

# Reactors II

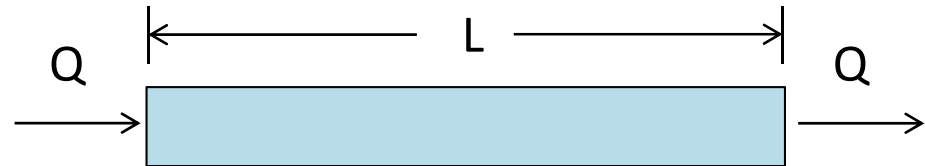
# Today's lecture

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- **Plug flow reactor**
  - Concept
  - PFR analysis for 1<sup>st</sup> order reaction
  - PFR analysis for Monod kinetics
  
- **Continuous-stirred tank reactor**
  - CSTR analysis for 1<sup>st</sup> order reaction
  - PFR vs. CSTR
  - CSTR analysis for Monod kinetics

# Reactor analysis: PFR

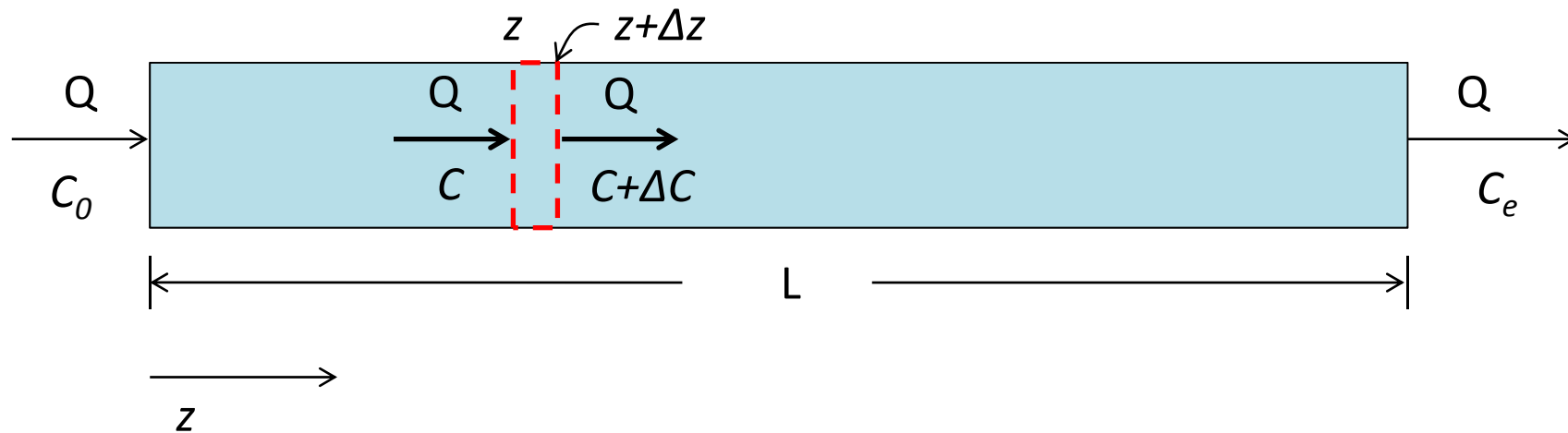
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- Plug flow reactor (PFR)
  - assumption: no mixing in the direction of flow & completely mixed in the direction perpendicular to the flow
  - reactors get close to PFR as the length gets longer than the width and depth (e.g., rivers)

# PFR, first-order reaction

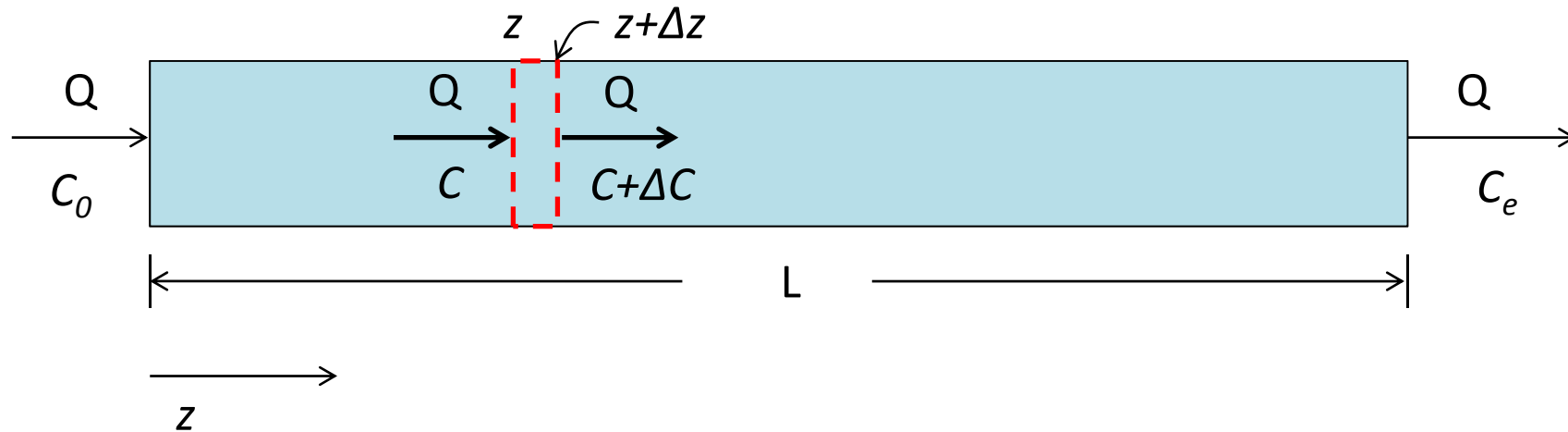
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- Take control volume as a thin plate perpendicular to the flow at  $z=z$  with a dimension of  $\Delta z$  in  $z$  dir.

# PFR, first-order reaction

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$$C/C_0 = e^{-k\theta}$$

Same form as the batch reactor (why??)

# PFR, first-order reaction

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*distance=0*

*time=0*

$$C=C_0$$

*distance=z*

*time=t*

$$C=C_0e^{-kt}$$
$$=C_0e^{-kz/u}$$

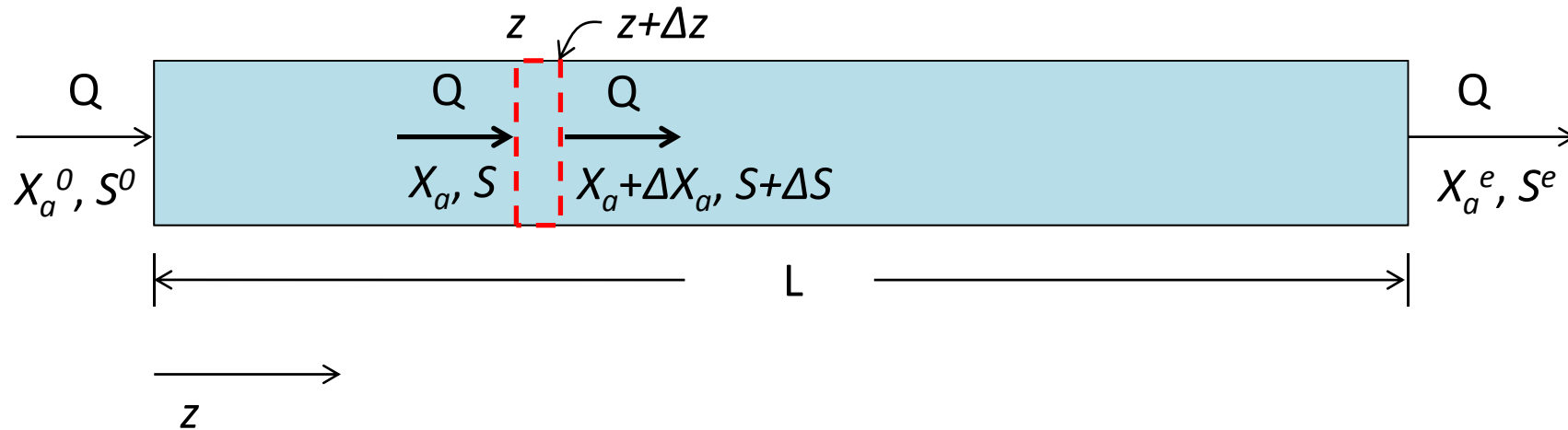
*distance=L*

*time=θ*

$$C=C_0e^{-kθ}$$
$$=C_0e^{-kL/u}$$

- We model plug flow reactor as a movement of a “plug”
- The plug has a cross sectional area same as the reactor dimension and an infinitesimal dimension in z-dir (a thin plate)
- Complete mixing within the plug → batch reactor moving in the direction of flow

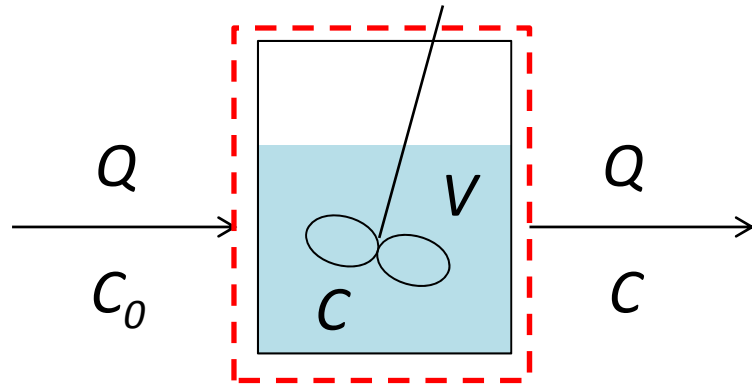
# PFR, Monod kinetics



$$u \frac{dS}{dz} = - \frac{\hat{q}S}{K + S} [X_a^0 + Y(S^0 - S)]$$

# Reactor analysis: CSTR, 1<sup>st</sup> order

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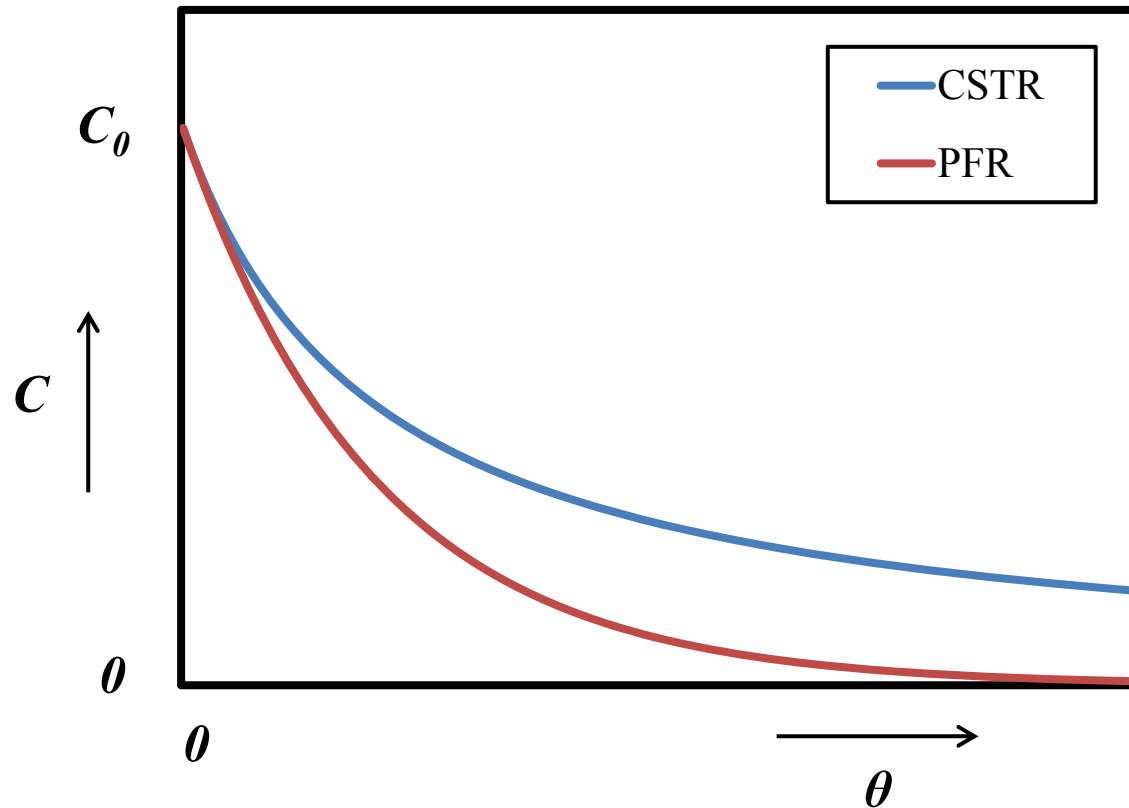
At steady state,

$$C = \frac{C_0}{1 + k\theta}$$



# PFR vs. CSTR

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PFR shows better performance esp.  
at high HRTs

For 1<sup>st</sup> order  
reaction,

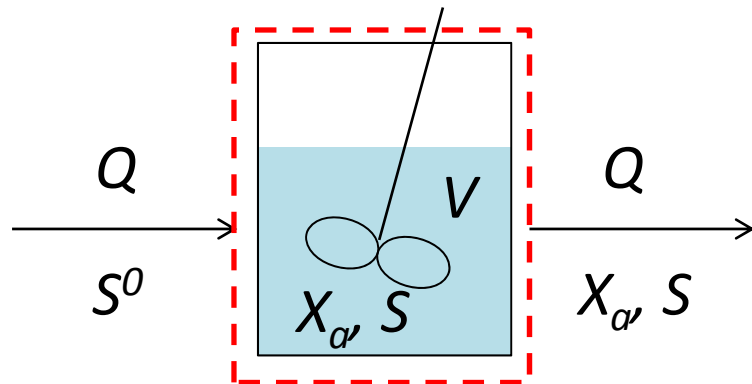
CSTR:

$$C = \frac{C_0}{1 + k\theta}$$

PFR:

$$C = C_0 e^{-k\theta}$$

# Reactor analysis: CSTR, Monod kinetics



Assumption:

- Steady state
- $X_a = 0$  in the influent  
(negligible influent biomass)

$$S = K \frac{1 + b\theta}{Y\hat{q}\theta - (1 + b\theta)}$$

No  $S_0$  or  $X_a$  in the equation!

$$X_a = Y \frac{S^0 - S}{1 + b\theta}$$