Optimum Design

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Contents

Ch. 7 Metaheuristic Optimization Method: Genetic Algorithms

7.1 Overview

Genetic Algorithms (GA) (1/3)

- ☑ Adaptive metaheuristic search algorithm based on the evolutionary ideas of natural selection and genetics
- ☑ One of guided random search algorithms based on the mechanics of biological evolution
- ☑ Part of evolutionary computing, a rapidly growing area of artificial intelligence
- ☑ Inspired by Darwin's theory about evolution ("Survival of the fittest")
- ☑ Represent an intelligent exploitation of a random search used to solve optimization problems.

* W. Williams, Genetic Algorithms: A Tutorial, 1995

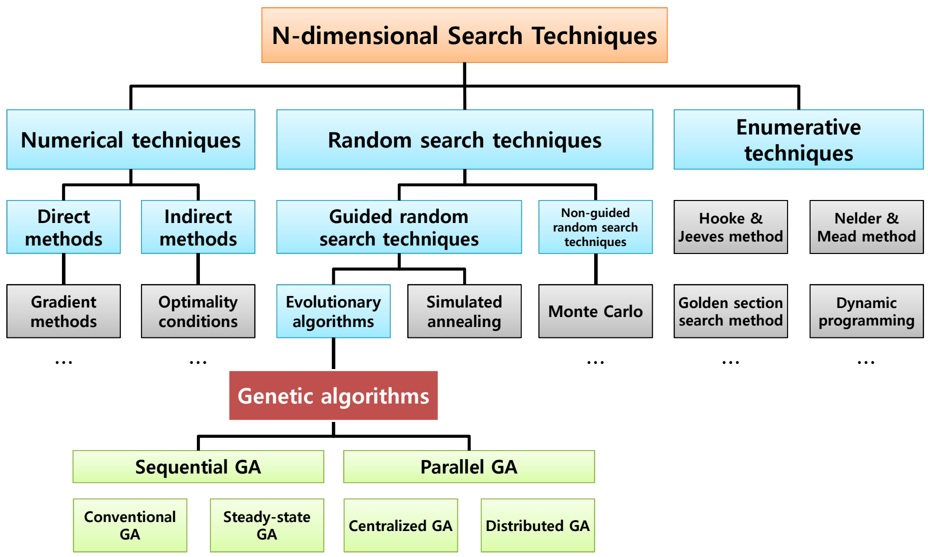
Genetic Algorithms (GA) (2/3)

- ☑ Although randomized, it exploits historical information to direct the search into the region of better performance within search space
- ☑ In nature, competition among individuals for scanty resources results in the fittest individuals dominating over the weaker ones.
- ☑ Find an optimum by repeating a series of genetic operators; selection, crossover, mutation, etc.
- ☑ Yield a global optimum for complex optimization problems having a number of local optima ➡ Global Optimization Method

Genetic Algorithms (GA) (3/3)

- ☑ Suitable for solving complex optimization problems and for applications that require adaptive problem solving strategies
- ☑ Provide efficient, effective techniques for optimization and machine learning applications
- ☑ Widely used today in engineering, business, and scientific fields
- ☑ Developed by John Holland at University of Michigan in 1975
  - To understand the adaptive processes of natural systems
  - To design artificial systems software that retains the robustness of natural systems

Classification of Genetic Algorithms



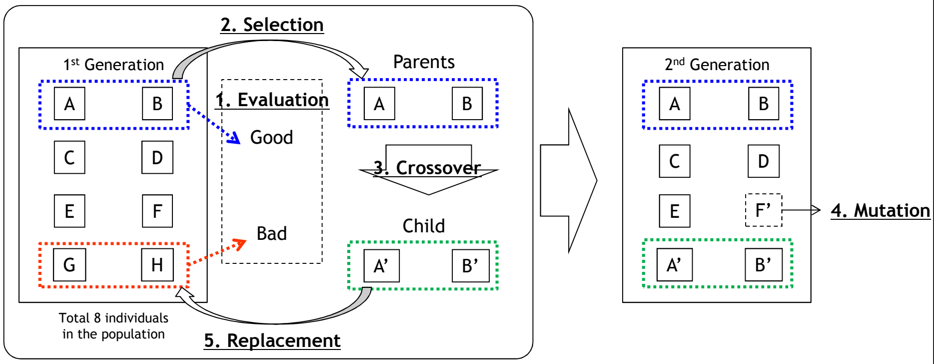
Components of Genetic Algorithms

☑ **To solve a problem with GA, the followings are needed.**

☑ **Encoding technique: Gene, chromosome**
☑ **Initialization procedure: Initialization (or creation)**
☑ **Evaluation function: Fitness calculation**
☑ **Selection: Selection of parents**
☑ **Modification: Crossover and mutation for the parents**
☑ **Deletion: Update of the population**
☑ **Parameter setting: Practice and experience**

* W. Williams, Genetic Algorithms: A Tutorial, 1995

Procedure of Finding an Optimum by Using GA



1. **Evaluation**: Evaluate the quality (fitness) of each individual in the population.
2. **Selection**: Select two individuals of good quality as parents.
3. **Crossover**: Generate two child from the parents.
4. **Mutation**: Generate a new child by introducing the property which the parents don't have. (Optional)
5. **Replacement**: Replace individuals of bad quality with the child.

The optimum is found by repeating the evaluation, selection, crossover, mutation, and replacement.

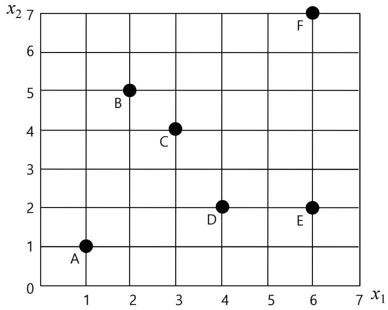
Example of Finding an Optimum by Using GA (1/5)

Minimize $f(\mathbf{x}) = x_1^2 + x_2^2 - 8x_1 - 8x_2$

1st Generation Evaluation

A	(1 , 1)	-14
B	(2 , 5)	-27
C	(3 , 4)	-31
D	(4 , 2)	-28
E	(6 , 2)	-24
F	(6 , 7)	-19

Parents
(3 , 4)
(4 , 2)



1. Evaluation: Evaluate the quality (fitness) of each individual in the population

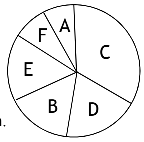
2. Selection: Select two individuals of good quality as parents.

- In this example, two individuals having higher fitness were selected as parents.
- **Selection by probability**. The better fitness, the higher selection probability.

$$P_{selection}(i) = \frac{Ft(i)_i}{\sum_{j=1}^{N} Ft(i)_i}$$

$Ft(i)$: Fitness of individual i in the population
N: Number of individuals in the population
$P_{selection}(i)$: Probability of being selected of individual i

Fitness proportionate selection (Roulette wheel selection)
- Genetic operator used in genetic algorithms for selecting potentially useful solutions for crossover
- The analogy to a roulette wheel can be envisaged by imagining a roulette wheel in which each candidate solution represents a pocket on the wheel; the size of the pockets are proportionate to the probability of selection of the solution.



Example of Finding an Optimum by Using GA (2/5)

Minimize $f(\mathbf{x}) = x_1^2 + x_2^2 - 8x_1 - 8x_2$

1st Generation Evaluation

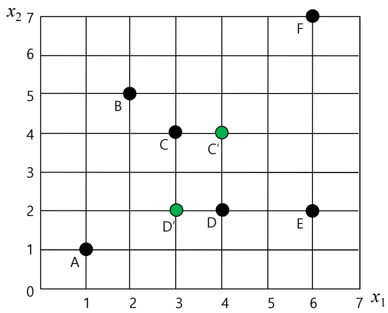
A	(1 , 1)	-14
B	(2 , 5)	-27
C	(3 , 4)	-31
D	(4 , 2)	-28
E	(6 , 2)	-24
F	(6 , 7)	-19

Parents
(3 , 4)
(4 , 2)

Crossover

Child
C' (3 , 2)
D' (4 , 4)

Replacement



3. Crossover: Generate two child from the parents.

- In this example, a method of switching x_2 was used as crossover.

4. Replacement: Replace individuals of bad quality with the child.

- In this example, two individuals having worse quality was replaced with the child.

Replacement methods of individuals
+ Ex 1) Evaluate the quality of the child. If its quality is better than that of parent, replace it with the parent, **or do not replace (it is dropped)**.
+ Ex 2) Evaluate the quality of the child. If its quality is better than that of parent, replace it with the parent, **or replace it with one of the parent having worse quality**.

Example of Finding an Optimum by Using GA (3/5)

Minimize $f(\mathbf{x}) = x_1^2 + x_2^2 - 8x_1 - 8x_2$

2nd Generation Evaluation

A	(3 , 2)	-27
B	(2 , 5)	-27
C	(3 , 4)	-31
D	(4 , 2)	-28
E	(6 , 2)	-24
F	(4 , 4)	-32

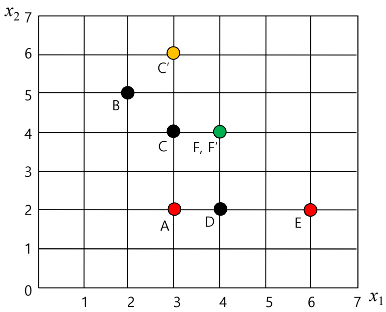
Replacement

Mutation

Parents: (3 , 4) ↕ (4 , 4)

Crossover

Child: C' (3 , 6) F' (4 , 4)



3rd Generation Evaluation

A	(3 , 6)	
B	(2 , 5)	
C	(3 , 4)	
D	(4 , 2)	
E	(4 , 4)	
F	(4 , 4)	

By repeating evaluation, selection, crossover, and replacement, the optimum can be found.

Mutation: Generate a new child by introducing the property which the parents don't have.

Stopping Criteria:

- Stop optimization after repeating the generation by a certain number.
  + Assumption that the solution will converge if the generation is repeated by a certain number.
- Stop optimization by check if most of individuals in the population (e.g., 70%) are almost same or not.

Example of Finding an Optimum by Using GA (4/5)

Minimize $f(\mathbf{x}) = x_1^2 + x_2^2 - 8x_1 - 8x_2$

3rd Generation Evaluation

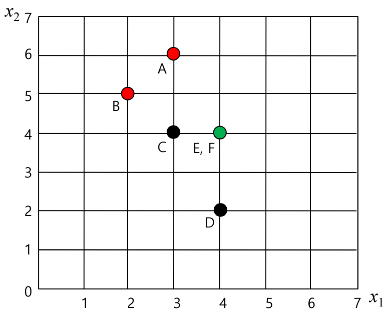
A	(3 , 6)	-27
B	(2 , 5)	-27
C	(3 , 4)	-31
D	(4 , 2)	-28
E	(4 , 4)	-32
F	(4 , 4)	-32

Replacement

Parents: (4 , 4) ↕ (4 , 4)

Crossover

Child: E' (4 , 4) F' (4 , 4)



4th Generation Evaluation

A	(4 , 4)	
B	(4 , 4)	
C	(3 , 4)	
D	(4 , 2)	
E	(4 , 4)	
F	(4 , 4)	

By repeating evaluation, selection, crossover, and replacement, the optimum can be found.

Stopping Criteria:

- Stop optimization after repeating the generation by a certain number.
  + Assumption that the solution will converge if the generation is repeated by a certain number.
- Stop optimization by check if most of individuals in the population (e.g., 70) are almost same or not.

Example of Finding an Optimum by Using GA (5/5)

Minimize $f(\mathbf{x}) = x_1^2 + x_2^2 - 8x_1 - 8x_2$

4th Generation Evaluation

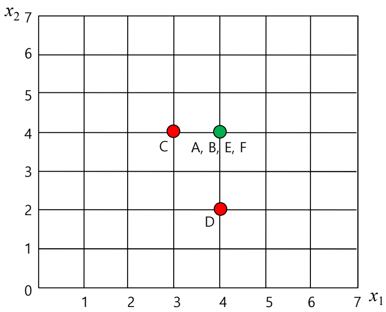
A	(4 , 4)	-32
B	(4 , 4)	-32
C	(3 , 4)	-31
D	(4 , 2)	-28
E	(4 , 4)	-32
F	(4 , 4)	-32

Replacement

Parents: (4 , 4), (4 , 4)

Crossover

Child: E' (4 , 4), F' (4 , 4)



5th Generation Evaluation

A	(4 , 4)	
B	(4 , 4)	
C	(4 , 4)	
D	(4 , 4)	
E	(4 , 4)	
F	(4 , 4)	

By repeating evaluation, selection, crossover, and replacement, the optimum can be found.

Stopping Criteria:

- Stop optimization after repeating the generation by a certain number.
  + Assumption that the solution will converge if the generation is repeated by a certain number.
- Stop optimization by check if most of individuals in the population (e.g., 70) are almost same or not.

Finish ➡ Optimum is (4, 4).

Differences from Other Search Techniques

☑ **Search from a population of points ("individuals"), not a single point.**

☑ **Use probabilistic (and not deterministic) transition rules and thus can be considered to be randomized algorithms.**

☑ **Use payoff information (fitness value) from an objective function, not intermediate information (such as derivatives or domain knowledge).**

☑ **Work with a coding of the parameter set (variables), not the parameters themselves.**

7.2 Generals

Benefits of Genetic Algorithms

- ☑ Concept is easy to understand.
- ☑ Modular, separate from application
- ☑ Support multi-objective optimization
- ☑ Good for noisy environments (hard problem to be solved)
- ☑ Always an answer; answer gets better with time
- ☑ Inherently parallel; easily distributed
- ☑ Many ways to speed up and improve a GA-based application as knowledge about problem domain is gained
- ☑ Easy to exploit previous or alternate solutions
- ☑ Flexible building blocks for hybrid applications
- ☑ Substantial history and range of use
- ☑ No gradient information needed for objective function and constraint

When to Use Genetic Algorithms

- ☑ Alternate solutions are too slow or overly complicated.
- ☑ Need an exploratory tool to examine new approaches.
- ☑ A problem is similar to one that has already been successfully solved by using a GA.
- ☑ Want to hybridize with an existing solution.
- ☑ Benefits of the GA technology meet key problem requirements.

Applications Genetic Algorithms in Various Fields

- ☑ Combinatorial optimization: Set covering, travelling salesman, routing (e.g., piping), bin packing, graph coloring and partitioning
- ☑ Design: Layout design (e.g., compartment layout, topside layout), aircraft design, keyboard configuration, communication networks
- ☑ Control: Gas pipeline, pole balancing, missile evasion, pursuit
- ☑ Scheduling: Manufacturing, facility scheduling (e.g., gantry crane, transporter), resource allocation
- ☑ Machine learning: Designing neural networks, improving classification algorithms, classifier systems
- ☑ Robotics: Trajectory planning
- ☑ Signal processing: Filter design
- ☑ Game playing: Poker, checkers, prisoner's dilemma

Performance Improvement of Genetic Algorithms

- ☑ Selection of Suitable Genetic Operators
  - Representation of chromosome: Binary-string coding vs. decimal coding
  - Selection: fitness proportionate Selection (roulette wheel selection) vs. tournament selection
  - Crossover, mutation
  - Application of elitism: Conserving superior individuals in the population without the replacement of them
- ☑ Selection of Suitable Optimization Parameters
  - Population no: Number of individuals in the population
  - Maximum generation or iteration no: How many iterations should be made for finding optimum
  - Crossover probability
  - Mutation probability
- ☑ It needs many trials and errors to select suitable genetic operations and optimization parameters for the corresponding problem.

Some Issues for Genetic Algorithms

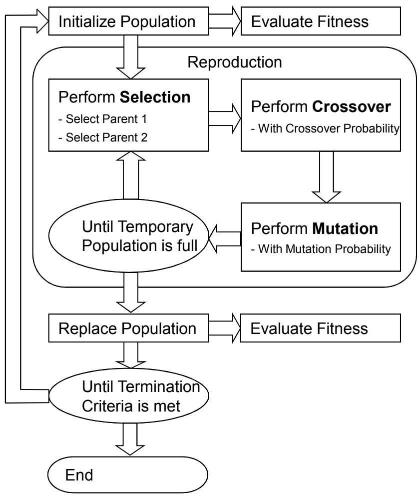
- ☑ Choosing Basic Implementation Issues
  - Representation of chromosome
  - Population size, crossover probability, mutation probability, ...
  - Genetic operators: Selection, crossover, mutation
- ☑ Termination Criteria
  - Number of generations
  - Convergence ratio
- ☑ Performance and Scalability
  - Objective function and Fitness value
- ☑ Optimum is only as good as the evaluation function (often hardest part). That is, the better evaluation function, the better optimum.

7.3 Cycles of Genetic Algorithms

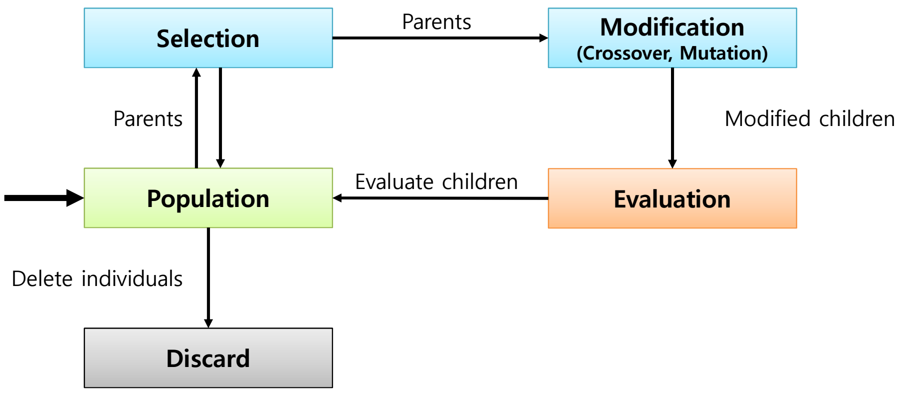
Pseudo Code for General Genetic Algorithms

```
Initialize population;

Evaluate population;

while StoppingCriteriaNotSatisfied
{
        Select parents for reproduction;
        Perform Crossover;
        Perform Mutation;
        Evaluate population;
        Replace population;
}
```

Cycle of Genetic Algorithms (Overview)



Reproduction Cycle of Genetic Algorithms



* W. Williams, Genetic Algorithms: A Tutorial, 1995

Population

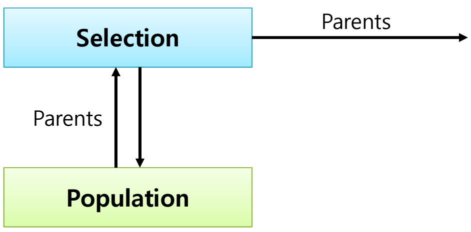
☑ Representation of chromosome

- Bit strings (Binary-string coding): e.g., 0101 … 1100
- Real numbers (Decimal coding): e.g., 37.1 0.0 … -19.4
- Permutations of element: e.g., S12 S7 … S9)
- Lists of rules: e.g., R1 R2 R3 … R22 R23
- Program elements: Tree structure (genetic programming)
- Any data structure…

Selection

☑ Parents are selected at random with selection chances biased in relation to chromosome evaluations.

☑ The selected parents are subjected to be used for generating new child by modification operators such as crossover and mutation.



Selection Operator

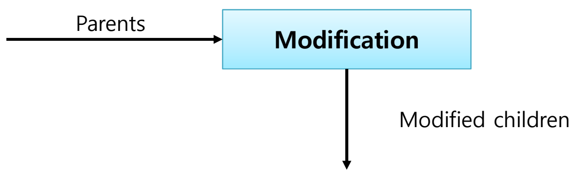
☑ Genetic operator used in genetic algorithms for **selecting potentially useful solutions for crossover**

☑ Some Selection Operators
- Fitness proportionate selection (roulette wheel selection)
- Tournament selection
- Stochastic universal sampling
- Reward-based selection

Modification

☑ After parents are selected, modifications for the parents are stochastically triggered.

☑ Operators for Modification
- Crossover
- Mutation



Crossover Operator

☑ **Genetic operator which is a critical feature of genetic algorithms**
☑ **Greatly accelerate search early in evolution of a population.**
☑ **Lead to effective combination of schemata (subsolutions on different chromosomes)**

	Before		**After**	
Parents				Children
P_1	(0 1 1 0 1 0 0 0)	➡	(0 1 0 0 1 0 0 0)	C_1
P_2	(1 1 0 1 1 0 1 0)		(1 1 1 1 1 0 1 0)	C_2

Example of binary-string coding

Mutation Operator

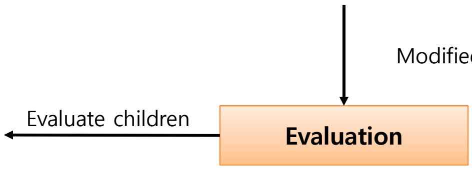
☑ **Cause local or global movement in the search space.**
☑ **Restore lost information to the population.**

	Before		**After**	
C_1	(0 1 0 0 1 0 0 0)	➡	(0 1 0 1 1 0 0 0)	C_1'

Example of binary-string coding

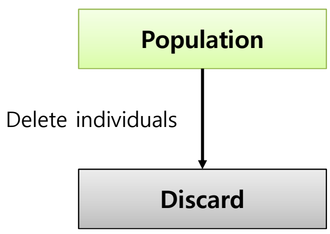
Evaluation

- ☑ The evaluator decodes a chromosome and assigns it a fitness measure.
- ☑ The evaluator is the only link between genetic algorithms and the problem to be solved.



Deletion

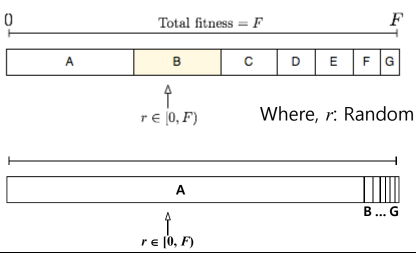
- ☑ Genetic operator for updating the current population
- ☑ In conventional (generational) GA, entire populations replaced with each iteration.
- ☑ In steady-state GA, a few members replaced each generation.



7.4 Genetic Operators

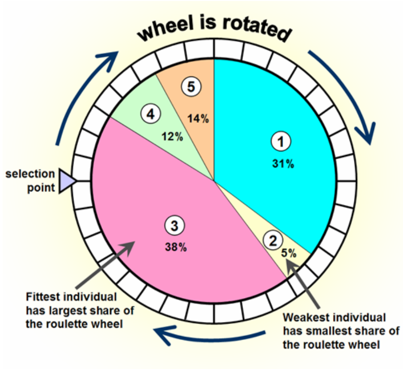
Selection Operator - Fitness Proportionate Selection (Roulette Wheel Selection) (1/2)

- ☑ The individuals are mapped to contiguous segments of a line, such that each individual's segment is equal in size to its fitness.
  ➡ The better fitness, the longer size.
- ☑ A random number is generated and the individual whose segment spans the random number is selected.
- ☑ The process is repeated until the desired number of individuals is obtained (called mating population).
- ☑ This technique is analogous to a roulette wheel with each slice proportional in size to the fitness.



Where, r: Random value

Selection Operator
- Fitness Proportionate Selection (Roulette Wheel Selection) (2/2)



$$p_{selection}(i) = \frac{Ft(i)_i}{\sum_{j=1}^{N} Ft(i)_i} \quad \Rightarrow \quad p_{selction}(i) \propto Ft(i)$$

where,
$Ft(i)$: Fitness of individual i in the population
N: Number of individuals in the population
$P_{iselection}(i)$: Probability of being selected of individual i

* Engineering Design Center, Newcastle University

Fitness Function

☑ A function to measure the quality of each individual

☑ It can be made from an objective function of the problem to be solved.

☑ In the case of a constrained optimization problem, the penalty function method can be used.

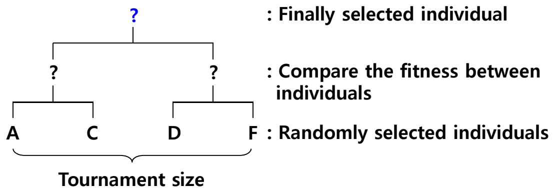
$$Ft(i) = M - F(i) \quad or \quad Ft(i) = \frac{1}{F(i)}, \; if \;\; F(i) > 0$$

$$F(i) = f(i) + \sum_{j=1}^{q} R_j \cdot \max\left\{g_j(i), 0\right\}$$ ➡ Penalty Function Method

where,
M: Large number
$f(i)$: Objective function value of individual i in the population
$g_j(i)$: j^{th} constraint value of individual i in the population
R_j: Penalty factor or penalty parameter

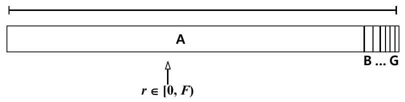
Selection Operator
- Tournament Selection

- ☑ Tournament selection involves running several tournaments among a few individuals chosen at random from the population.
- ☑ The winner of each tournament (the one with the best fitness) is selected for crossover.
- ☑ Selection pressure is easily adjusted by changing the tournament size.
- ☑ If the tournament size is larger, weak individuals have a smaller chance to be selected.
- ☑ It is efficient to code, works on parallel architectures and allows the selection pressure to be easily adjusted.



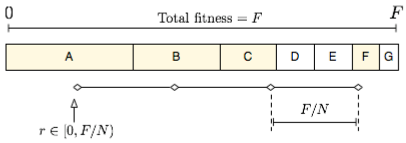
Selection Operator
- Stochastic Universal Sampling (SUS) (1/2)

- ☑ SUS is a development of fitness proportionate selection (FPS) which exhibits no bias and minimal spread.
- ☑ Where the fitness proportionate selection chooses several individuals from the population by repeated random sampling, SUS uses a single random value to sample several individuals by choosing them at evenly spaced intervals.
- ☑ This gives weaker members of the population (according to their fitness) a chance to be chosen and thus reduces the unfair nature of fitness-proportional selection methods.
- ☑ Other methods like roulette wheel can have bad performance when an individual of the population has a really large fitness in comparison with other individuals.



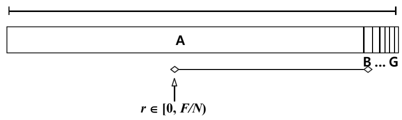
Selection Operator
- Stochastic Universal Sampling (SUS) (2/2)

☑ **Using a comb-like ruler, SUS starts from a small random number, and chooses the next candidates from the rest of population remaining, not allowing the fittest members to saturate the candidate space.**



Where,
r: Random value
N: Number of individuals to select. In this figure, N = 4.

Example for N = 2



Selection Operator
- Reward-based Selection

☑ **The probability of being selected for an individual is proportional to the cumulative reward from the fitness function, obtained by the individual.**

☑ **The cumulative reward can be computed as a sum of the individual reward and the reward, inherited from parents.**

$$p_{selection}(i) = \frac{Ft(i)_i}{\sum_{j=1}^{N} Ft(i)_i} \quad \Rightarrow \quad p_{selction}(i) \propto Ft(i)$$

$$Ft(i) = Ft(i) + \sum Ft(parent_i)$$

where,
$Ft(i)$: Fitness of individual i in the population
$Ft(parent_i)$: Fitness of individual i in the population, inherited from parents
N: Number of individuals in the population
$P_{iselection}(i)$: Probability of being selected of individual i

Crossover Operator
- Order 1 Crossover (1/2)

- ☑ Order 1 crossover is a fairly simple permutation crossover.
- ☑ Basically, a swath of consecutive genes from Parent 1 drops down, and remaining values are placed in the child in the order which they appear in Parent 2.

- ☑ Performance
  - Order 1 crossover is perhaps the fastest of all crossover operators because it requires virtually no overhead operations.
  - On a generation by generation basis, edge recombination typically outperforms Order 1, but the fact that Order 1 runs between 100 and 1000 times faster usually allows the processing of more generations in a given time period.

Crossover Operator
- Order 1 Crossover (2/2)

- ☑ General Steps
  - Step 1: Select a random swath of consecutive genes from Parent 1. (red box)
  - Step 2: Drop the swath down to Child 1 and mark out these genes in Parent 2.
  - Step 3: Starting on the right side of the swath, grab genes from Parent 2 and insert them in Child 1 at the right edge of the swath. Since 5 is in that position in Parent 2, it is inserted into Child 1 first at the right edge of the swath. Notice that genes 7, 2, 1, and 6 are skipped because they are marked out and 8 is inserted into the 1st spot in Child 1.
  - Step 4: If you desire a second child from the two parents, flip Parent 1 and Parent 2 and go back to Step 1.

	P_1	(3	5	7	2	1	6	4	8)
	P_2	(~~2~~	4	~~1~~	~~6~~	8	3	5	~~7~~)
	C_1	(8	3	7	2	1	6	5	4)
Other variation:	C_1	(4	8	7	2	1	6	3	5)

Crossover Operator
- Order Multiple Crossover

☑ **This crossover is identical to the Order 1 Crossover, except multiple swaths are participants in the genetic exchange.**

P_1 (3 $\boxed{5 \quad 7 \quad 2}$ 1 $\boxed{6 \quad 4}$ 8)
P_2 ($\cancel{2}$ $\cancel{4}$ 1 $\cancel{6}$ 8 3 $\cancel{5}$ $\cancel{7}$)

C_1 (1 $\boxed{5 \quad 7 \quad 2}$ 8 $\boxed{6 \quad 4}$ 3)

Crossover Operator
- Cycle Crossover (1/2)

☑ **The cycle crossover operator identifies a number of so-called cycles between two parent chromosomes. Then, to form Child 1, cycle one is copied from Parent 1, cycle 2 from Parent 2, cycle 3 from Parent 1, and so on. Two child can be made at the same time.**

☑ **General Steps**

- **Step 1: Identify all cycles.**

P_1 (3 5 7 2 1 6 4 8)

P_2 (2 4 1 6 8 3 5 7)

Cycle 1 values: **3 2 6**

Cycle 2 values: **5 4**

Cycle 3 values: **7 1 8**

Crossover Operator
- Cycle Crossover (2/2)

☑ **General Steps (continued)**
- **Step 2: Copy alternate cycles to children.**

P_1	(3	5	7	2	1	6	4	8)
P_2	(2	4	1	6	8	3	5	7)

Cycle 1 values: **3 2 6**
Cycle 2 values: **5 4**
Cycle 3 values: **7 1 8**

C_1	(3	4	7	2	1	6	5	8)
C_2	(2	5	1	6	8	3	4	7)

Copy Cycle 1: Cycle 1 values from Parent 1 and copied to Child 1, and values from Parent 2 will be copied to Child 2. Cycle 2 will by different.

Copy Cycle 2: Cycle 2 values from Parent 1 will be copied to Child 2, and values from Parent 2 will be copied to Child 1.

Copy Cycle 3: Cycle 3 is like Cycle 1, Parent 1 goes to Child 1, Parent 2 goes to Child 2.

Crossover Operator
- Edge Recombination (1/4)

☑ **This is a crossover techniques for permutation (ordered) chromosomes.**

☑ **It strives to introduce the fewest paths possible.**

☑ **In problems such as the traveling salesman, introducing a stray edge between two nodes is usually very bad for a chromosome's fitness.**

☑ **The idea here is to use as many existing edges, or node-connections, as possible to generate children.**

☑ **Edge recombination typically outperforms PMX and Ordered crossover, but usually takes longer to compute.**

Crossover Operator
- Edge Recombination (2/4)

☑ **General Steps**

- **Step 1: Generate neighbor list.**

P_1 (A B F E D G C)
P_2 (G F A B C D E)

Neighbor list of A: **B C F**

P_1 (A → B F E D G C)
P_2 (G F ← A → B C D E)

Neighbor lists:

A: B C F B: A F C C: G A B D
D: E G C E: F D G F: B E G A
G: D C E F

Crossover Operator
- Edge Recombination (3/4)

P_1 (A B F E D G C)
P_2 (G F A B C D E)

Neighbor lists:
A: B C F / B: A F C / C: G A B D
D: E G C / E: F D G / F: B E G A / G: D C E F

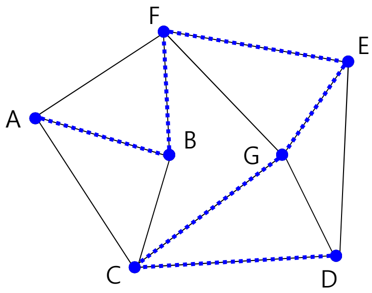
☑ **General Steps (continued)**

- **Step 2: Generate a child.**
  - **First, we randomly select the first node of a parent. Child C_1: A**
  - **Next, after crossing A out from all neighbor lists, we see that B is the least full list. So, Child C_1: A B**
  - **Next, after crossing B out from all neighbor lists, F and C both have only 2 neighbors, so we randomly choose between the two: Child C_1: A B F**
  - **Next, after crossing F out from all neighbor lists, E is the neighbor of F that has the fewest neighbors. Child C_1: A B F E**
  - **Next, after crossing E out from all neighbor lists, D and G both have only 2 neighbors, so we randomly choose between the two: Child C_1: A B F E G**
  - **Next, after crossing G out from all neighbor lists, D and C both have only 1 neighbor, so we randomly choose between the two: Child C_1: A B F E G C**
  - **Next, after crossing C out from all neighbor lists, D has only one neighbor and it has no neighbors left, so we randomly choose between the nodes that aren't yet included in the child. In this case, D is the only one left, so we're done: Child C_1: A B F E G C D**
  - **The child that we produced introduced only one new edge: A to D. This algorithm makes excellent use of existing edges and is much less likely to introduce stray edges during crossover.**

Crossover Operator
- Edge Recombination (4/4)

Neighbor lists:
A: B C F / B: A F C / C: G A B D
D: E G C / E: F D G / F: B E G A / G: D C E F

Adjacency graph



Process 1: Child C_1: **A**

Process 2: Child C_1: **A B**

Process 3: Child C_1: **A B F**

Process 4: Child C_1: **A B F E**

Process 5: Child C_1: **A B F E G**

Process 6: Child C_1: **A B F E G C**

Process 7: Child C_1: **A B F E G C D**

P_1 (A B F E D G C)
P_2 (G F A B C D E)
➡ C_1 (A B F E G C D)

Crossover Operator
- PMX (Partially Mapped Crossover) (1/3)

☑ **PMX crossover is a genetic algorithm operator for crossover.**
☑ **For some problems it offers better performance than most other crossover techniques.**
☑ **Basically, Parent 1 donates a swath genetic material and the corresponding swath from the other parent is sprinkled about in the child.**
☑ **Once that is done, the remaining genes are copied direct from Parent 2.**

Crossover Operator
- PMX (Partially Mapped Crossover) (2/3)

☑ **General Steps**

- **Step 1: Randomly select a swath of genes from Parent 1 and copy them directly to the child. Note the indexes of the segment.**
- **Step 2: Looking in the same segment positions in Parent 2, select each value that hasn't already been copied to the child. For each of these values:**
  - **Step 2-1: Note the index of this value in Parent 2. Locate the value from Parent 1 in this same position.**
  - **Step 2-2: Locate this same value in Parent 2.**
  - **Step 2-3: If the index of this value in Parent 2 is part of the original swath, go to Step 2-1 using this value.**
  - **Step 2-4: If the position isn't part of the original swath, insert Step 1's value into the child in this position.**
- **Step 3: Copy any remaining positions from Parent 2 to the child.**

Crossover Operator
- PMX (Partially Mapped Crossover) (3/3)

☑ **Example**

1. We copy a random swath of consecutive genes from Parent 1 to the Child.

P_1 (3 5 [7 2 1 6] 4 8)
P_2 (2 4 1 6 8 3 5 7)
C_1 ( )

2. '8' is the first value in the swath of Parent 2 that isn't in the child. We identify '1' as the value in the same position in Parent 1. We locate the value '1' in Parent 2 and notice that it is still in the swath. So, we go back to Step 2-1 using '1' as the value.

3. Repeating Step 2-1: Once again, we see that '7' is in the same position in Parent 1, and we locate '7' in Parent 2 in the last position. Finally, we have obtained a position in the child for the value '8' from Step 2.

P_1 (3 5 [7 2 1 6] 4 8)
P_2 (2 4 1 6 8 3 5 7)
C_1 (7 2 1 6)

4. '3' is the next value in the swath in Parent 2 that isn't already included in the child. So, we check the same index in Parent 1 and see a '6' in that position. Next, we check for '6' in Parent 2 and notice that it is still in the swath. So, we go back to Step 2-1 using '6' as the value.

5. Repeating Step 2-1: Once again, we see that '2' is in the same position in Parent 1, and we locate '2' in Parent 2 in the first position. Finally, we have obtained a position in the child for the value '3' from Step 2.

P_1 (3 5 [7 2 1 6] 4 8)
P_2 (2 4 1 6 8 3 5 7)
C_1 (7 2 1 6 8)

6. Now the easy part, we've taken care of all swath values, so everything else from Parent 2 drops down to the child.

P_1 (3 5 [7 2 1 6] 4 8)
P_2 (2 4 1 6 8 3 5 7)
C_1 (3 7 2 1 6 8)

If we wish to create a 2nd child with the same set of parents, simply swap the parents and start over.

Mutation Operator
- Inversion Mutation Operator

☑ General Steps
- Step 1: Determine start and stop of swath.
- Step 2: Put the values of the swath back in reverse order.
- Step 3: Copy the values which are not in the swath.

C_1 (4 8 7 2 1 6 3 5)

C_1' (4 8 6 1 2 7 3 5)

Mutation Operator
- Insertion Mutation Operator

☑ General Steps
- Step 1: Select some genes for insertion.
- Step 2: Put the values of the genes in order.
- Step 3: Copy the values which are not selected for insertion.

C_1 (4 8 7 2 1 6 3 5)

C_1' (4 8 7 1 3 2 6 5)

Mutation Operator
- Single Swap (or Exchange) Operator

☑ General Steps
- Step 1: Select two genes to be swapped.
- Step 2: Swap two genes and put them.
- Step 3: Copy the values which are not selected for swap.

C_1 (4 8 $\boxed{7}$ 2 $\boxed{1}$ 6 3 5)

C_1' (4 8 $\boxed{1}$ 2 $\boxed{7}$ 6 3 5)

Mutation Operator
- Random Swap Operator

☑ This is like single swap, but swaps a random string of consecutive values instead.

☑ General Steps
- Step 1: Select two swaths to be swapped.
- Step 2: Swap two swaths and put them.
- Step 3: Copy the values which are not selected for swap.

C_1 (4 $\boxed{8 \quad 7}$ 2 $\boxed{1 \quad 6}$ 3 5)

C_1' (4 $\boxed{1 \quad 6}$ 2 $\boxed{8 \quad 7}$ 3 5)

Mutation Operator
- Scramble Mutation Operator

☑ General Steps
- Step 1: Determine start and stop of swath.
- Step 2: Scramble and put the values of the swath back.
- Step 3: Copy the values which are not in the swath.

C_1 (4 8 $\boxed{7\ 2\ 1\ 6}$ 3 5)

C_1' (4 8 $\boxed{2\ 6\ 7\ 1}$ 3 5)

Mutation Operator
- Random Slide (or Displacement) Mutation Operator

☑ General Steps
- Step 1: Determine start and stop of swath.
- Step 2: Determine the length of the slide.
- Step 3: Calculate new location of swath after slide.
- Step 4: Put the values of the swath back.
- Step 5: Copy the values which are not in the swath.

C_1 (4 8 $\boxed{7\ 2\ 1}$ 6 3 5)

Slide length = 3

C_1' (4 8 6 3 5 $\boxed{7\ 2\ 1}$)

Mutation Operator
- Displaced Inversion Mutation Operator

- ☑ This mutation operator is a combination of "Inversion" and "Random Slide" operators.
- ☑ General Steps
  - Step 1: Determine start and stop of swath.
  - Step 2: Determine the length of the slide.
  - Step 3: Calculate new location of swath after slide.
  - Step 4: Put the values of the swath back in reverse order.
  - Step 5: Copy the values which are not in the swath.

C_1 (4 8 7 2 1 6 3 5)

Slide length = 3

C_1' (4 8 6 3 5 1 2 7)

7.5 Example of Traveling Salesman Problem

Traveling Salesman Problem (TSP)

☑ A problem for finding a tour of a given set of cities so that
- each city is visited only once
- the total distance traveled is minimized

* W. Williams, Genetic Algorithms: A Tutorial, 1995

Representation of Chromosome for TSP

☑ Representation is an ordered list of city numbers known as an order-based GA.

☑ Example of City Numbers
- 1: Seoul, 2: Tokyo, 3: Beijing, 4: Washington, 5: London, 6: Ottawa, 7: Canberra, 8: New Delhi

CityList1	(3	5	7	2	1	6	4	8)
CityList2	(2	4	1	6	8	3	5	7)

Crossover Operator for TSP

- ☑ Crossover combines inversion and recombination.
- ☑ "Order 1" crossover can be used in this example.

P_1	(3	5	7	2	1	6	4	8)
P_2	(~~2~~	4	~~1~~	~~6~~	8	3	5	~~7~~)
C_1	(8	3	7	2	1	6	5	4)

Mutation Operator for TSP

- ☑ Mutation involves reordering of the list.

Before	C_1	(8	3	7	2	1	6	5	4)
After	C_1'	(8	3	6	2	1	7	5	4)

7.6 Multi-Objective Genetic Algorithms

Classification of Optimization Problems According to Number of Objective Functions

☑ Single-objective Optimization Problem
- The problem has only one objective function.
- The problem has only a single (global) optimum.
- The optimum can be maximum or minimum according to the type of objective function.

☑ Multi-objective Optimization Problem
- The problem has at least two or more objective functions
- The problem has not a single optimum but multiple optima called Pareto optimal set.
- The optima can be subdivided into dominant solution and non-dominant solution.
  - Dominant solution: When objective function value of the solution is better than those of others
  - Non-dominant solution: When objective function value of the solution can not be improved without the increase of those of others. It is also called Pareto optimal set or Pareto front or Pareto frontier.

General Solving Method of Multi-Objective Problem - Weighting Method (1/2)

☑ A method for finding optimum after **transforming multi-objective problem into single problem by multiplying objective functions by weight factors**

☑ Various optima (**Pareto optimal set**) can be found according to weight factors for objective functions.

$$\textit{Minimize} \quad f(x) = w_1 f_1(x) + w_2 f_2(x) + w_3 f_3(x) + \cdots$$

w_n: weight factors
f_n: Objective functions

☑ Limitations
- Optimum can change according to weight factors.
- In some cases for the selected weight factors, optimum can not be found.
- It requires much computing resources to find optima for all weight factors.
- It is difficult to find a suitable combination of weight factors.

General Solving Method of Multi-Objective Problem - Weighting Method (2/2)

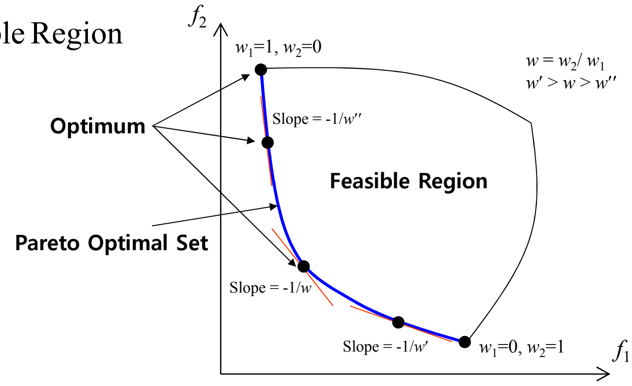
☑ Example of Multi-objective Problem Having 2 Objective Functions

Minimize

$$f(x_1,\cdots,x_n;w) = w_1 f_1(x_1,\cdots,x_n) + w_2 f_2(x_1,\cdots,x_n)$$

Subject to

$$(x_1,\cdots,x_n) \in \text{Feasible Region}$$



General Solving Method of Multi-Objective Problem - Constraint Method (1/2)

☑ A method for performing optimization by regarding the other objective functions except for one as additional constraints of the problem to be solved

☑ Various optima (Pareto optimal set) can be found according to limiting values for the additional constraints.

☑ Limitations
- It is difficult to find optimum when the problem has many objective functions due to the increase of additional constraints.

General Solving Method of Multi-Objective Problem - Constraint Method (2/2)

☑ Example of Multi-objective Problem Having 2 Objective Functions

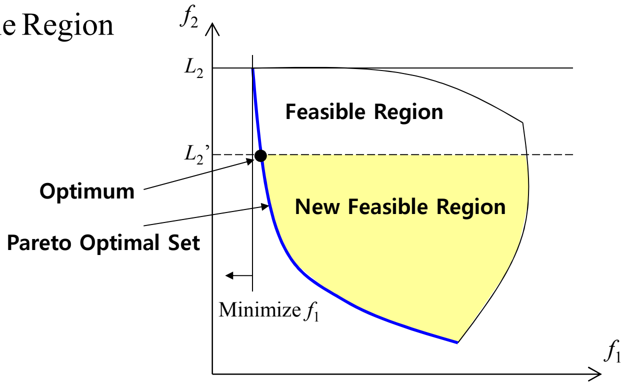
Minimize

$$f(x_1,\cdots,x_n) = f_1(x_1,\cdots,x_n)$$

Subject to

$$(x_1,\cdots,x_n) \in \text{Feasible Region}$$

$$f_2(x_1,\cdots,x_n) \le L_2$$



Multi-Objective GA (MOGA) (1/3)

☑ **Most optimization problems in real world can not be represented with one objective function, but with various objective functions which are opposed to each other (e.g., Maximize strength vs. Minimize cost).**
➡ Multi-objective optimization problem

☑ **It is called genetic algorithms for solving multi-objective optimization problems.**

☑ **In 1985, it was developed by Schaffer and is being widely used in various fields.**

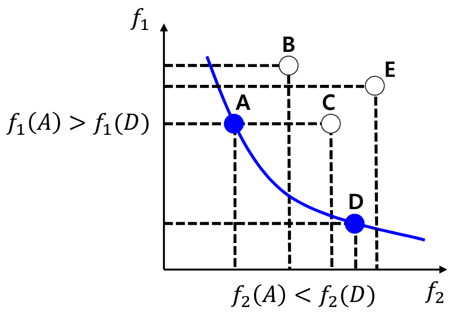
Multi-Objective GA (MOGA) (2/3)

Dominant Solution

- When objective function value of the solution is better than those of others
- A ⇔ B, C, E or D ⇔ E

Non-dominant Solution

- When objective function value of the solution can not be improved without the increase of those of others
- It is also called Pareto optimal set or Pareto front or Pareto frontier.
- A ⇔ D



- ✓ **Point B is dominated by point A. (The values of f_1 and f_2 are all small.)**
- ✓ **Point C is dominated by point A. (The value of f_1 is same and the value of f_2 is small.)**
- ✓ **Point E is not on the Pareto Frontier because it is dominated by both point A and point D.**
- ✓ **Points A and D are not strictly dominated by any other, and hence do lie on the frontier.**
- ✓ **A and D are called 'Pareto Optimal Set'.**

Multi-Objective GA (MOGA) (3/3)

☑ MOGA = Multi-Objective optimization problem + Genetic Algorithms

☑ Various methods have been developed to find Pareto optimal set based on genetic algorithms.

Multi-objective optimization problem ⇐ Genetic algorithms

⇓

Multi-Objective Genetic Algorithms ⇒

NPGA (Niched Pareto Genetic Algorithm)

PAES (Pareto Archived Evolution Strategy)

NSGA (Non-dominated Sorting Genetic Algorithm)

⋮