In-Class Exercise – Solutions

1 i)

$$\theta_x = \theta = \frac{V}{Q} = \frac{10\ m^3}{2\ m^3/hr} = 5\ hr$$

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x - (1+b\theta_x)} = (20\ mg/L)\cdot\frac{1+(0.1/d)(5\ hr)\times 1/24\ d/hr}{0.5\times 15/d\times 5hr\times 1/24\ d/hr - (1+\underbrace{0.1/d\times 5hr\times 1/24\ d/hr}_{0.02083})}$$

$$= 37.7\ mg/L$$

$$X_a = \frac{Y(S^0-S)}{1+b\theta_x} = \frac{0.5\times(200-37.7)\ mg/L}{1+0.02083} = 79.5\ mg/L$$

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x$$

$$= 30\ mg/L + (79.5\ mg/L)\cdot(1-0.8)\cdot 0.02083$$

$$= 30.3\ mg/L$$

$$X_v = X_a + X_i = 79.5\ mg/L + 30.3\ mg/L = 109.8\ mg/L$$

SMP — ?

$$r_{ut} = -\frac{S^0-S}{\theta} = -\frac{(200-37.7)\ mg/L}{5/24\ d} = -779\ mg/L\text{-}d$$

$$\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta = 1.5\times 79.5\times\frac{5}{24} + 100 + 0.12\times(-779)\times\frac{5}{24}$$

$$= 105.37\ mg/L$$

$$-4K_{UAP}k_1 r_{ut}\theta = -4\times 100\times 0.12\times(-779)\times\frac{5}{24}$$

$$= 7790\ (mg/L)^2$$

$$K_{BAP} + (\hat{q}_{BAP} - k_2) X_a \theta = 50 + (0.1 - 0.08) \times 79.5 \times \frac{5}{24}$$

$$= 50.33 \text{ mg/L}$$

$$4 K_{BAP} k_2 X_a \theta = 4 \times 50 \times 0.08 \times 79.5 \times \frac{5}{24}$$

$$= 265 \text{ (mg/L)}^2$$

$$UAP = \frac{-106.5 + \sqrt{106.5^2 + 7770}}{2} = 16.0 \text{ mg/L}$$

$$BAP = \frac{-50.4 + \sqrt{50.4^2 + 265}}{2} = 1.3 \text{ mg/L}.$$

$$\left.\right\} SMP = 17.3 \text{ mg/L}$$

$$\text{Effluent COD} = S + SMP + 1.42 X_v$$

$$= 37.7 + 17.3 + 1.42 \times 109.8$$

$$= 211 \text{ mg/L}$$

$$\text{Effluent } BOD_L = S + SMP + 1.42 f_d X_a$$

$$= 37.7 + 17.3 + 1.42 \times 0.8 \times 79.5$$

$$= 145 \text{ mg/L}.$$

ii) Using $C_5H_7O_2N$ as cell formula,

$$\gamma_N = 0.124 \text{ g N/g VSS}.$$

$$r_N = \gamma_N \cdot Y \cdot r_{ut} \frac{1 + (1 - f_d) b \theta_x}{1 + b \theta_x}$$

$$= (0.124) \cdot (0.5) \cdot (-779) \cdot \frac{1 + (1 - 0.8) \cdot 0.02083}{1 + 0.02083}$$

$$= -47.5 \text{ mg N/L-d}$$

$$C_N = C_N^0 + r_N \theta$$

$$= 40 \text{ mg N/L} + (-47.5 \text{ mg N/L-d}) \cdot (5/24 \text{ d})$$

$$= 30.1 \text{ mg N/L}$$

iii) $Q(S_a^0 - S_a) + R_a = \gamma_a Q[(S^0 - S - SMP) + 1.42(X_v^0 - X_v)]$

$$R_a = \{1 \text{ g } O_2/\text{g COD} \times (2\times24) \text{ m}^3/\text{d} \times [(200 - 37.7 - 17.3) \text{ mg/L}$$

$$+ 1.42 \times (30 - 109.8) \text{ mg/L}] - (2\times24) \text{ m}^3/\text{d} \times (8-3) \text{ mg/L}\}$$

$$\times 10^3 \text{ L/m}^3 \times 10^{-3} \text{ g/mg}$$

$$= 1280 \text{ g } O_2/\text{d}$$

$$= 53.4 \text{ g } O_2/\text{hr}$$

2

i) inlet, $S = 100\ mg/L$

from Excel Spreadsheet,

$$S_s = 0.011\ mg/cm^3 = 11\ mg/L$$

$$J = 1.84\ mg/cm^2\text{-}d$$

$$L_f = \frac{YJ}{X_f b'} = \frac{0.2 \times 1.84\ mg/cm^2\text{-}d}{60\ mg/cm^3 \times 0.2\ d^{-1}} = 0.031\ cm$$

$$\tau_1 = \sqrt{\frac{D_f \cdot K}{\hat{q} \cdot X_f}} = \sqrt{\frac{(1.2 \times 10^{-5}\ cm/s) \cdot 86400\ s/d \cdot 0.8 \cdot 0.02\ mg/cm^3}{15/d \cdot 60\ mg/cm^3}}$$

$$= 4.3 \times 10^{-3}\ cm$$

$\frac{L_f}{\tau_1} > 1$ → deep biofilm assumption valid

ii) outlet, $S = 20\ mg/L$

from Excel spreadsheet,

$$S_s = 0.0020\ mg/cm^3 = 2.0\ mg/L$$

$$J = 0.374\ mg/cm^2\text{-}d$$

$$L_f = \frac{0.2 \times 0.374\ mg/cm^2\text{-}d}{60\ mg/cm^3 \times 0.2\ d^{-1}} = 6.2 \times 10^{-3}\ cm$$

$\frac{L_f}{\tau_1} > 1$ → deep biofilm assumption valid

∴ $J = 1.84\ mg/cm^2\text{-}d$ (inlet)

$J = 0.374\ mg/cm^2\text{-}d$ (outlet)