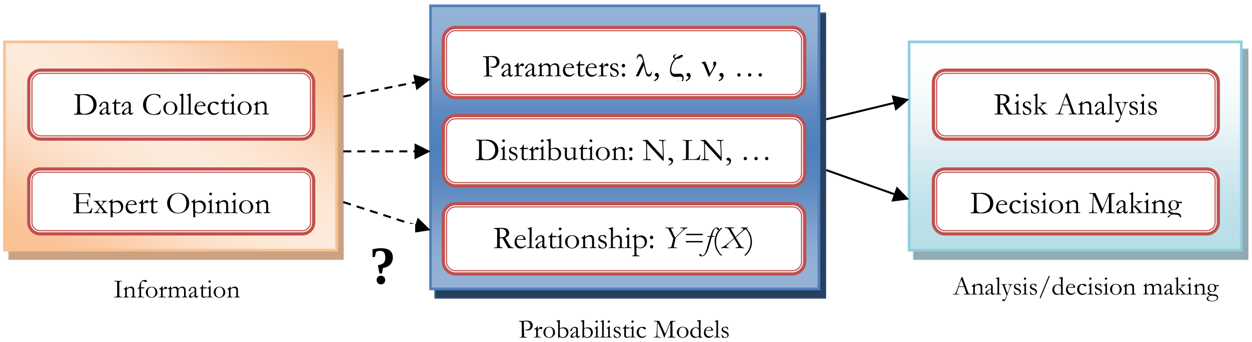
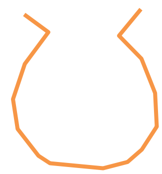
457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 19
Point Parameter Estimation (1) (A&T: 6.1-6.2)

1. **Statistical Inference**



2. **Point Estimation of Parameters**



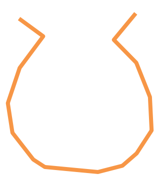
(a) Sample statistics $\hat{\theta}$: Estimator of true parameter θ

Example: Sample mean $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ is an estimator of true mean μ, i.e. $\hat{\mu} = \bar{X}$

Sample standard deviation $\hat{\sigma} = s$

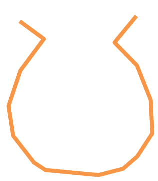
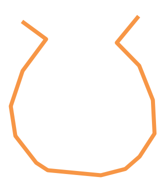
(b) Desirable properties of a point estimator

(i) Unbiased: $E[\hat{\theta}] = \theta$ ~ Average of point estimates is the same as the true parameter.



*Recall: “biased” (1/*N*) and “unbiased” (1/(*N*-1)) sample standard deviation

(ii) Consistent: $\lim_{N\to\infty} \text{Var}[\hat{\theta}] = 0$ ~ As the size of the sample increases, it converges.

(iii) Efficient: $\text{Var}[\hat{\theta}]$ as small as possible (for the same N).

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Example 1: Is the sample mean $\overline{X}$ an "unbiased" and "consistent" estimator?

3. **Point Estimation by "Method of Moments"**

Step 1: Find the relationship between the true parameter and moments

$$\theta = g(\mathrm{E}[X], \mathrm{E}[X^2], \ldots)$$

Step 2: Estimate the moments by

$$\hat{\mathrm{E}}[X^m] = \frac{1}{N}\sum_{i=1}^{N} x_i^m$$

Step 3: Substitute the estimated moments into the relationship in Step 1

$$\hat{\theta} = g(\hat{\mathrm{E}}[X], \hat{\mathrm{E}}[X^2], \ldots)$$

(a) Point estimate of mean (μ) by Method of Moments

$\mu =$

$\hat{E}[X] =$

$\hat{\mu} =$

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(b) Point estimate of variance by M.M.

$\sigma^2 =$

$\hat{\mathrm{E}}[X] =$

$\hat{\mathrm{E}}[X^2] =$

$$\hat{\sigma}^2 = \hat{\mathrm{E}}[X^2] - \hat{\mathrm{E}}[X]^2 = \frac{1}{N}\left(\sum_{i=1}^{N} x_i^2 - N\overline{X}^2\right) = s_{\text{biased}}^2$$

*Note: use "unbiased" one:

(c) Point estimate on other distribution parameters by M.M.

~ Use the relationship between parameters and moments (Table 6.1 A&T; Supp. #6)

(i) Lognormal distribution parameters λ and ζ

$$\hat{\lambda} = \ln\hat{\mu} - 0.5\ln\left(1 + \frac{\hat{\sigma}^2}{\hat{\mu}^2}\right),\ \hat{\zeta} = \sqrt{\ln\left(1 + \frac{\hat{\sigma}^2}{\hat{\mu}^2}\right)}$$

(ii) Exponential distribution parameter ν: $\mu = \dfrac{1}{\nu}$

Therefore, $\hat{\nu} = \dfrac{1}{\hat{\mu}}$

Example 2: Table E6.1 A&T: Concrete crushing strength of 25 test data $\{x_1, \ldots, x_{25}\}$

$$\sum_{i=1}^{25} x_i = 140,\quad \sum_{i=1}^{25} x_i^2 = 794.52$$

(a) Point estimate of mean by M.M.: $\hat{\mu}$

(b) Point estimate of standard deviation by M.M.: $\hat{\sigma}$

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(c) Point estimates of lognormal distribution parameters by M.M.: $\hat{\lambda}$ and $\hat{\zeta}$

(d) Point estimates of Gamma distribution parameters by M.M.: $\hat{\nu}$ and $\hat{k}$

Hint: $\mu = k/\nu$ and $\sigma^2 = k/\nu^2$ for a Gamma distribution.

Example 3 (Modified A&T Example 6.2): Data for the fatigue life of 75 S-T Aluminum available. Its sample mean and variances are

$\bar{X} = 26.75$ million cycles

$s^2 = 360.0$ million cycles

If the fatigue life follows a Gumbel distribution (See Table 6.1 A&T), what are the point estimates on the distribution parameters u and α by M.M.?

Hint: $\mu = u + \dfrac{0.5772}{\alpha}$, $\sigma^2 = \dfrac{\pi^2}{6\alpha^2}$