457.204 Elementary Fluid Mechanics and Lab.
Elementary Test

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ET 4: Free and Forced Vortex

1. Objective

A vortex can be seen in the spiraling motion of air or liquid around a center of rotation. Circular current of water of conflicting tides form vortex shapes. Turbulent flow makes many vortices. A good example of a vortex is the atmospheric phenomenon of a whirlwind or a tornado or dust devil. This whirling air mass mostly takes the form of a helix, column, or spiral. In this experiment, through the generating of free and forced vortex, we can compare one vortex with the other.

2. Theory

1) Forced vortex

In a forced vortex the fluid essentially rotates as a solid body (there is no shear). The motion can be realized by placing a dish of fluid on a turntable rotating at ω radians/sec; the fluid has vorticity of 2ω everywhere, and the free surface (if present) is a parabola. The tangential velocity is given by:

$$q = \omega r \tag{1}$$

Where ω is the angular velocity and r is the radial distance from the center of the vortex.

In this forced vortex, equation of centripetal force is given by:

$$\frac{\delta p}{\rho g} + \delta z = \frac{q^2}{r}\frac{\delta r}{g} \tag{2}$$

Substitute eq. (1) into eq. (2), and then integrate,

$$\frac{p}{\rho g} + z = \frac{\varpi^2 r^2}{2g} + \text{constant} \tag{3}$$

Because of free surface condition, $p = 0$. Therefore,

$$z = \frac{\varpi^2 r^2}{2g} + \text{constant} \tag{4}$$

At the boundary condition ($z = z_0 \; at \; r = 0$),

$$z = \frac{\varpi^2 r^2}{2g} + z_0 \tag{5}$$

z_0 is center of the forced vortex.

2) Free vortex

When fluid is drawn down a plug-hole, one can observe the phenomenon of a free vortex. The tangential velocity q varies inversely as the distance r from the center of rotation, so the angular momentum, qr, is constant; the vorticity is zero everywhere (except for a singularity at the center) and the circulation about a contour containing $r=0$ has the same value everywhere. The free surface (if present) dips sharply (as r^{-2}) as the center is approached.

The tangential velocity is given by:

$$q=\frac{k}{r} \tag{6}$$

In the steady state flow condition, Bernoulli's equation is applied:

$$\frac{p}{\rho g}+\frac{q^2}{2g}+z=\text{constant} \tag{7}$$

Because of free surface condition, $p=0$. Then substitute eq. (6) into eq. (7):

$$z=c-\frac{k^2}{r^2 2g} \tag{8}$$

Flow velocity is given by:

$$q=\sqrt{2gh} \tag{9}$$

3. Experimental Set-up

1) Hydraulics Bench

The bench is constructed from lightweight corrosion resistant plastic and is mounted on wheels for mobility. The bench top incorporates an open channel with side channels to support the accessory on test. The volumetric measuring tank is stepped to accommodate low or high flow rates. A stilling baffle reduces turbulence and a remote sight tube with scale gives an instantaneous indication of water level. A measuring cylinder is included in the supply for measurement of very small flow rates. A dump valve in the base of the volumetric tank is operated by a remote actuator. Opening the dump valve returns the measured volume of water to the sump in the base of the bench for recycling.

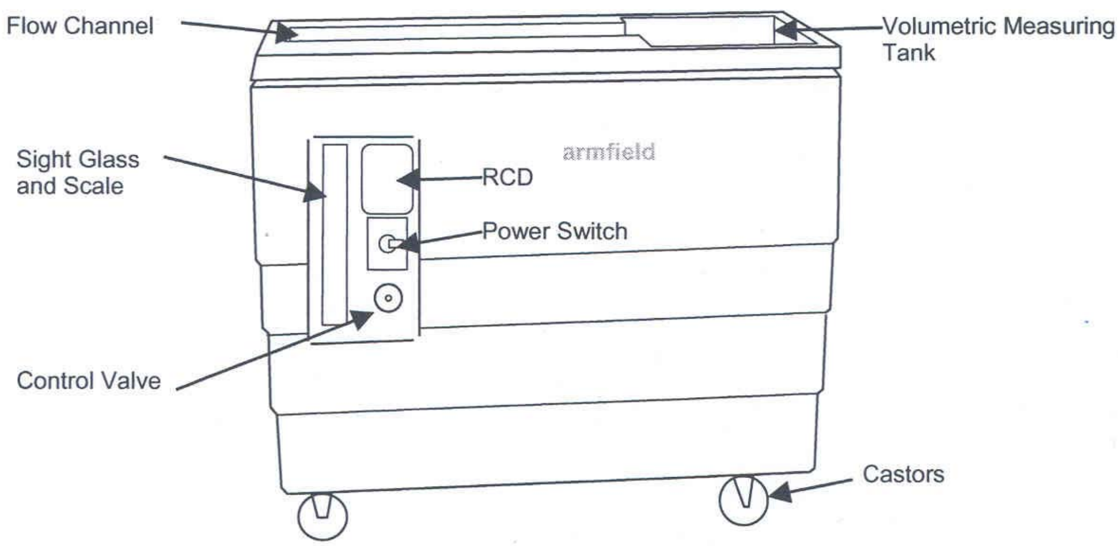

Fig. 1 Hydraulics Bench

2) Forced and Free Vortices Apparatus

The apparatus comprises a clear acrylic cylinder on a plinth designed to produce and measure free and forced vortices. The free vortex is generated by water discharging through an interchangeable orifice in the base of the cylinder and the resulting profile is measured using a combined caliper and depth scale. The forced vortex is induced by a paddle in the base of the cylinder which is rotated by jets of water. The profile of the forced vortex is determined using a series of depth gauges. Velocity at any point in the free or forced vortices may be measured using the appropriate pitot tube.

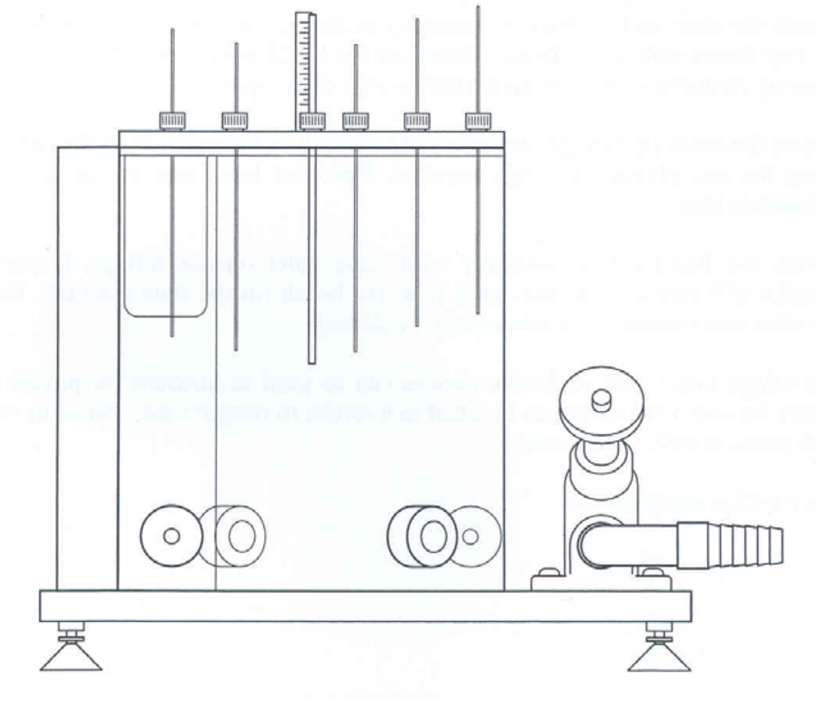

Fig. 2 Free & Forced Vortices apparatus

4. Procedure

1) Experiment of forced vortex

① Set the vortex apparatus on the Hydraulics Bench and keep it level by using the bubble level.

② Connect the Y-hose to the 9 mm inlet of the cylinder, and to the outlet of the Hydraulics Bench so the water can flow from the bench to the cylinder

③ Connect the hose to the 12 mm inlet of the cylinder, and to the outlet of the Hydraulics Bench so the water can flow to the volumetric tank.

④ Attach the paddle to the bottom center of the cylinder.

⑤ Attach a series depth gage on the cylinder.

⑥ Turn on the Hydraulics Bench and use the control valve to regulate the amount of water inflow to keep the water elevation steady.

⑦ Use the stopwatch and paddle to measure the angular velocity.

⑧ The profile is determined using a series of depth gauges.

⑨ Repeat the experiment by changing the angular velocity.

*Notice: A tube must be filled with flowing-water. Height of tube must low to prevent from a siphon phenomenon.

2) Experiment of free vortex

① Set the vortex apparatus on the Hydraulics Bench and keep it level by using the bubble level.

② Connect the Y-hose to the 12mm inlet of the cylinder, and to the outlet of the Hydraulics Bench so the water can flow from the bench to the cylinder

③ Close the valve on the outlet of the apparatus so the water flows outward only through the orifice of the cylinder.

④ Attach one of the orifices with different diameters to the bottom center of the cylinder.

⑤ Attach the vortex measure gage on the cylinder.

⑥ Turn on the Hydraulics Bench and use the control valve to regulate the amount of water inflow to keep the water elevation steady.

⑦ Use the gage to measure the shape of the free vortex.

⑧ If the vortex is not configured in the center of the cylinder and is difficult to measure with a gage, use only one needle of the gage to measure the shape.

⑨ Use the pitot tube to measure the hydraulic head. Use three pitot tubes for measurement.

⑩ Repeat the experiment by changing the diameter of the orifice.

5. Result

1) Forced vortex

• During the experiment of forced vortex, note the resultant table below.

• Repeat the experiment by changing the angular velocity as many as possible.

Table. 1 Test Result (Case #1: $n = 47$)

	Number of revolutions n	Time t (sec)	Revs Per sec r.p.s. (ω)	Radius r (m)	Measured Needle Length L_m (m)	Height from Datum z_m (m)	Calculated Height z_t (m)
1	47	60	4.922	0.110	0.1070	0.1700	0.1700
2	47	60	4.922	0.090	0.1135	0.1635	0.1650
3	47	60	4.922	0.070	0.1165	0.1605	0.1611
4	47	60	4.922	0.050	0.1185	0.1585	0.1581
5	47	60	4.922	0.030	0.1205	0.1565	0.1561
6	47	60	4.922	0.000	0.1220	0.1550	0.1550

• Plot a graph to compare calculated result with experimental result.

• Observe the characteristic of flow using little buoy on the water surface.

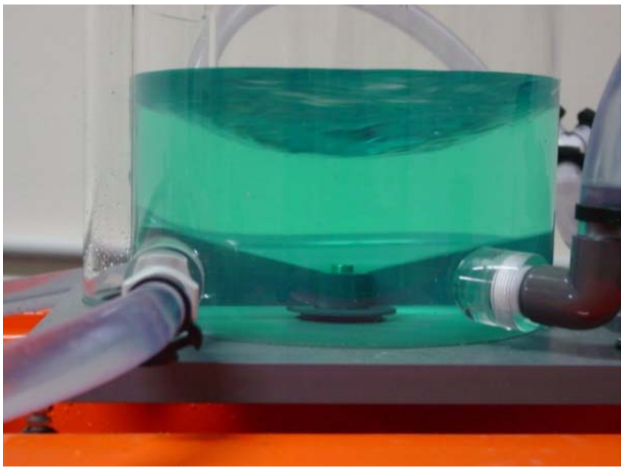

Fig. 3 Forced Vortex

2) Free vortex

• Plot a graph to compare calculated result with experimental result.

• Study the reason of difference between calculated result with measured result.

Table 2. Test Result (Case #1; Orifice radius=0.0024 m)

	Pitot tube arm length (m)	Pitot tube water height (m)	$q=\sqrt{2gh}$ (m/sec)	Datum Height z_0 (m)	Calculated Height z_t (m)
1	0.015	0.021	0.642	0.172	0.134
2	0.025	0.013	0.545	0.172	0.158
3	0.030	0.004	0.280	0.172	0.162

Fig. 4 Free Vortex (Case #1)