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457.212 Statistics for Civil & Environmental Engineers

In-Class Material: Class 07

Elements of Probability Theory – Part II (A&T: 2.3)

4. Conditional probability and statistical independence (Continued)

(d) **Multiplication rule**

- $P(E_1E_2) =$

hint: $P(E_1 \mid E_2) = \dfrac{P(E_1E_2)}{P(E_2)}$

(e) All the probability rules should apply to conditional probabilities defined within the same reconstituted (   )(   ).

- $P(E_1 \cup E_2 \mid E_3) =$
- $P(E_1E_2 \mid E_3) =$
- $P(\overline{E}_1 \mid E_2) = 1-$

Thus,

- $P(E_1E_2E_3) =$
- $P(E_1E_2 \cdots E_n) =$

(f) **Statistical independence –** the occurrence (or non-occurrence) of one event does not affect the probability of the other event.

Three equivalent expressions for "E_1 and E_2 are statistically independent."

- $P(E_1 \mid E_2) =$
- $P(E_1E_2) =$
- $P(E_2 \mid E_1) =$

Extended to n events:

- $P\left(\bigcap_{i=1}^{n} E_i\right) =$

(g) "E_1 and E_2 are **statistically independent** (s.i.)"

$\Leftrightarrow$

$\Leftrightarrow$

$\Leftrightarrow$

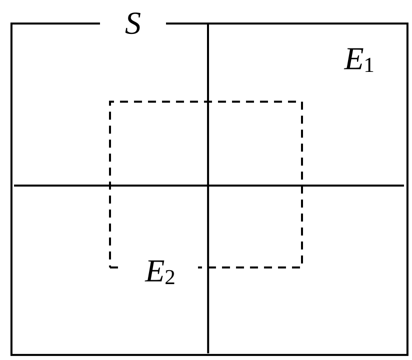
(proof for the first case: as a practice regarding probability rules)

$$
\begin{aligned}
P(\overline{E}_1\overline{E}_2) &= P(\overline{\qquad\qquad}) \\
&= 1 - P(\qquad\quad) \\
&= 1 - P(\ ) - P(\ ) + P(\quad) \\
&= [1 - P(\ \ )][1 - P(\ \ )] \\
&= P(\ \ )P(\ \ )
\end{aligned}
$$

Note: In general, it is not straightforward to visualize "s.i." using a Venn diagram, but here is a special example:

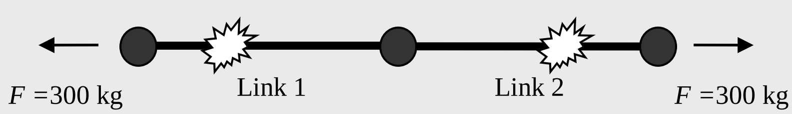
$P(E_1) =$

$P(E_1 \mid E_2) =$



Example: a baseball hitter whose batting average vs. lefty is the same as the total avg.

Example 1 (A&T 2.19): Consider a chain system consisting of two links subjected to a force F =300 kg. Let E_1 and E_2 respectively denote the events that "Link 1 fails" and "Link 2 fails." It is known that $P(E_1) = P(E_2) = 0.05$.



Compute the probability of the failure of the chain when

(a) the failures of the links are statistically independent of each other:

(b) the failures of the links have complete or total dependence between each other, i.e. if one fails, the other one always fails:

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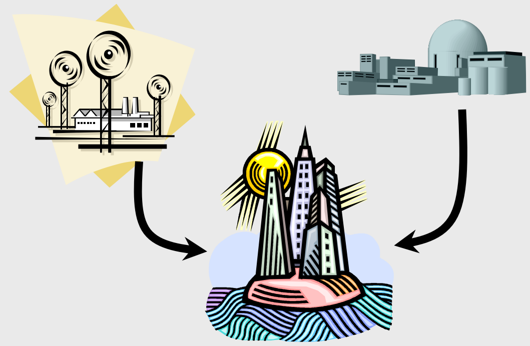
Example 2 (A&T 2.22): A city is powered by two generating plants – *Plant a* and *Plant b*. The city needs the capacities of both plants during peak hours. Otherwise, it will have a brownout or blackout. Events *A* and *B* denote the failure of *Plant a* and *Plant b*, respectively.



Given: $P(A) = 0.05,\ P(B) = 0.07,$ and $P(AB) = 0.01$

(a) If *Plant a* fails, what is the probability that *Plant b* fails?

(b) If *Plant b* fails, what is the probability that *Plant a* fails?

(c) What is the probability that there will be a brown/blackout in the city during the peak hour?

When there is a brown/blackout in the city during the peak hour, compute the following probabilities:

(d) the brown/blackout caused solely by the failure of *Plant a*.

(e) the brown/blackout caused solely by the failure of *Plant b*.

(f) the brown/blackout caused by both plants.

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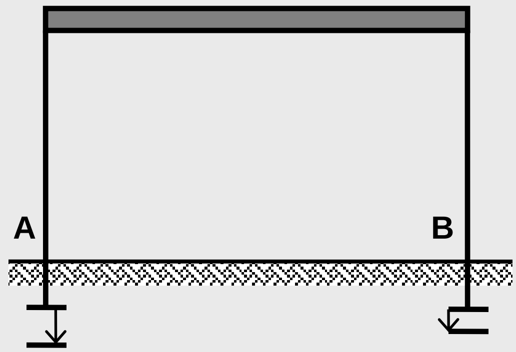
Example 3: Consider a steel frame in the figure. E_A and E_B respectively denote the settlements at location A and B, respectively.



Given: $P(E_A) = 0.1$

$P(E_B) = 0.1$

$P(E_A \mid E_B) = 0.8$

$P(E_B \mid E_A) = 0.8$

(a) Probability that there will be any settlements:

(b) Probability of a "differential" settlement:

Example 4: Suppose the foundation of a structure can fail either when the structural demand exceeds its bearing capacity (Event B) and/or when the settlement (Event S_t) occurs.

Given: $P(B) = 0.001$, $P(S_t) = 0.008$ and

$P(B \mid S_t) = 0.1$

(a) Probability of the failure of foundation:

(What if s.i. assumed?)

(b) Probability that building has settlement, but no excessive demands in terms of bearing capacity?