Seoul National University
Dept. of Civil and Environmental Engineering

Instructor: Junho Song
junhosong@snu.ac.kr

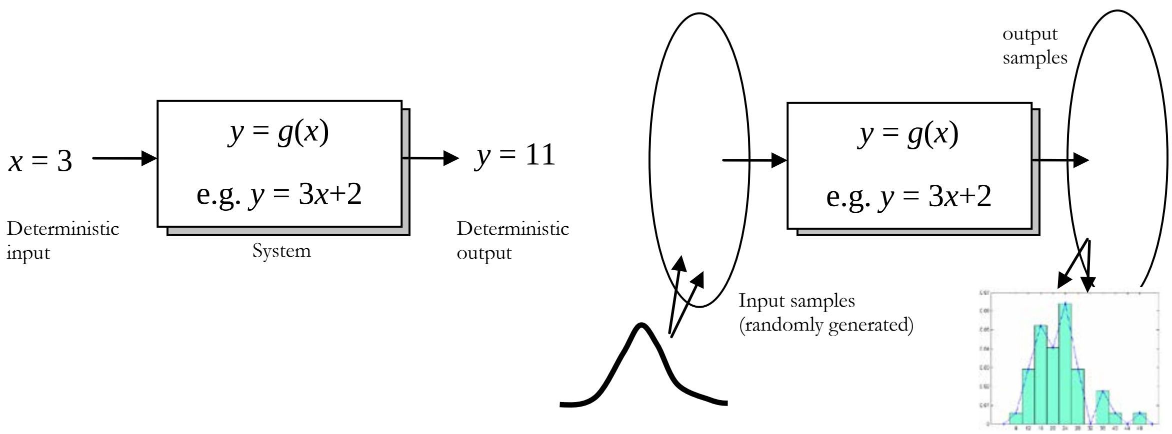
457.212 Statistics for Civil & Environmental Engineers

In-Class Material: Class 15

Monte Carlo Simulations; Propagation of Uncertainty; and Central Limit Theorem (A&T 5.2; Supplement #3-6)

1. **Monte Carlo** Simulations

- Generate artificial samples randomly according to probability distribution models (e.g. PDF, CDF, etc.)
- Perform (deterministic) system analysis using each random input sample to obtain a set of artificial outputs



(a) How to generate random numbers – See **Supplement # 3** (using Matlab®)

- Generating single random variable: `shortnamernd(parameters)`
  e.g. normal: `normrnd(mu,sigma,M,N)`
  lognormal: `lognrnd(lambda,zeta,M,N)`

- Generating correlated (normal) random variables: `mvnrnd(mu,sigma,N)`

(b) Convergence of the estimate on the probability of an event by MCS

- Example: $P(Y > 10)$ in the example above.

- For each sample x_i, the event of interest occurs with probability p (unknown).

- Generate N samples, i.e. $\{x_1,..., x_N\}$ independently (i.e. s.i.)

- This is a B___________ trial. The number of occurrences ~ B(,   )

- Introduce Q_i, Bernoulli random variable

$$Q_i = \begin{cases} 1 & \text{the event occurs (prob., } p) \\ 0 & \text{the event does not occur (prob., } 1-p) \end{cases}$$

Instructor: Junho Song
junhosong@snu.ac.kr

- An estimate on p after N simulations:

$$\tilde{p} = \frac{\sum_{i=1}^{N} q_i}{N}$$

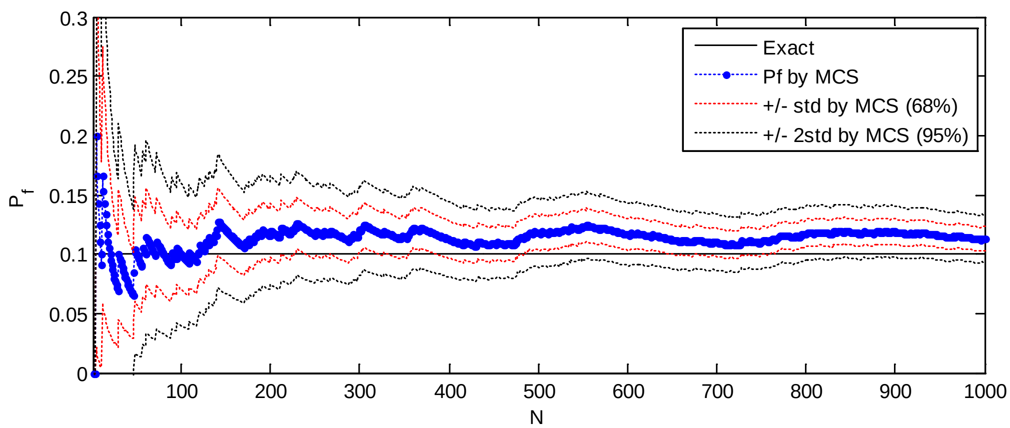
- $\tilde{p}$ is __________ variable.
- It is known that

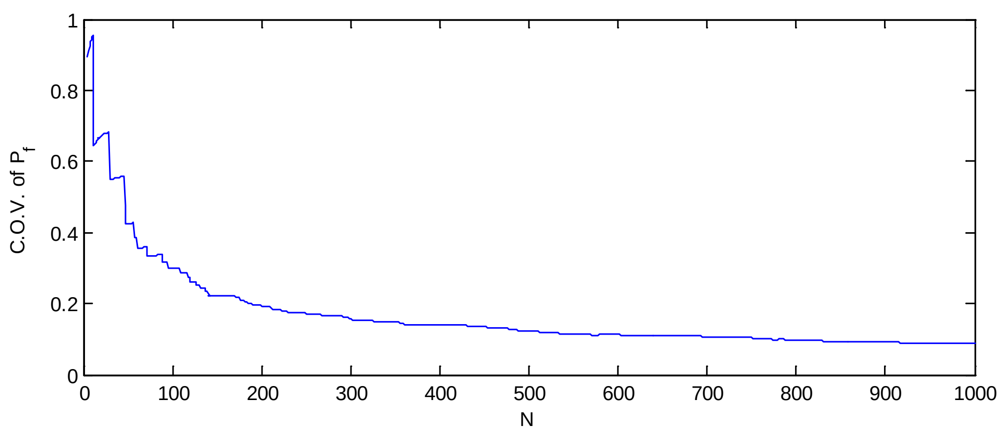
$$\text{E}[\tilde{p}] = p \qquad \text{(Un\_\_\_\_\_\_ estimate)}$$

$$\text{Var}[\tilde{p}] = \frac{p(1-p)}{N}$$ The more samplings, the smaller _____________

$$\delta_{\tilde{p}} = \frac{1}{\sqrt{N}}\sqrt{\frac{1-p}{p}}$$ C.O.V. of the randomness in the estimate

- Example: $p = 0.01$

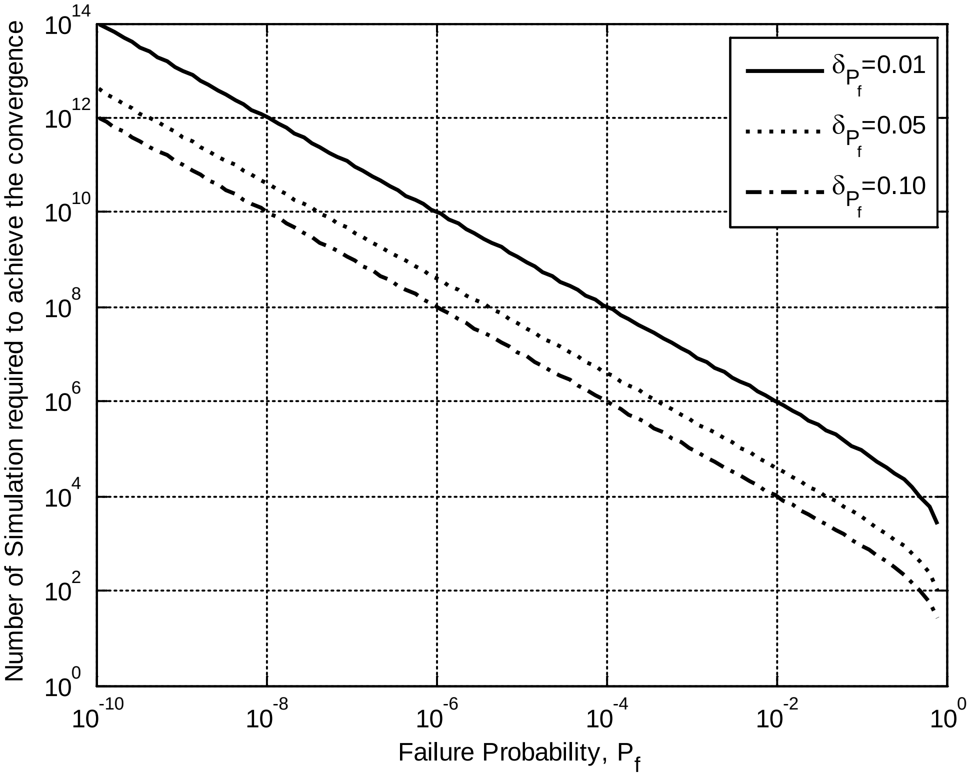




Instructor: Junho Song
junhosong@snu.ac.kr

- Minimum number of MCS to achieve a target level of C.O.V., $\bar{\delta}$

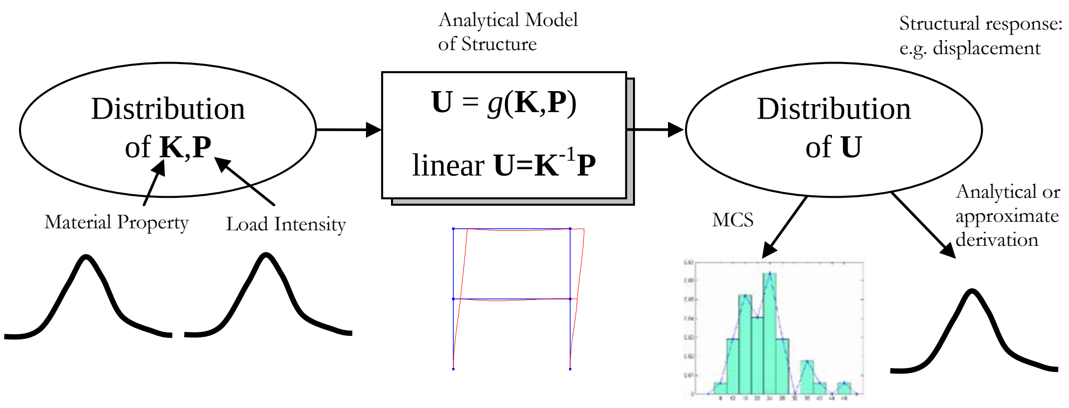
$$N_{\bar{\delta}} = \frac{1-p}{\bar{\delta}^2 p}$$



(e.g. $p = 0.01$ and $\bar{\delta} = 0.01$, $N_{\bar{\delta}} \cong 10^6$)

2. **Propagation of Uncertainty**

   (See **Supplement #4** – Matlab® demonstration by FRAME93)



3. **Central Limit Theorem**

(See **Supplement # 5** – Matlab® demonstration by randomwalk.m)

(a) Sum of many random variables asymptotically converges to (   ) r.v.

$$Y = \sum_{i=1}^{\infty} X_i \rightarrow N(\mu_Y, \sigma_Y)$$

(b) Product of many (positive) random variables asymptotically converges to (   ) r.v.

$$Y = \prod_{i=1}^{\infty} X_i \rightarrow ?$$

$$\ln Y = \sum_{i=1}^{\infty} \ln X_i \rightarrow N(\mu_{\ln Y}, \sigma_{\ln Y})$$ (Central Limit Theorem)

Therefore, $Y = \exp(\ln Y) \rightarrow LN(\lambda = \mu_{\ln Y}, \zeta = \sigma_{\ln Y})$

(c) Screen shots of randomwalk.m