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457.212 Statistics for Civil & Environmental Engineers

In-Class Material: Class 22

Hypothesis Testing (A&T: 6.3)

1. Hypothesis Testing

Decide which of two competing claims or statements about a parameter is true based on a sample.

(a) **Step 1**: state **Null Hypothesis** H_0
- usually represents *status quo* about a parameter
- a statement one wishes to test based on a new sample and finding

(b) **Step 2**: formulate **Alternative Hypothesis** H_1
- usually represents *the question to be answered* or *theory to be tested*.
- a conjecture one can make based on a sample.
- the statement we should accept if the null hypothesis is rejected.
- note H_0 and H_1 are mutually (   )

(c) **Step 3**: try to reach one of the following conclusions:

- **Reject** H_0 in favor of H_1 because of sufficient evidence in the data or
- **Fail to reject** H_0 because of insufficient evidence in the data

Example: It has been believed that *p* = 0.10 and one conjectures that *p* might be "higher than 0.10" or "different from 0.10" from what he/she has seen in a sample.

H_0: $p = 0.10$

H_1: $p > 0.10$ (one-sided) or $p \neq 0.10$ (two-sided)

(d) Test **errors**: Due to the limited size of a sample, there is a probability of making a wrong conclusion.

- **Type I Error:** Rejecting H_0 even though it is true.
  The probability of Type I Error: α, "level of significance"
- **Type II Error:** H_0 because of insufficient evidence in the data.
  The probability of Type II Error: β

	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Type II error
Reject H_0	Type I error	Correct Decision

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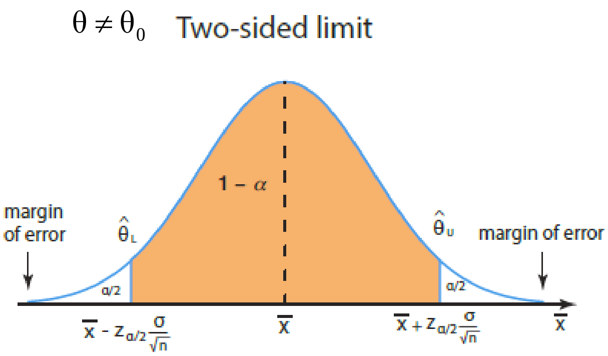

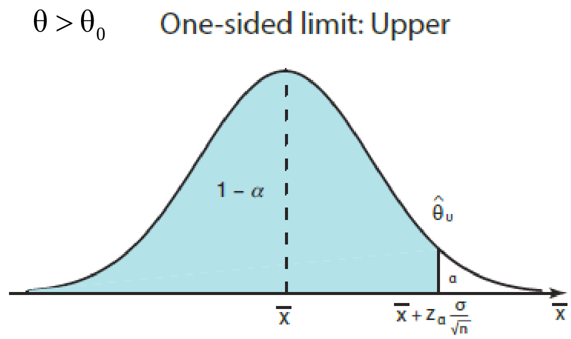

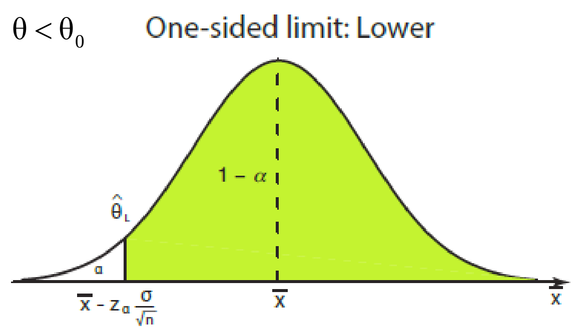

Example 1: A certain type of cold vaccine is known to be only 25% effective after a period of two years. To determine if a new and somewhat more expensive vaccine is superior in providing protection, 20 people are chosen at random.

Proposed testing method: If more than 8 of those receiving the new vaccine surpass the 2-year period without contracting the virus, the new vaccine will be considered superior to the one presently in use.

Let *p* denote the proportion of the population of receiving the new vaccine that surpass the 2-year period without contracting the virus.

(a) Null hypothesis and alternative hypothesis.

(b) Type I error α, or "level of significance"?

- The null hypothesis is being tested at α = level of significance.

(c) Type II error β (for an alternative hypothesis p = 0.5)?

(d) **New testing method**: If more than 7 surpass, the new vaccine is considered superior

α =

β =

Note: The probability of committing both types of error can be reduced by increasing the sample size

Example 2 [Source: *Applied Statistics and Probability for Engineers* by Montgomery & Runger, 5th edition, 2011]

Suppose that an engineer is designing an air crew escape system that consists of an ejection seat and a rocket motor that powers the seat. The rocket motor contains a propellant, and in order for the ejection seat to function properly, the propellant should have a mean burning rate of 50 cm/sec.

So, we are interested in deciding whether or not the mean burning rate is 50 cm/sec.

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For hypothesis testing, we may express this formally as:

$H_0: \mu = 50$ cm/sec
$H_1: \mu \neq 50$ cm/sec

Suppose that a sample of n = 10 specimens is tested and that the sample mean burning rate $\overline{X}$ is observed. Let us assume that if $48.5 < \overline{X} < 51.5$, we will not reject the null hypothesis H_0, and if either $\overline{X} < 48.5$ or $\overline{X} > 51.5$, we will reject the null hypothesis in favor of the alternative hypothesis H_1.

Assume that the standard deviation of burning rate is $\sigma = 2.5$ cm/sec and the burning rate has a distribution for which the conditions of the central limit theorem apply.

(a) What is the probability of Type I error?

(b) What is the probability of Type II error when the true mean $\mu = 52$ cm/sec?

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Example 3 (A&T 6.5): The specification for the yield strength of rebars requires a mean value of 38 psi. From the rebars delivered to the construction site, the engineer randomly selects 25 rebars and tested for yield strengths.

The sample mean is 37.5 psi. The (exact) standard deviation of rebar strength is known to be 3.0 psi.

Since the engineer would be concerned only with rebars having mean yield strength lower than 38 psi, a ____-sided test is appropriate.

(a) What are the null and alternative hypotheses?

(b) With 5 % significance level, would the null hypothesis be accepted?

Suppose the exact standard deviation is not known, so the testing should rely on the sample standard deviation 3.50 psi. Answer the above questions under this condition.