

# Reactors II

# Today's lecture

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- Reactor analysis (continued)
  - Plug flow reactor
  - Completely mixed flow reactor
  - Detention time

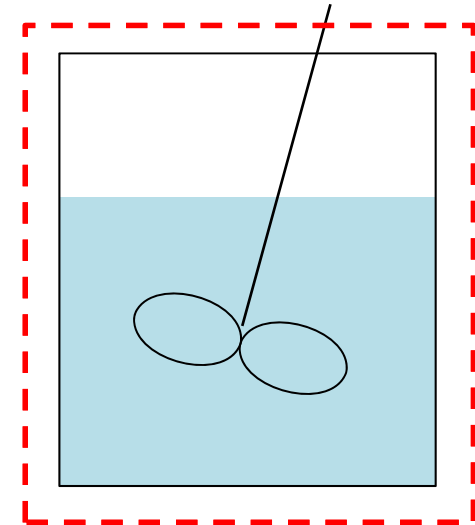
# Review – CMBR, 1<sup>st</sup> order reaction

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- **Completely mixed batch reactor (CMBR)**

- Fill-and-draw type
- No flow in or flow out

- 1) define control volume
- 2) write a mass balance eq.



$$V \frac{dC}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + R$$

$-kCV$

# Review - CMBR, 1<sup>st</sup> order reaction

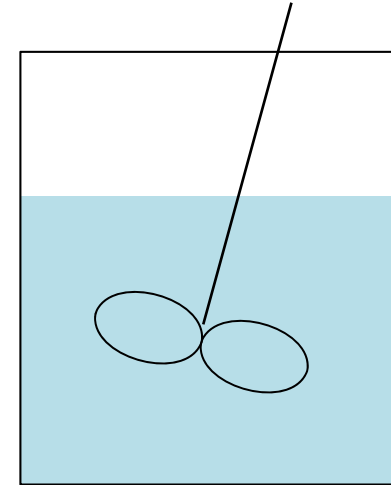
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3) solve the equation

$$\frac{dC}{dt} = -kC$$

integrating over  $t=0$  to  $t_{final}$ :

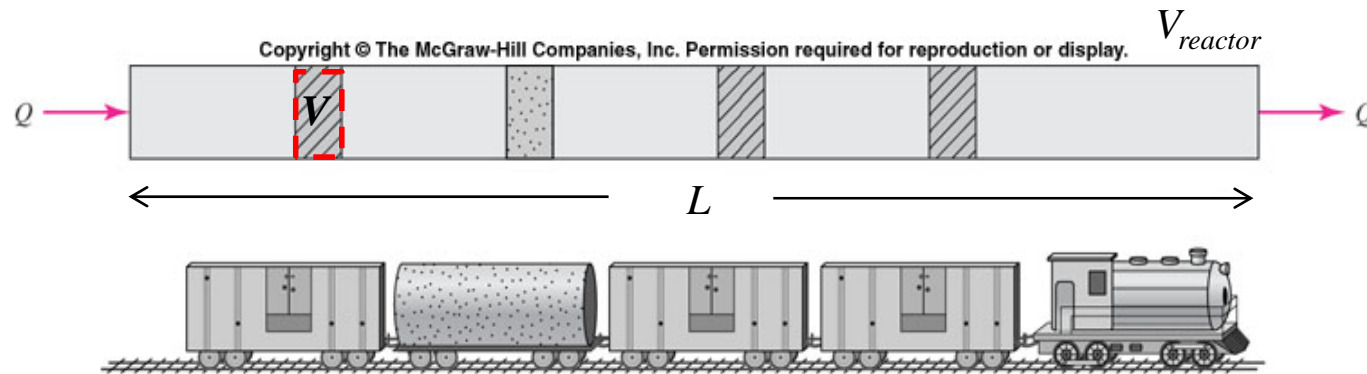
$$\frac{C_{final}}{C_{initial}} = e^{-kt_{final}}$$



# Reactor analysis – PFR, 1<sup>st</sup> order reaction

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- Plug-flow reactor (PFR)



1) define control volume:

the moving “plug”: a very thin, homogeneous plate moving in the direction of flow

# Reactor analysis – PFR, 1<sup>st</sup> order reaction

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2) write a mass balance eq.:

$$V \frac{dC}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + R$$

$-kCV$  (1<sup>st</sup> order)

3) solve the equation:  $\frac{dC}{dt} = -kC$

integrating over  $t=0$  to  $t_0$  ( $= L/v_{flow} = V_{reactor}/Q$ ):

$$\frac{C_{out}}{C_{in}} = e^{-kt_0} \rightarrow \text{same form as the batch reactor!}$$

(why??)

# Reactor analysis – PFR, 1<sup>st</sup> order reaction

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- Examples of PFRs



Disinfection



Rivers and streams

# Reactor analysis - PFR

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**Q:** In the U.S., a wastewater treatment plant must disinfect its effluent before discharging the wastewater to a stream. The wastewater contains  $4.5 \times 10^5$  CFU/L of fecal coliform. The effluent standard for fecal coliform is  $2 \times 10^3$  CFU/L. Assuming that the disinfection facility is a PFR, determine the length of pipe required if the velocity of the wastewater in the PFR is 0.75 m/s. Assume that the PFR is at steady state and the first-order reaction rate constant for destruction of the fecal coliforms is  $0.23 \text{ min}^{-1}$ .

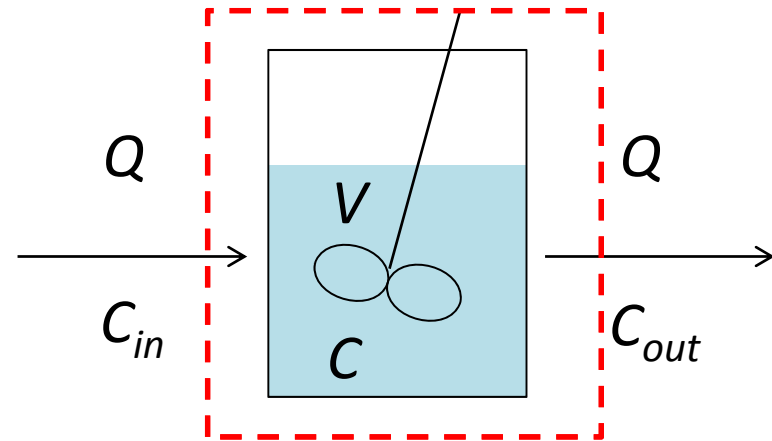


# Reactor analysis – CMFR, 1<sup>st</sup> order reaction

- Completely mixed flow reactor (CMFR)  
(continuous-flow stirred tank reactor, CSTR)

- 1) define control volume
- 2) write a mass balance eq.

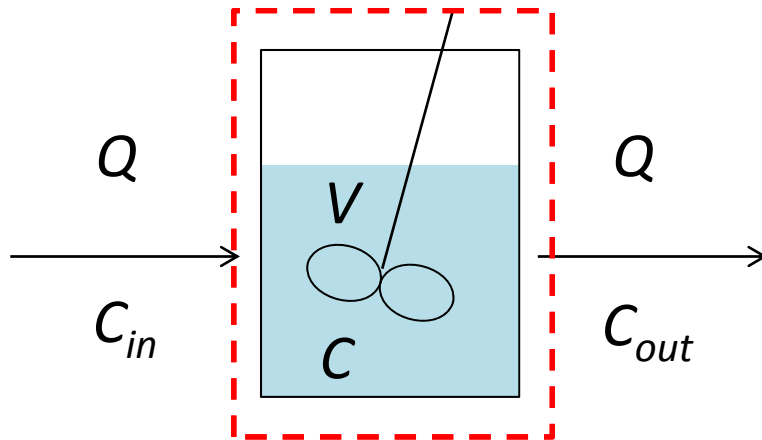
$$V \frac{dC}{dt} = Q \cdot C_{in} - Q \cdot C_{out} - kCV \text{ (1<sup>st</sup> order)}$$



# Reactor analysis – CMFR, 1<sup>st</sup> order reaction

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3) solve the equation:



Because of homogeneous mixing,  $C = C_{out}$

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

# Reactor analysis – CMFR, 1<sup>st</sup> order reaction

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$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

**Special case I:** No reaction, initial concentration =  $C_0$

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$
$$\frac{dC_{out}}{dt} = \frac{1}{t_0} (C_{in} - C_{out}) \quad (t_0 = V/Q)$$

integrating over  $t=0$  to  $t$ :

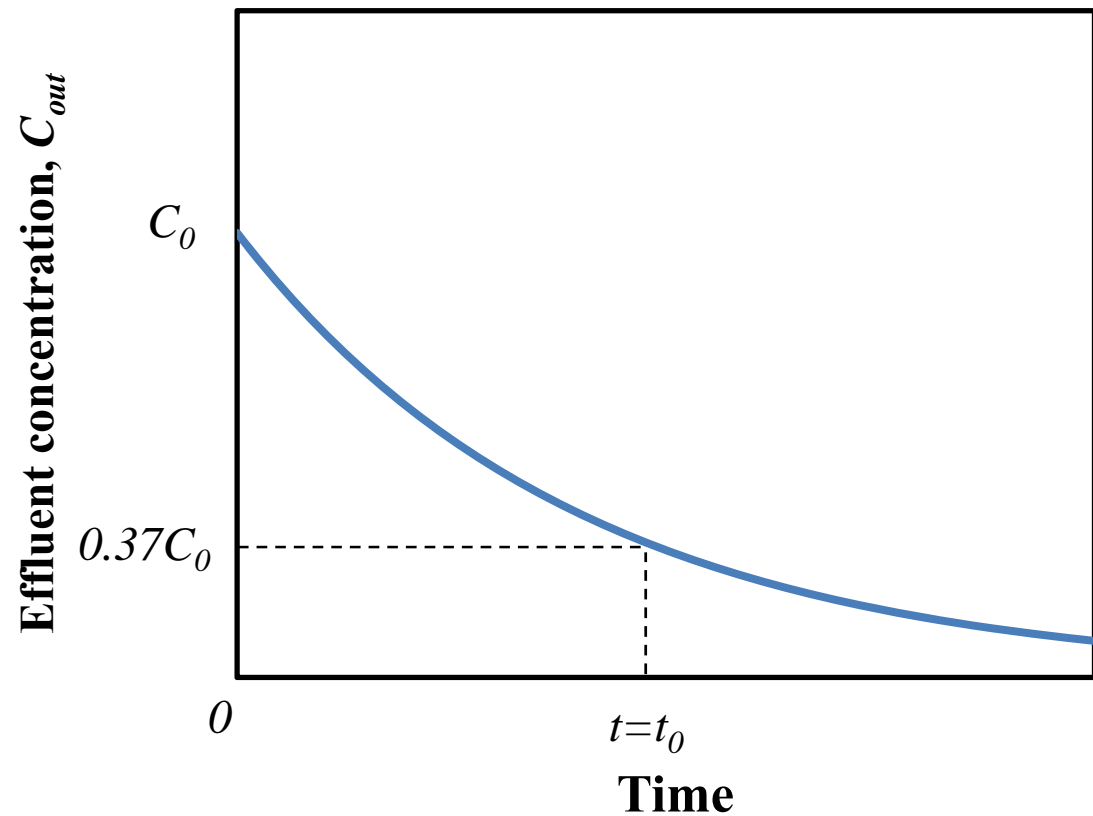
$$C_{out,t} = C_0 \left[ \exp\left(-\frac{t}{t_0}\right) \right] + C_{in} \left[ 1 - \exp\left(-\frac{t}{t_0}\right) \right]$$

# Reactor analysis – CMFR, 1<sup>st</sup> order reaction

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when  $C_{in} = 0$ ,

$$C_{out,t} = C_o \left[ \exp\left(-\frac{t}{t_0}\right) \right]$$



# Reactor analysis – CMFR, 1<sup>st</sup> order reaction

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$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

**Special case II:** Steady state

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

$$C_{out} = \frac{C_{in}}{1 + kt_0} \quad (t_0 = V/Q)$$

→ influent concentration ( $C_{in}$ ) is reduced in the effluent by a factor of  $(1+kt_0)$

# Reactor analysis – CMFR, 1<sup>st</sup> order reaction

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- Examples of CMFRs



Biological wastewater  
treatment



Lake

# Reactor analysis - CMFR

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**Q:** Activated sludge is a key process for most wastewater treatment facilities. The process is often run as a CMFR. Assume a 400 m<sup>3</sup>-sized CMFR for an activated sludge process receiving 2000 m<sup>3</sup>/d of wastewater. If a terrorist dumped 10 kg of a non-biodegradable toxic chemical to the reactor, how long will it take for the toxic chemical concentration in the reactor to a safe level (1 mg/L)?

# Reactor analysis - CMFR

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**Q:** A chemical degrades in a steady-state CMFR according to first-order reaction kinetics. The upstream concentration of the chemical is 10 mg/L and the downstream concentration is 2 mg/L. Water is being treated at a rate of 29 m<sup>3</sup>/min. The volume of the tank is 580 m<sup>3</sup>. What is the rate of decay? What is the rate constant?



# Retention time

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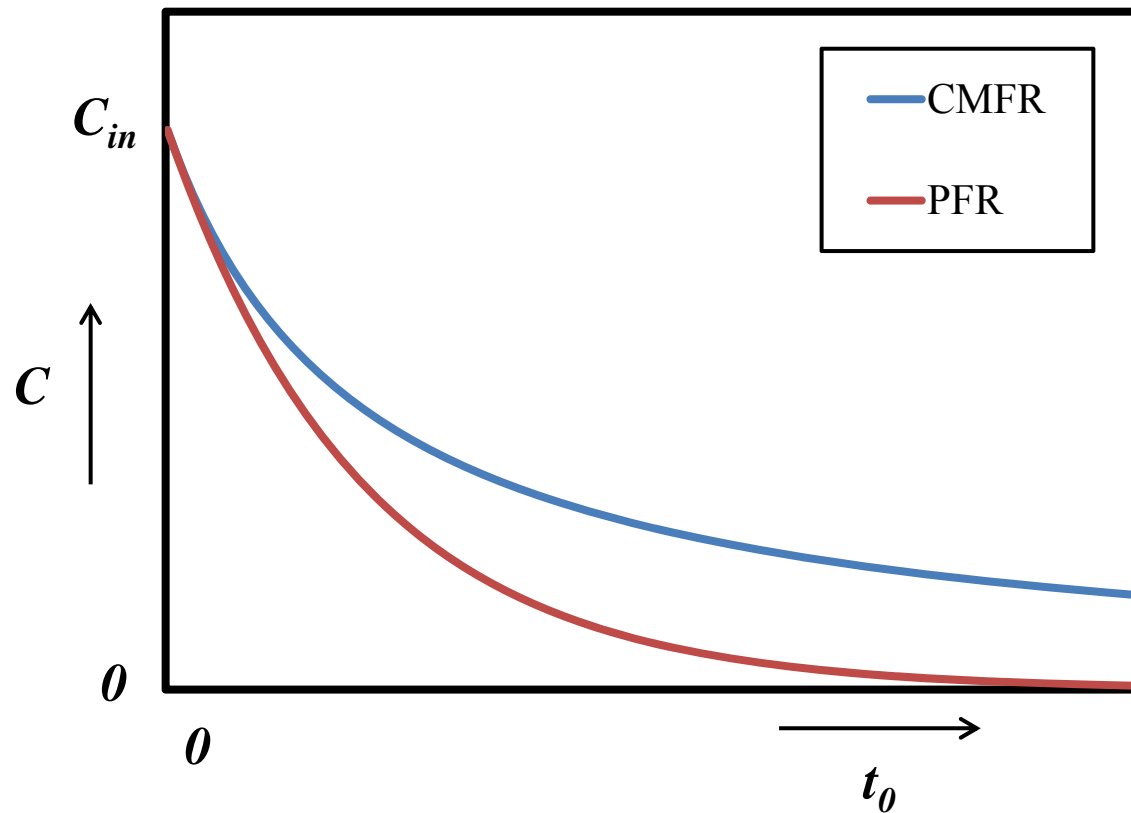
- Retention time (detention time),  $t_0$

$$t_0 = V / Q$$

- The average time the fluid particles spend in the reactor
- PFR: the time that fluid particles spend in a reactor is the same ( $=t_0$ ) for all particles
- CMFR: the time that fluid particles spend in a reactor is different

# PFR vs. CMFR

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- At the same  $t_0$ , PFR shows better performance
- Advantage of using CMFR: less sensitive to shock loads and toxic compounds

# Reading assignment

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- Textbook Ch4 p. 162-168