

457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 14
Multiple Random Variables – Part II (A&T: 3.3, Supp #2)

1. **Covariance and Correlation Coefficient** (Recall sample COV and sample corr. coeff.)

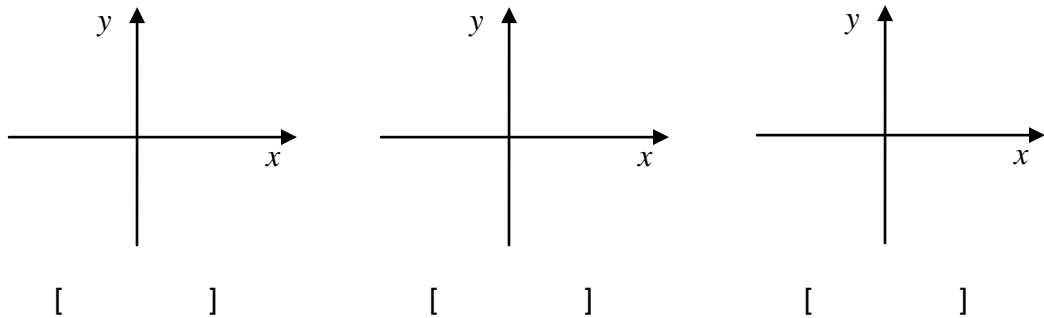
(a) Covariance

$$\begin{aligned} \text{Cov}[X, Y] &= E[(X - \mu_X)(Y - \mu_Y)] && \text{“mean of product minus product of means”} \\ &= E[XY] - \mu_X \mu_Y \end{aligned}$$

(b) Covariance of discrete r.v.'s: $\text{Cov}[X, Y] =$

(c) Covariance of continuous r.v.'s: $\text{Cov}[X, Y] =$

(d) Sign of covariance:

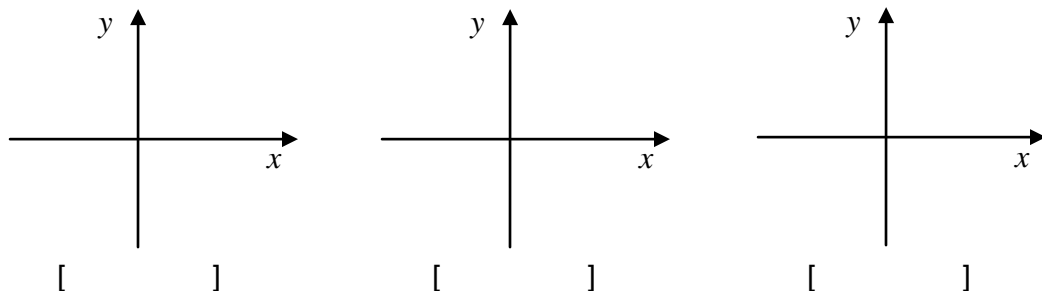


(e) Correlation coefficient:

$$\rho_{XY} = \frac{\text{Cov}[X, Y]}{\sigma_X \sigma_Y}$$

(f) Unlike covariance, correlation coefficient is bounded, i.e. $-1 \leq \rho_{XY} \leq 1$

(g) Correlation coefficient can be used as a relative measure of the strength of statistical ()

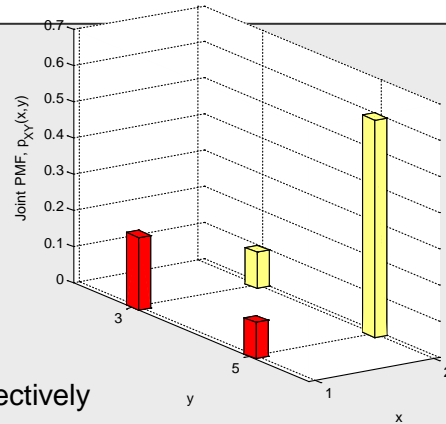


2. **Example** of joint PDF ~ two normal random variables (bi-variate joint PDF)

→ See Supplement # 2 on the course website

Example 1: Consider two discrete random variables X and Y . Their joint PMF is given as

	x		
		1	2
y			
	3	0.2	0.1
	5	0.1	0.6



(a) Determine the marginal PMF's of X and Y , respectively

x	$p_X(x)$

y	$p_Y(y)$

(b) Conditional probability mass function of X given $Y = 3$?

x	$p_{X Y}(x 3)$

(c) Find the means of X and Y , respectively.

(d) Find the standard deviations of X and Y , respectively

(e) Find the covariance between X and Y .

(f) Find the correlation coefficient between X and Y .

Example 2: Consider two random variables X and Y within the domain shown in the figure. It is assumed that the probability of each point in the domain is the same.

(a) Determine the joint PDF and plot.

(b) Determine the marginal PDFs of X and Y , respectively.

(c) Find the means of X and Y , respectively.

(d) Find the standard deviations of X and Y , respectively.

(e) Find the covariance of X and Y .

(f) Find the correlation coefficient between X and Y . Does the sign make sense to you? Why (not)?

