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457.212 Statistics for Civil & Environmental Engineers

In-Class Material: Class 04

Elements of Set Theory – Part I (A&T: 2.1-2.2)

1. Why do we need 'set theory' in uncertainty analysis?

   - **Uncertainty:** a multiplicity of possible ( )
   - **Probability:** numerical measure of the ( ) of an event (i.e. a group of outcomes) relative to the other possible events

	Possible outcomes	Event of Interest	Probability
Tossing a dice	{ }	"Even number"	
		"Prime & odd"	
Watching a baseball game	{ }		텍스트

   - Uncertainty analysis requires considering the collection of all ( ) ( ).
   - Principles of set theory are essential tool for this task.

2. Definitions

(a) **Sample space** (S): the set of all possible outcomes, e.g. { }

Sample point (x): an individual outcome, e.g. " ", " "

Obviously, any sample point belongs to the sample space, that is, $x \in S$
Note that there is no sample point outside ( )

Criteria	Sample space	Examples
Continuous?	"Continuous"	Peak flow rate of Addison Creek (ft^3/sec)
	"Discrete"	Number of tornadoes in U.S. in 2008
Can count sample points?	"Finite"	( ) and ( ) $S = \{1.2, 2.2, ..., 1000.2\}$
	"Infinite"	( ) or ( ) $S = \{x \mid 2.73 \le x \le 5.25\}$, $S = \{0,1,2,...\}$

(b) **Event** (E): any collection of sample ( ); any subset of sample ( )

Example 1: Sample space of annual peak flow rate of Addison Creek?

$$S = \{x \mid \quad\}$$

Event: $E_1 =$ "Peak flow rate of Addison creek is higher than 1000 ft^3/sec"

$$E_1 = \{x \mid \quad\}$$

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Example 2: Albert Pujols at the plate. The sample space of his outcomes?

$S = \{$ $\}$

Event: $E_2 =$ "Pujols reaches a base"

$E_2 = \{$ $\}$

(c) Some notable events

- **Certain** event: $E = S$
  - An event containing all the sample ( )
  - Occurs with certainty
- **Null** event: $E = \phi$
  - An event containing ( ) sample points
  - No possibility of occurrence
    (e.g. *E* = "get an odd number greater than 5 from a dice")
- **Complementary** event of E: $\overline{E}$
  - An event that contains all the sample points that are ( ) in E
  - e.g. $S = \{0,1,2,3,4,5\}$, $E = \{3,4,5\}$, then $\overline{E} = \{$ $\}$
  - e.g. $\overline{S} =$, $\overline{\phi} =$

(d) **Venn diagram**: graphical representation of the sample space, sample points and events

Example 3 (A&T 2.1): A contractor plans the acquisition of three bulldozers. After 6 months, each bulldozer can be *operational* (O) or *nonoperational* (N).

(*) Sample space of the possible conditions of the three bulldozers after 6 months?

(*) Event that the *i*-th bulldozer is *operational* after 6 months, $i = 1,2,3$?

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(*) Event there are two operational bulldozers after 6 months and its complementary event?

(*) Venn diagrams of the sample space and the events above?

(e) **"Conditioned"** sample space

- A new sample space after observing/assuming the occurrence of a related event.
- In general, the sample space ( ) when new information becomes available or assumed.

Example 3 (contd.): Venn diagram of the new sample space when we know the Bulldozer 2 survived six months?

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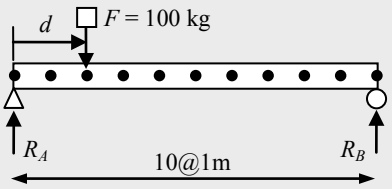
Example 4 (A&T 2.5): A simply supported beam



(a) 100 kg at any of the 1-meter interval points.
The sample space of the reaction force R_A?
(hint: $R_A = 100 - 10d$ where d is distance from A)

(b) Visualize the sample space of (R_A, R_B)?
(hint: $R_A + R_B = 100$)

(c) Now the 100-kg load can be placed anywhere along the beam.

- Sample space of R_A
- Sample space of (R_A, R_B)
- Show the event "$20 < R_A < 40$"

(d) The load can be 100, 200, or 300 kg and can be placed anywhere along the beam.
Sample space of (R_A, R_B)?

(e) The load can be any value between 100 and 300 kg and can be placed anywhere along the beam.

- Sample space of (R_A, R_B)?
- Show the event "$R_A < 100$ and $R_B > 100$"
- Show the event "$R_A > 200$"