Microbial kinetics in reactors II

Today's lecture

- Nutrient & e$^-$ acceptor consumption

- Hydrolysis

- Alternate rate expressions

- Example question

Nutrient consumption

- For the consumption of nutrients for biomass production:

$$r_n = \gamma_n \cdot Y_{obs} \cdot r_{ut} = \gamma_n \cdot Y \cdot r_{ut} \frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

r_n = rate of nutrient consumption [$M_n L^{-3} T^{-1}$]

γ_n = the stoichiometric ratio of nutrient mass to VSS for biomass [$M_n M_x^{-1}$]

Using $C_5H_7O_2N$ as cell formula:

γ_N = 14 g N/113 g VSS = 0.124 g N/g VSS

γ_P = 0.2 x 0.124 = 0.025 g P/g VSS (generally P = 0.2N)

Nutrient consumption in a CSTR

- Steady-state mass balance:

$$0 = QC_n{}^0 - QC_n + r_n V$$

$$\boxed{C_n = C_n{}^0 + r_n \theta}$$

if $C_n < 0$, nutrient-limiting

e⁻-acceptor consumption

(e⁻ acceptor used in a reactor)
= [(total O.D. in the influent) - (total O.D. in the effluent)]
  x (conversion factor)

In terms of the use rate for a reactor ($\Delta S_a/\Delta t$):

$$\frac{\Delta S_a}{\Delta t} = \gamma_a \left[Q\left(S^0 + 1.42 X_v{}^0\right) - Q\left(S + SMP + 1.42 X_v\right) \right]$$

$$= \gamma_a Q \left[S^0 - S - SMP + 1.42\left(X_v{}^0 - X_v\right) \right]$$

γ_a = the stoichiometric ratio of acceptor mass to oxygen demand

  for oxygen: 1 g O_2/g COD

  for nitrate: 0.35 g $NO_3{}^-$N/g COD

e⁻-acceptor consumption

To estimate the required mass rate of acceptor supply
(ex: aeration [O_2] requirement),

the calculated e⁻ acceptor use rate, $\Delta S_a/\Delta t$ can be
written as:

$$\frac{\Delta S_a}{\Delta t} = \gamma_a \left[Q\left(S^0 + 1.42 X_v{}^0\right) - Q\left(S + SMP + 1.42 X_v\right) \right]$$

$$= Q\left(S_a{}^0 - S_a\right) + R_a$$

(mass flow rate in) −
(mass flow rate out)

Requirement of e⁻
acceptor addition

Hydrolysis of particulates & polymers

- Particulates and polymeric substances account for a significant portion of BOD in wastewater

- >50% of BOD in typical sewage is particulates (SS)

- Particulates and large-MW compounds cannot penetrate the cell membrane

  → needs to be hydrolyzed to smaller molecules

- Catalyzed by extracellular enzymes

- The mechanism and kinetics of hydrolysis it not fully understood

Hydrolysis of particulates & polymers

One simple way of describing hydrolysis is to assume first-order kinetics for particulates (or polymers):

$$r_{hyd} = -k_{hyd}S_p$$

r_{hyd} = rate of accumulation of particulates (=dS_p/dt) [$M_sL^{-3}T^{-1}$]

k_{hyd} = first-order hydrolysis rate coefficient [T^{-1}]

S_p = concentration of particulates [M_sL^{-3}]

In a steady-state CSTR,

$$0 = Q\left(S_p{}^0 - S_p\right) - k_{hyd}S_pV$$

$$\longrightarrow \quad S_p = \frac{S_p{}^0}{1 + k_{hyd}\theta}$$

Hydrolysis of particulates & polymers

- Effect of hydrolysis on dissolved substrates

  mass balance in a steady-state CSTR

$$0 = Q(S^0 - S) - \frac{\hat{q}S}{K + S}X_a V + k_{hyd}S_p V$$

$$0 = (S^0 - S) - \frac{\hat{q}S}{K + S}X_a \theta + k_{hyd}S_p \theta$$

→ Increase in S^0 by $k_{hyd}S_p\theta$

→ Increased biomass, but no change in dissolved substrates in the reactor

Alternate rate expressions

- Contois equation

$$r_{ut} = -\frac{\hat{q}S}{BX_a + S}X_a$$

B = constant [M_s/M_x]

When $X_a \to \infty$, $r_{ut} = -\frac{\hat{q}}{B}S$

(at high biomass concentrations substrate utilization depends on S, not X_a)

Alternate rate expressions

- Moser equation

$$r_{ut} = -\frac{\hat{q}S}{K + S^{-\gamma}}X_a \qquad \gamma = \text{constant [unitless]}$$

- Tessier equation

$$r_{ut} = -\hat{q}\left(1 - e^{S/K}\right)X_a$$

*Just **REMEMBER** that Monod Eq. is **NOT** the only option!!!*

Dual Monod equation

$$r_{ut} = -\hat{q} \frac{S}{K+S} \frac{A}{K_A+A} X_a$$

A = e$^-$ acceptor concentration [M_A/L^3]

K_A = half-saturation coeff. for e$^-$ acceptor [M_A/L^3]

- e$^-$ acceptor can also be limiting!
- Can be reduced to single Monod Eq. if $A >> K_A$
- Terms for other limiting substances can be added as well

Analyzing CSTR (Chemostat) Performance

Q: A chemostat having V=2,000 m³ receives a flow rate of Q = 1,000 m³/d of wastewater containing S^0 = 500 mg BOD$_L$/L. Also included in the wastewater is the inert biomass X_i^0 = 50 mg VSS/L. The following parameters are found for aerobic biodegradation:

$$\hat{q} = 20 \; g \; BOD_L \, / \, g \; VSS_a - d$$

$$Y = 0.42 \; g \; VSS_a \, / \, g \; BOD_L$$

$$K = 20 \; mg \; BOD_L \, / \, L$$

$$b = 0.15 \, / \, d$$

$$f_d = 0.8$$

$$k_1 = 0.12 \; g \; COD_p \, / \, g \; BOD_L$$

$$k_2 = 0.09 \; g \; COD_p \, / \, g \; VSS_a - d$$

$$\hat{q}_{UAP} = 1.8 \; g \; COD_p \, / \, g \; VSS_a - d$$

$$K_{UAP} = 100 \; mg \; COD_p \, / \, L$$

$$\hat{q}_{BAP} = 0.1 \; g \; COD_p \, / \, g \; VSS_a - d$$

$$K_{BAP} = 85 \; mg \; COD_p \, / \, L$$

Analyzing CSTR (Chemostat) Performance

Questions:

1. Calculate S_{min}, Θ_x^{min}, and Θ_x of the chemostat
2. Calculate effluent VSS, COD, and BOD_L.
3. Calculate the effluent N and P concentrations when influent concentrations are 50 mg NH_4^+-N/L and 10 mg PO_4^{3-}-P/L, respectively.
4. Calculate the amount of O_2 that should be supplied to the reactor when influent and effluent DO are 6 and 2 mg/L, respectively.
5. Assuming that the influent also contains biodegradable particulate organic matter with a concentration of 100 mg COD/L and the hydrolysis rate coefficient is k_{hyd} = 0.2/d, recalculate the effluent VSS, COD, and BOD_L.