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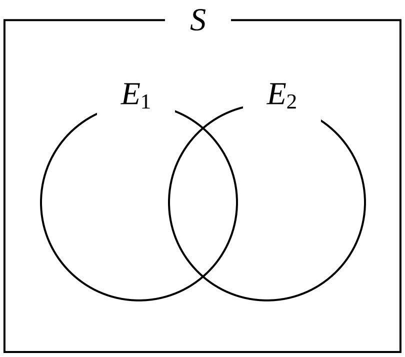
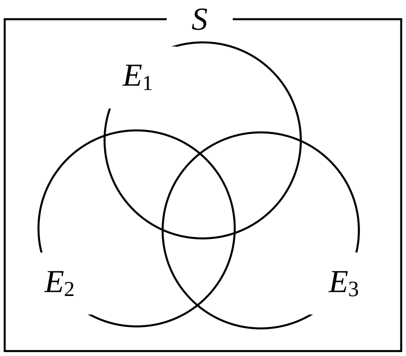
457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 05
Elements of Set Theory – Part II (A&T: 2.1-2.2)

3. **Operations** of events

(a) **Union** of events





- **Union** of E_1 and E_2, ( ): An event that contains all the sample points that are in E_1 (/) E_2.
- Can be extended to the cases with more than two events - **Union** of $E_1, E_2, ..., E_n$, ( ) or ( ):

  An event that contains all the sample points that are in ( ) one of $E_1, E_2, ..., E_n$.
- Some notable cases:

  (1) $E \cup S =$

  (2) $E \cup \phi =$

  (3) $E \cup E =$

  (4) If $E_1 \subset E_2$, $E_1 \cup E_2 =$

- CEE examples:

  (1) Concrete production may be hampered by shortage of water (E_1), sand (E_2), gravel (E_3) or cement (E_4).

  The event that concrete can not be produced due to material shortage, *E*

  E =

  (2) During a hurricane event, a wall/roof panel can fail due to wind pressure (E_1) or missile-like flying objects (E_2). *Animation

  The event that a wall panel fails, *E*

  E =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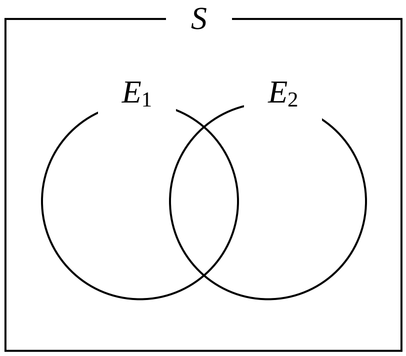
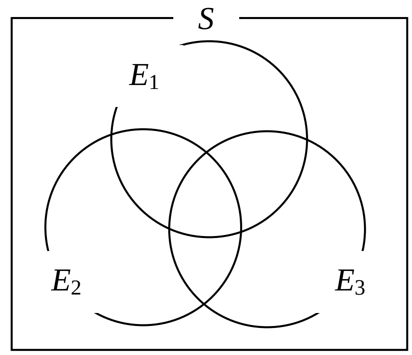
(b) **Intersection** of events





- **Intersection** of E_1 and E_2, (   ): An event that contains all the sample points that are both in E_1 (   ) E_2.
- Can be extended to the cases with more than two events - **Intersection** of $E_1, E_2, ..., E_n$, (   ) or (   ): An event that contains all the sample points that belongs to (   ) one of $E_1, E_2, ..., E_n$.
- Some notable cases:

  (1) $E \cap S = E \cdot S =$

  (2) $E \cap \phi = E \cdot \phi =$

  (3) $E \cap E = E \cdot E =$

  (4) If $E_1 \subset E_2$, $E_1 \cap E_2 = E_1 \cdot E_2 =$
- CEE examples:

  (1) A city has *n* evacuation routes. Let E_i, $i = 1,\ldots,n$ denote the event that the i-th route is not available after a hazardous event.

  The event that people can not evaculate, *E*

  E =

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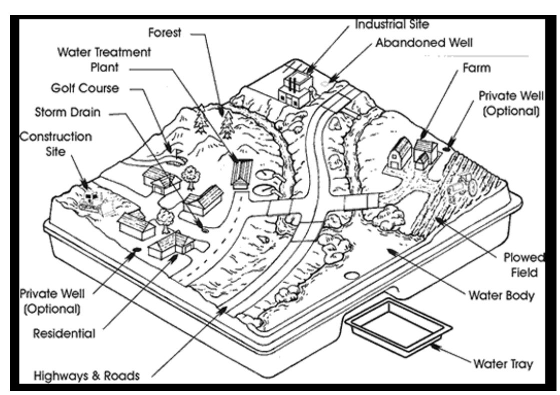
  (2) The industrial site could accidentally discharge the pollutant X (Event E_1). The water treatment plant may not properly work (Event E_2). Joe will be exposed to the pollutant if he does not have his own filtering system (Event E_3).

  The event that Joe will be exposed to the pollutant X, *E*

  E =



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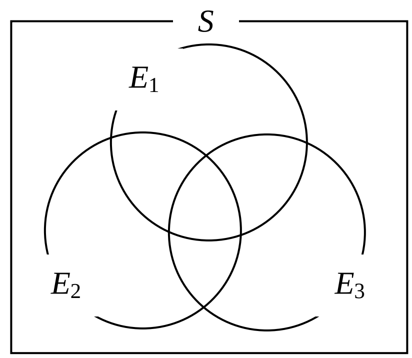
Note: Treat compound events obtained by union or intersection operations just like elementary events in the following operations.

Example: Intersection of the compound event $E_1 \cup E_2$ and E_3, i.e.



4. **Four basic rules** of operations of events

(a) **Commutative Rule**: the order of the events in union or intersection operations is (   ).

- $E_1 \cup E_2 =$
- $E_1 \cdot E_2 =$

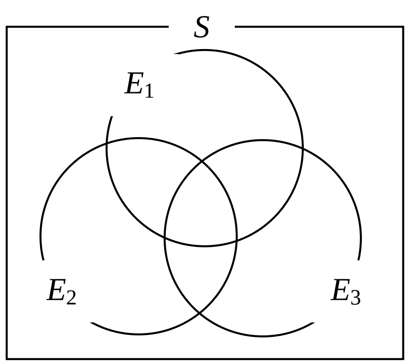
Question: The events in which pair can be interchanged in $E_1 \cup E_2 \cap E_3$ and $(E_1 \cup E_2) \cap E_3$?

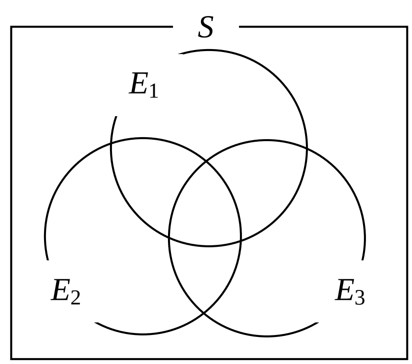
(b) **Associative Rule**: the union and intersection of three or more events are independent of the (   ).

- $(E_1 \cup E_2) \cup E_3 =$ =
- $(E_1 E_2) E_3 =$ =

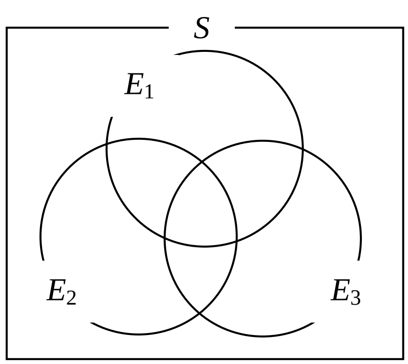
(c) **Distributive Rule**:

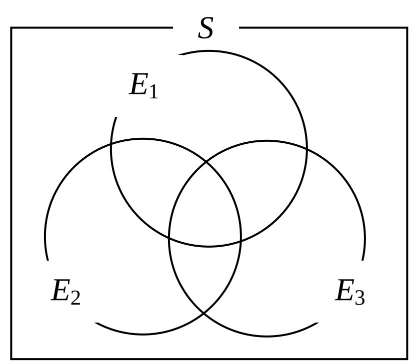
- $(E_1 \cup E_2) E_3$
  =





- $(E_1 E_2) \cup E_3$
  =





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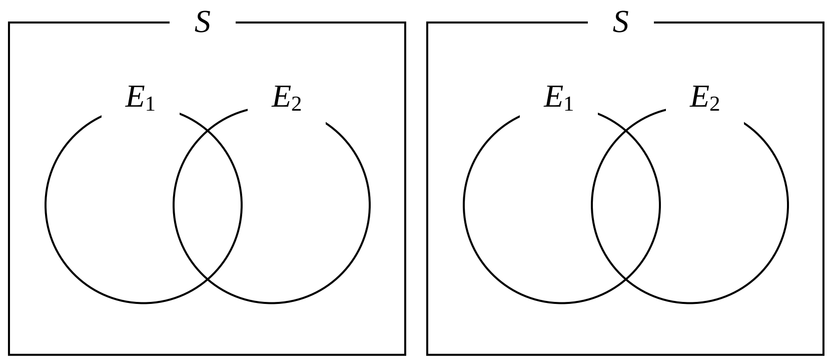
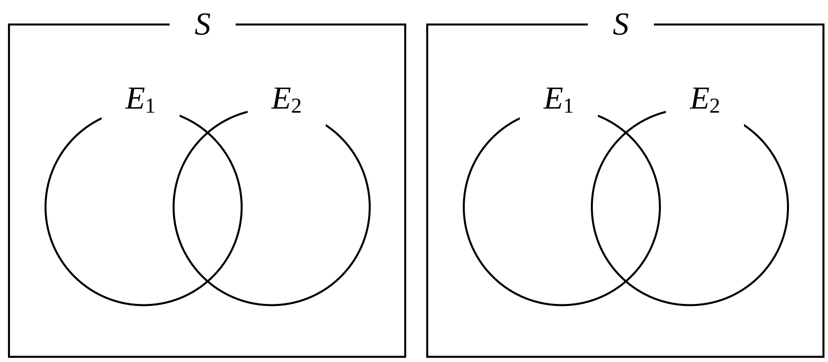
(d) **De Morgan's Rule**:

- $\overline{(E_1 \cup E_2)}$
  =



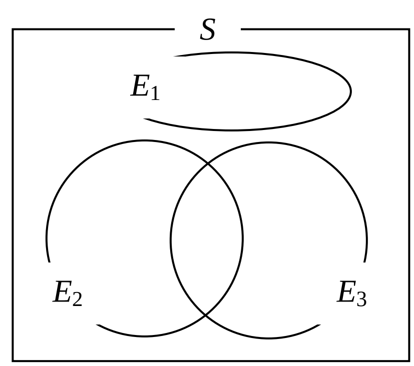
- $\overline{E_1 \cap E_2} = \overline{E_1 E_2}$
  =



Note: Extension to cases with *n* events:

5. **Notable relationship** between events

(a) **Mutually exclusive** events: when the intersection of the events are (   ) events.

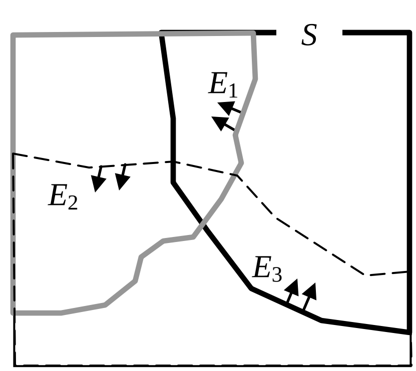


$E_1 \cap E_2 = E_1 E_2 =$

- The events "can not occur together."
- An obvious example:

Question: Which pairs are mutually exclusive?

(b) **Collective exhaustive** events: when the union of the events constitutes the (   )



$E_1 \cup E_2 \cup \cdots \cup E_n = \bigcup_{i=1}^{n} E_i =$

- Every sample point should belong to (   ) of these events.
- An obvious example:

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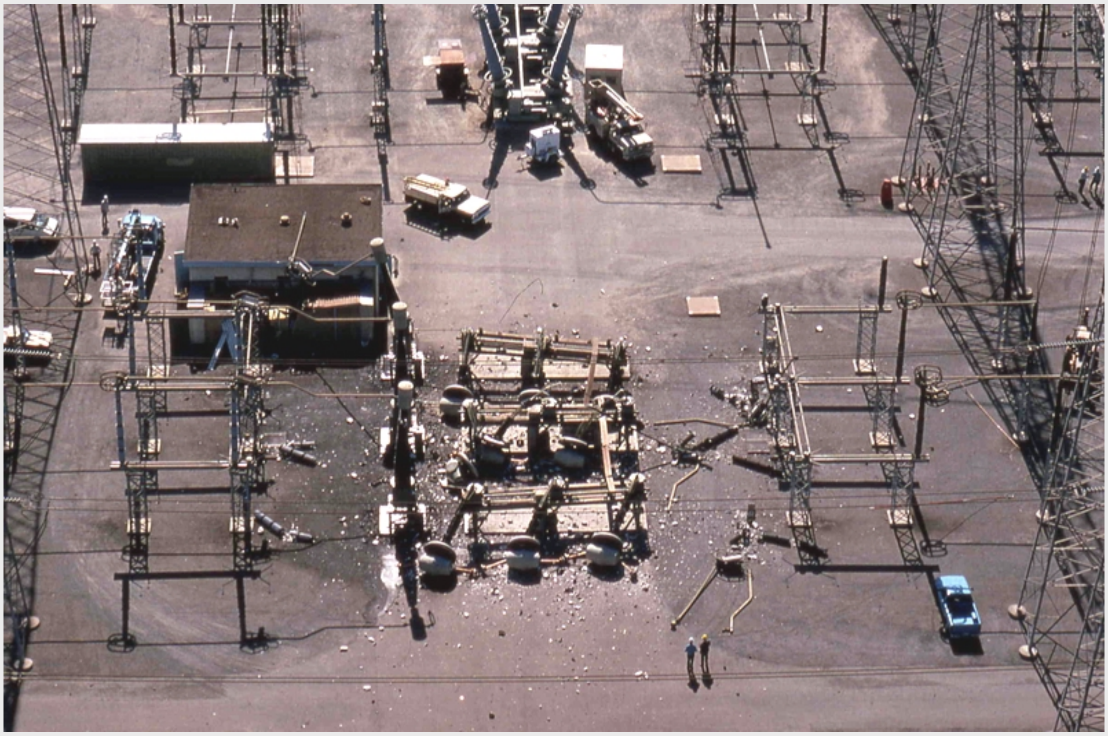
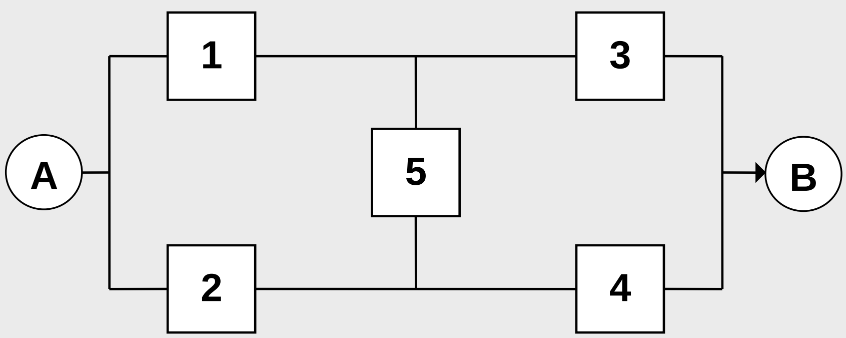
Example: Connectivity of an electrical substation after an earthquake event



Consider a simplified example of substation:



Let E_i, i = 1,…,5 denote the event that the i-th equipment fails during an earthquake event. Describe the event that the electricity can not travel from the point A to B in terms of E_i's.

$E_{\text{disconnect}}$ =