Biofilm kinetics I

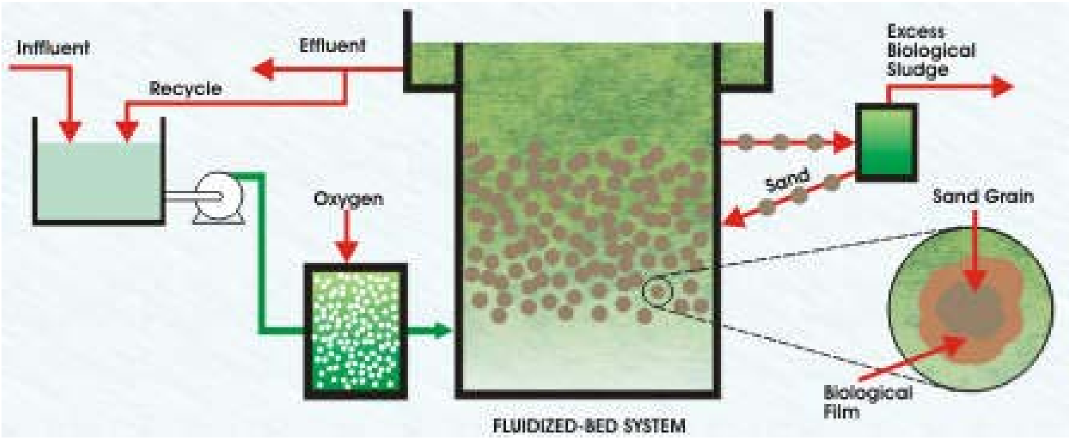
Today's lecture

- Biofilm processes
- Concept, assumptions, theory
- Steady state biofilm analysis

Biofilm processes

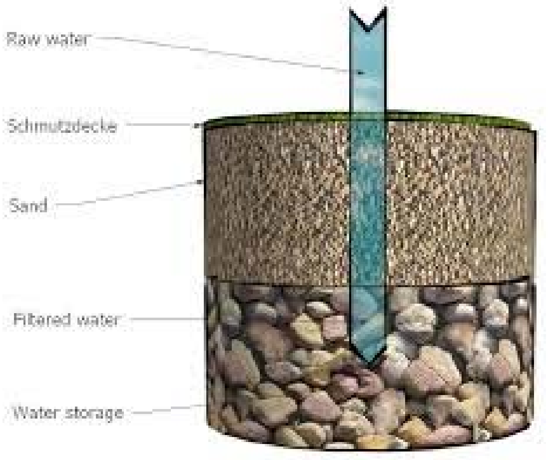
Trickling filter

Rotating biological contactor

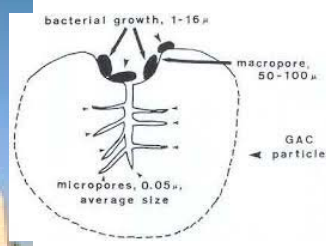


Fluidized bed bioreactor

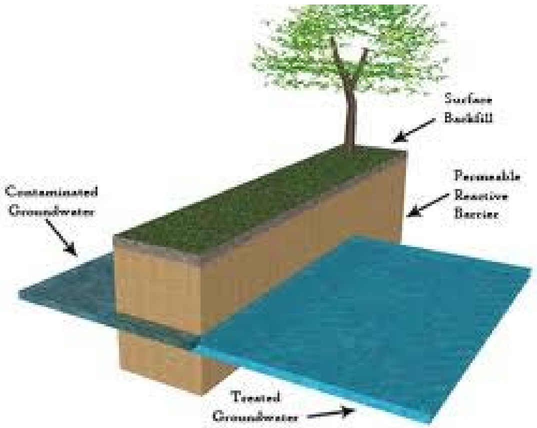
Biofilm processes



Slow sand filtration
(Schmutzdecke)



Biological GAC treatment



In-situ bioremediation
of groundwater

Biofilm kinetics – key concept

- *Addition of a key mechanism: DIFFUSION*
  - Diffusion of substrates
  - Diffusion of e^- acceptors
  - Diffusion of nutrients

- Fick's Law of diffusion
  - Fick's 1st law: $J = -D\frac{\partial C}{\partial x}$
  - Fick's 2nd law: $\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$

J = flux of a substance $[ML^{-2}T^{-1}]$

D = diffusion coefficient $[L^2T^{-1}]$

C = concentration of a substance $[ML^{-3}]$

Assumptions for biofilm analysis

Idealizing a biofilm:

- The biofilm has a uniform biomass density X_f ($M_x L^{-3}$)
- It has a locally uniform thickness of L_f.
- Mass transport resistance can be important inside the biofilm and to the biofilm.
  - External mass transport (bulk liquid → surface of a biofilm): represented by an effective diffusion layer of thickness L (film theory)
  - Internal mass transport (within the biofilm): molecular diffusion (Fick's 2nd law)

Film Theory

- The entire resistance to mass transport resides in a stagnant film at the phase interface.
- Equilibrium is obtained at the interface
- The bulk fluids are sufficiently well-mixed so that the concentration gradients in the bulk fluid are negligible
- The concentration gradient in the film is linear, following the steady state diffusion in a stagnant fluid.

Film Theory

The flux in the film is:

$$J = -\frac{D}{\delta}\left(C_{bulk} - C_{interface}\right)$$

J = flux of a substance [$ML^{-2}T^{-1}$]

D = diffusion coefficient [$L^{2}T^{-1}$]

δ = film (effective diffusion layer) thickness [L]

C_{bulk} = concentration in the bulk fluid [ML^{-3}]

$C_{interface}$ = concentration at the interface [ML^{-3}]

Deep & shallow biofilm

- Deep biofilm: the substrate concentration approaches zero at some point in the film
  - Further increase in biofilm thickness does not increase the overall rate of substrate utilization
- Shallow biofilm: the substrate concentration remains above zero at all points in the film
  - Fully penetrated biofilm: a special case of shallow biofilm where the substrate concentration at the outer surface and the attachment surface are almost identical

Substrate analysis

The substrate utilization follows Monod kinetics:

$$r_{ut} = -\frac{\hat{q} X_f S_f}{K + S_f}$$

X_f = active biomass density within the biofilm $[M_x L^{-3}]$

S_f = substrate concentration at a point of the biofilm $[M_s L^{-3}]$

The molecular diffusion of the substrate (Fick's 2nd law):

$$r_{diff} = D_f \frac{d^2 S_f}{dz^2}$$

r_{diff} = rate of substrate accumulation due to diffusion $[M_s L^{-3} T^{-1}]$

D_f = molecular diffusion coeff. of the substrate in the biofilm $[M_s L^{-3}]$

z = depth dimension normal to the biofilm surface [L]

Substrate analysis

Combining the substrate utilization and diffusion, and assuming steady state,

$$0 = D_f \frac{d^2 S_f}{dz^2} - \frac{\hat{q} X_f S_f}{K + S_f}$$

Boundary condition I

no flux to the attachment surface $\left.\frac{dS_f}{dz}\right|_{z=L_f} = 0$

L_f = biofilm thickness [L]

Boundary condition II

Flux at the biofilm/water interface determined according to the film theory

$$J = \frac{D}{L}(S - S_s) = D_f \left.\frac{dS_f}{dz}\right|_{z=0} = D \left.\frac{dS}{dz}\right|_{z=0}$$

D = molecular diffusion coefficient in water

L = effective diffusion layer thickness [L]

S, S_s = substrate concentrations in the bulk liquid and at the biofilm/liquid interface, respectively [$M_s L^{-3}$]

Substrate analysis – analytical solution

$$0 = D_f \frac{d^2 S_f}{dz^2} - \frac{\hat{q} X_f S_f}{K + S_f}$$ (+ two B.C.s)

The first integration yields *J* and the second integration yields S_f, but closed-form solutions cannot be obtained for this non-linear form

Substrate analysis – analytical solution

- Dimensionless parameter for the deepness of the biofilm:
  $L_f/\tau_1 > 1$: deep biofilm
  $L_f/\tau_1 << 1$: fully penetrated biofilm

$\tau_1 = \sqrt{(D_f \cdot K)/(\hat{q} \cdot X_f)}$ = standard biofilm depth dimension [L]

Special Case: Deep biofilm

For a deep biofilm, we have an additional B.C.:

$$S_f\Big|_{z=0} = 0$$

Substrate analysis – analytical solution

In this case, we can get an analytical solution in a closed form:

$$J_{deep} = \left[2\hat{q}X_f D_f \left(S_s + K\, ln\left(\frac{K}{K + S_s}\right)\right)\right]^{1/2}$$

For shallow biofilm:

A complicated procedure is needed → FYI, refer to pp. 217-220 of the textbook

Biofilm analysis

- The active biomass mass balance at a position inside the biofilm with a thickness of *dz*:

$$\frac{d(X_f dz)}{dt} = Y \frac{\hat{q} S_f}{K + S_f} (X_f dz) - b' X_f dz$$

b' = overall biofilm specific loss rate [T^{-1}]

: <u>*b* (decay coeff.) + b_{det} (detachment)</u>

Biofilm analysis

- Integrating over the entire biofilm depth:

$$\int_0^{L_f} \frac{d(X_f dz)}{dt} = \int_0^{L_f} Y \frac{\hat{q} S_f}{K + S_f} X_f dz - \int_0^{L_f} b' X_f dz$$

Steady state assumption

- (Pseudo) Steady state assumption:
  - The biofilm thickness, L_f, and the active biomass density, X_f, do not change with time
  - Then, the biomass per unit surface area, $X_f L_f$, do not change with time
  - Steady state as a whole: at any given point of the biofilm steady state is not achieved, the whole biofilm is at steady state
  - Dynamic steady state: near the outer surface, the substrate concentrations are high, and active biomass growth is positive; near the attachment surface, substrate concentrations are low, and active biomass growth is negative. The active biomass exchanges within the biofilm to maintain uniform X_f

Steady state biofilm analysis

- By steady state assumption:

$$\int_0^{L_f} \frac{d(X_f dz)}{dt} = \int_0^{L_f} \frac{d(X_f L_f)}{dt} = 0$$

- The active biomass mass balance over the whole biofilm depth at steady state:

$$0 = \int_0^{L_f} Y \frac{\hat{q} S_f}{K + S_f} X_f dz - \int_0^{L_f} b' X_f dz$$

Steady state biofilm analysis

- The growth term:

$$\int_0^{L_f} Y \frac{\hat{q} S_f}{K + S_f} X_f dz = Y \int_0^{L_f} r_{ut} dz = Y\, J$$

substrate utilization rate over the entire depth

Substrate transport from bulk liquid to the biofilm

- The loss term:

$$\int_0^{L_f} b' X_f dz = b' X_f L_f$$

(the loss process is averaged across the biofilm)

Steady state biofilm analysis

Now, the equation reduces to:

$$0 = YJ - b'X_f L_f$$

Biomass per unit area,

$$X_f L_f = \frac{YJ}{b'}$$

The biofilm thickness,

$$L_f = \frac{YJ}{X_f b'}$$