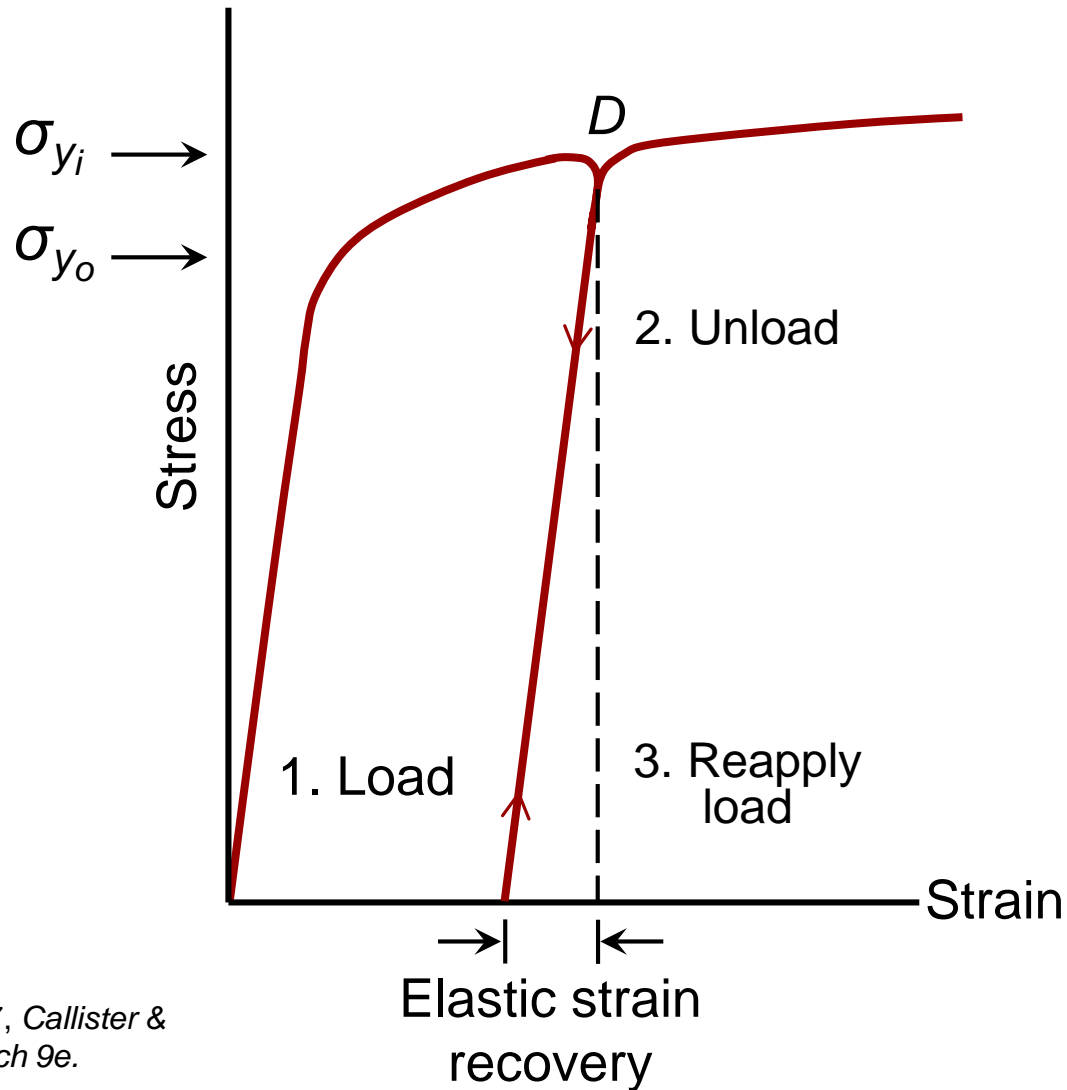
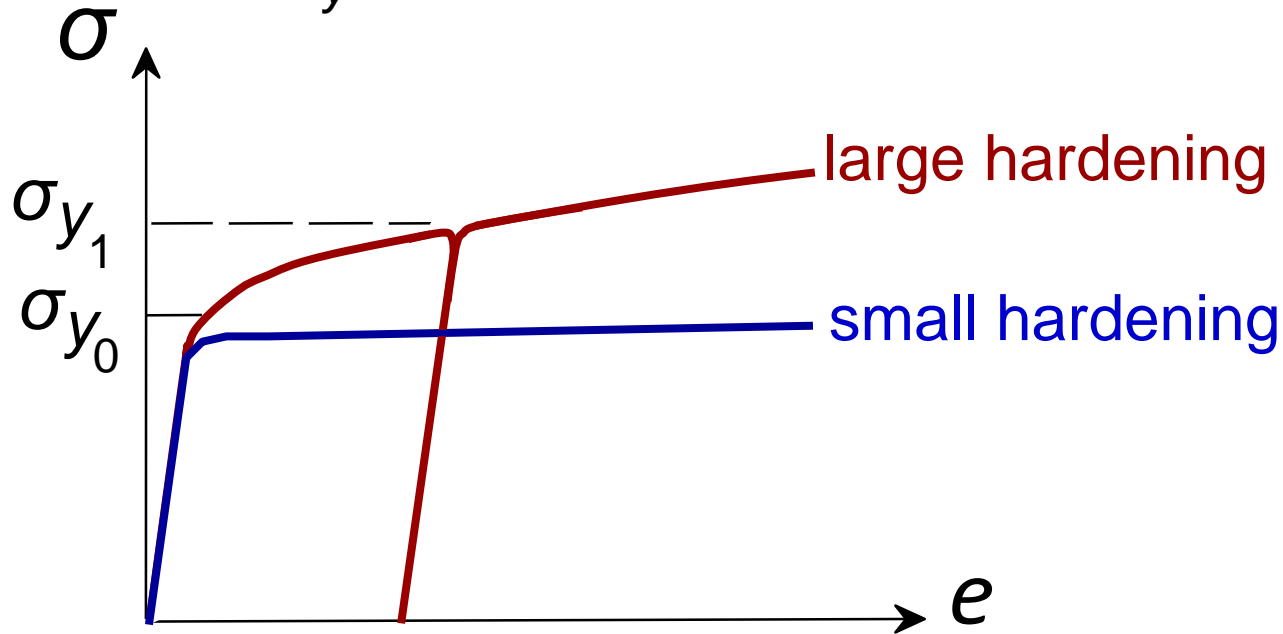


Fig. 8.17, Callister & Rethwisch 9e.

VII. Hardening

- An increase in σ_y due to plastic deformation.



- Curve fit to the stress-strain response:

$$\sigma_T = K(e_T)^n$$

“true” stress (F/A)

“true” strain: $\ln(l/l_0)$

hardening exponent:
 $n = 0.15$ (some steels)
to $n = 0.5$ (some coppers)

VIII. True Stress & Strain

Note: Cross-sectional area changes when sample stretched

- True stress

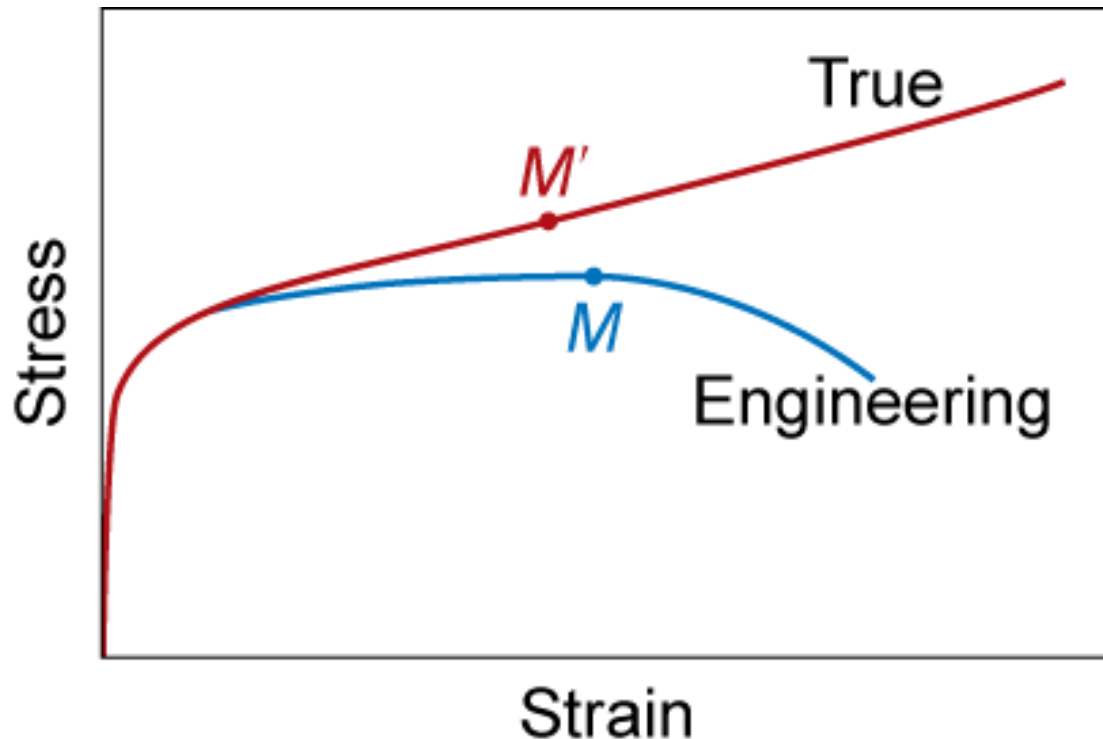
$$\sigma_T = F/A_i$$

$$\sigma_T = \sigma(1 + \epsilon)$$

- True strain

$$\epsilon_T = \ln(\ell_i/\ell_o)$$

$$\epsilon_T = \ln(1 + \epsilon)$$

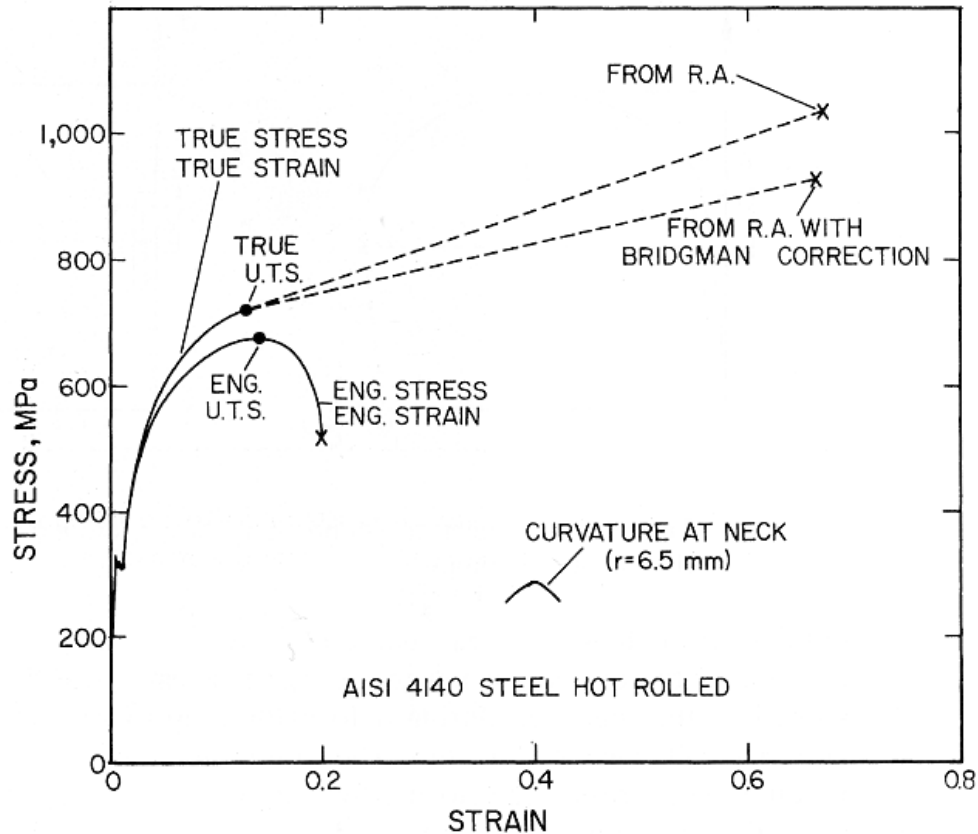


Adapted from Fig. 8.16,
Callister & Rethwisch 9e.

Necking – Work hardening

Represented as $\sigma = \sigma_0 + K\varepsilon^n$

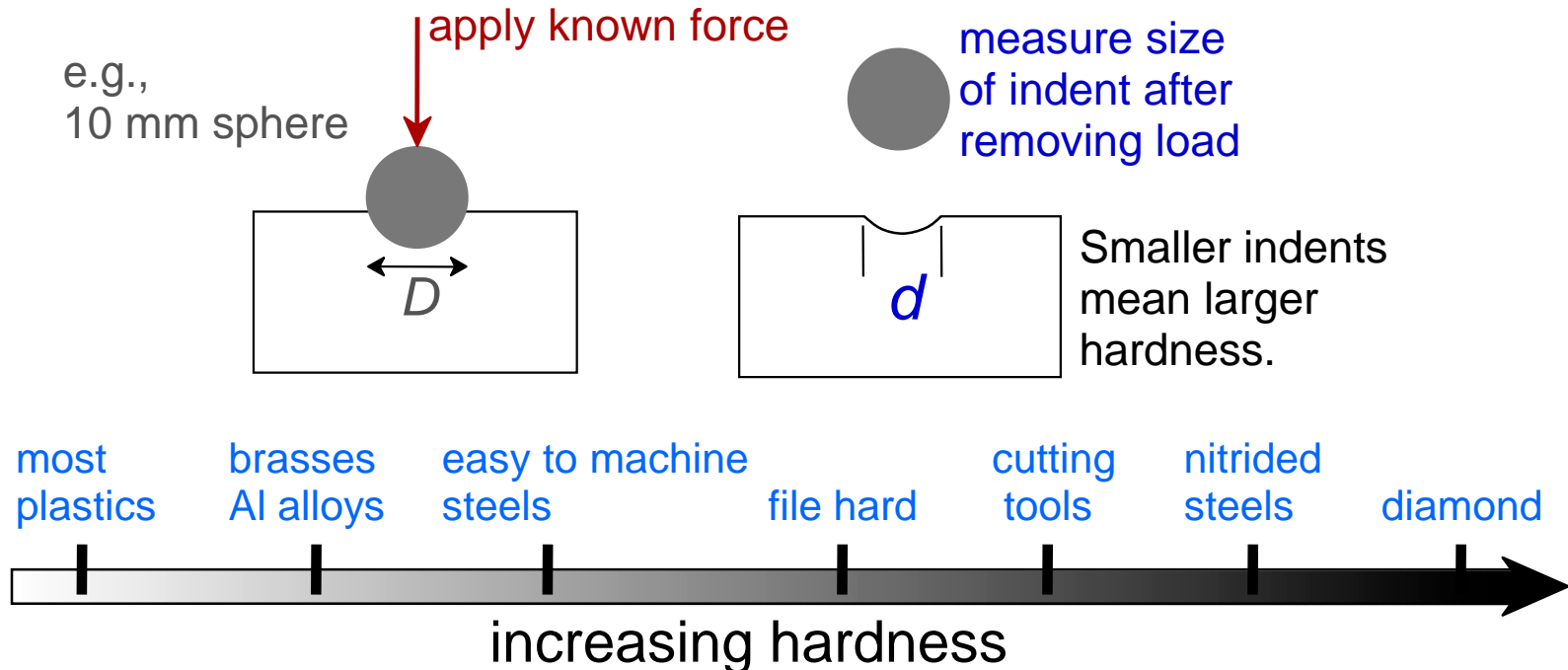
Hollomon's eq



~~What is n ? > meaning from the relationship of stress strain~~

IX. Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
 - resistance to plastic deformation or cracking in compression.
 - better wear properties.



Hardness: Measurement

- Rockwell

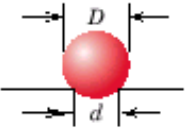

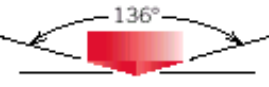

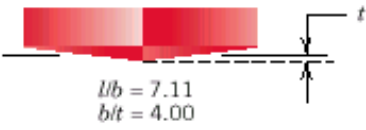
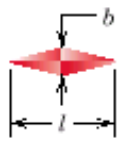
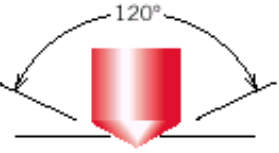



- No major sample damage
- Each scale runs to 130 but only useful in range 20-100.
- Minor load 10 kg
- Major load 60 (A), 100 (B) & 150 (C) kg
 - A = diamond, B = 1/16 in. ball, C = diamond

- HB = Brinell Hardness

- TS (psia=pounds per square inch) = 500 x HB
- TS (MPa) = 3.45 x HB

Hardness: Measurement

Table 8.5 Hardness Testing Techniques

Test	Indenter	Shape of Indentation		Load	Formula for Hardness Number ^a
		Side View	Top View		
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	<ul style="list-style-type: none"> Diamond cone $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres 	 	 	<ul style="list-style-type: none"> 60 kg 100 kg 150 kg } Rockwell <ul style="list-style-type: none"> 15 kg 30 kg 45 kg } Superficial Rockwell	

^a For the hardness formulas given, P (the applied load) is in kg, while D , d , d_1 , and l are all in mm.

Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

X. Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics

– Mean

$$\bar{X} = \frac{\sum^n X_n}{n}$$

– Standard Deviation

$$S = \left[\frac{\sum^n (X_i - \bar{X})^2}{n - 1} \right]^{\frac{1}{2}}$$

where n is the number of data points

XI. Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, N

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is between 1.2 and 4

- Example: Calculate a diameter, d , to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$\sigma_{working} = \frac{\sigma_y}{N}$

$\frac{220,000\text{N}}{\pi(d^2/4)}$

5

1045 plain carbon steel:
 $\sigma_y = 310\text{ MPa}$
 $TS = 565\text{ MPa}$

$F = 220,000\text{N}$

d

L_0

$d = 0.067\text{ m} = 6.7\text{ cm}$

Summary

Chapter 8: Mechanical Properties of Metals

- **Stress** and **strain**: These are size-independent measures of load and displacement, respectively.
- **Elastic** behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).
- **Plastic** behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y .
- **Toughness**: The energy needed to break a unit volume of material.
- **Ductility**: The plastic strain at failure.

Deformation

➤ Elastic vs. Plastic region

➤ Key points

1. Elastic modulus (=Young's modulus), E

2. 0.2% (0.002 strain) yield stress, σ_y

3. Ultimate yield stress, σ_{UTS}

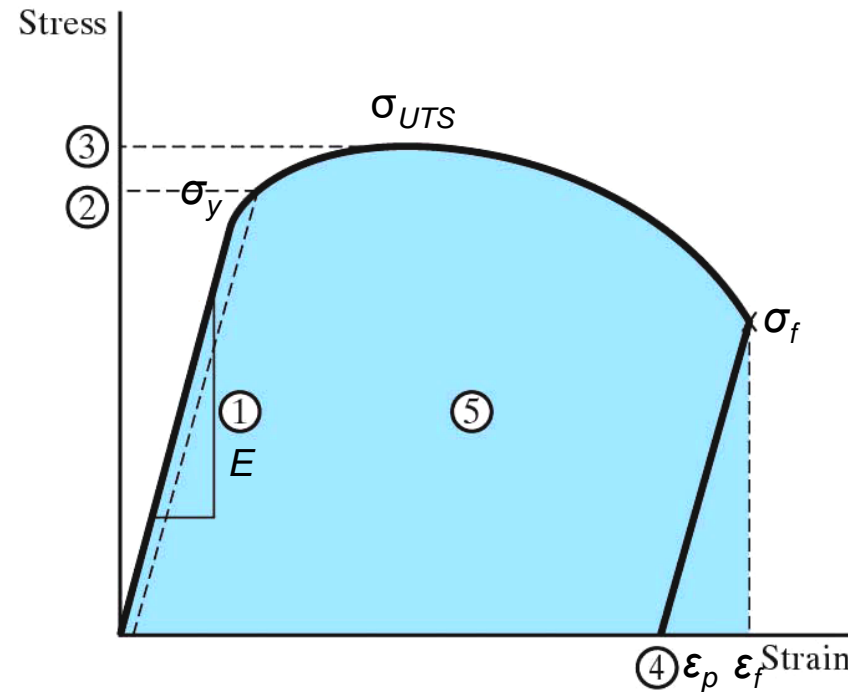
4. Ductility, ϵ_p

5. Toughness

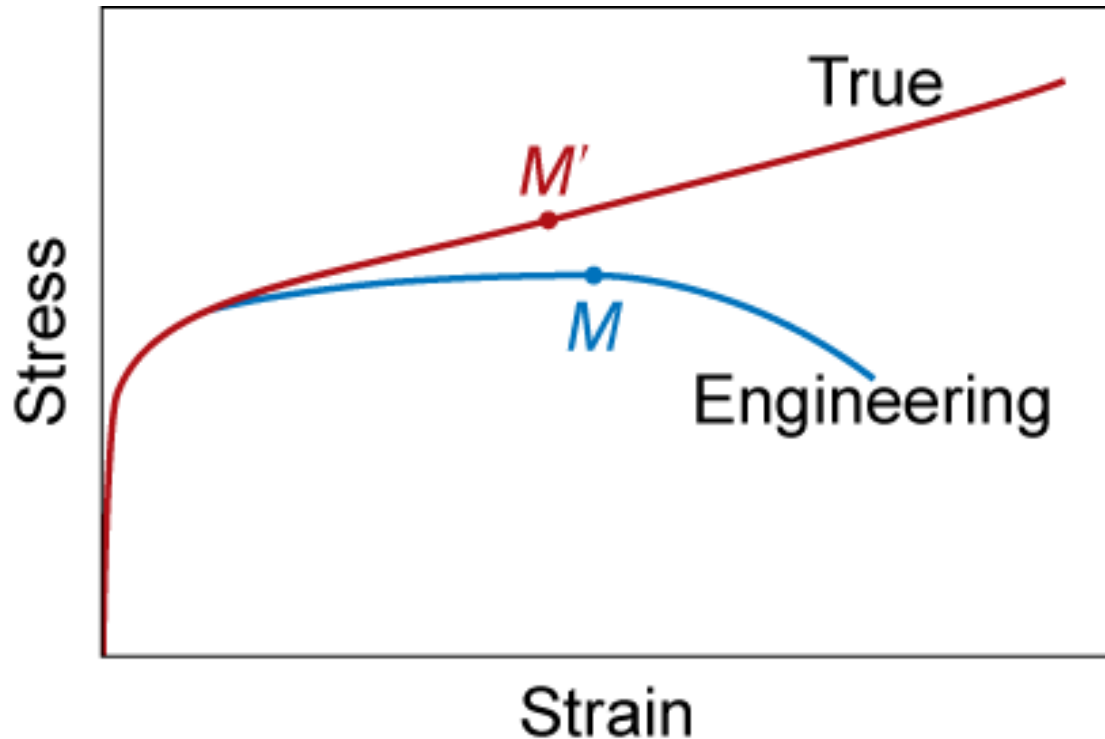
$$\int \sigma d\epsilon$$

6. Fracture stress, σ_f

7. Fracture strain, ϵ_f



Engineering vs True Stress & Strain



Adapted from Fig. 8.16,
Callister & Rethwisch 9e.

Variability in Material Properties : need to Statistics

Material design considering
Safety factor:

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is
between
1.2 and 4