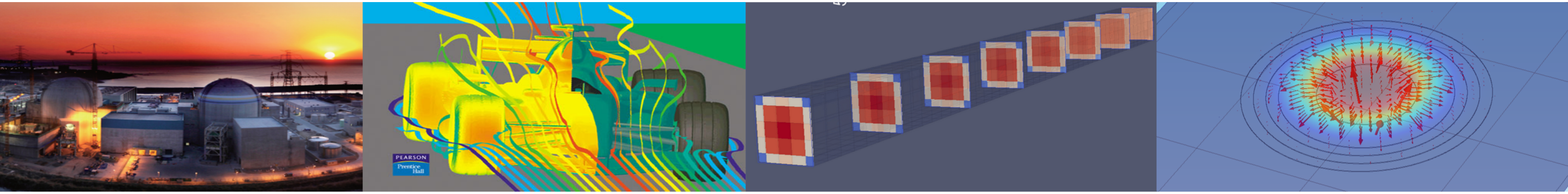

COMPUTATIONAL NUCLEAR THERMAL HYDRAULICS

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TWO-FLUID MODEL SOLVER-1: SMAC

Contents

Introduction

- SMAC
  - Simplified Marker and Cell
  - One of the most well-known numerical algorithms for incompressible fluid flow
  - MAC (Marker and Cell) method
    - Developed by Harlow and Welch in 1965
    - Extended to the SMAC method by Amsden and Harlow in 1970
  - Proper algorithm for the calculation of transient fluid flow

et al., 1990; Blunsdon *et al.*, 1992, 1993). Kim and Benson (1992) compared the PISO method with the SMAC algorithms for the prediction of unsteady flows and reported that SMAC was more efficient, faster and more accurate than PISO. The MAC/ICE class of methods are, however, mathematically complex and not widely used in general-purpose CFD procedures.

Introduction

- Other Methods
  - Numerical Algorithms for Incompressible Navier-Stokes equation
  - SIMPLE
    - Semi-IMplicit Pressure Linked Equation
    - Developed by Patankar and Spalding in 1972
    - Extended to SIMPLER (SIMPLE-Revised) and SIMPLEC (SIMPLE Consistent)
  - PISO
    - Pressure Implicit with Splitting of Operators
    - Developed by Issa in 1986

SMAC for Single Phase

- Governing Equations of Incompressible Flow
  - Continuity Equation

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

  - Momentum Equations

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \nabla \cdot (\vec{u}\vec{u})\right) = -\nabla P + \nabla \cdot \vec{\tau} + \rho \vec{g} \qquad (2)$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho uu}{\partial x} + \frac{\partial \rho uv}{\partial x} = \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{dp}{dx} + \rho g_x$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho uv}{\partial x} + \frac{\partial \rho vv}{\partial x} = \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) - \frac{dp}{dy} + \rho g_y$$

SMAC for Single Phase

- Numerical procedure
  - Explicit time discretization of momentum equations
    - For x-direction

$$\frac{\partial \rho u}{\partial t}+\frac{\partial \rho uu}{\partial x}+\frac{\partial \rho uv}{\partial x}=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{dp}{dx}+\rho g_x$$

– Explicit treatment of whole terms except pressure

$$\int_V \frac{\partial \rho u}{\partial t}dV=\rho\frac{u_P^*-u_P}{\Delta t}\Delta V$$

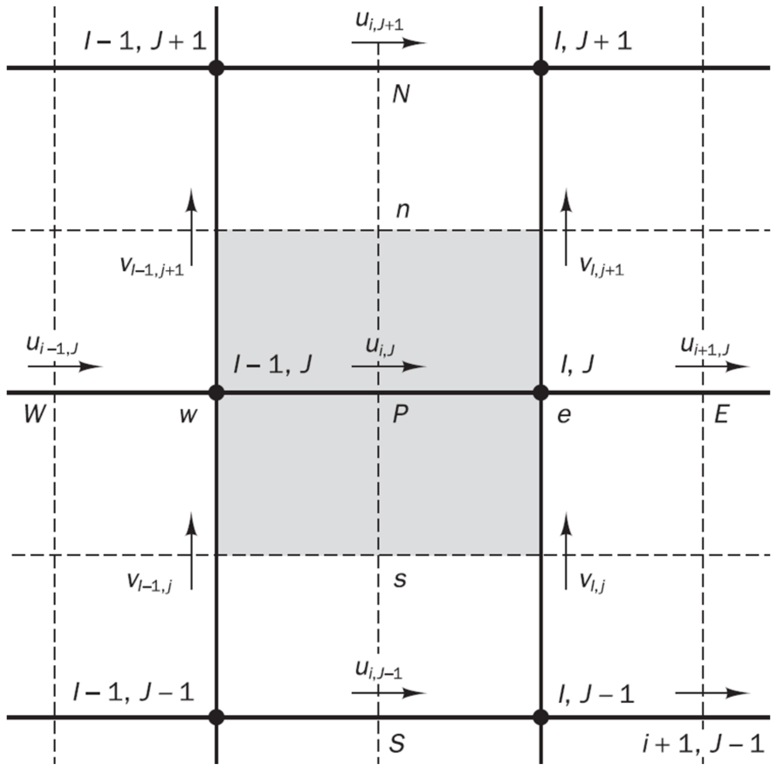
No implicit term!

$$\int \nabla\cdot(\rho\vec{u}u)dV=\int(\rho\vec{u}u)\cdot\vec{n}dA=\sum_f(\rho\vec{u}u)\cdot\vec{A}$$

$$=(\rho uu)_e A_e-(\rho uu)_w A_w+(\rho vu)_n A_n-(\rho vu)_s A_s$$

$$=(\rho u)_e u_e A_e-(\rho u)_w u_w A_w+(\rho v)_n u_n A_n-(\rho v)_s u_s A_s$$

$$=F_e u_e A_e-F_w u_w A_w+F_n u_n A_n-F_s u_s A_s$$



SMAC for Single Phase

- Numerical procedure
  - Explicit time discretization of momentum equations
    - For x-direction

$$\int \nabla \cdot \mu(\nabla u) dV = \int \mu(\nabla u) \cdot \vec{n} dA = \sum_f (\mu \nabla u) \cdot \vec{A}$$

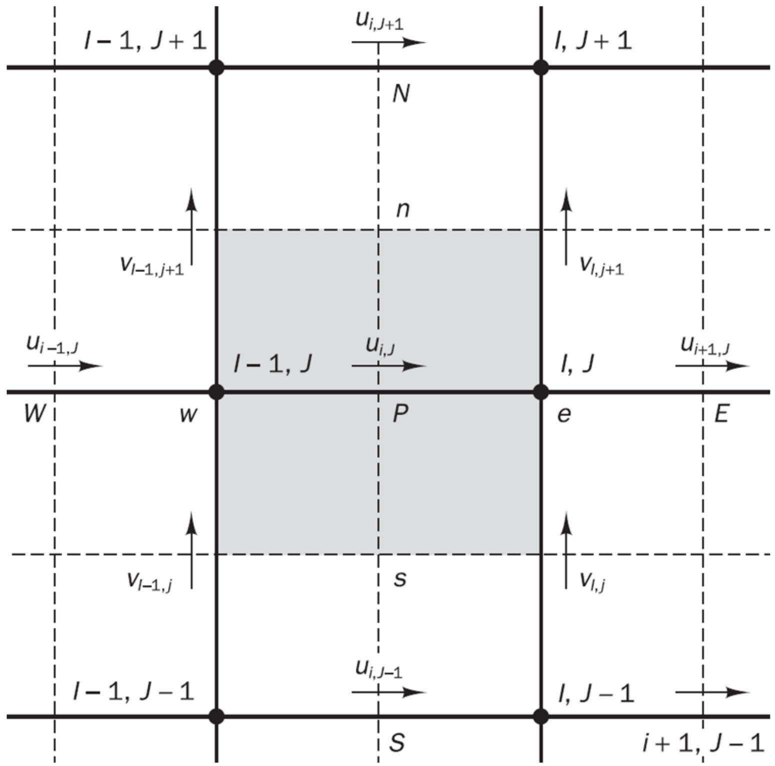
$$= \left(\mu \frac{du}{dx}\right)_e A_e - \left(\mu \frac{du}{dx}\right)_w A_w + \left(\mu \frac{du}{dy}\right)_n A_n - \left(\mu \frac{du}{dy}\right)_s A_s$$

$$= \left(\mu_e \frac{u_E - u_P}{\delta x_e}\right) A_e - \left(\mu_w \frac{u_P - u_W}{\delta x_w}\right) A_w$$

$$+ \left(\mu_n \frac{u_N - u_P}{\delta x_n}\right) A_n - \left(\mu_s \frac{u_P - u_S}{\delta x_s}\right)_w A_s$$

$$= \left(\mu_e \frac{u_{i+1,J} - u_{i,J}}{x_{i+1} - x_i}\right) A_e - \left(\mu_w \frac{u_{i,j} - u_{i-1,J}}{x_i - x_{i-1}}\right) A_w$$

$$+ \left(\mu_n \frac{u_{i,J+1} - u_{i,J}}{x_{i,J+1} - x_{i,J}}\right) A_n - \left(\mu_s \frac{u_{i,J} - u_{i,J-1}}{x_{i,J} - x_{i,J-1}}\right) A_s$$



No implicit term!

SMAC for Single Phase

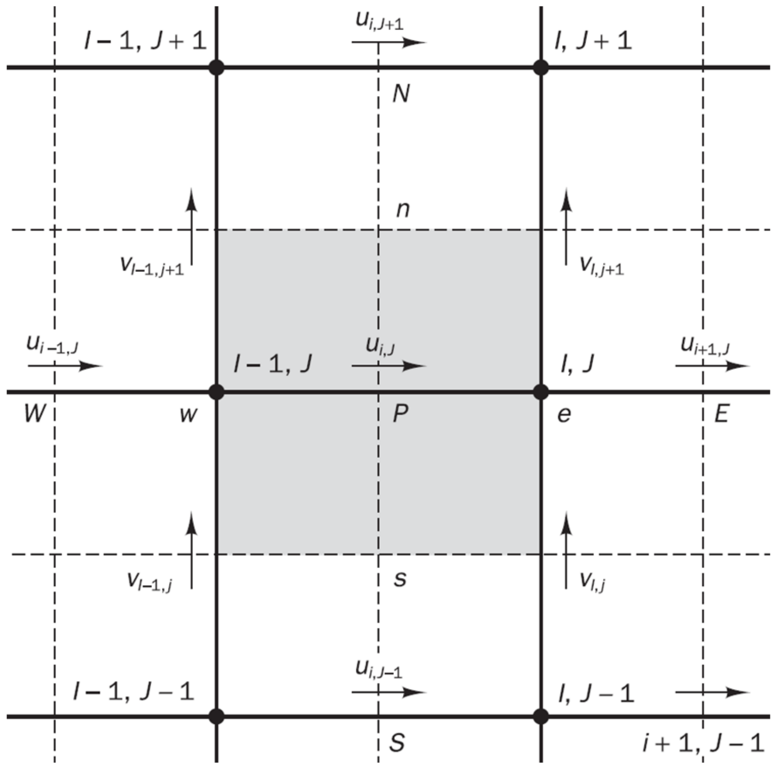
- Numerical procedure
  - Explicit time discretization of momentum equations
    - For x-direction

$$\frac{\partial \rho u}{\partial t}+\frac{\partial \rho uu}{\partial x}+\frac{\partial \rho uv}{\partial x}=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{dp}{dx}+\rho g_x$$

$$\int\left(\frac{\partial p}{\partial x}\right)dV=\frac{p_e-p_w}{x_{I,J}-x_{I-1,J}}\Delta V=\frac{p_{I,J}-p_{I-1,J}}{x_{I,J}-x_{I-1,J}}\Delta V$$

$$\int \rho g_x dV=\rho g_x \Delta V$$

No implicit term!



Intermediate velocities for x- and y- can be obtained!

SMAC for Single Phase

❖ Numerical procedure

- u^* vs. u^{n+1}

$$\rho\left(\frac{\vec{u}^{n+1}-\vec{u}^{n}}{\Delta t}+\nabla\cdot(\vec{u}^{n}\vec{u}^{n})\right)=-\nabla P^{n+1}+\nabla\cdot\vec{\tau}^{n}+\rho\vec{g}^{n} \qquad (3)$$

$$\rho\left(\frac{\vec{u}^{*}-\vec{u}^{n}}{\Delta t}+\nabla\cdot(\vec{u}^{n}\vec{u}^{n})\right)=-\nabla P^{n}+\nabla\cdot\vec{\tau}^{n}+\rho\vec{g}^{n} \qquad (4)$$

$$P^{n+1}=P^{n}+P' \qquad (5)$$

$$\vec{u}^{n+1}=\vec{u}^{*}+\vec{u}' \qquad (6)$$

- It does not satisfy mass continuity, so some correction is required.

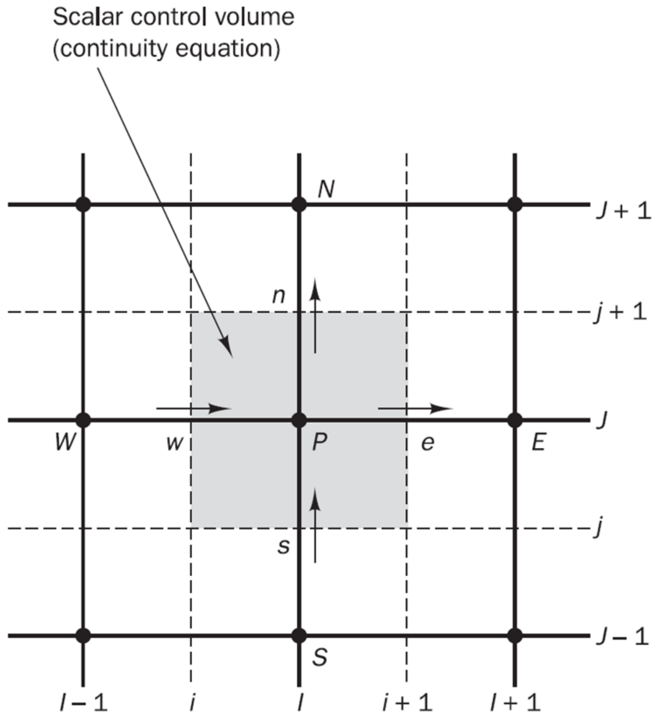
$$\rho\left(\frac{(\vec{u}^{*}+\vec{u}')-\vec{u}^{n}}{\Delta t}+\nabla\cdot(\vec{u}^{n}\vec{u}^{n})\right)=-(\nabla P^{n}+\nabla P')+\nabla\cdot\vec{\tau}^{n}+\rho\vec{g}^{n} \qquad (7)$$

$$\rho\left(\frac{\vec{u}^{*}-\vec{u}^{n}}{\Delta t}+\nabla\cdot(\vec{u}^{n}\vec{u}^{n})\right)+\rho\frac{\vec{u}'}{\Delta t}=\left(-\nabla P^{n}+\nabla\cdot\vec{\tau}^{n}+\rho\vec{g}^{n}\right)-\nabla P' \qquad (8)$$

$$\rho\frac{\vec{u}'}{\Delta t}=\rho\frac{\vec{u}^{n+1}-\vec{u}^{*}}{\Delta t}=-\nabla P' \qquad (9)$$

SMAC for Single Phase

- Numerical procedure
  - Continuity equation



$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

$$\sum_f \rho \vec{u} \cdot \vec{A} = \left[(\rho u A)_e - (\rho u A)_w\right] + \left[(\rho v A)_n - (\rho v A)_s\right]$$

$$= \left[(\rho u A)_{i+1,J} - (\rho u A)_{i,J}\right] + \left[(\rho v A)_{I,j+1} - (\rho v A)_{I,j}\right] = 0$$

$$\rho \frac{\vec{u}\,'}{\Delta t} = \rho \frac{\vec{u}^{\,n+1} - \vec{u}^{\,*}}{\Delta t} = -\nabla P' \qquad (9) \qquad \vec{u}^{\,n+1} = \vec{u}^{\,*} - \frac{\Delta t}{\rho} \nabla P'$$

$$\sum_f \rho \vec{u} \cdot \vec{A} = \rho\left[(u^{n+1}A)_e - (u^{n+1}A)_w\right] + \left[(v^{n+1}A)_n - (v^{n+1}A)_s\right] = 0$$

$$\left[\left(u^* - \frac{\Delta t}{\rho}\frac{dP'}{dx}\right)_e A_e - \left(u^* - \frac{\Delta t}{\rho}\frac{dP'}{dx}\right)_w A_w\right] + \left[\left(v^* - \frac{\Delta t}{\rho}\frac{dP'}{dy}\right)_n A_n - \left(v^* - \frac{\Delta t}{\rho}\frac{dP'}{dy}\right)_s A_s\right] = 0$$

$$\left[(u^*A)_e - (u^*A)_w + (v^*A)_n - (v^*A)_s\right] - \frac{\Delta t}{\rho}\left[\left(\frac{\Delta P'}{\Delta x}A\right)_e - \left(\frac{\Delta P'}{\Delta x}A\right)_w + \left(\frac{\Delta P'}{\Delta y}A\right)_n - \left(\frac{\Delta P'}{\Delta y}A\right)_s\right] = 0$$

SMAC for Single Phase

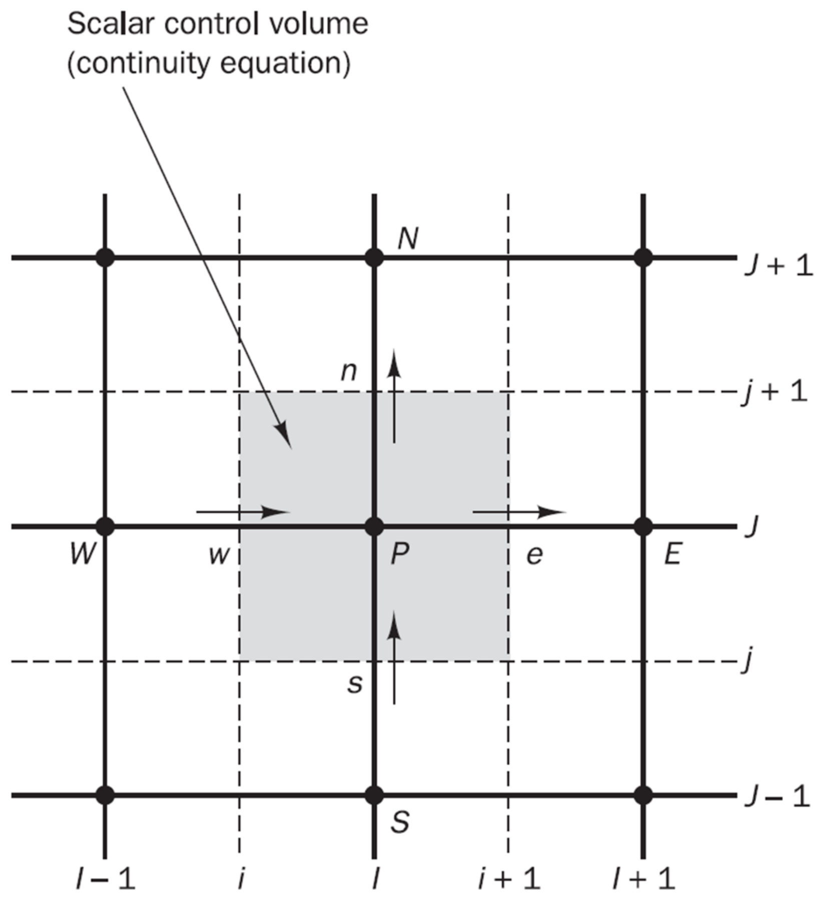
- Numerical procedure
  - Continuity equation

$$\frac{\Delta t}{\rho}\left[\left(\frac{\Delta P'}{\Delta x}A\right)_e-\left(\frac{\Delta P'}{\Delta x}A\right)_w+\left(\frac{\Delta P'}{\Delta y}A\right)_n-\left(\frac{\Delta P'}{\Delta y}A\right)_s\right]$$
$$=\left[(u^*A)_e-(u^*A)_w+(v^*A)_n-(v^*A)_s\right]$$

$$\frac{\Delta t}{\rho}\left[\left(\frac{P'_{I+1,J}-P'_{I,J}}{x_{I+1,J}-x_{I,J}}A\right)_e-\left(\frac{P'_{I,J}-P'_{I-1,J}}{x_{I,J}-x_{I-1,J}}A\right)_w+\left(\frac{P'_{I,J+1}-P'_{I,J}}{y_{I,J+1}-y_{I,J}}A\right)_n-\left(\frac{P'_{I,J}-P'_{I,J-1}}{y_{I,J}-y_{I,J-1}}A\right)_s\right]$$
$$=\left[(u^*A)_{i+1,J}-(u^*A)_{i,J}+(v^*A)_{I,j+1}-(v^*A)_{I,j}\right]$$

$$\vec{u}^{\,n+1}=\vec{u}^{\,*}-\frac{\Delta t}{\rho}\nabla P'$$

$$u'_{i,J}=-\frac{\Delta t}{\rho}\left(\frac{P'_{I,J}-P'_{I-1,J}}{x_{I,J}-x_{I-1,J}}\right)=d_{i,J}\left(P'_{I,J}-P'_{I-1,J}\right)$$



$$u'_{i,J}=d_{i,J}(p'_{I-1,J}-p'_{I,J})$$
$$v'_{I,j}=d_{I,j}(p'_{I,J-1}-p'_{I,J})$$

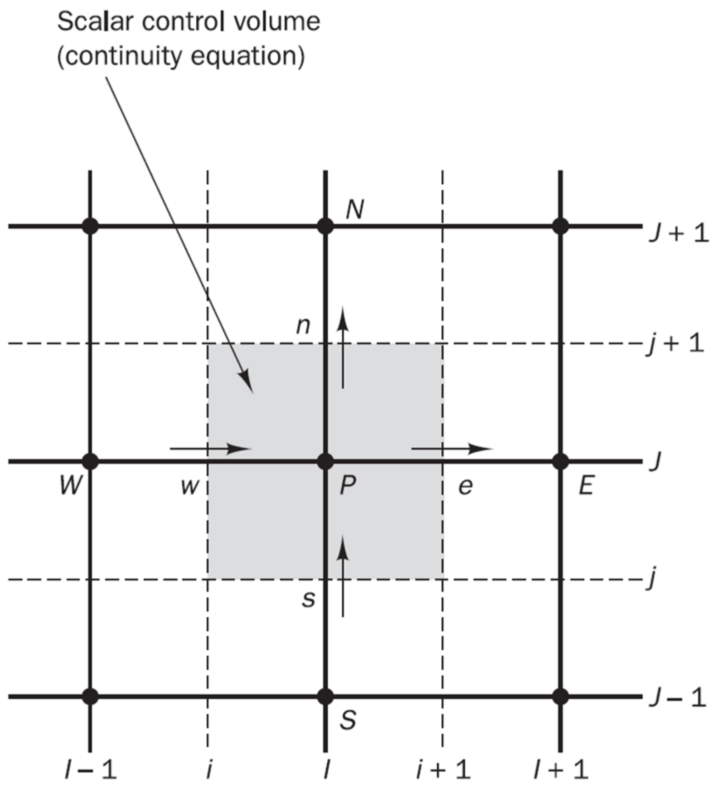
$$\text{where } d_{i,J}=\frac{A_{i,J}}{a_{i,J}} \text{ and } d_{I,j}=\frac{A_{I,j}}{a_{I,j}}$$

SMAC for Single Phase

- Numerical procedure
  - Continuity equation

$$\begin{bmatrix} A_{i+1,J}d_{i+1,J}\left(P'_{I+1,J}-P'_{I,J}\right)-A_{i,J}d_{i,J}\left(P'_{I,J}-P'_{I-1,J}\right) \\ +A_{I,j+1}d_{I,j+1}\left(P'_{I,J+1}-P'_{I,J}\right)-A_{I,j}d_{I,j}\left(P'_{I,J}-P'_{I,J-1}\right) \end{bmatrix}$$
$$=\left[\left(u^{*}A\right)_{i+1,J}-\left(u^{*}A\right)_{i,J}+\left(v^{*}A\right)_{I,j+1}-\left(v^{*}A\right)_{I,j}\right]$$

$$\left[A_{i+1,J}d_{i+1,J}+A_{i,J}d_{i,J}+A_{I,j+1}d_{I,j+1}+A_{I,j}d_{I,j}\right]P'_{I,J}$$
$$=\left[A_{i+1,J}d_{i+1,J}P'_{I+1,J}+A_{i,J}d_{i,J}P'_{I-1,J}+A_{I,j+1}d_{I,j+1}P'_{I,J+1}+A_{I,j}d_{I,j}P'_{I,J-1}\right]$$
$$+\left[\left(u^{*}A\right)_{i,J}-\left(u^{*}A\right)_{i+1,J}+\left(v^{*}A\right)_{I,j}-\left(v^{*}A\right)_{I,j+1}\right]$$



$$[(\rho dA)_{i+1,J}+(\rho dA)_{i,J}+(\rho dA)_{I,j+1}+(\rho dA)_{I,j}]p'_{I,J}$$
$$=(\rho dA)_{i+1,J}p'_{I+1,J}+(\rho dA)_{i,J}p'_{I-1,J}+(\rho dA)_{I,j+1}p'_{I,J+1}+(\rho dA)_{I,j}p'_{I,J-1}$$
$$+[(\rho u^{*}A)_{i,J}-(\rho u^{*}A)_{i+1,J}+(\rho v^{*}A)_{I,j}-(\rho v^{*}A)_{I,j+1}]$$

SMAC for Single Phase

- Numerical procedure
  - Cell pressure correction equation

$$\left[A_{i+1,J}d_{i+1,J} + A_{i,J}d_{i,J} + A_{I,j+1}d_{I,j+1} + A_{I,j}d_{I,j}\right]P'_{I,J}$$
$$= \left[A_{i+1,J}d_{i+1,J}P'_{I+1,J} + A_{i,J}d_{i,J}P'_{I-1,J} + A_{I,j+1}d_{I,j+1}P'_{I,J+1} + A_{I,j}d_{I,j}P'_{I,J-1}\right]$$
$$+ \left[(u^*A)_{i,J} - (u^*A)_{i+1,J} + (v^*A)_{I,j} - (v^*A)_{I,j+1}\right]$$

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,J+1}p'_{I,J+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J}$$

$$a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1}$$

$a_{I+1,J}$	$a_{I-1,J}$	$a_{I,J+1}$	$a_{I,J-1}$	$b'_{I,J}$
$(\rho dA)_{i+1,J}$	$(\rho dA)_{i,J}$	$(\rho dA)_{I,j+1}$	$(\rho dA)_{I,j}$	$(\rho u^*A)_{i,J} - (\rho u^*A)_{i+1,J} + (\rho v^*A)_{I,j} - (\rho v^*A)_{I,j+1}$

SMAC for Single Phase

- Numerical procedure
  - System pressure correction equation

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,J+1}p'_{I,J+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J}$$

$$\begin{pmatrix} & & & & & & & & \\ \cdots & -a_{I,J-1} & \cdots & -a_{I-1,J} & a_{I,J} & -a_{I+1,J} & \cdots & -a_{I,J-1} & \cdots \\ & & & & & & & & \end{pmatrix} \begin{pmatrix} \vdots \\ p'_{I,J-1} \\ \vdots \\ p'_{I-1,J} \\ p'_{I,J} \\ p'_{I+1,J} \\ \vdots \\ p'_{I,J+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} \\ b'_{I,J} \\ \\ \end{pmatrix}$$

SMAC for Single Phase

- Numerical procedure
  - Pressure and velocity correction

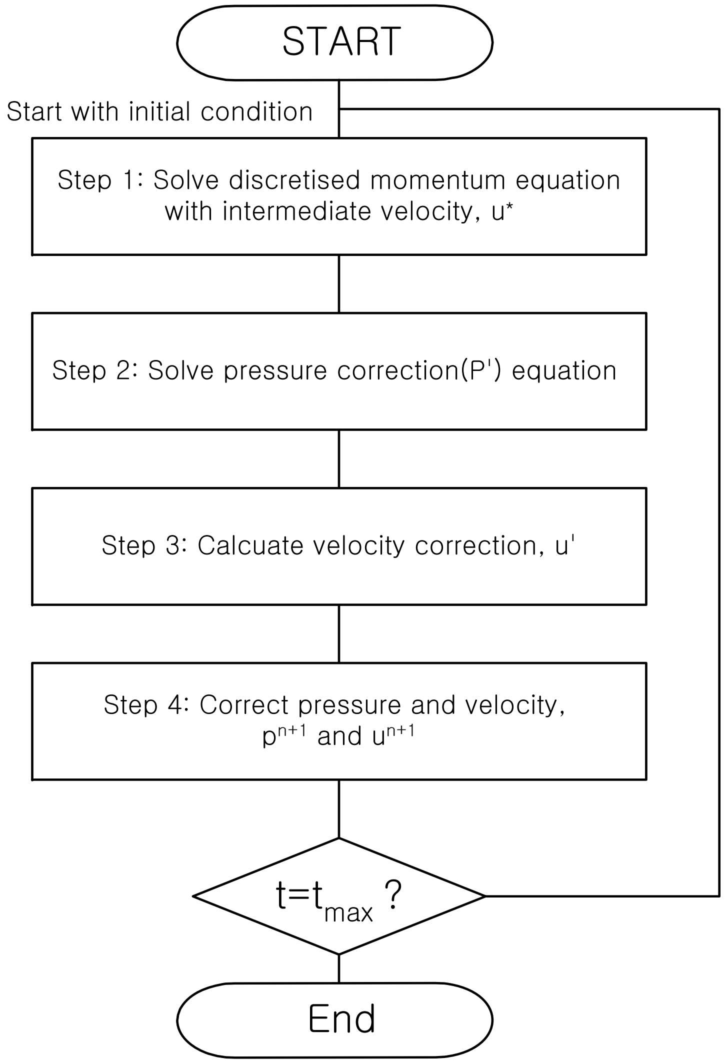
$$p = p^* + p'$$
$$u = u^* + u'$$
$$v = v^* + v'$$

$$u'_{i,J} = -\frac{\Delta t}{\rho}\left(\frac{P'_{I,J} - P'_{I-1,J}}{x_{I,J} - x_{I-1,J}}\right) = d_{i,J}\left(P'_{I,J} - P'_{I-1,J}\right)$$

$$v'_{I,j} = -\frac{\Delta t}{\rho}\left(\frac{P'_{I,J} - P'_{I,J-1}}{x_{I,J} - x_{I,J-1}}\right) = d_{I,j}\left(P'_{I,J} - P'_{I,J-1}\right)$$

Final solution for one-time step!

  - New time step calculation with updated values



SMAC for Single Phase

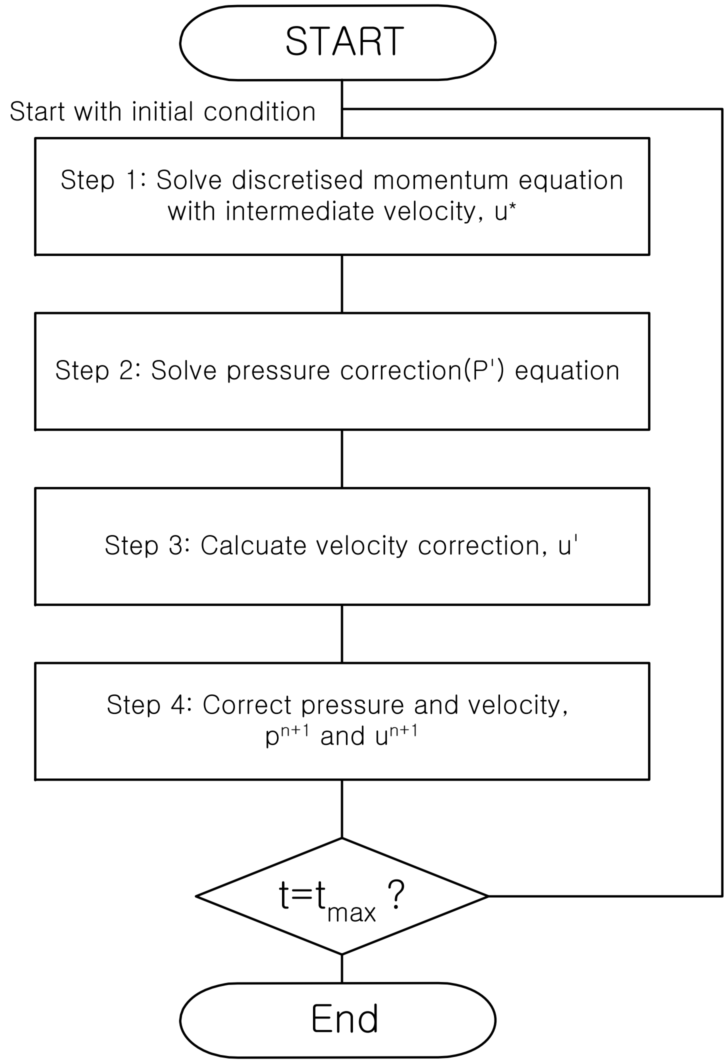
- Numerical procedure
  - Time step restriction
    - Courant number
      - Portion of a cell that a solute will traverse by advection in one time step

$$C_r = \frac{V\delta t}{\delta x} < 1$$

$$\delta t < \frac{\delta x}{V}$$

    - In practice,

$$C_r = \frac{V\delta t}{\delta x} < 0.3 \qquad \delta t < 0.3\frac{\delta x}{V}$$

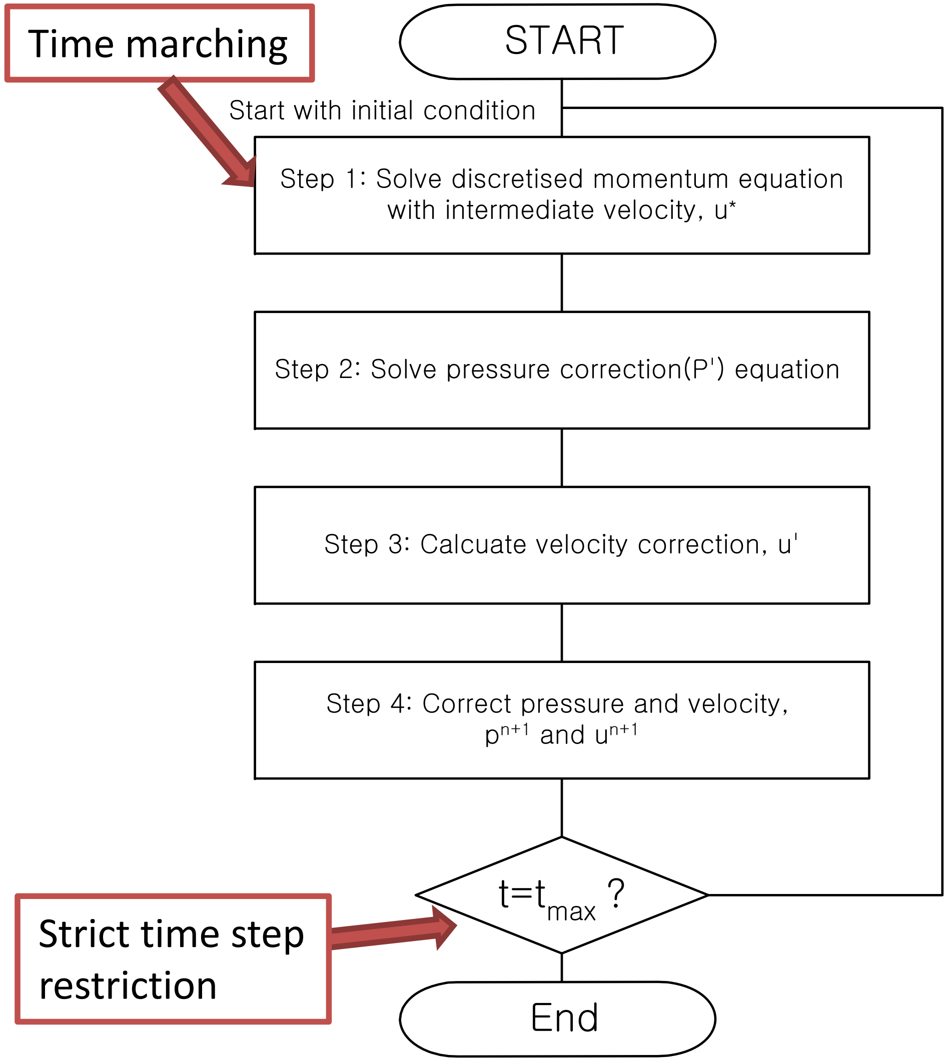


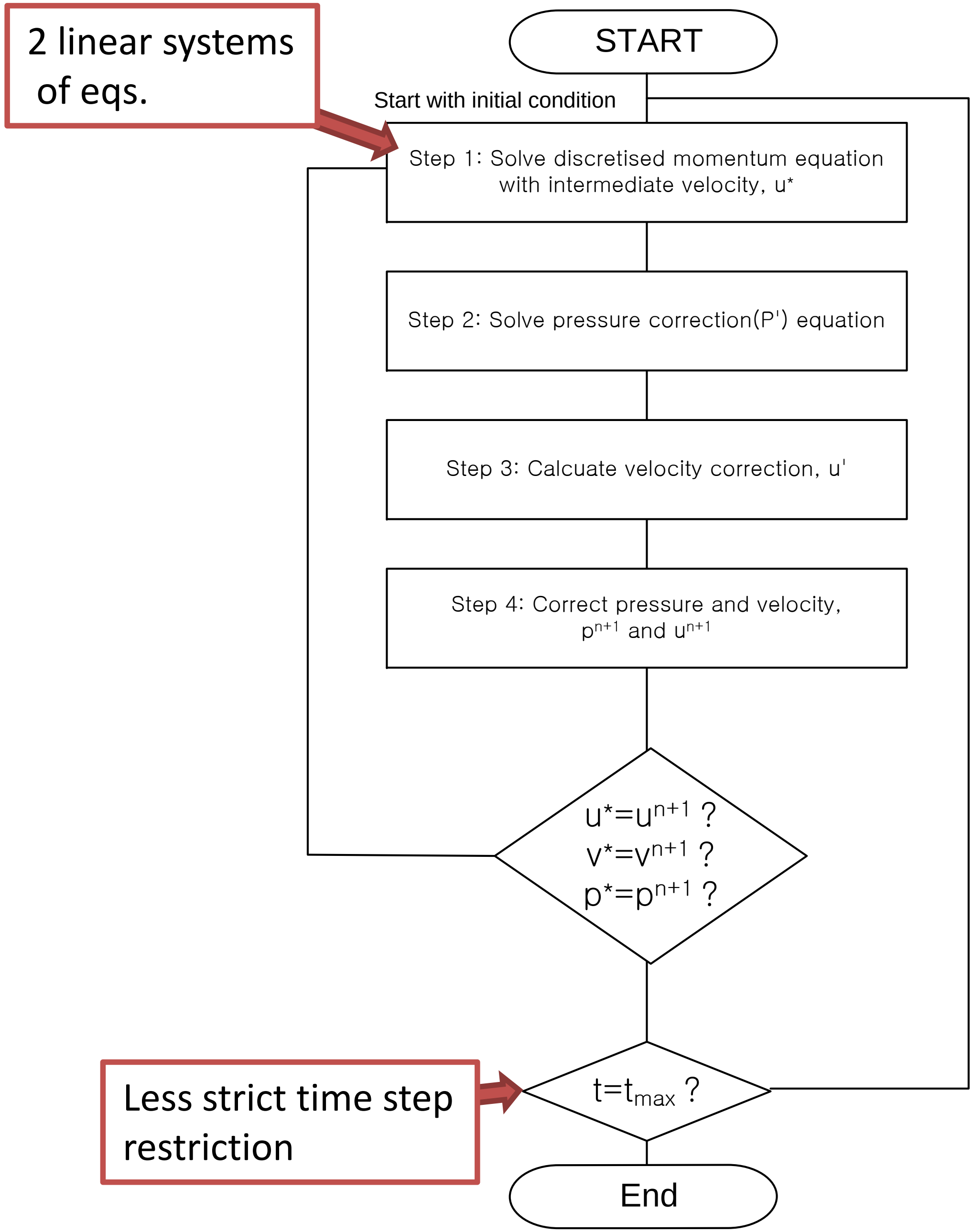
SMAC for Single Phase

- HW4
  - Derive the discretized equations
  - Modify your simple code to transient SMAC
  - Check the effect of time step size
  - Repeat same calculation from initial velocity 0 m/s for whole domain.
  - 2D Laminar flow

SMAC for Single Phase

- SMAC vs. SIMPLE





Contents

SMAC for two-phase flow

- Governing equation for two-phase flow
  - Two-fluid model
    - Most widely accepted mixture model for two-phase flow
    - For nuclear reactor T/H analysis
    - From system analysis codes (MARS, RELAP, SPACE) to CFD codes
  - Assumption for this chapter
    - Adiabatic and incompressible flow

SMAC for two-phase flow

- Governing equation for two-phase flow
  - Two-fluid model
    - Two continuity equations

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

$$\frac{\partial \alpha_g \rho_g}{\partial t}+\nabla\cdot\left(\alpha_g \rho_g \vec{u}_g\right)=0 \qquad (15)$$

$$\frac{\partial \alpha_l \rho_l}{\partial t}+\nabla\cdot\left(\alpha_l \rho_l \vec{u}_l\right)=0 \qquad (16)$$

    - Two momentum equations
      - With interfacial momentum transfer term: M_k

$$\frac{\partial \rho u}{\partial t}+\frac{\partial \rho uu}{\partial x}+\frac{\partial \rho uv}{\partial x}=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{dp}{dx}+\rho g_x$$

$$\frac{\partial(\alpha_g \rho_g \vec{u}_g)}{\partial t}+\nabla\cdot\left(\alpha_g \rho_g \vec{u}_g \vec{u}_g\right)=-\alpha_g \nabla P+\nabla\cdot(\alpha_g \tau_g)+\nabla\cdot(\alpha_g \tau_g^t)+\alpha_g \rho_g \vec{g}+\vec{M}_g \qquad (17)$$

$$\frac{\partial(\alpha_l \rho_l \vec{u}_l)}{\partial t}+\nabla\cdot\left(\alpha_l \rho_l \vec{u}_l \vec{u}_l\right)=-\alpha_l \nabla P+\nabla\cdot(\alpha_l \tau_l)+\nabla\cdot(\alpha_l \tau_l^t)+\alpha_l \rho_l \vec{g}+\vec{M}_l \qquad (18)$$

SMAC for two-phase flow

- Governing equation for two-phase flow
  - Two-fluid model
    - Interfacial momentum transfer term
      - Interfacial drag or interfacial friction
      - Virtual mass force
      - Lift force
      - Wall lubrication force
      - Turbulent dispersion force

$$M_{g,x} = -M_{l,x} = M_{d,g,x} + M_{vm,g,x}$$

$$M_{g,y} = -M_{l,y} = M_{d,g,y} + M_{vm,g,y}$$

$$\frac{\partial(\alpha_g \rho_g \vec{u}_g)}{\partial t} + \nabla \cdot \left(\alpha_g \rho_g \vec{u}_g \vec{u}_g\right) = -\alpha_g \nabla P + \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_g \qquad (17)$$

$$\frac{\partial(\alpha_l \rho_l \vec{u}_l)}{\partial t} + \nabla \cdot \left(\alpha_l \rho_l \vec{u}_l \vec{u}_l\right) = -\alpha_l \nabla P + \nabla \cdot (\alpha_l \tau_l) + \nabla \cdot (\alpha_l \tau_l^t) + \alpha_l \rho_l \vec{g} + \vec{M}_l \qquad (18)$$

SMAC for two-phase flow

- Governing equation for two-phase flow
  - Two-fluid model
    - Interfacial momentum transfer term
      - Interfacial drag or interfacial friction

$$M_{d,g,x} = -\frac{1}{8} A_i \rho_c C_D |\vec{u}_g - \vec{u}_l| (u_g - u_l)$$

$$M_{d,g,y} = -\frac{1}{8} A_i \rho_c C_D |\vec{u}_g - \vec{u}_l| (v_g - v_l)$$

$$M_{d,l,x} = \frac{1}{8} A_i \rho_c C_D |\vec{u}_g - \vec{u}_l| (u_g - u_l)$$

$$M_{d,l,y} = \frac{1}{8} A_i \rho_c C_D |\vec{u}_g - \vec{u}_l| (v_g - v_l)$$

      - Interfacial drag coefficient: simple assumption for this chapter

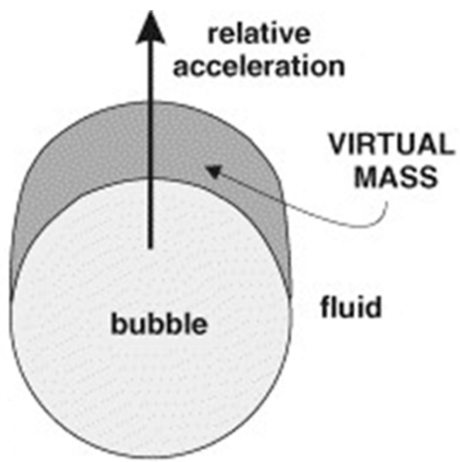
$$C_D = 0.44$$

      - Interfacial area concentration

$$A_i = \frac{6\alpha}{D_b}$$

SMAC for two-phase flow

- Governing equation for two-phase flow
  - Two-fluid model
    - Virtual mass force term
      - An accelerating or decelerating body must move some volume of surrounding fluid as it moves through it. Added mass is a common issue because the object and surrounding fluid cannot occupy the same physical space simultaneously.
      - For ships, the added mass can easily reach ¼ or ⅓ of the mass of the ship and therefore represents a significant inertia, in addition to frictional and wavemaking drag forces.
      - Since added mass is a virtual mass and not a real mass, it is not taken into account for structural designs.
      - In aircraft, the added mass is not usually taken into account because the density of the air is so small.



$$\vec{M}_{g,vm} = C_{vm}\alpha_g\alpha_l\rho_m\left[\left\{\frac{\partial \vec{u}_l}{\partial t} + (\vec{u}_l \cdot \nabla)\vec{u}_l\right\} - \left\{\frac{\partial \vec{u}_g}{\partial t} + (\vec{u}_g \cdot \nabla)\vec{u}_g\right\}\right] = -\vec{M}_{l,vm}$$

SMAC for two-phase flow

- Numerical procedure
  - Two-fluid model in the expanded form

$$\frac{\partial(\alpha_g\rho_g\vec{u}_g)}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\vec{u}_g\vec{u}_g\right)=-\alpha_g\nabla P+\nabla\cdot(\alpha_g\tau_g)+\nabla\cdot(\alpha_g\tau_g^t)+\alpha_g\rho_g\vec{g}+\vec{M}_g \qquad (17)$$

$$\frac{\partial(\alpha_l\rho_l\vec{u}_l)}{\partial t}+\nabla\cdot\left(\alpha_l\rho_l\vec{u}_l\vec{u}_l\right)=-\alpha_l\nabla P+\nabla\cdot(\alpha_l\tau_l)+\nabla\cdot(\alpha_l\tau_l^t)+\alpha_l\rho_l\vec{g}+\vec{M}_l \qquad (18)$$

$$\rho_g\left(\alpha_g\frac{\partial\vec{u}_g}{\partial t}+\vec{u}_g\frac{\partial\alpha_g}{\partial t}\right)+\nabla\cdot\left(\alpha_g\rho_g\vec{u}_g\vec{u}_g\right)$$

$$=\left[\alpha_g\rho_g\frac{\partial\vec{u}_g}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\vec{u}_g\vec{u}_g\right)\right]+\rho_g\vec{u}_g\frac{\partial\alpha_g}{\partial t}$$

$$=\left[\alpha_g\rho_g\frac{\partial\vec{u}_g}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\vec{u}_g\vec{u}_g\right)\right]-\rho_g\vec{u}_g\nabla\cdot\left(\alpha_g\vec{u}_g\right)$$

$$=\alpha_g\rho_g\frac{\partial\vec{u}_g}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\vec{u}_g\vec{u}_g\right)-\rho_g\vec{u}_g\nabla\cdot\left(\alpha_g\vec{u}_g\right)$$

$$\frac{\partial\alpha_g\rho_g}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\vec{u}_g\right)=0 \qquad (15)$$

SMAC for two-phase flow

- Numerical procedure
  - Two-fluid model in the expanded form

$$\frac{\partial(\alpha_g \rho_g \vec{u}_g)}{\partial t} + \nabla \cdot \left(\alpha_g \rho_g \vec{u}_g \vec{u}_g\right) = -\alpha_g \nabla P + \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_g \qquad (17)$$

$$\frac{\partial(\alpha_l \rho_l \vec{u}_l)}{\partial t} + \nabla \cdot \left(\alpha_l \rho_l \vec{u}_l \vec{u}_l\right) = -\alpha_l \nabla P + \nabla \cdot (\alpha_l \tau_l) + \nabla \cdot (\alpha_l \tau_l^t) + \alpha_l \rho_l \vec{g} + \vec{M}_l \qquad (18)$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + (\alpha_g \rho_g \vec{u}_g \cdot \nabla)\vec{u}_g = -\alpha_g \nabla P + \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_g$$

$$\alpha_l \rho_l \frac{\partial \vec{u}_l}{\partial t} + (\alpha_l \rho_l \vec{u}_l \cdot \nabla)\vec{u}_l = -\alpha_l \nabla P + \nabla \cdot (\alpha_l \tau_l) + \nabla \cdot (\alpha_l \tau_l^t) + \alpha_l \rho_l \vec{g} + \vec{M}_l$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot \left(\alpha_g \rho_g \vec{u}_g \vec{u}_g\right) - \rho_g \vec{u}_g \nabla \cdot \left(\alpha_g \vec{u}_g\right)$$

SMAC for two-phase flow

- Numerical procedure

  - Applying the virtual mass model,

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + (\alpha_g \rho_g \vec{u}_g \cdot \nabla)\vec{u}_g = -\alpha_g \nabla P + \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_{g,vm} + \vec{M}_{g,d}$$

$$\vec{M}_{g,vm} = C_{vm} \alpha_g \alpha_l \rho_m \left[\left\{ \frac{\partial \vec{u}_l}{\partial t} + (\vec{u}_l \cdot \nabla)\vec{u}_l \right\} - \left\{ \frac{\partial \vec{u}_g}{\partial t} + (\vec{u}_g \cdot \nabla)\vec{u}_g \right\} \right] = -\vec{M}_{l,vm}$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + (\alpha_g \rho_g \vec{u}_g \cdot \nabla)\vec{u}_g = -\alpha_g \nabla P + \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_{g,d}$$
$$+ C_{vm} \alpha_g \alpha_l \rho_m \left[\left\{ \frac{\partial \vec{u}_l}{\partial t} + (\vec{u}_l \cdot \nabla)\vec{u}_l \right\} - \left\{ \frac{\partial \vec{u}_g}{\partial t} + (\vec{u}_g \cdot \nabla)\vec{u}_g \right\} \right]$$

$$(\alpha_g \rho_g + C_{vm} \alpha_g \alpha_l \rho_m) \frac{\partial \vec{u}_g}{\partial t} - (C_{vm} \alpha_g \alpha_l \rho_m) \frac{\partial \vec{u}_l}{\partial t} = -(\alpha_g \rho_g \vec{u}_g \cdot \nabla)\vec{u}_g - \alpha_g \nabla P$$
$$+ \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_{g,d}$$
$$+ C_{vm} \alpha_g \alpha_l \rho_m \left[(\vec{u}_l \cdot \nabla)\vec{u}_l - (\vec{u}_g \cdot \nabla)\vec{u}_g \right] = -\alpha_g \nabla P + \vec{R}_g$$

SMAC for two-phase flow

- Numerical procedure
  - Applying the virtual mass model,

$$\left(\alpha_g\rho_g + C_{vm}\alpha_g\alpha_l\rho_m\right)\frac{\partial\vec{u}_g}{\partial t} - \left(C_{vm}\alpha_g\alpha_l\rho_m\right)\frac{\partial\vec{u}_l}{\partial t} = -\alpha_g\nabla P + \vec{R}_g$$

$$-\left(C_{vm}\alpha_g\alpha_l\rho_m\right)\frac{\partial\vec{u}_g}{\partial t} + \left(\alpha_l\rho_l + C_{vm}\alpha_g\alpha_l\rho_m\right)\frac{\partial\vec{u}_l}{\partial t} = -\alpha_l\nabla P + \vec{R}_l$$

$$\begin{pmatrix} \left(\rho_g + C_{vm}\alpha_l\rho_m\right) & -\left(C_{vm}\alpha_g\rho_m\right) \\ -\left(C_{vm}\alpha_l\rho_m\right) & \left(\rho_l + C_{vm}\alpha_g\rho_m\right) \end{pmatrix} \begin{pmatrix} \alpha_g\dfrac{\partial\vec{u}_g}{\partial t} \\ \alpha_l\dfrac{\partial\vec{u}_l}{\partial t} \end{pmatrix} = \begin{pmatrix} -\alpha_g\nabla P + \vec{R}_g \\ -\alpha_l\nabla P + \vec{R}_l \end{pmatrix}$$

$$\begin{pmatrix} \alpha_g\dfrac{\partial\vec{u}_g}{\partial t} \\ \alpha_l\dfrac{\partial\vec{u}_l}{\partial t} \end{pmatrix} = A^{-1}\begin{pmatrix} -\alpha_g\nabla P + \vec{R}_g \\ -\alpha_l\nabla P + \vec{R}_l \end{pmatrix}$$

SMAC for two-phase flow

- Numerical procedure
  - Intermediate velocity

$$\begin{pmatrix} \alpha_g \dfrac{\partial \vec{u}_g}{\partial t} \\ \alpha_l \dfrac{\partial \vec{u}_l}{\partial t} \end{pmatrix} = A^{-1} \begin{pmatrix} -\alpha_g \nabla P + \vec{R}_g \\ -\alpha_l \nabla P + \vec{R}_l \end{pmatrix}$$

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,*} - \vec{u}_g^{\,n}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,*} - \vec{u}_l^{\,n}}{\Delta t} \end{pmatrix} = A^{-1} \begin{pmatrix} -\alpha_g \nabla P^n + \vec{R}_g \\ -\alpha_l \nabla P^n + \vec{R}_l \end{pmatrix}$$

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,*}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,*}}{\Delta t} \end{pmatrix} = \begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n}}{\Delta t} \end{pmatrix} + A^{-1} \begin{pmatrix} -\alpha_g \nabla P^n + \vec{R}_g \\ -\alpha_l \nabla P^n + \vec{R}_l \end{pmatrix}$$

  - New time step velocity

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n+1}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n+1}}{\Delta t} \end{pmatrix} = \begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n}}{\Delta t} \end{pmatrix} + A^{-1} \begin{pmatrix} -\alpha_g \nabla P^{n+1} + \vec{R}_g \\ -\alpha_l \nabla P^{n+1} + \vec{R}_l \end{pmatrix}$$

SMAC for two-phase flow

- Numerical procedure

  - Intermediate velocity vs. new times step velocity

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,*}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,*}}{\Delta t} \end{pmatrix} = \begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n}}{\Delta t} \end{pmatrix} + A^{-1} \begin{pmatrix} -\alpha_g \nabla P^n + \vec{R}_g \\ -\alpha_l \nabla P^n + \vec{R}_l \end{pmatrix}$$

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n+1}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n+1}}{\Delta t} \end{pmatrix} = \begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n}}{\Delta t} \end{pmatrix} + A^{-1} \begin{pmatrix} -\alpha_g \nabla P^{n+1} + \vec{R}_g \\ -\alpha_l \nabla P^{n+1} + \vec{R}_l \end{pmatrix}$$

  - Pressure correction vs. velocity correction

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,'}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,'}}{\Delta t} \end{pmatrix} = A^{-1} \begin{pmatrix} -\alpha_g \nabla P^{'} \\ -\alpha_l \nabla P^{'} \end{pmatrix}$$

SMAC for two-phase flow

- Numerical procedure
  - Continuity equations

$$\frac{\partial \alpha_g \rho_g}{\partial t} + \nabla \cdot \left(\alpha_g \rho_g \vec{u}_g \right) = 0 \qquad (15) \qquad \frac{\partial \alpha_l \rho_l}{\partial t} + \nabla \cdot \left(\alpha_l \rho_l \vec{u}_l \right) = 0 \qquad (16)$$

$$\frac{\partial \alpha_g}{\partial t} + \nabla \cdot \left(\alpha_g \vec{u}_g \right) + \frac{\partial \alpha_l}{\partial t} + \nabla \cdot \left(\alpha_l \vec{u}_l \right) = \nabla \cdot \left(\alpha_g \vec{u}_g \right) + \nabla \cdot \left(\alpha_l \vec{u}_l \right) = 0$$

$$\nabla \cdot \left(\alpha_g \vec{u}_g^{\,n+1} \right) + \nabla \cdot \left(\alpha_l \vec{u}_l^{\,n+1} \right) = 0$$

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n+1}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n+1}}{\Delta t} \end{pmatrix} = \begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,*}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,*}}{\Delta t} \end{pmatrix} + A^{-1} \begin{pmatrix} -\alpha_g \nabla P' \\ -\alpha_l \nabla P' \end{pmatrix} \Rightarrow \begin{pmatrix} \alpha_g \vec{u}_g^{\,n+1} \\ \alpha_l \vec{u}_l^{\,n+1} \end{pmatrix} = \begin{pmatrix} \alpha_g \vec{u}_g^{\,*} \\ \alpha_l \vec{u}_l^{\,*} \end{pmatrix} - \Delta t \begin{pmatrix} A_{11} \cdot \alpha_g \nabla P' + A_{12} \cdot \alpha_l \nabla P' \\ A_{21} \cdot \alpha_g \nabla P' + A_{22} \cdot \alpha_l \nabla P' \end{pmatrix}$$

SMAC for two-phase flow

❖ Numerical procedure

- Pressure correction equation

$$\nabla \cdot \left(\alpha_g \vec{u}_g^{n+1}\right) + \nabla \cdot \left(\alpha_l \vec{u}_l^{n+1}\right) = 0 \quad \Leftarrow \quad \begin{pmatrix} \alpha_g \vec{u}_g^{n+1} \\ \alpha_l \vec{u}_l^{n+1} \end{pmatrix} = \begin{pmatrix} \alpha_g \vec{u}_g^{*} \\ \alpha_l \vec{u}_l^{*} \end{pmatrix} - \Delta t \begin{pmatrix} A_{11} \cdot \alpha_g \nabla P' + A_{12} \cdot \alpha_l \nabla P' \\ A_{21} \cdot \alpha_g \nabla P' + A_{22} \cdot \alpha_l \nabla P' \end{pmatrix}$$

$$\nabla \cdot \left(\alpha_g \vec{u}_g^{*} - \Delta t \left(A_{11} \cdot \alpha_g \nabla P' + A_{12} \cdot \alpha_l \nabla P'\right)\right) + \nabla \cdot \left(\alpha_l \vec{u}_l^{*} - \Delta t \left(A_{21} \cdot \alpha_g \nabla P' + A_{22} \cdot \alpha_l \nabla P'\right)\right) = 0$$

$$\nabla \cdot \left(\alpha_g \vec{u}_g^{*}\right) + \nabla \cdot \left(\alpha_l \vec{u}_l^{*}\right) = \Delta t \nabla \cdot \left[\left(A_{11} + A_{21}\right)\alpha_g + \left(A_{12} + A_{22}\right)\alpha_l\right] \nabla P'$$

SMAC for two-phase flow

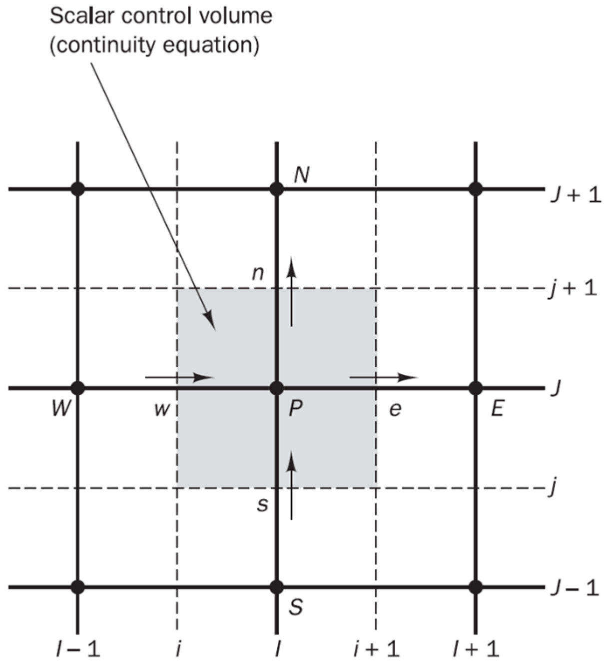
- Numerical procedure
  - Discretized pressure correction equation

$$\nabla \cdot \left(\alpha_g \vec{u}_g^*\right) + \nabla \cdot \left(\alpha_l \vec{u}_l^*\right) = \Delta t \nabla \cdot \left[\left(A_{11} + A_{21}\right)\alpha_g + \left(A_{12} + A_{22}\right)\alpha_l\right] \nabla P'$$

$$\int_V \nabla \cdot \left(\alpha_g \vec{u}_g^*\right) dV + \int_V \nabla \cdot \left(\alpha_l \vec{u}_l^*\right) dV = \int_V \Delta t \nabla \cdot \left[\left(A_{11} + A_{21}\right)\alpha_g + \left(A_{12} + A_{22}\right)\alpha_l\right] \nabla P' dV$$

$$\int_A \left(\alpha_g \vec{u}_g^*\right) \cdot \vec{n} dA + \int_A \left(\alpha_l \vec{u}_l^*\right) \cdot \vec{n} dA = \int_A \Delta t \left[\left(A_{11} + A_{21}\right)\alpha_g + \left(A_{12} + A_{22}\right)\alpha_l\right] \nabla P' \cdot \vec{n} dA$$

$$\sum_j \left(\alpha_g \vec{u}_g^*\right) \cdot \vec{A} + \sum_j \left(\alpha_l \vec{u}_l^*\right) \cdot \vec{A} = \sum_j \Delta t \left[\left(A_{11} + A_{21}\right)\alpha_g + \left(A_{12} + A_{22}\right)\alpha_l\right] \nabla P' \cdot \vec{A}$$

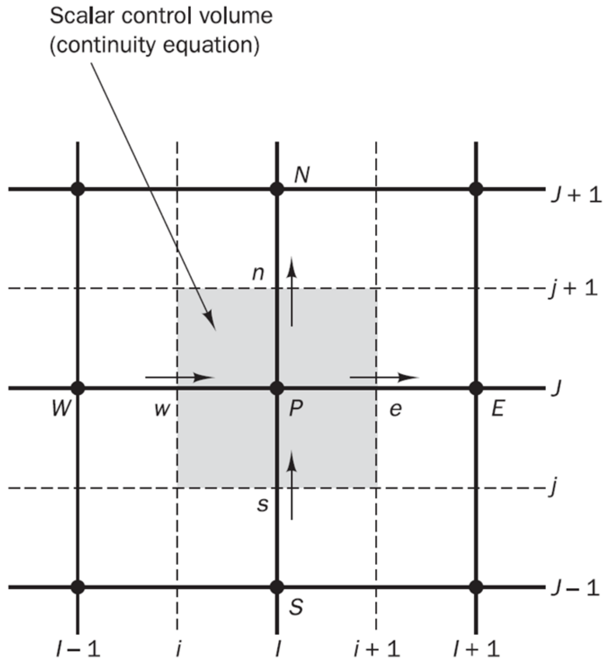


SMAC for two-phase flow

- Numerical procedure
  - Discretized pressure correction equation

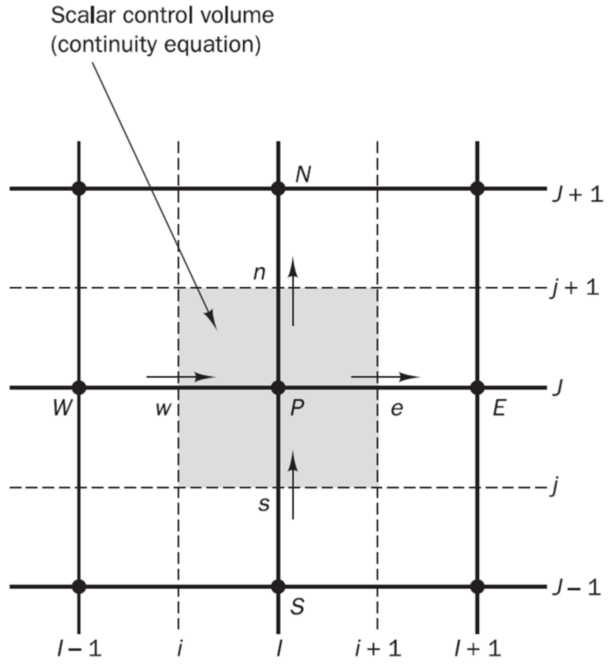
$$\sum_j \left(\alpha_g \vec{u}_g^*\right)\cdot \vec{A} + \sum_j \left(\alpha_l \vec{u}_l^*\right)\cdot \vec{A} = \sum_j \Delta t\left[\left(A_{11}+A_{21}\right)\alpha_g + \left(A_{12}+A_{22}\right)\alpha_l\right]\nabla P' \cdot \vec{A}$$

$$\begin{aligned}
&\left(\alpha_g u_g^* A\right)_e - \left(\alpha_g u_g^* A\right)_w + \left(\alpha_g v_g^* A\right)_n - \left(\alpha_g v_g^* A\right)_s \\
&+ \left(\alpha_l u_l^* A\right)_e - \left(\alpha_l u_l^* A\right)_w + \left(\alpha_l v_l^* A\right)_n - \left(\alpha_l v_l^* A\right)_s \\
&= \Delta t\left[\left(A_{11}+A_{21}\right)\alpha_g + \left(A_{12}+A_{22}\right)\alpha_l\right]_e \frac{P_E' - P_P'}{x_E - x_P} A_e \\
&- \Delta t\left[\left(A_{11}+A_{21}\right)\alpha_g + \left(A_{12}+A_{22}\right)\alpha_l\right]_w \frac{P_P' - P_W'}{x_P - x_W} A_w \\
&+ \Delta t\left[\left(A_{11}+A_{21}\right)\alpha_g + \left(A_{12}+A_{22}\right)\alpha_l\right]_n \frac{P_N' - P_P'}{y_N - y_P} A_n \\
&- \Delta t\left[\left(A_{11}+A_{21}\right)\alpha_g + \left(A_{12}+A_{22}\right)\alpha_l\right]_s \frac{P_P' - P_S'}{y_P - y_S} A_s
\end{aligned}$$



SMAC for two-phase flow

- Numerical procedure
  - Discretized pressure correction equation



$$
\begin{aligned}
&\left(\alpha_g u_g^* A_g\right)_e - \left(\alpha_g u_g^* A_g\right)_w + \left(\alpha_g v_g^* A_g\right)_n - \left(\alpha_g v_g^* A_g\right)_s \\
&+ \left(\alpha_l u_l^* A_l\right)_e - \left(\alpha_l u_l^* A_l\right)_w + \left(\alpha_l v_l^* A_l\right)_n - \left(\alpha_l v_l^* A_l\right)_s \\
&= (dA)_e\left(P_E' - P_P'\right) - (dA)_w\left(P_P' - P_W'\right) \\
&\;+ (dA)_n\left(P_P' - P_N'\right) - (dA)_s\left(P_P' - P_S'\right)
\end{aligned}
$$

$$
\begin{aligned}
&a_P P_P' = a_E P_E' + a_W P_W' + a_N P_N' + a_S P_S' + b_p \\
&a_E = (dA)_e,\; a_W = (dA)_w,\; a_N = (dA)_n,\; a_S = (dA)_s \\
&a_P = a_E + a_W + a_N + a_S \\
&b_p = \left(\alpha_g u_g^* A\right)_e - \left(\alpha_g u_g^* A\right)_w + \left(\alpha_g v_g^* A\right)_n - \left(\alpha_g v_g^* A\right)_s + \left(\alpha_l u_l^* A\right)_e - \left(\alpha_l u_l^* A\right)_w + \left(\alpha_l v_l^* A\right)_n - \left(\alpha_l v_l^* A\right)_s
\end{aligned}
$$

SMAC for two-phase flow

- Numerical procedure

  - Discretized pressure correction equation

$$a_P P_P^{'} = a_E P_E^{'} + a_W P_W^{'} + a_N P_N^{'} + a_S P_S^{'} + b_p$$

$$a_E = (dA)_e,\ \ a_W = (dA)_w,\ \ a_N = (dA)_n,\ \ a_S = (dA)_s$$

$$a_P = a_E + a_W + a_N + a_S$$

$$b_p = (\alpha_g u_g^* A)_e - (\alpha_g u_g^* A)_w + (\alpha_g v_g^* A)_n - (\alpha_g v_g^* A)_s + (\alpha_l u_l^* A)_e - (\alpha_l u_l^* A)_w + (\alpha_l v_l^* A)_n - (\alpha_l v_l^* A_l)_s$$

$$a_{I,J} p'_{I,J} = a_{I+1,J} p'_{I+1,J} + a_{I-1,J} p'_{I-1,J} + a_{I,J+1} p'_{I,J+1} + a_{I,J-1} p'_{I,J-1} + b'_{I,J}$$

$a_{I+1,J}$	$a_{I-1,J}$	$a_{I,J+1}$	$a_{I,J-1}$	$b'_{I,J}$
$(\rho dA)_{i+1,J}$	$(\rho dA)_{i,J}$	$(\rho dA)_{I,j+1}$	$(\rho dA)_{I,j}$	$(\rho u^* A)_{i,J} - (\rho u^* A)_{i+1,J}$ $+ (\rho v^* A)_{I,j} - (\rho v^* A)_{I,j+1}$

SMAC for two-phase flow

- Numerical procedure
  - Discretized pressure correction equation

$$a_P P_P^{'} = a_E P_E^{'} + a_W P_W^{'} + a_N P_N^{'} + a_S P_S^{'} + b_p$$
$$a_E = (dA)_e,\ a_W = (dA)_w,\ a_N = (dA)_n,\ a_S = (dA)_s$$
$$a_P = a_E + a_W + a_N + a_S$$
$$b_p = (\alpha_g u_g^* A)_e - (\alpha_g u_g^* A)_w + (\alpha_g v_g^* A)_n - (\alpha_g v_g^* A)_s + (\alpha_l u_l^* A)_e - (\alpha_l u_l^* A)_w + (\alpha_l v_l^* A)_n - (\alpha_l v_l^* A)_s$$

$$A^{-1} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} (\rho_g + C_{vm}\alpha_l\rho_m) & -(C_{vm}\alpha_g\rho_m) \\ -(C_{vm}\alpha_l\rho_m) & (\rho_l + C_{vm}\alpha_g\rho_m) \end{pmatrix}^{-1}$$

$$\vec{u}^{\,n+1} = \vec{u}^{\,*} - \frac{\Delta t}{\rho}\nabla P'$$

$$d_e = \frac{\Delta t\left\lfloor (A_{11} + A_{21})\alpha_g + (A_{12} + A_{22})\alpha_l \right\rfloor_e}{x_E - x_P}$$

$$u_{i,J}^{'} = -\frac{\Delta t}{\rho}\left(\frac{P_{I,J}^{'} - P_{I-1,J}^{'}}{x_{I,J} - x_{I-1,J}}\right) = d_{i,J}\left(P_{I,J}^{'} - P_{I-1,J}^{'}\right)$$

SMAC for two-phase flow

- Numerical procedure
  - Pressure and velocity correction, update void fraction

$$P^{n+1} = P^n + P' \qquad (34)$$

$$\begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,n+1}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,n+1}}{\Delta t} \end{pmatrix} = \begin{pmatrix} \alpha_g \dfrac{\vec{u}_g^{\,*}}{\Delta t} \\ \alpha_l \dfrac{\vec{u}_l^{\,*}}{\Delta t} \end{pmatrix} + A^{-1} \begin{pmatrix} -\alpha_g \nabla P' \\ -\alpha_l \nabla P' \end{pmatrix} \qquad \begin{pmatrix} \alpha_g \vec{u}_g^{\,n+1} \\ \alpha_l \vec{u}_l^{\,n+1} \end{pmatrix} = \begin{pmatrix} \alpha_g \vec{u}_g^{\,*} \\ \alpha_l \vec{u}_l^{\,*} \end{pmatrix} - \Delta t \begin{pmatrix} A_{11} \cdot \alpha_g \nabla P' + A_{12} \cdot \alpha_l \nabla P' \\ A_{21} \cdot \alpha_g \nabla P' + A_{22} \cdot \alpha_l \nabla P' \end{pmatrix}$$

$$\frac{\partial \alpha_g}{\partial t} + \nabla \cdot \left(\alpha_g \vec{u}_g \right) = 0 \;\Rightarrow\; \frac{\alpha_g^{n+1} - \alpha_g^n}{\Delta t} V + u_{g,e}^{n+1} A_e - u_{g,w}^{n+1} A_w + v_{g,n}^{n+1} A_n - v_{g,s}^{n+1} A_s = 0$$

$$\frac{\partial \alpha_l}{\partial t} + \nabla \cdot \left(\alpha_l \vec{u}_l \right) = 0 \;\Rightarrow\; \frac{\alpha_l^{n+1} - \alpha_l^n}{\Delta t} + u_{l,e}^{n+1} A_e - u_{l,w}^{n+1} A_w + v_{l,n}^{n+1} A_n - v_{l,s}^{n+1} A_s = 0$$

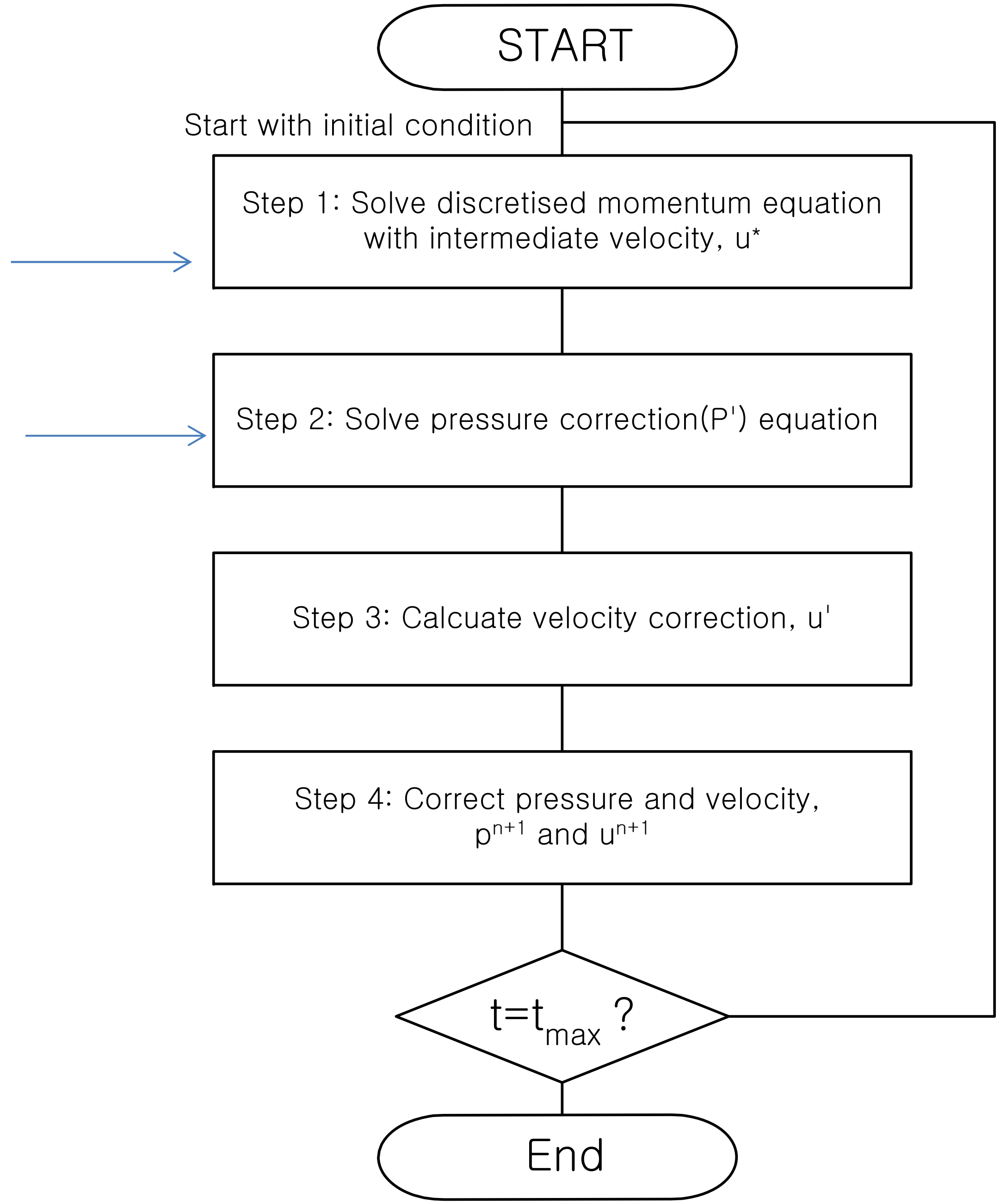
  - Check $\alpha_g + \alpha_l = 1$? You need to make the time step smaller? Or bug!

SMAC for two-phase flow

❖ Numerical procedure

2x2 matrix
because of the virtual mass

NxN matrix



SMAC for two-phase flow

- Sample problem
  - Solve the two-fluid model with SMAC scheme
  - Incompressible flow
  - Consider gravity effect
  - Bubble diameter 0.005 m
    - Use this for interfacial area concentration
  - Outlet pressure: 0.1 MPa
  - Inlet velocity: 1 m/s for both phases
  - rhof=1000 kg/m3, rhog=0.5 kg/m3
  - Report the void fraction, liquid and gas velocity profiles at the exit
  - Report axial pressure distribution