Chapter 2

Fluid Statics

Chapter 2 Fluid Statics

Contents

Objectives

- Study hydrostatic pressure relationship
- Study forces on surfaces by hydrostatic pressure
- Study buoyant forces

2.1 Pressure-Density-Height Relationship

Fluid statics

~ study of fluid problems in which there is no relative motion between fluid elements

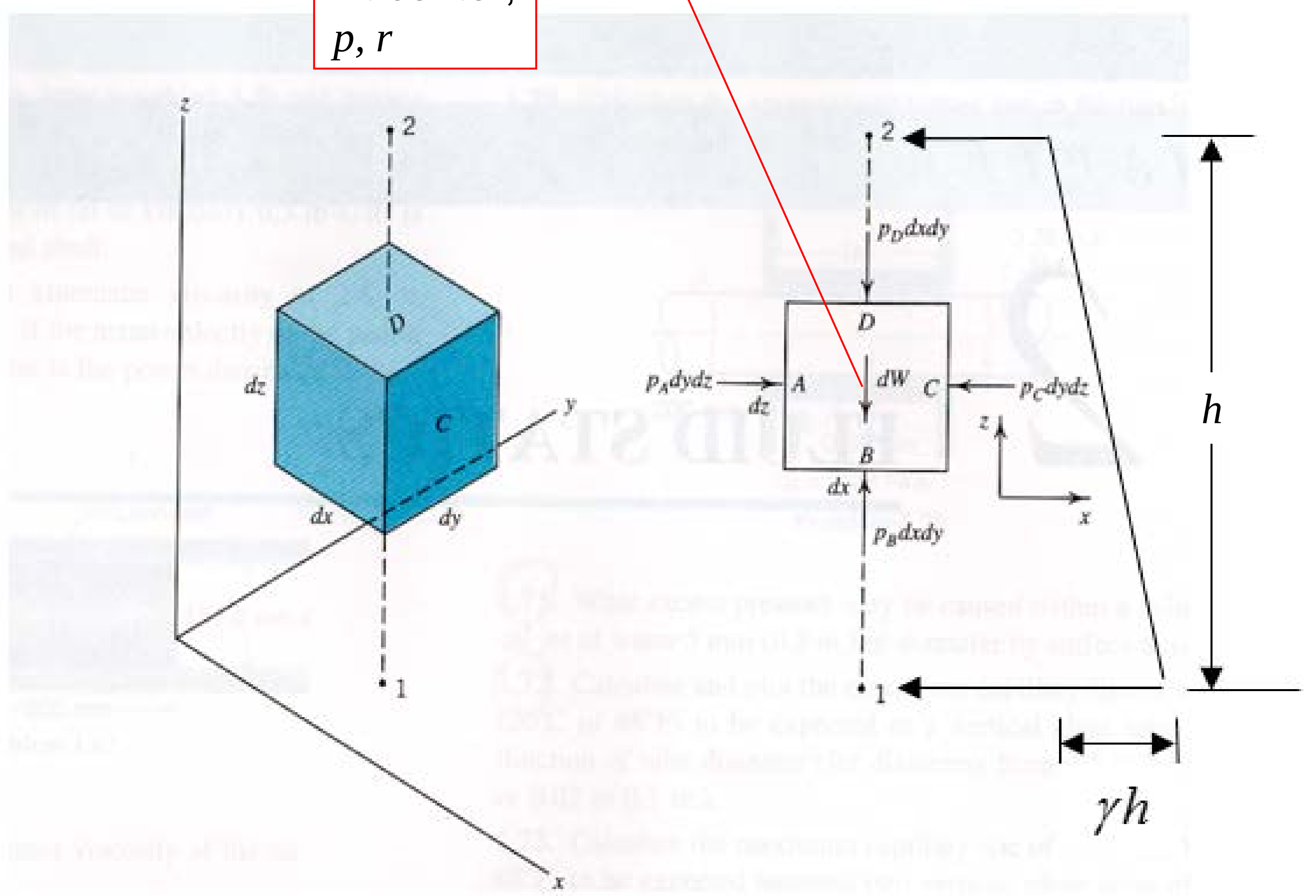
→ no velocity gradients

→ no shear stress

→ only normal pressure forces are present

2.1 Pressure-Density-Height Relationship

At center;
p, r



2.1 Pressure-Density-Height Relationship

• Static equilibrium of a typical differential element of fluid

- vertical axis = Z - axis = direction parallel to the gravitational force field

- Newton's first law

$$\sum F = 0$$

F = external force:

pressure, shear, gravity

$$\sum F_x = 0: \quad \sum F_x = p_A dz - p_c dz = 0 \qquad (2.1)$$

Assume unit thickness

in y direction; =1

$$\sum F_z = 0: \quad \sum p_B dx - p_D dx - dW = 0 \qquad (2.2)$$

in which $p = f(x,z)$

2.1 Pressure-Density-Height Relationship

$\dfrac{\partial p}{\partial x}$ = variation of pressure with direction

$$p_A = p - \frac{\partial p}{\partial x}\frac{dx}{2} \quad p_c = p + \frac{\partial p}{\partial x}\frac{dx}{2} \qquad (1)$$

$$p_B = p - \frac{\partial p}{\partial z}\frac{dz}{2} \quad p_D = p + \frac{\partial p}{\partial z}\frac{dz}{2} \qquad (2)$$

$$dW = \rho g dxdz = \gamma dxdz \qquad (3)$$

Substituting (1) and (3) into (2.1) yields

$$dF_x = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)dz - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)dz = -\frac{\partial p}{\partial x}dxdz = 0$$

$$\rightarrow \frac{\partial p}{\partial x} = 0 \qquad (A)$$

2.1 Pressure-Density-Height Relationship

Substituting (2) and (3) into (2.2) yields

$$dF_z = \left(p - \frac{\partial p}{\partial z}\frac{dz}{2} \right) dx - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2} \right) dx - \gamma dxdz = -\frac{\partial p}{\partial z} dzdx - \gamma dxdz = 0$$

$$\rightarrow \frac{\partial p}{\partial z} = \frac{dp}{dz} = -\gamma = -\rho g$$ (partial derivative → total derivative because of (A))

$$\boxed{\frac{\partial p}{\partial x} = 0 \ \left(\therefore p = fn(z \ only) \right)}$$

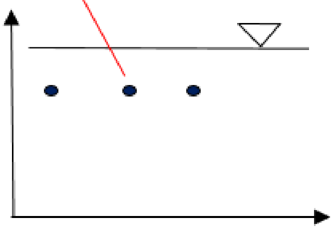
(1) $\frac{\partial p}{\partial x} = 0$

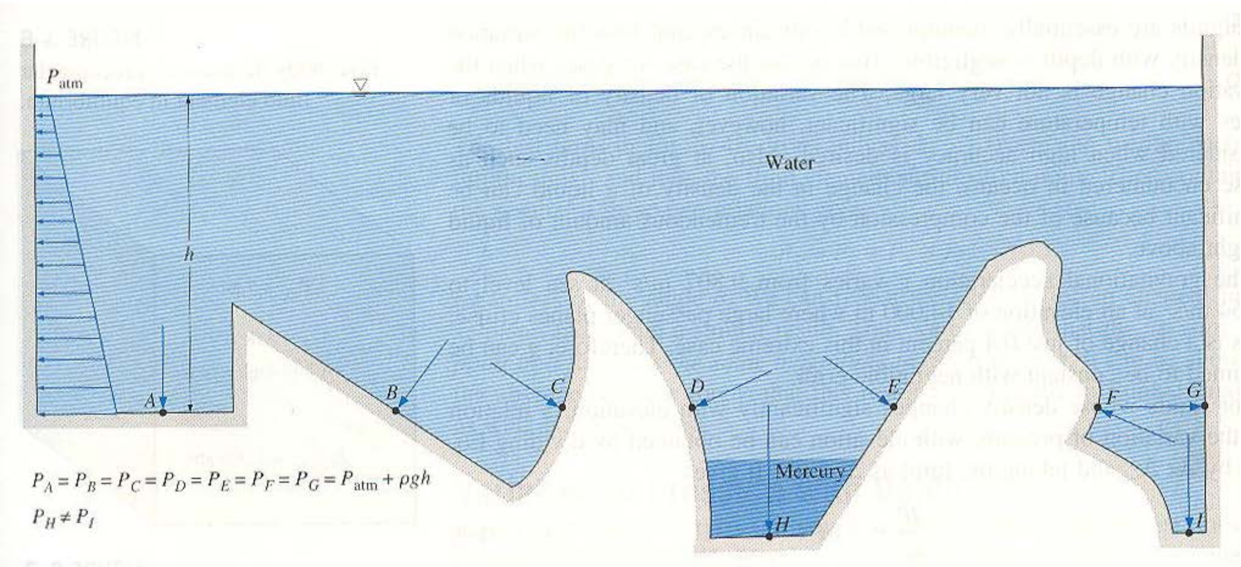
~ no variation of pressure with horizontal distance

~ pressure is constant in a horizontal plane in a static fluid

2.1 Pressure-Density-Height Relationship

$$p_1 = p_2 = p_3$$

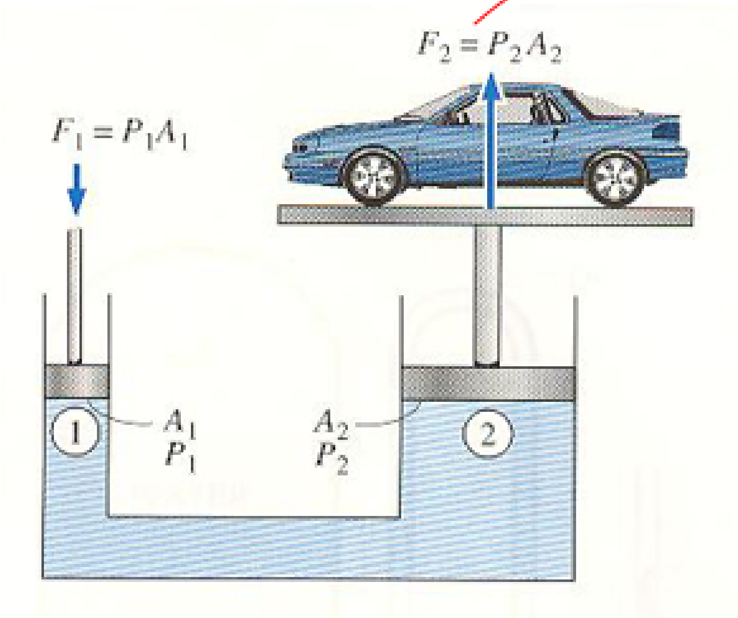




$P_A = P_B = P_C = P_D = P_E = P_F = P_G = P_{atm} + \rho g h$

$P_H \neq P_I$

2.1 Pressure-Density-Height Relationship



$$p_1 = p_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

$$\frac{A_2}{A_1} = mechanical\ advantage$$

of hydraulic lift

2.1 Pressure-Density-Height Relationship

(2) $\frac{dp}{dz} = -\gamma$ minus sign indicates that as z gets larger, the pressure gets smaller)

$$\rightarrow -dz = \frac{dp}{\gamma}$$

$$\int_{z_1}^{z_2} -dz = \int_{p_1}^{p_2} \frac{dp}{\gamma} \qquad (2.3)$$

Integrate over depth

$$(z_2 - z_1) = -\int_{p_1}^{p_2} \frac{dp}{\gamma} = \int_{p_2}^{p_1} \frac{dp}{\gamma} \qquad (2.4)$$

2.1 Pressure-Density-Height Relationship

For fluid of constant density (incompressible fluid; γ const.)

$$z_2 - z_1 = h = \frac{p_1 - p_2}{\gamma}$$

$$\therefore p_1 - p_2 = \gamma(z_2 - z_1) = \gamma h$$

$$\therefore p_1 = p_2 + \gamma h \qquad (2.5)$$

~ increase of pressure with depth in a fluid of constant density

→ <u>linear increase</u>

~ expressed as a head h of fluid of specific weight γ

~ heads in millimeters of mercury, meters of water; $\frac{\Delta p}{\gamma} = h$ (m)

2.1 Pressure-Density-Height Relationship

[Cf] For compressible fluid, $\gamma = fn(z \; or \; p)$

[Re] External forces

no contact

1) body force - forces acting on the fluid element

- gravity force, centrifugal force, Corioli's force (due to Earth's rotation)

2) surface force - forces transmitted from the surrounding fluid and acting at right angles against sides of the fluid element

- pressure, shear force

2.1 Pressure-Density-Height Relationship

- Manometer or Piezometer

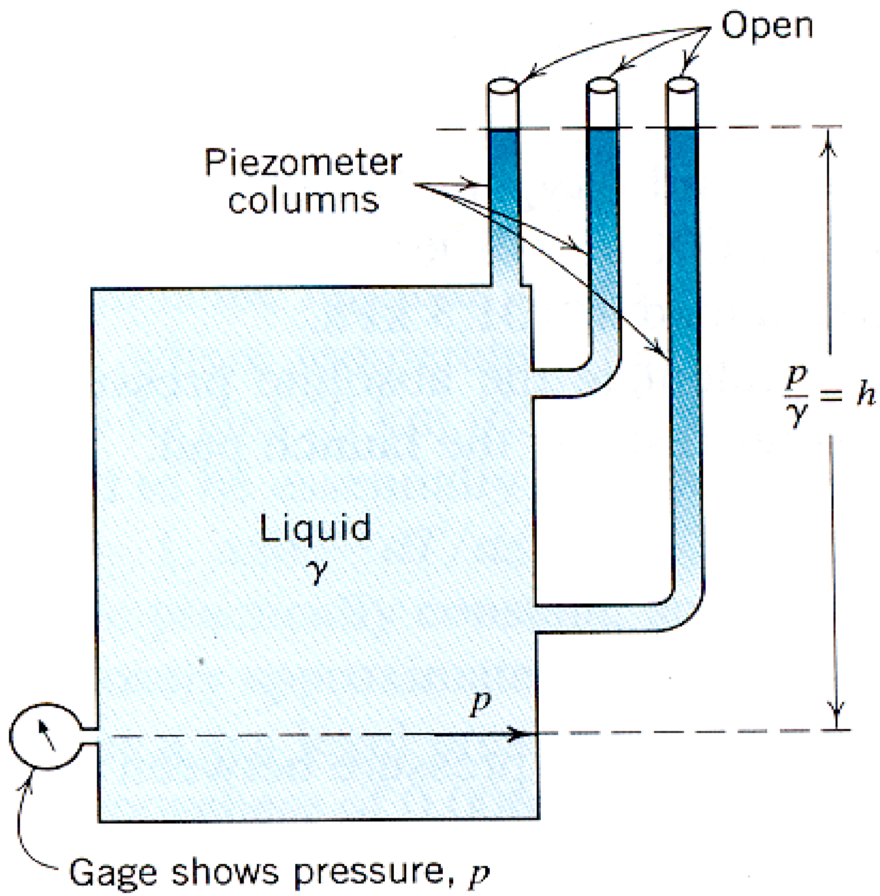
h = height of a column of any fluid

$$h \text{ (m of } H_2O) = \frac{p\,(\mathrm{kN/m^2})}{\underbrace{9.81\ \mathrm{kN/m^3}}_{\boxed{\gamma_w}}} = 0.102 \times p\,(\mathrm{kN/m^2})$$

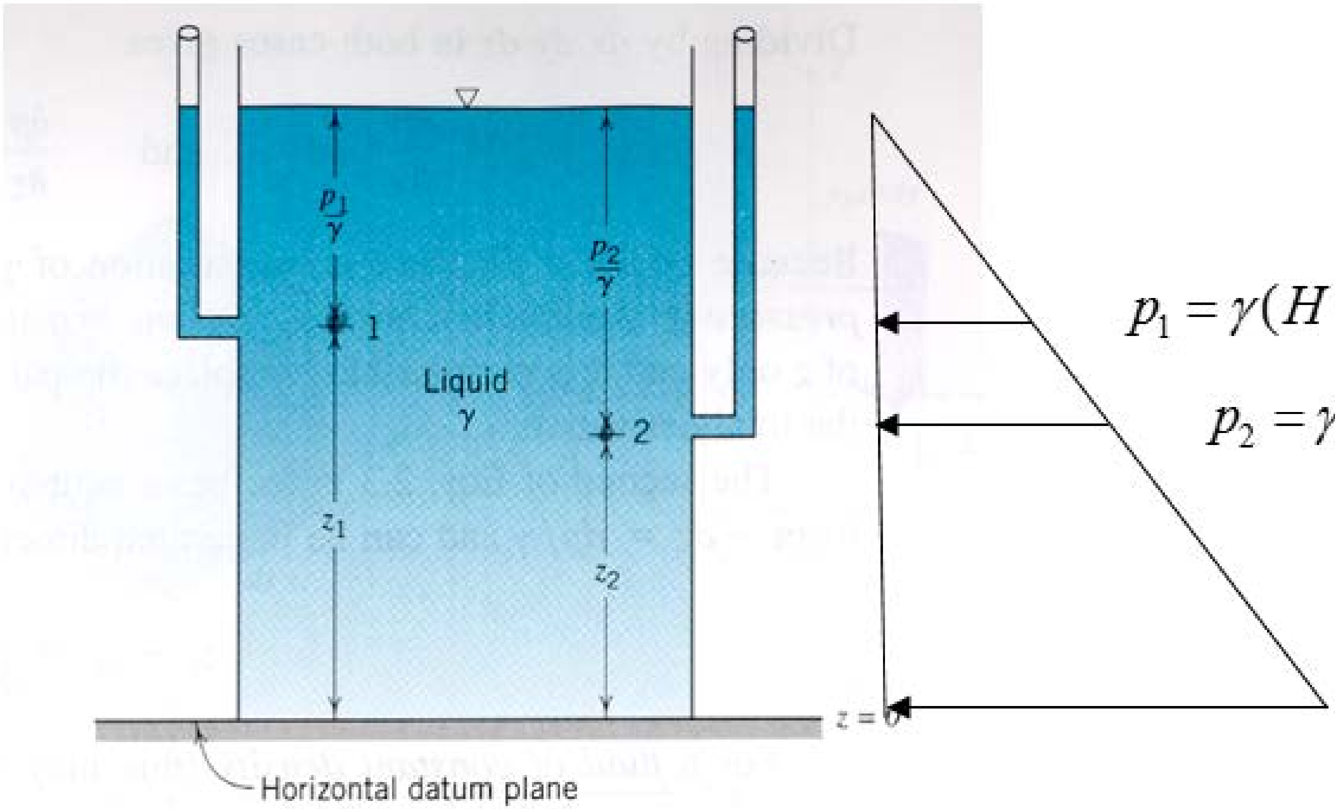
- For a static fluid

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{const.} \tag{2.6}$$

2.1 Pressure-Density-Height Relationship



2.1 Pressure-Density-Height Relationship



$p_1 = \gamma(H - z_1)$

$p_2 = \gamma(H - z_2)$

$p_B = \gamma H$

2.1 Pressure-Density-Height Relationship

• For a fluid of variable density (compressible fluid)

~ need to know a relationship between p and γ

~ oceanography, meteorology

[IP 2.1] The liquid oxygen (LOX) tank of space shuttle booster is filled to a depth of 10 m with LOX at -196°C. The absolute pressure in the vapor above the liquid surface is 101.3 kPa. Calculate absolute pressure at the inlet valve.

2.1 Pressure-Density-Height Relationship

[Sol]

From App. 2 (Table A2.1)

ρ of LOX at -196°C = 1,206 kg/m^3

$$p_2 = p_{atm} + \gamma_{LOX} h$$

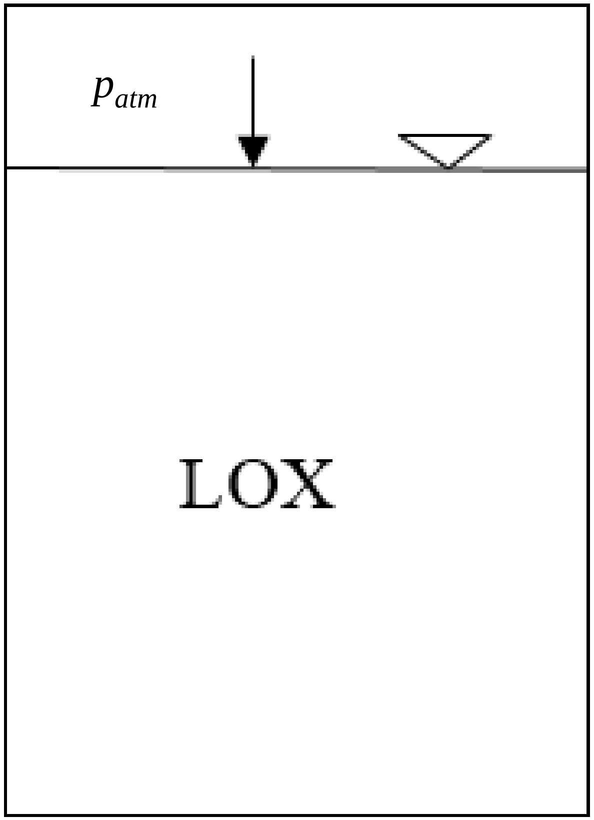
p_2 = 101.3 kPa+ (1,206 kg/m^3) (9.81 m/s^2) (10 m)

= 101.3 kPa+ 118,308 kg·m/s^2/m^2

= 101.3 kPa+ 118,308 kPa

= 219.6 kPa absolute

2.1 Pressure-Density-Height Relationship



2.2 Absolute and Gage Pressure

1) absolute pressure = atmospheric pressure + gage pressure for $p > p_{atm}$

atmospheric pressure - vacuum for $p < p_{atm}$

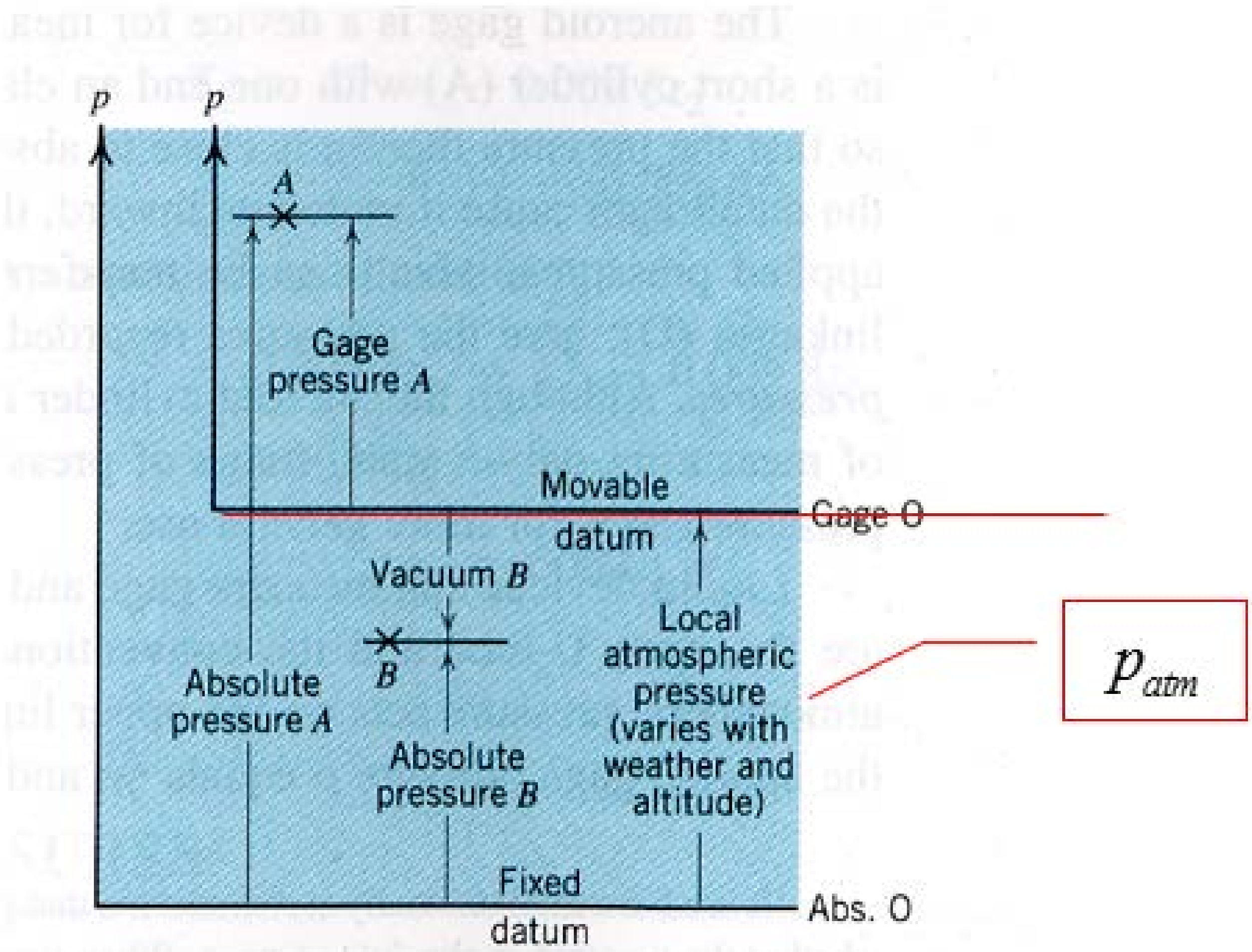
2) relative (gage) pressure → $p_{atm} = 0$

Bourdon pressure gage ~ measure gage pressure ⇒ open U-tube manometer

Aneroid pressure gage ~ measure absolute pressure ⇒ mercury barometer

- gage pressure is normally substituted by "pressure"

2.2 Absolute and Gage Pressure



2.2 Absolute and Gage Pressure

- Mercury barometer (Fig. 2.5)

  ~ invented by Torricelli (1643) → measure absolute pressure/local atmospheric pressure

  ~ filling tube with air-free mercury

  ~ inverting it with its open end beneath the mercury surface in the receptacle

[IP 2.4] A Bourdon gage registers <u>a vacuum</u> of 310 mm of mercury;

Gage pressure

$$p_{atm} = 100 \text{ kPa, absolute.}$$

Find absolute pressure.

2.2 Absolute and Gage Pressure

[Sol] absolute pressure = $100 \text{ kPa} - 310 \text{ mmHg}$

$$= 100 \text{ kPa} - 310\left(\frac{101.3 \text{ kPa}}{760}\right) = 58.7 \text{ kPa}$$

[Re] App. 1

760 mmHg = 101.3 kPa = 1,013 mb → 1 mmHg = 101,300 / 760

= 133.3 Pa

1 bar = 100 kPa = 10^3 mb

760 mmHg = 760×10^{-3} m×13.6×9,800 N/m^3 = 101.3 kN/m^2

= 101,300 N/m^2 / 9,800 N/m^3 = 10.3 m of H_2O

2.3 Manometry

~ more precise than Bourdon gage (mechanical gage)

(i) U-tube manometer

~ Over horizontal planes within continuous columns of the same fluid, pressures are equal.

$$\left(\because \frac{\partial p}{\partial x} = 0 \right)$$

$$\rightarrow p_1 = p_2$$

$$p_1 = p_x + \gamma l$$

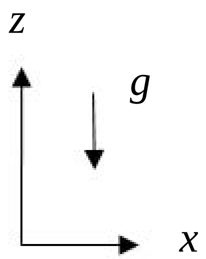
$$p_2 = 0 + \gamma_1 h$$

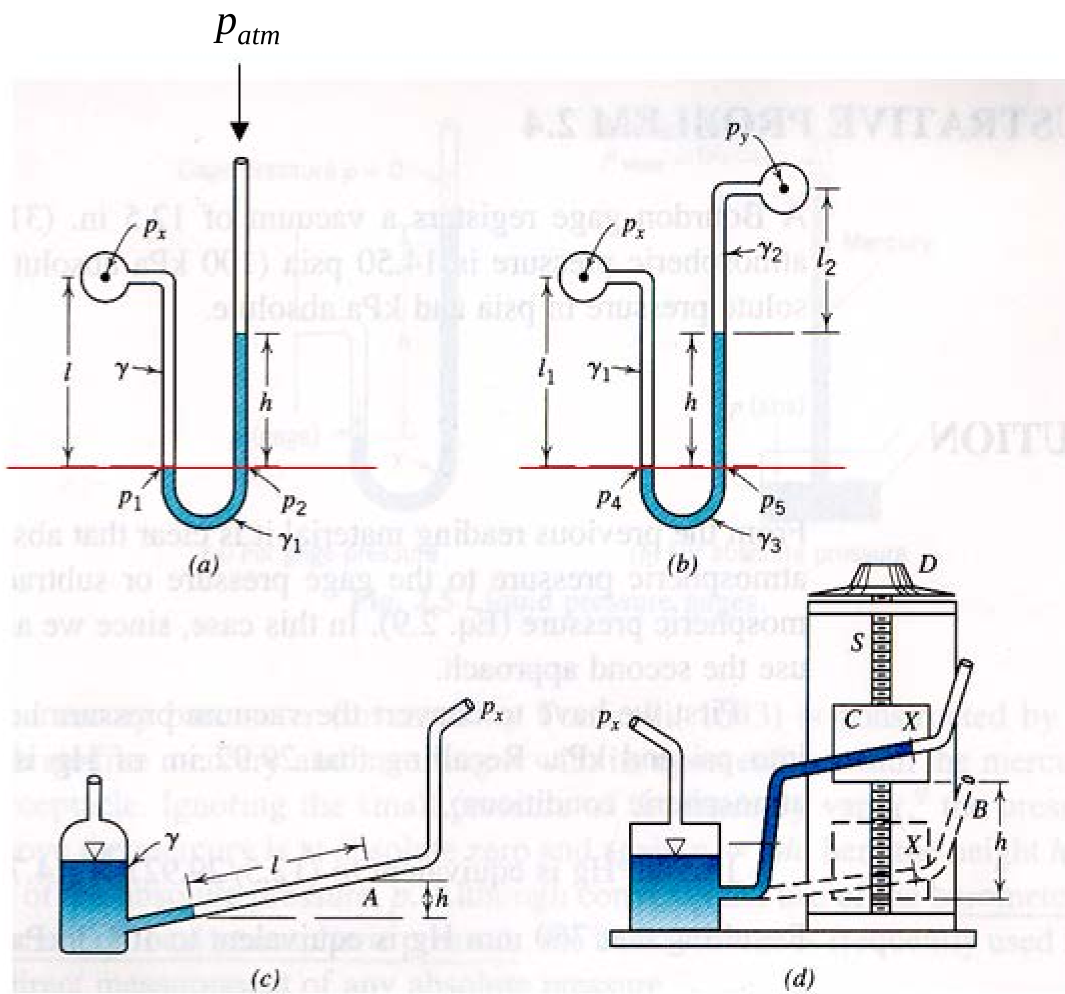
$$\boxed{P_{atm} \rightarrow 0}$$

$$p_1 = p_2;\ p_x + \gamma l = 0 + \gamma_1 h$$

$$\therefore p_x = \gamma_1 h - \gamma l$$

2.3 Manometry





2.3 Manometry

(ii) Differential manometer

~ measure difference between two unknown pressures

$$p_4 = p_5$$

$$p_4 = p_x + \gamma_1 l_1 \quad p_5 = p_y + \gamma_2 l_2 + \gamma_3 h$$

$$p_x + \gamma_1 l_1 = p_y + \gamma_2 l_2 + \gamma_3 h$$

$$\therefore p_x - p_y = \gamma_2 l_2 + l_3 h - \gamma_1 l_1$$

If $\gamma_1 = \gamma_2 = \gamma_w$ and x and y are horizontal

$$p_x - p_y = \gamma_3 h + \gamma_w \underbrace{(l_2 - l_1)}_{\boxed{-h}}$$

$$= \gamma_3 h + \gamma_w (-h) = (\gamma_3 - \gamma_w) h$$

head: $\dfrac{p_x - p_y}{\gamma_w} = \left(\dfrac{\gamma_3}{\gamma_w} - 1 \right) h$

2.3 Manometry

(iii) Inclined gages

~ measure the comparatively small pressure in low-velocity gas flows

$$p_x = \gamma h = \gamma l \sin\theta$$

reading of l > reading of h → accurate

(iv) Open-end manometer

$$p_D = p_B = p_C$$

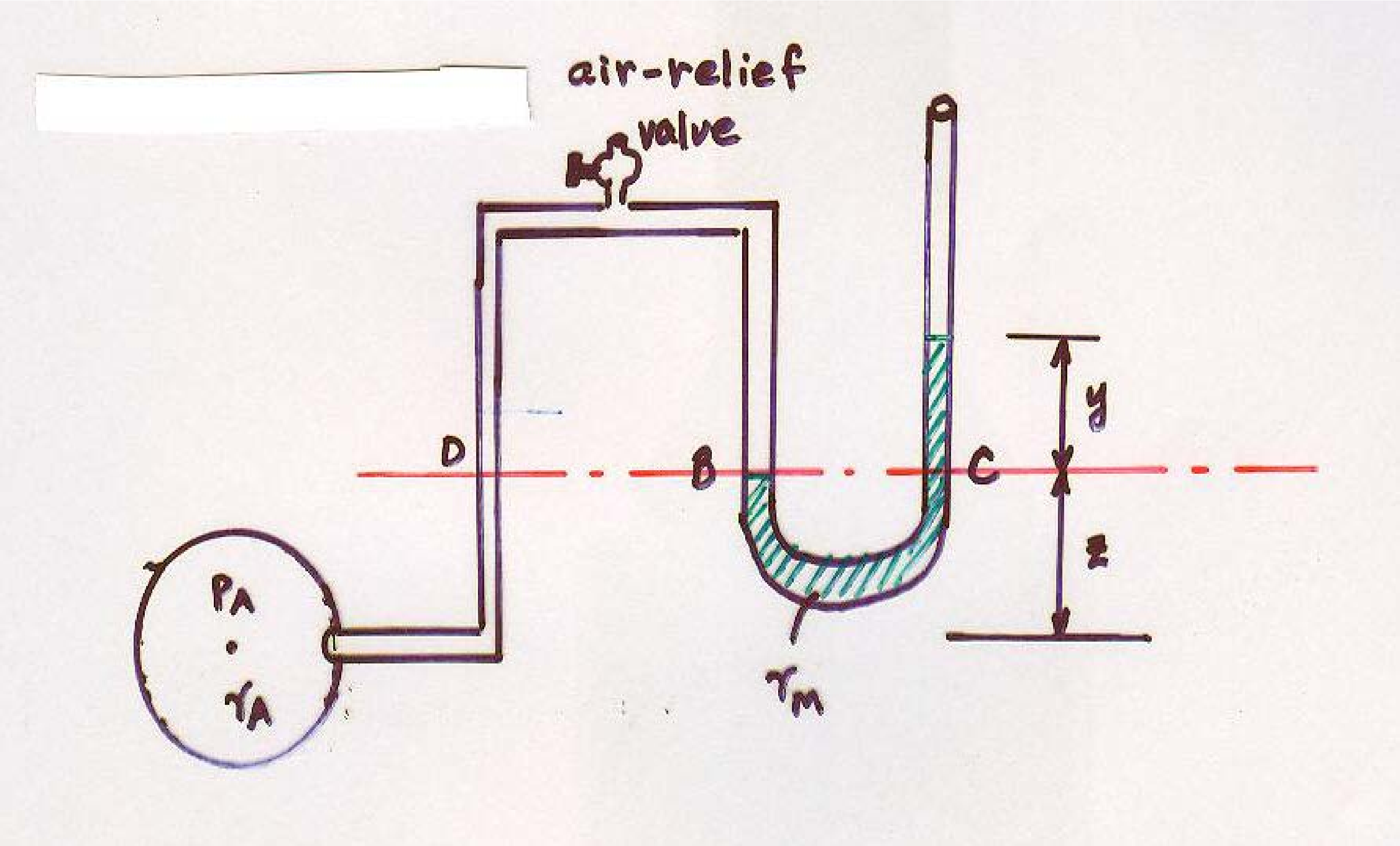
$$p_D = p_A - \gamma_A z$$

$$p_C = p_{atm} + \gamma_M y$$

$$p_A = p_{atm} + \gamma_M y + \gamma_A z$$

head : $\dfrac{p_A}{\gamma_A} = \dfrac{p_{atm}}{\gamma_A} + \dfrac{\gamma_M}{\gamma_A} y + z$

2.3 Manometry



2.3 Manometry

(v) Measure vacuum

$$p_1 = p_2$$

$$p_1 = p_A + \gamma_A z + \gamma_M y$$

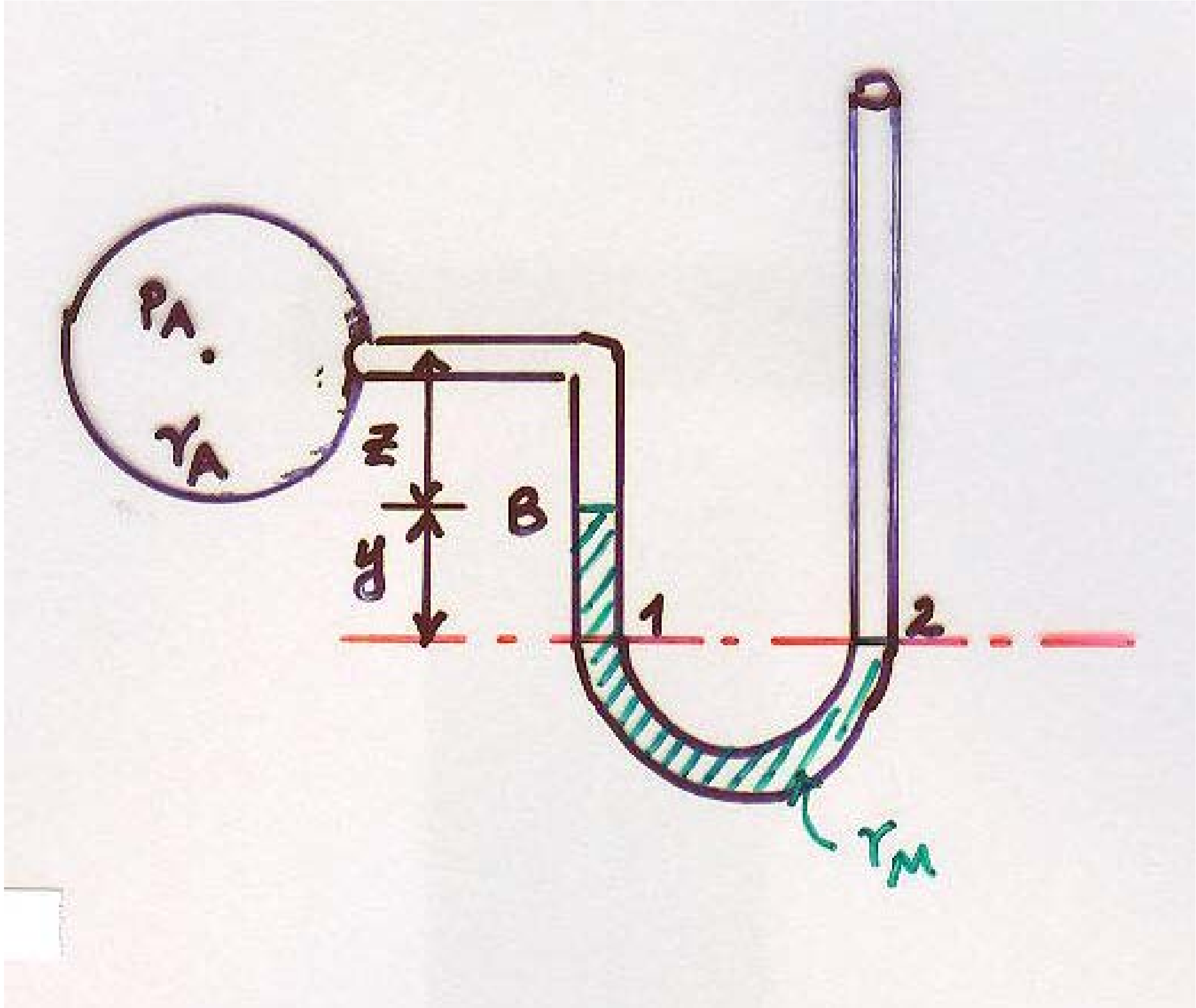
$$p_2 = p_{atm}$$

$$p_A + \gamma_A z + \gamma_M y = p_{atm}$$

$$p_A = p_{atm} - \gamma_A z - \gamma_M y$$

$$\boxed{p_A < p_{atm} \rightarrow vacuum}$$

2.3 Manometry



2.3 Manometry

(vi) Differential manometer

$$p_1 = p_2$$

$$p_1 = p_A - \gamma z_A$$

$$p_2 = p_B - \gamma z_B + \gamma_M y$$

$$\therefore p_A - \gamma z_A = p_B - \gamma z_B + \gamma_M y$$

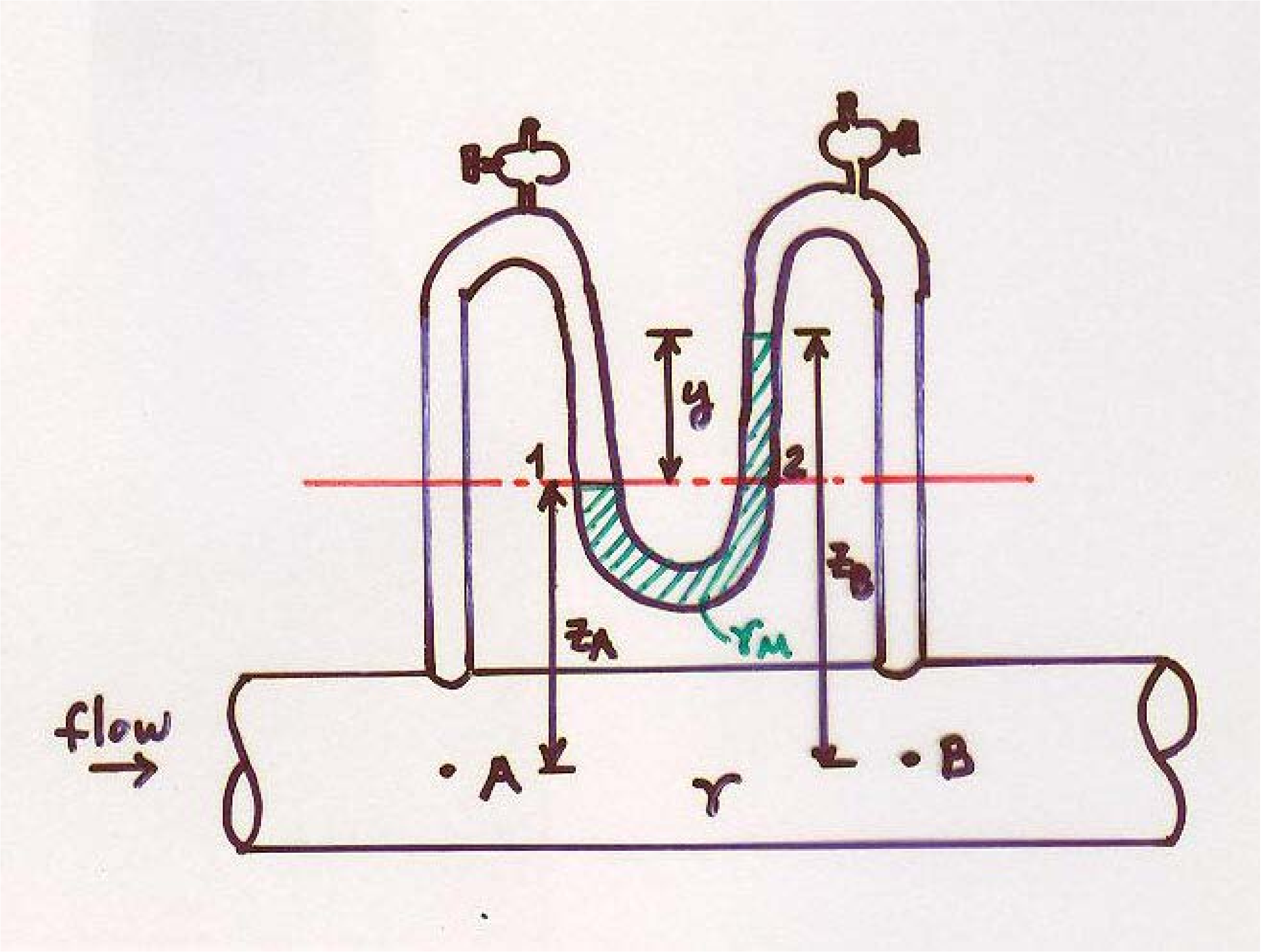
$$p_A - p_B = \gamma(z_A - z_B) + \gamma_M y$$

$$= -\gamma y + \gamma_M y = (\gamma_M - \gamma) y$$

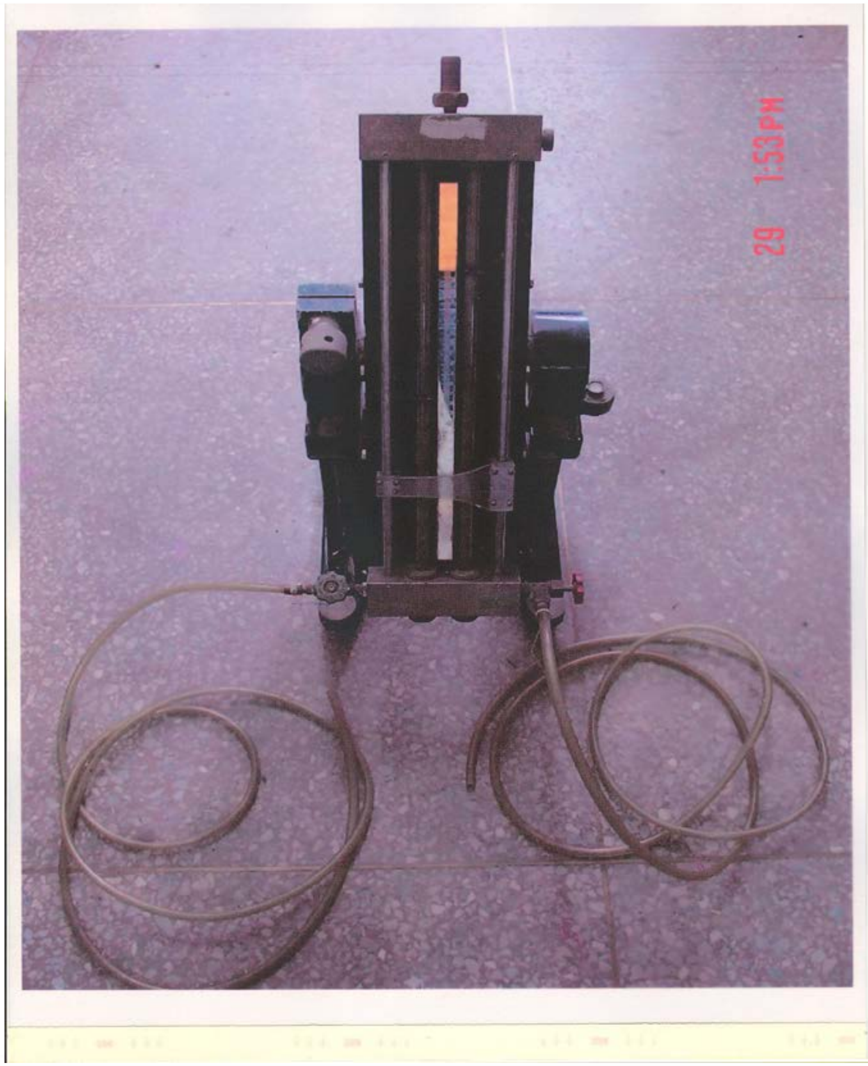
$$\frac{p_A - p_B}{\gamma} = \left(\frac{\gamma_M}{\gamma} - 1 \right) y$$

If $\gamma = \gamma_w \rightarrow \dfrac{p_A - p_B}{\gamma_w} = (s.g._M - 1) y$

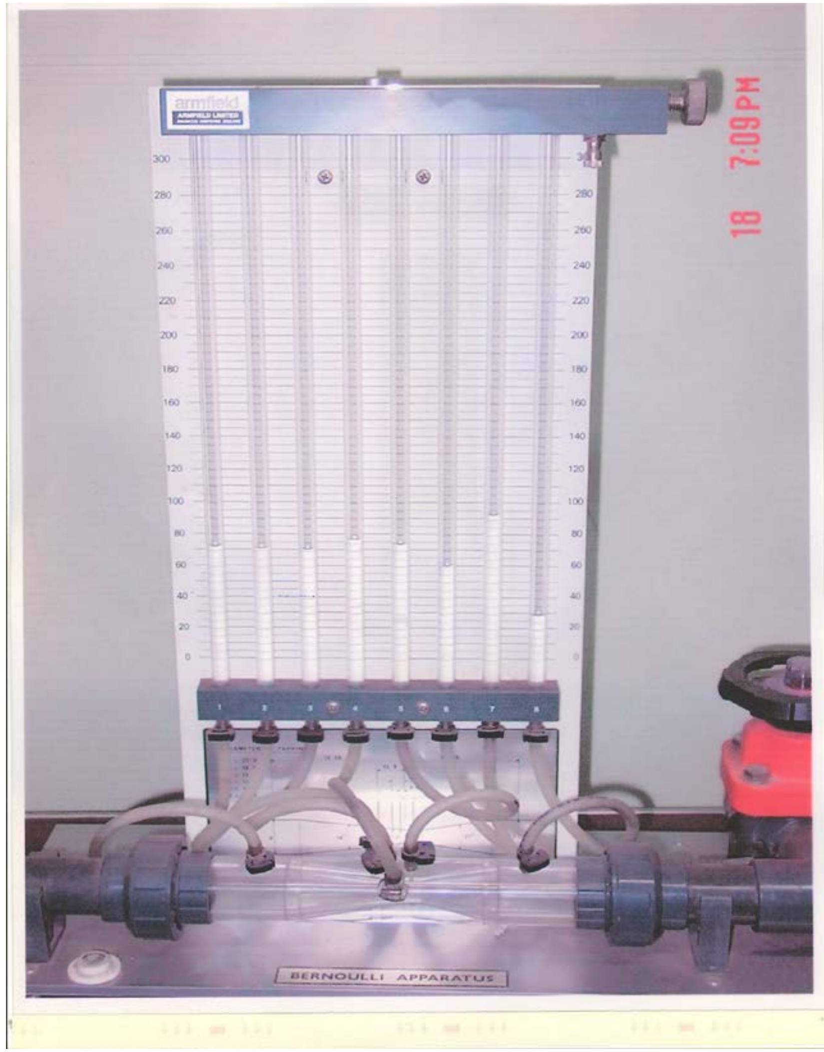
2.3 Manometry



2.3 Manometry



2.3 Manometry



2.3 Manometry

For measuring large pressure difference,

→ use heavy measuring liquid, such as mercury $s.g. = 13.55$ → makes y small

For a small pressure difference, $s.g. < 1$

→ use a light fluid such as oil, or even air

• Practical considerations for manometry

① Temperature effects on densities of manometer liquids should be appreciated.

② Errors due to capillarity may frequently be canceled by selecting manometer tubes of uniform sizes.

2.3 Manometry

[IP 2.5] The vertical pipeline shown contains oil of specific gravity 0.90. Find p_x

[Sol]

$$p_l = p_r$$

$$\boxed{\gamma_f = s.g. \times \gamma_w}$$

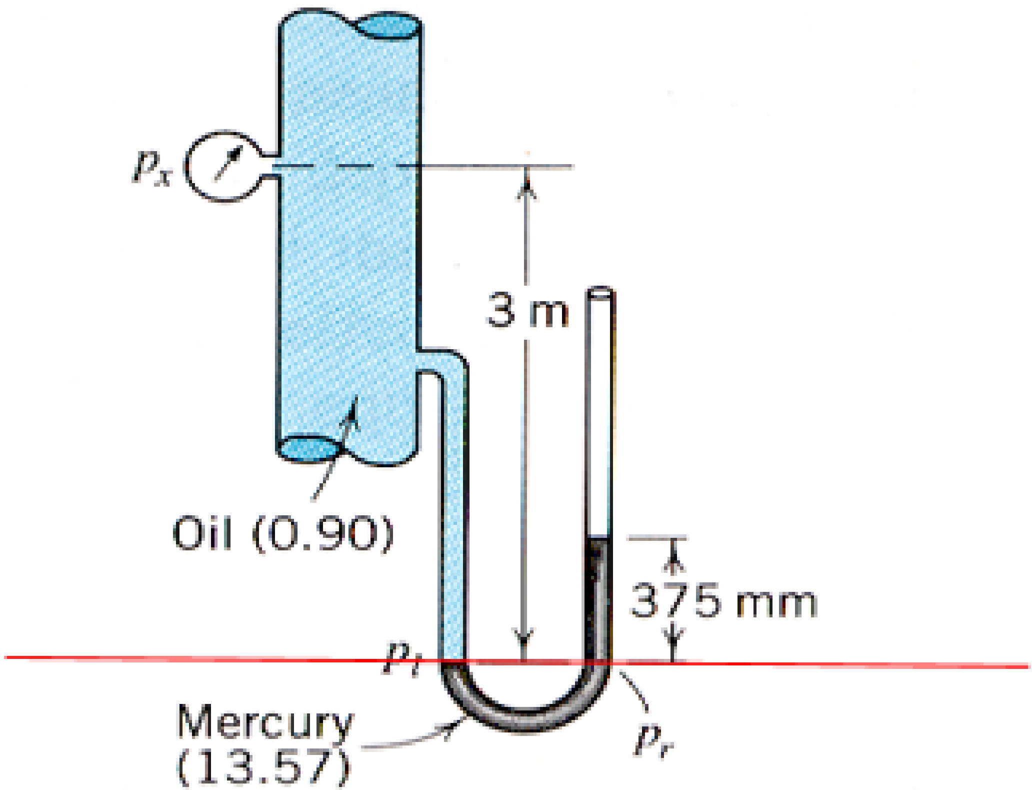
$$p_l = p_x + (0.90 \times 9.8 \times 10^3) \times 3$$

$$\boxed{p_{atm} = 0}$$

$$p_r = (13.57 \times 9.8 \times 10^3) \times 0.375$$

$\therefore p_x = 23.4$ kPa (kN/m^2)

2.3 Manometry



2.4 Forces on Submerged Plane Surfaces

- Calculation of magnitude, direction, and location of the total forces on surfaces submerged in a liquid is essential.

→ design of dams, bulkheads, gates, tanks, ships

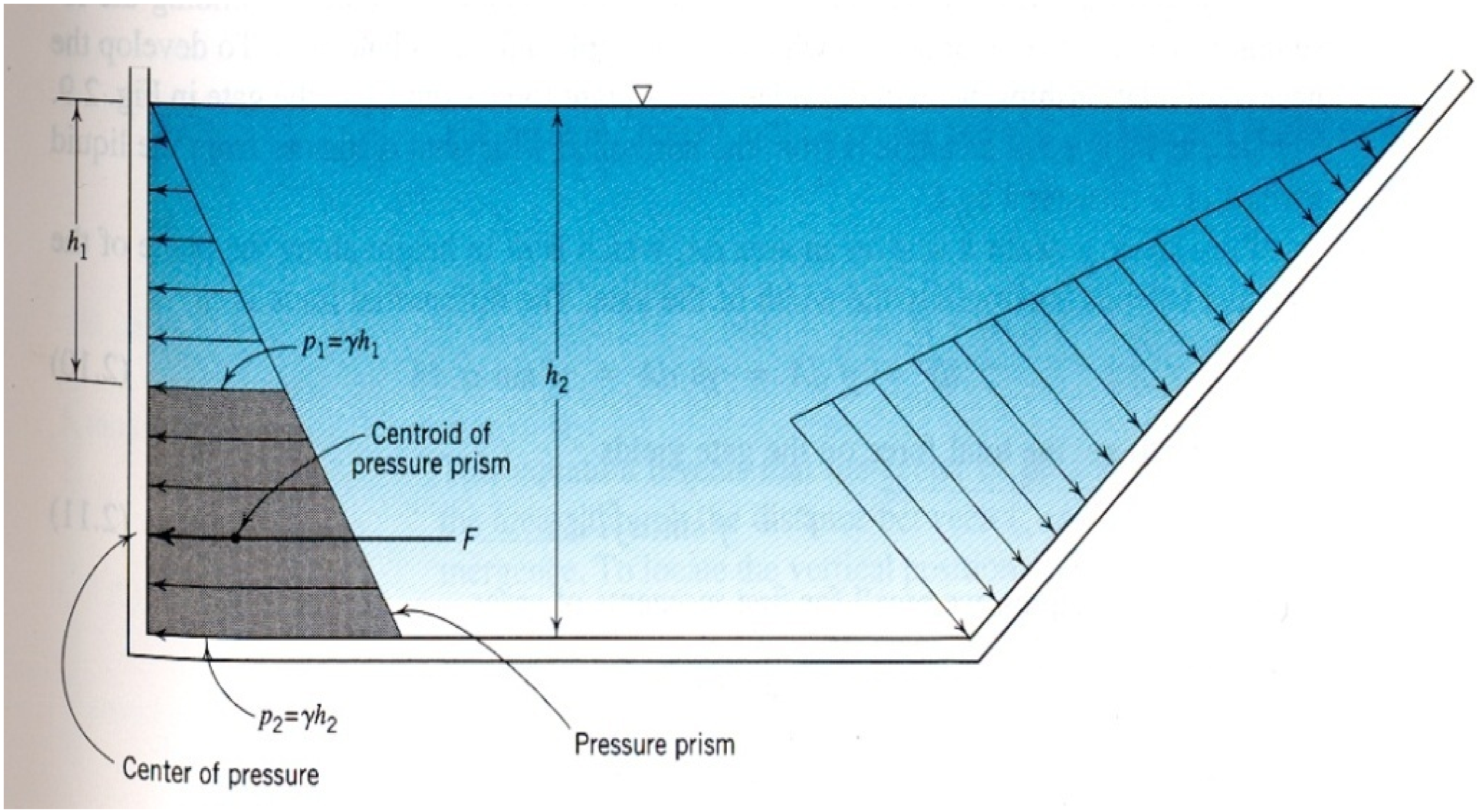
- Pressure variation for non-horizontal planes

$$\frac{\partial p}{\partial z} = -\gamma$$

$$\therefore p = \gamma h \qquad (2.7)$$

→ The pressure varies linearly with depth.

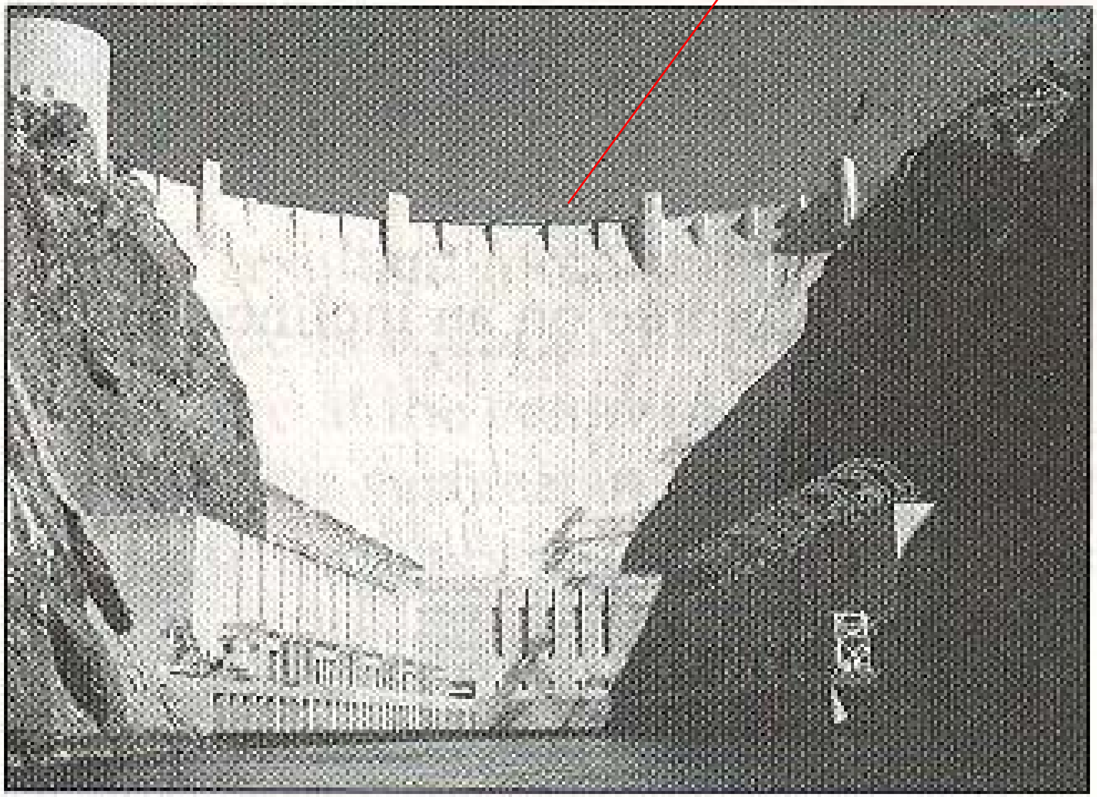
2.4 Forces on Submerged Plane Surfaces



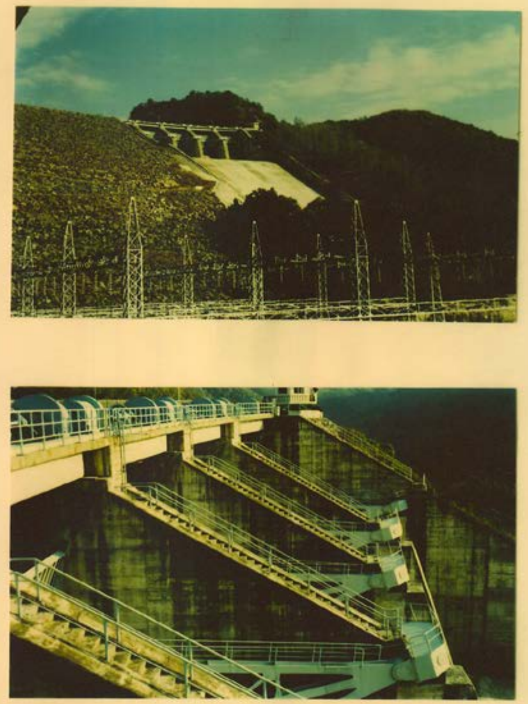
2.4 Forces on Submerged Plane Surfaces

Spillway

2.4 Forces on Submerged Plane Surfaces

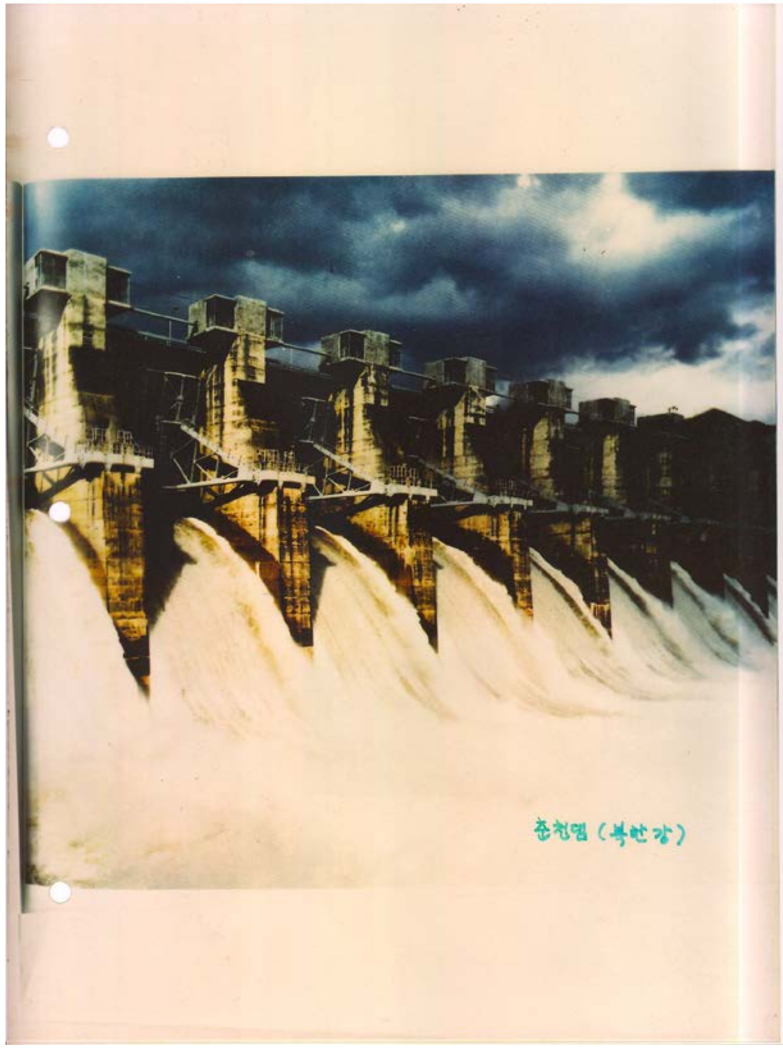


2.4 Forces on Submerged Plane Surfaces



소양강댐 여수로 (테인터게이트)

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

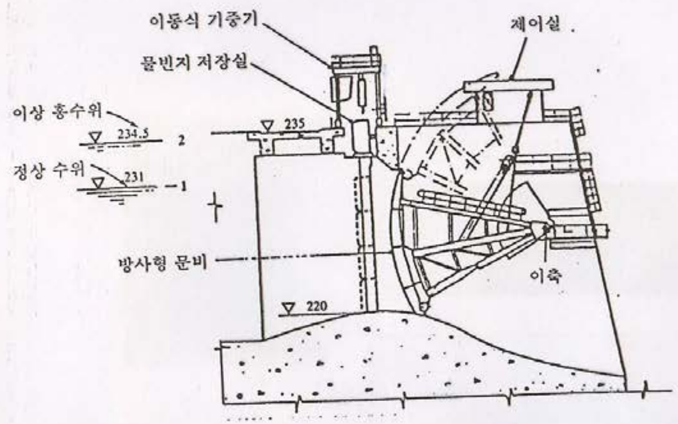


그림 6–15 테인터 문비(Self & Steel, 1987)

2.4 Forces on Submerged Plane Surfaces

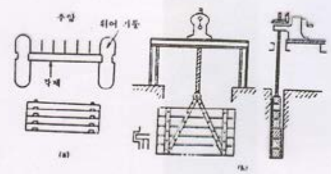
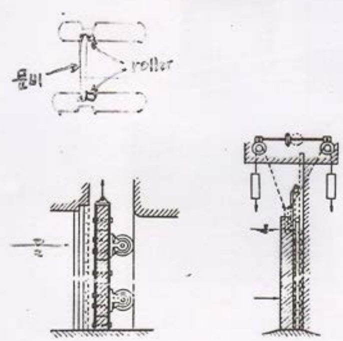


그림 4. 58 물빈지



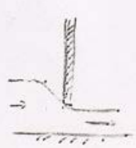


그림 4. 59 슬루스 게이트

2.4 Forces on Submerged Plane Surfaces

잠실수중보 (한강 잠실대교)

2.4 Forces on Submerged Plane Surfaces

- Pressure on the inclined plane

- Centroid of area A ~ at a depth h_c

  ~ at a distance l_c from the line of intersection 0-0

(i) Magnitude of total force

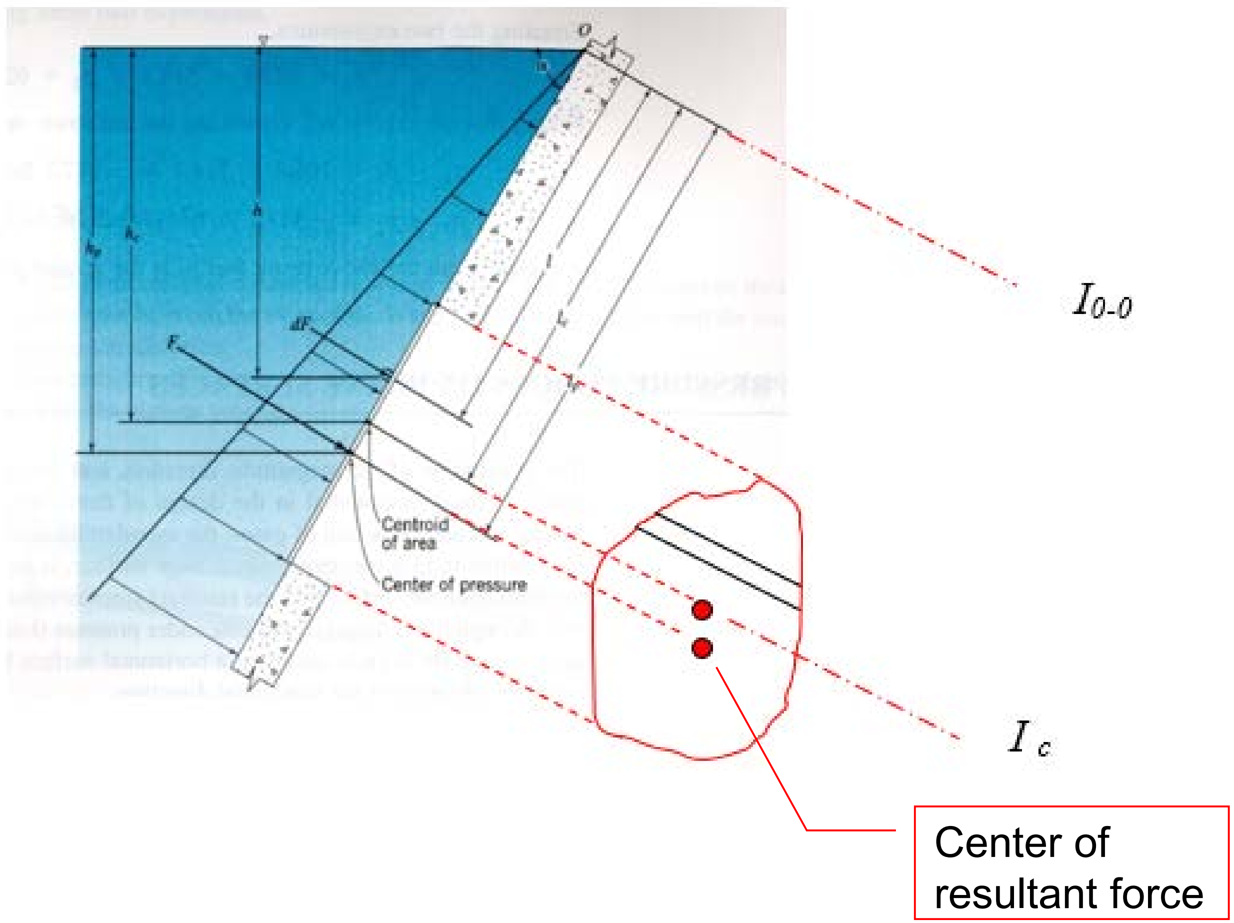
First, consider differential force dF

$$dF = pdA = \gamma h dA \quad (2.8)$$

$$h = l \sin\alpha \quad (2.9)$$

$$\rightarrow dF = \gamma l \sin\alpha \, dA \quad (2.10)$$

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

Then, integrate *dF* over area *A*

$$F = \int^{A} dF = \gamma \sin\alpha \int^{A} l\,dA \qquad (2.11)$$

in which $\int^{A} l\,dA$ = <u>1st moment of the area</u> *A* about the line 0-0

$$= A \cdot l_c$$

in which l_c = perpendicular distance from 0-0 to the centroid of area

$$\therefore F = \gamma A l_c \sin\alpha$$

Substitute $h_c = l_c \sin\alpha$

(pressure at centroid) × (area of plane)

$$\boxed{F = \gamma h_c A} \qquad (2.12)$$

2.4 Forces on Submerged Plane Surfaces

(ii) Location of total force

$\boxed{dF = \gamma l \sin\alpha dA}$

Consider moment of force about the line 0-0

$$dM = dF \cdot l = \gamma l^2 dA \sin\alpha$$

$$M = \int^A dM = \gamma \sin\alpha \int^A l^2 dA \qquad (2.13)$$

where $\int^A l^2 dA$ = <u>second moment of the area *A*</u>, about the line 0-0 $= I_{0-0}$

$$\therefore M = \gamma \sin\alpha I_{0-0} \qquad (a)$$

By the way,

unknown

$M = F \cdot l_p$ (total force × moment arm) (b)

2.4 Forces on Submerged Plane Surfaces

Combine (a) and (b)

$$Fl_p = \gamma I_{0-0} \sin\alpha \qquad \text{(c)}$$

Substitue $F = \gamma l_c \sin\alpha A$ into (c)

$$\gamma l_c \sin\alpha A l_p = \gamma I_{0-0} \sin\alpha$$

$$\therefore l_p = \frac{I_{0-0}}{l_c A} = \frac{I_c + l_c^2 A}{l_c A} = l_c + \frac{I_c}{l_c A} \qquad (2.14)$$

→ Center of pressure is always below the centroid by $\frac{I_c}{l_c A}$

$$l_p - l_c = \frac{I_c}{l_c A}$$

→ as l_c (depth of centroid) increases $l_p - l_c$ decreases

2.4 Forces on Submerged Plane Surfaces

- Second moment transfer equation

$$\boxed{I_{0-0} = I_c + l_c^2 A}$$

I_c = 2nd moment of the area *A* about an axis through the centroid,

parallel to 0-0

→ Appendix 3

2.4 Forces on Submerged Plane Surfaces

1) Rectangle

$$A = bh, \;\; y_c = \frac{h}{2}, \;\; I_c = \frac{bh^3}{12}$$

$$\therefore h_c = a + (h - y_c) = a + \frac{h}{2}$$

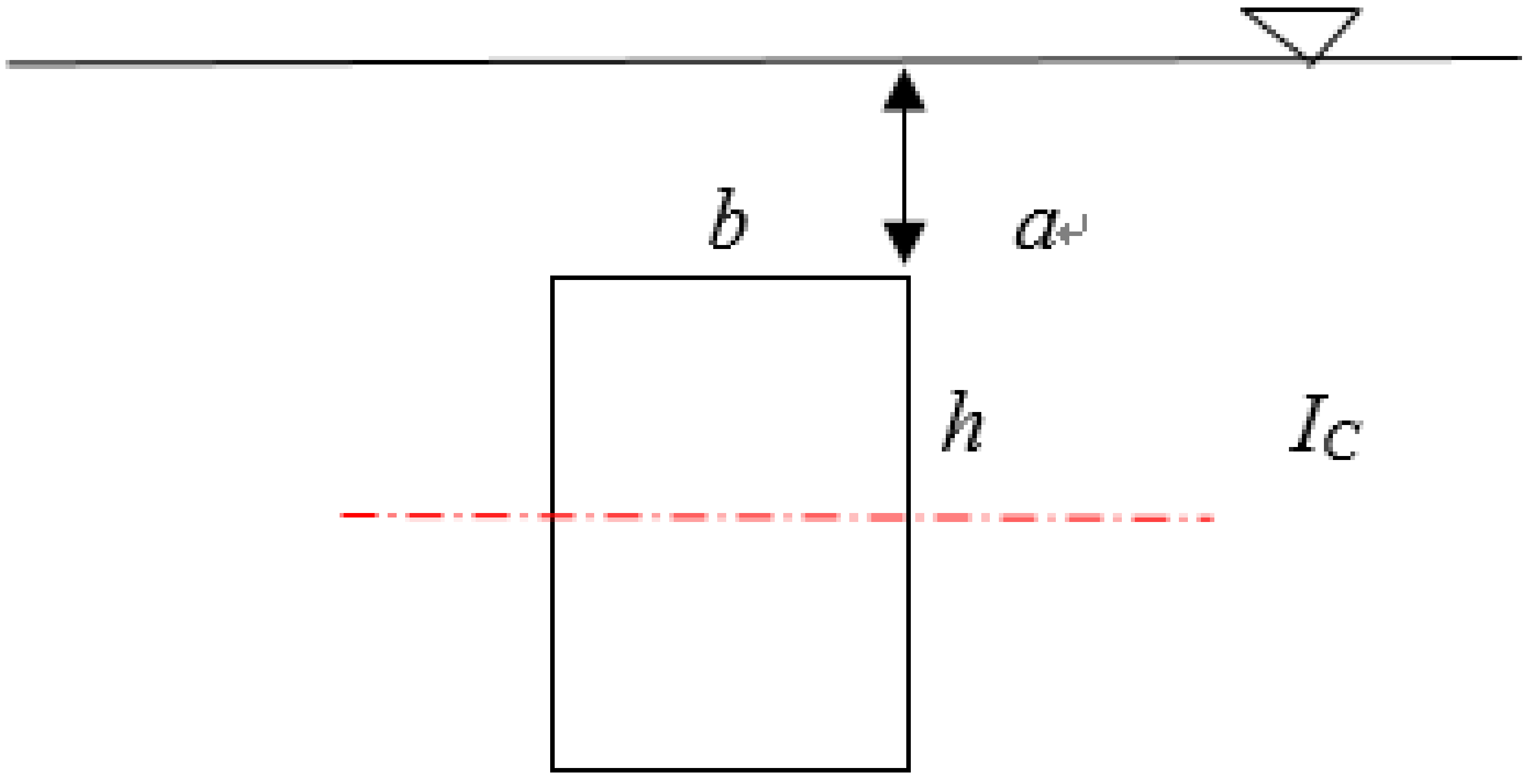
$$F = \gamma h_c A = \gamma \left(a + \frac{h}{2} \right)(bh)$$

$$h_p = h_c + \frac{I_c}{h_c A}$$

If $a = 0; \;\; h_c = \frac{h}{2}$

$$h_p = \frac{h}{2} + \frac{\frac{bh^3}{12}}{\frac{h}{2}bh} = \frac{h}{2} + \frac{h}{6} = \frac{2}{3}h$$

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

2) Semicircle

$$I = \frac{\pi d^4}{128},\ y_c = \frac{4r}{3\pi}$$

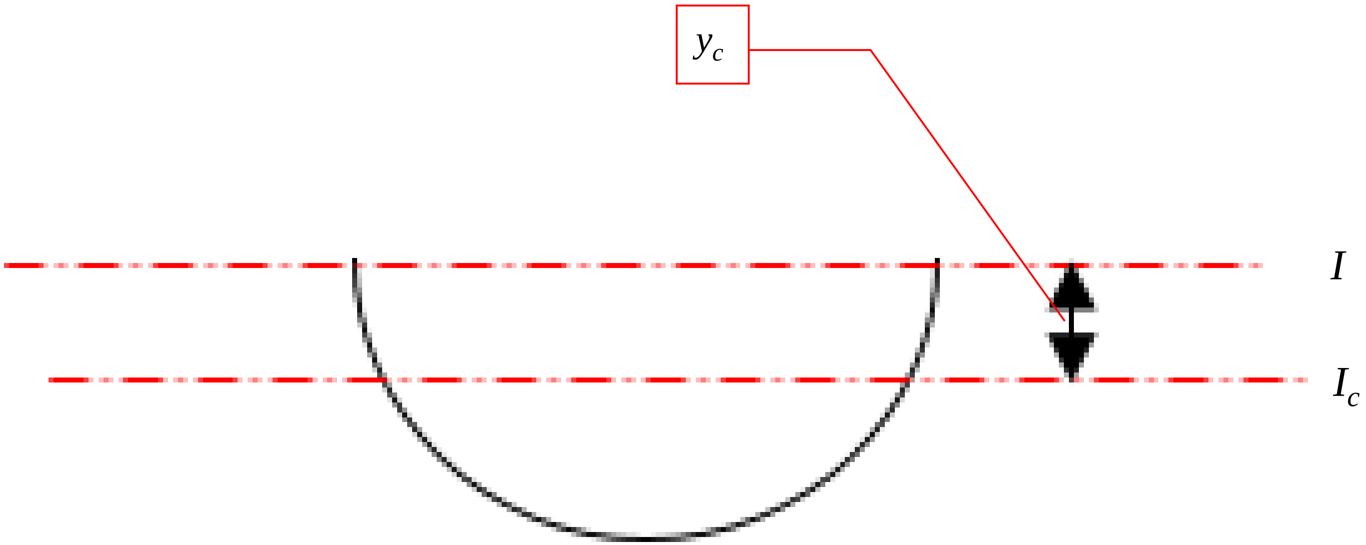
$$I = I_c + y_c^2 A$$

$$\therefore I_c = I - y_c^2 A$$

$$= \frac{\pi d^4}{128} - \left(\frac{4r}{3\pi}\right)^2 \left(\frac{\pi d^2}{8}\right)$$

$$= \left(\frac{\pi}{128} - \frac{1}{18\pi}\right) d^4 = 0.10976 r^4$$

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

3) Quadrant

$$I = \frac{\pi d^4}{256}, y_c = \frac{4r}{3\pi}$$

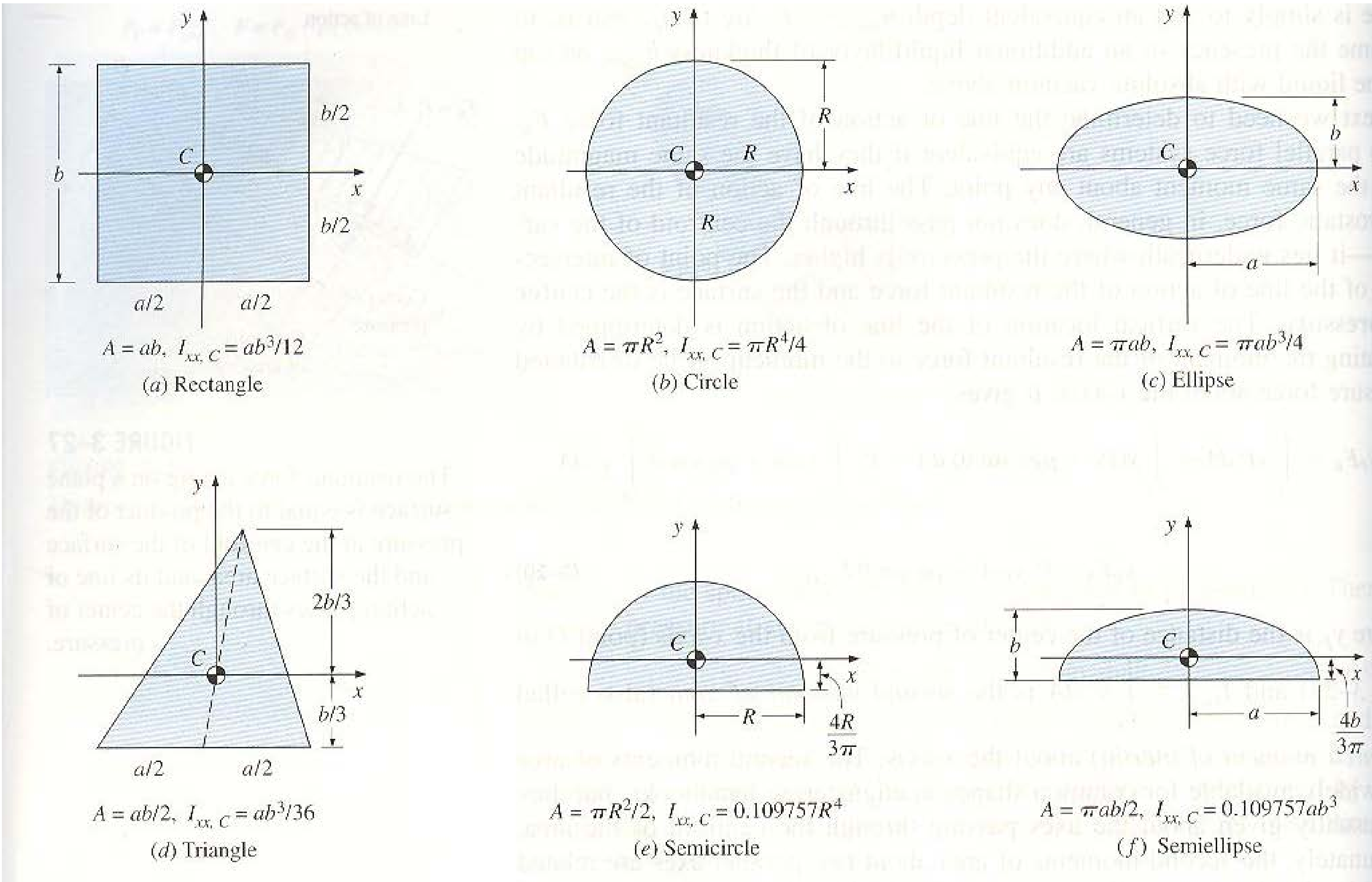
$$I_c = I + y_c^2 A$$

$$= \frac{\pi d^4}{256} - \left(\frac{4r}{3\pi}\right)^2 \left(\frac{\pi d^2}{16}\right)$$

$$= \left(\frac{\pi}{256} - \frac{1}{36\pi}\right) d^4$$

$$= 0.05488 r^4$$

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

(iii) Lateral location of the center of pressure for asymmetric submerged area

a. For regular plane

(i) divide whole area into a series of elemental horizontal strips of area dA

(ii) center of pressure for each strip would be at the midpoint of the strip

(the strip is a rectangle in the limit)

(iii) apply moment theorem about a vertical axis 0-0

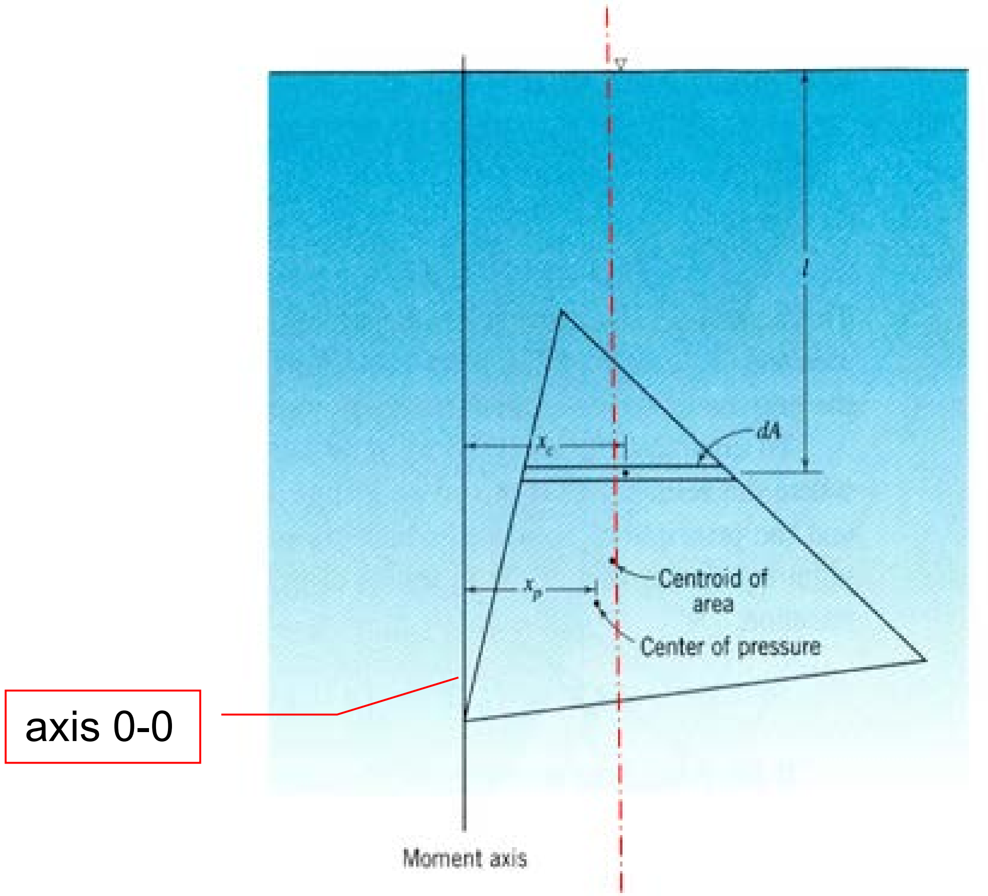
$$dF = \gamma h_c dA = \gamma l \sin\alpha dA \qquad \text{(a)}$$

$$dM = x_c dF = x_c \gamma l \sin\alpha dA$$

Integrate (a)

$$M = \int_A dM = \int x_c \gamma l \sin\alpha dA \qquad \text{(b)}$$

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

By the way, $M = x_p F$ (c)

Equate (b) and (c)

$$x_p F = \int x_c \gamma l \sin\alpha \, dA$$

$$\boxed{x_p = \frac{1}{F}\gamma \sin\alpha \int x_c l \, dA} \qquad (2.15)$$

b. For irregular forms

~ divide into simple areas

~ use methods of statics

[Re] Moment theorem

→ The moment of the resultant force is equal to the sum of the moments of the individual forces.

2.4 Forces on Submerged Plane Surfaces

[IP 2.9] A vertical gate: quarter circle

[Sol]

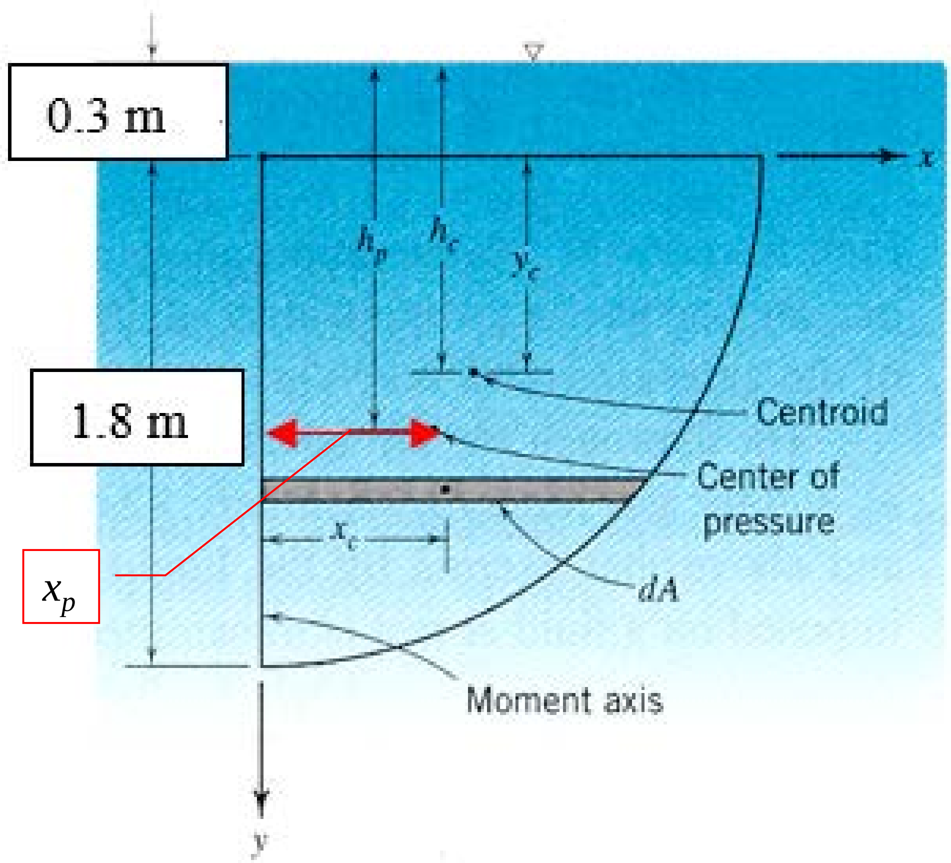
(i) Magnitude

$$y_c)_{quadrant} = \frac{4r}{3\pi} = \frac{4}{3\pi}(1.8) = 0.764;$$

$$h_c = 0.3 + 0.764 = 1.064$$

$$F_{quad} = \gamma h_c A = 9{,}800(1.064)\left(\frac{\pi}{4}(1.8)^2\right) = 26.53\ \text{kN}$$

2.4 Forces on Submerged Plane Surfaces



2.4 Forces on Submerged Plane Surfaces

(ii) Vertical location of resultant force

$$\left(\frac{I_c}{l_c A}\right)_{quad} = \frac{0.05488(1.8)^4}{(1.064)\left(\frac{\pi}{4}(1.8)^2\right)} = 0.213\,\mathrm{m}$$

$$\rightarrow l_p = 1.064 + 0.213 = 1.277\,\mathrm{m}$$

(iii) Lateral location of the center of pressure

Divide quadrant into horizontal strips

Take a moment of the force on dA about *y-axis*

2.4 Forces on Submerged Plane Surfaces

$$dM = \gamma h dA \cdot \text{ (moment arm) } = 9800(y+0.3)(xdy)\left(\frac{x}{2}\right)$$

$$x^2 + y^2 = (1.8)^2$$

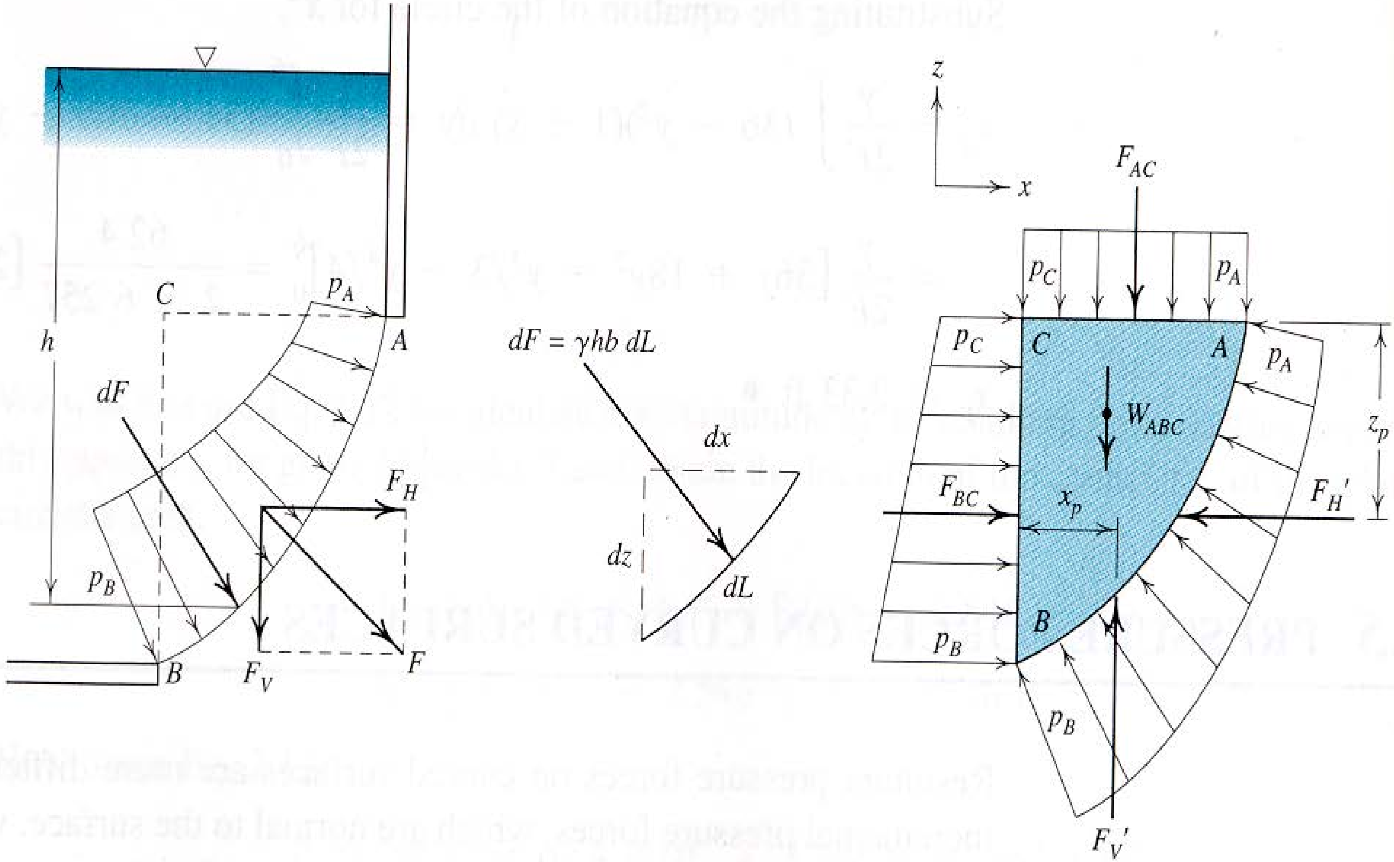
$$\frac{9800}{2}(y+0.3)x^2 dy = \frac{9800}{2}(y+0.3)(1.8^2 - y^2)dy$$

$$\therefore M = \int_0^{1.8} \frac{9800}{2}(y+0.3)(1.8^2 - y^2)dy = 18575\,\mathrm{N \cdot m}$$

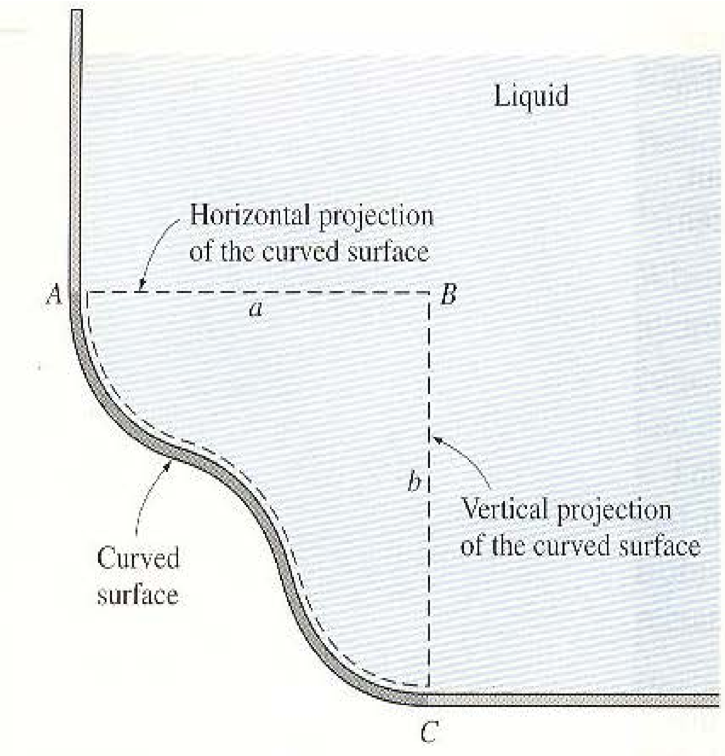
By the way, $M = F_{quad}\, x_p$

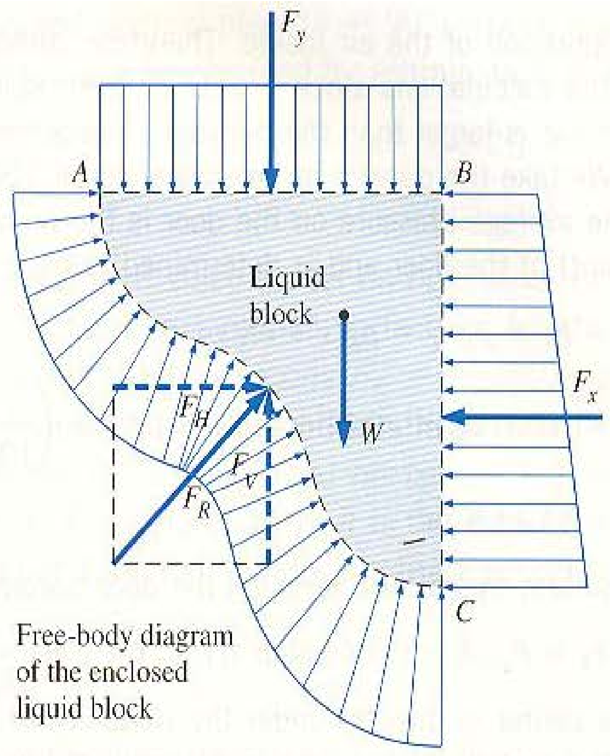
$x_p = 18575 / 26.53 \times 10^3 = 0.7m$ right to the y-axis

2.5 Forces on Submerged Curved Surfaces



2.5 Forces on Submerged Curved Surfaces





2.5 Forces on Submerged Curved Surfaces

• Resultant pressure forces on curved surfaces are more difficult to deal with because the incremental pressure forces vary continually in direction.

→ Direct integration

Method of basic mechanics

1) Direct integration

- Represent the curved shape functionally and integrate to find horizontal and vertical components of the resulting force

i) Horizontal component

$$F_H = \int dF_H = \int \gamma h b \, dz$$

where b = the width of the surface; dz = the vertical projection of the surface element dL

2.5 Forces on Submerged Curved Surfaces

location of F_H: take moments of dF about convenient point, e.g., point C

$$z_p F_H = \int z\, dF_H = \int z \gamma h b\, dz$$

where z_p = the vertical distance from the moment center to F_H

ii) Vertical component

$$F_V = \int dF_V = \int \gamma h b\, dx$$

where dx = the horizontal projection of the surface element dL

2.5 Forces on Submerged Curved Surfaces

location of F_V: take moments of dF about convenient point, e.g., point C

$$x_p F_V = \int x\, dF_V = \int x \gamma h b\, dx$$

where x_p = the horizontal distance from the moment center to F_V

2) Method of basic mechanics

- Use the basic mechanics concept of a free body and the equilibrium of a fluid mass
- Choose a convenient volume of fluid in a way that one of the fluid element boundaries coincide with the curved surface under consideration
- Isolate the fluid mass and show all the forces acting on the mass to keep it in equilibrium

2.5 Forces on Submerged Curved Surfaces

- Static equilibrium of free body ABC

$$\sum F_x = F_{BC} - F_H^{'} = 0 \qquad \therefore F_H^{'} = F_{BC} = \gamma h_c A_{BC}$$

$$\sum F_z = F_V^{'} - W_{ABC} - F_{AC} = 0 \qquad \therefore F_V^{'} = F_{AC} + W_{ABC}$$

$$F_{AC} = \gamma h_c A_{AC} = \gamma H A_{AC} = W_{ACDE}$$

$W_{ABC} =$ weight of free body ABC

$\therefore F_V^{'} =$ weight of $ABDE$

- Location

From the inability of the free body of fluid to support shear stress,

→ $F_{H'}$ must be colinear with F_{BC}

→ $F_{V'}$ must be colinear with the resultant of W_{ABC} and F_{AC}.

2.5 Forces on Submerged Curved Surfaces

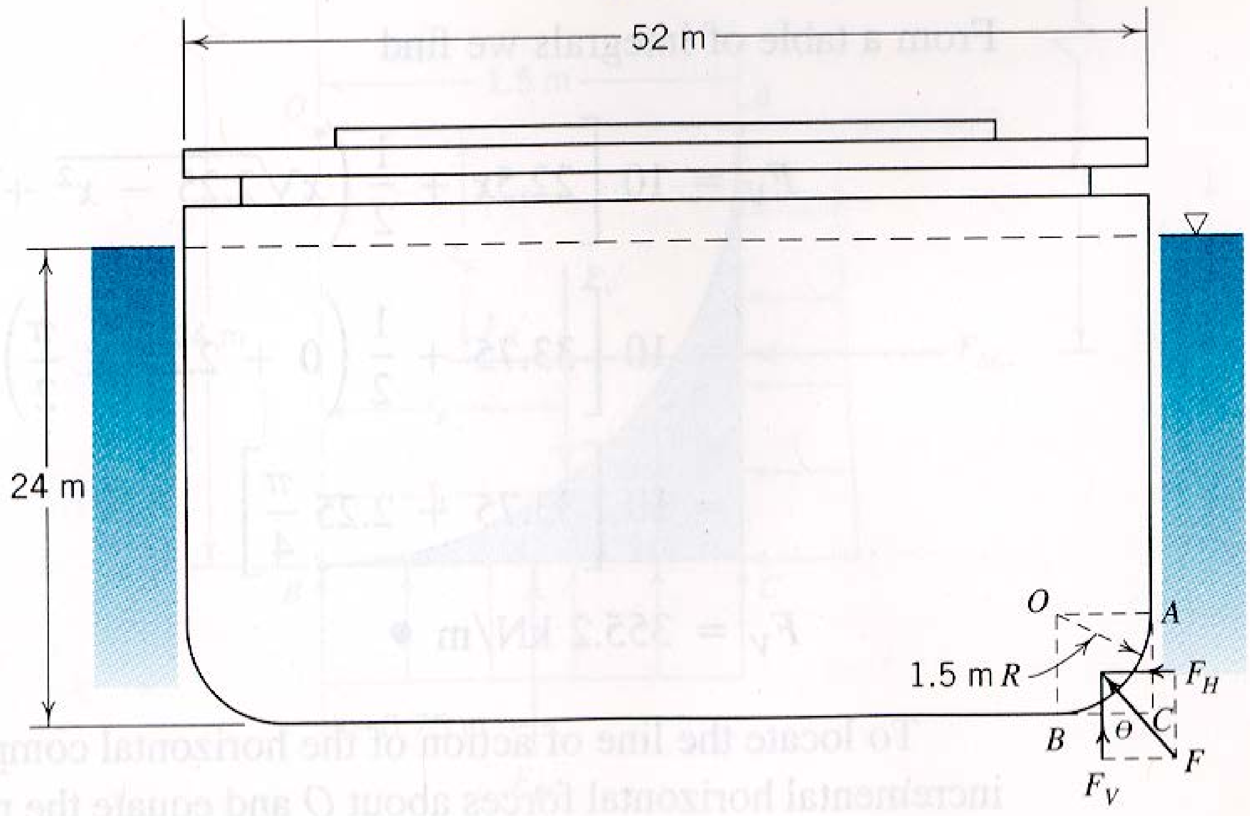
[IP 2.10] p. 59

Oil tanker $W = 330,000 \text{ tone} = 330,000 \times 10^3 \text{kg}$

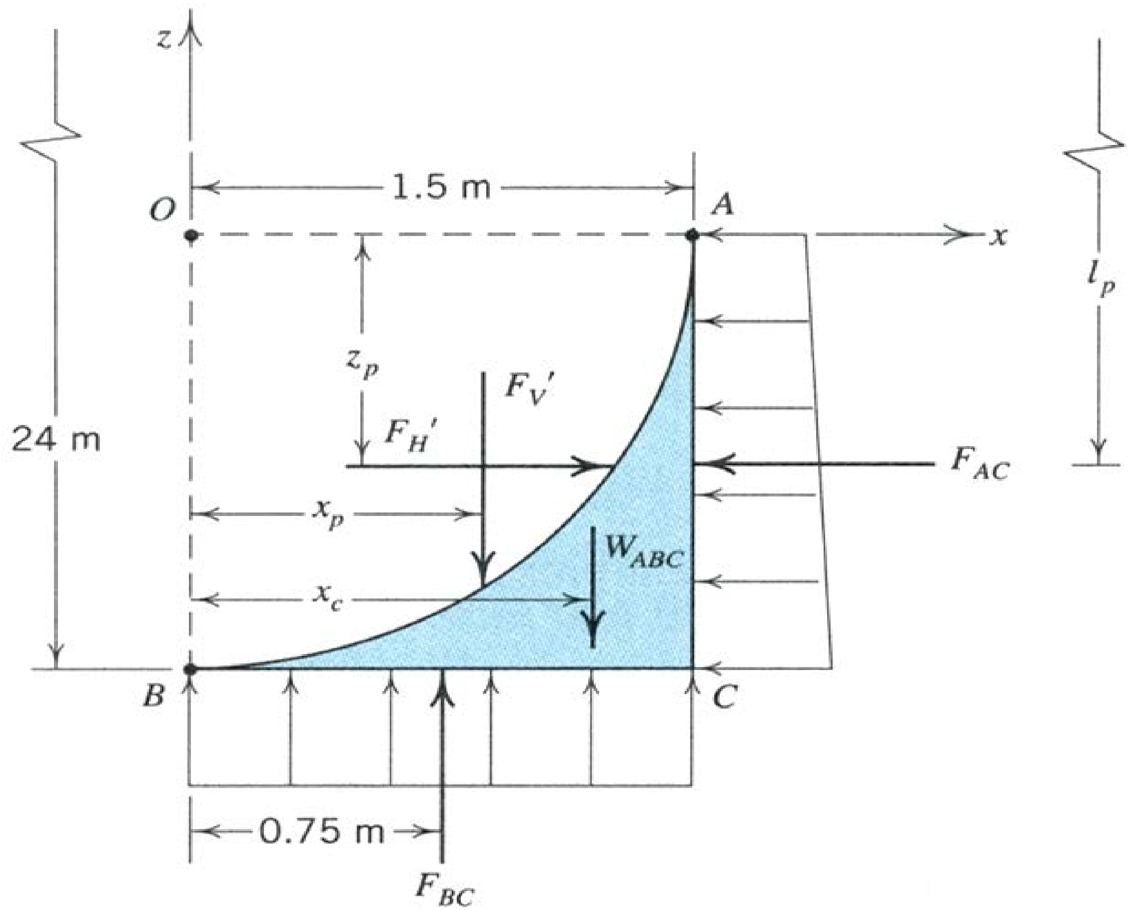
Calculate magnitude, direction, and location of resultant force/meter exerted by seawater ($\gamma = 10 \times 10^3$ N/m^3) on the curved surface AB (quarter cylinder) at the corner.

[Sol] Consider a free body ABC

2.5 Forces on Submerged Curved Surfaces



2.5 Forces on Submerged Curved Surfaces



2.5 Forces on Submerged Curved Surfaces

(i) Horizontal Comp.

$$F_H^{'} = F_{AC} = \gamma h_c A = 10^4 \times \left(22.5 + \frac{1.5}{2} \right) \times (1.5 \times 1) = 348.8 \text{ kN/m}$$

$$l_p = l_c + \frac{I_c}{l_c A} = 23.25 + \frac{\frac{1 \times (1.5)^3}{12}}{23.25 \times 1.5} = 23.25 + 0.0081 = 23.258 \text{ m}$$

$$\therefore z_p = 23.258 - 22.5 = 0.758 \text{ m below line OA}$$

$$= 24 - 23.258 = 0.742 \text{ m above line BC}$$

2.5 Forces on Submerged Curved Surfaces

(ii) Vertical Comp.

$$\sum F_z = F_{BC} - F_V^{'} - W_{ABC} = 0$$

$$\therefore F_V^{'} = F_{BC} - W_{ABC} = \gamma h_c A - \gamma Vol.$$

$$= 10^4 \times 24 \times (1.5 \times 1) - 10^4 \left(1.5 \times 1.5 - \frac{1}{4} \pi (1.5)^2 \right) \times 1 = 355.2\, kN/m$$

- To find the location of $F_v{}'$, we should first find center of gravity of ABC using statics OB

Take a moment of area about line

$$\frac{4(1.5)}{3\pi} \times \frac{1}{4} \pi (1.5)^2 + x_c \times 0.483 = 2.25 \times \frac{1.5}{2}$$

x_c = 1.1646 m

2.5 Forces on Submerged Curved Surfaces

[Cf] From App. 3, for segment of square

$$x_c = \frac{2}{3}\frac{r}{4-\pi} = \frac{2}{3}\frac{1.5}{4-\pi} = 1.165\,m$$

Now, find location of force $F_V^{'}$

Take a moment of force about point O

$$F_V^{'} \times x_p = F_{BC} \times 0.75 - W_{ABC} \times 1.1646$$

$$355.2 \times x_p = 360 \times 0.75 - 4.83 \times 1.1646$$

$\therefore x_p = 0.744$ m right of OB

2.5 Forces on Submerged Curved Surfaces

[Summary]

i) Magnitude of Resultant force F

$$F = \sqrt{(348.8)^2 + (355.2)^2} = 497.8 \text{ kN/m}$$

ii) Direction θ

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right) = \tan^{-1}\left(\frac{355.2}{348.8}\right) = 45.5^\circ$$

iii) Location

Force acting through a point 0.742 m above line BC and 0.744 m right of B

2.5 Forces on Submerged Curved Surfaces

$$\alpha_1 = \tan^{-1}\left(\frac{0.744}{0.758}\right) = 44.47^\circ$$

$$\alpha_2 = \tan^{-1}\left(\frac{348.8}{355.2}\right) = 44.47^\circ$$

$\alpha_1 = \alpha_2 \rightarrow F$ act through point O.

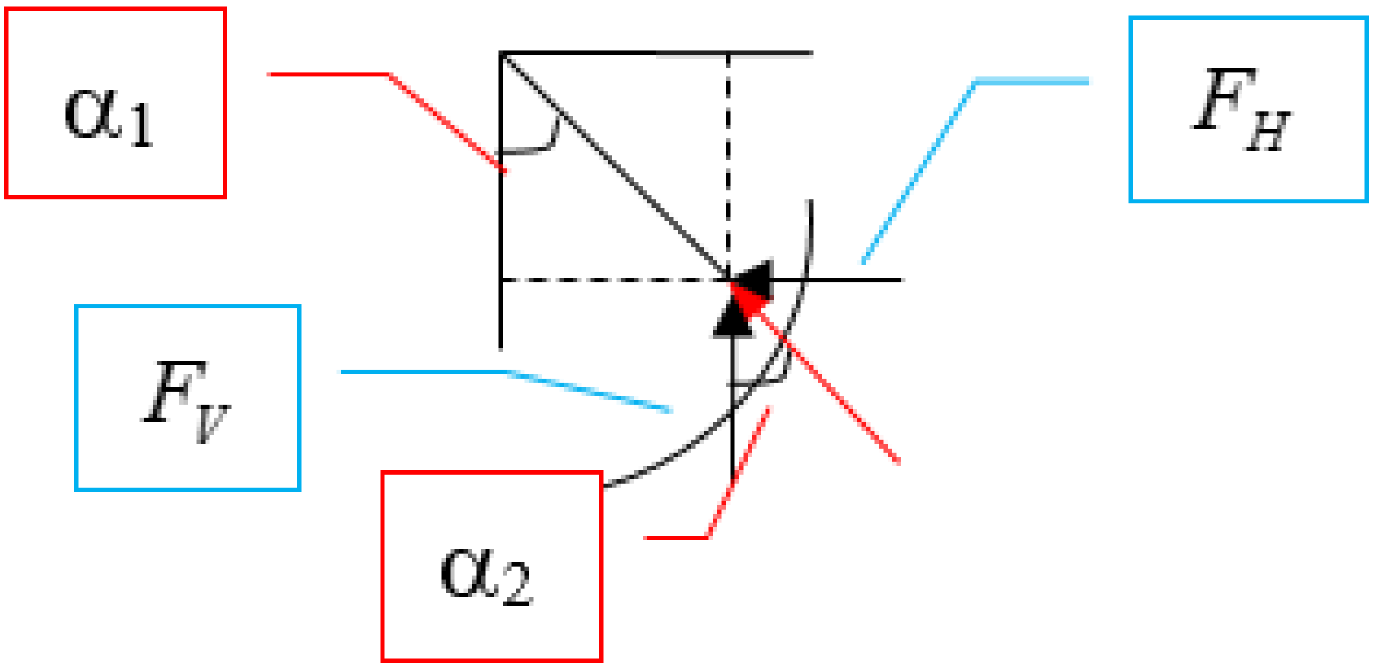
- Pressure acting on the cylindrical or spherical surface

  - The pressure forces are all normal to the surface.

  - For a circular arc, all the lines of action would pass through the center of the arc.

  → Hence, the resultant would also pass through the center.

2.5 Forces on Submerged Curved Surfaces



2.5 Forces on Submerged Curved Surfaces

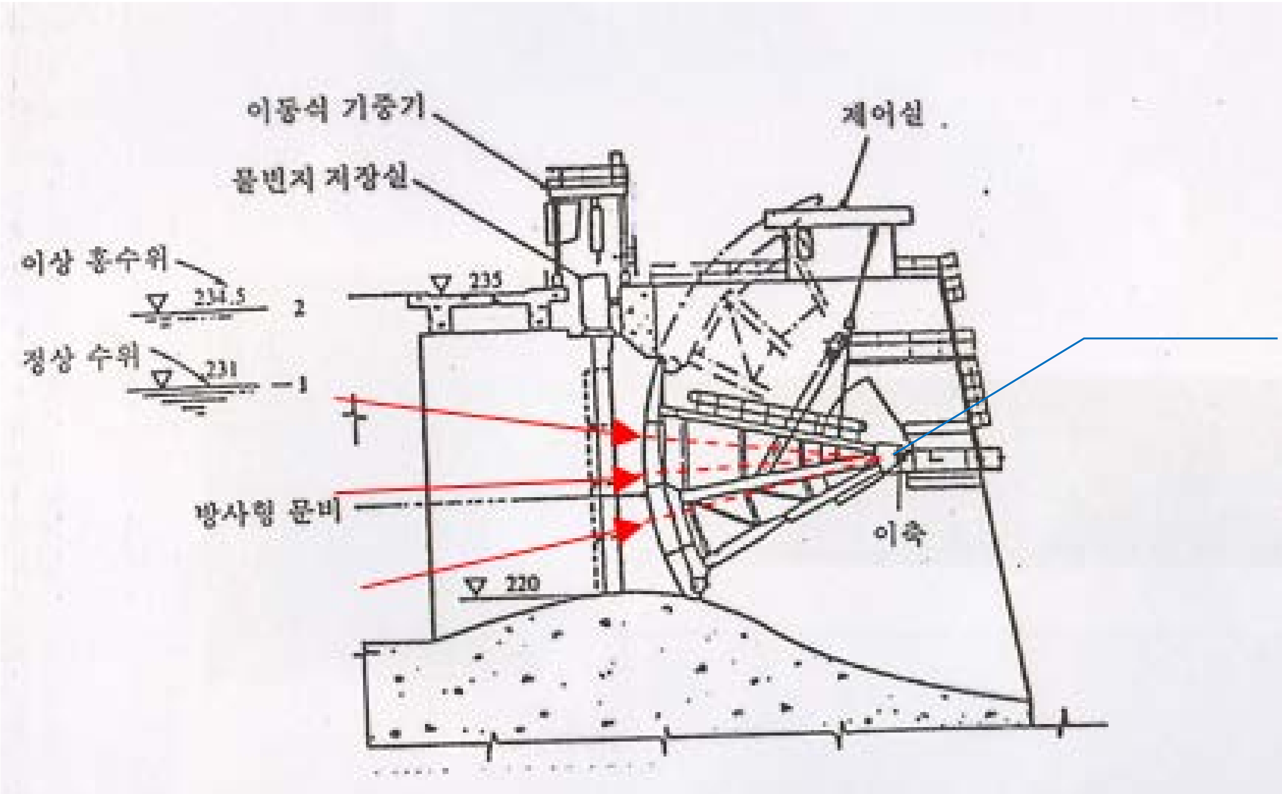
- Tainter gate (Radial gate) for dam spillway

  All hydrostatic pressures are radial, passing through the trunnion bearing.

→ only pin friction should be overcome to open the gate

pin friction (radial gate) < roller friction (lift gate)

2.5 Forces on Submerged Curved Surfaces



Trunnion pin

2.6 Buoyancy and Floatation

• Archimedes' principle

I. A body immersed in a fluid is buoyed up by a force equal to the weight of fluid displaced.

II. A floating body displaces its own weight of the liquid in which it floats.

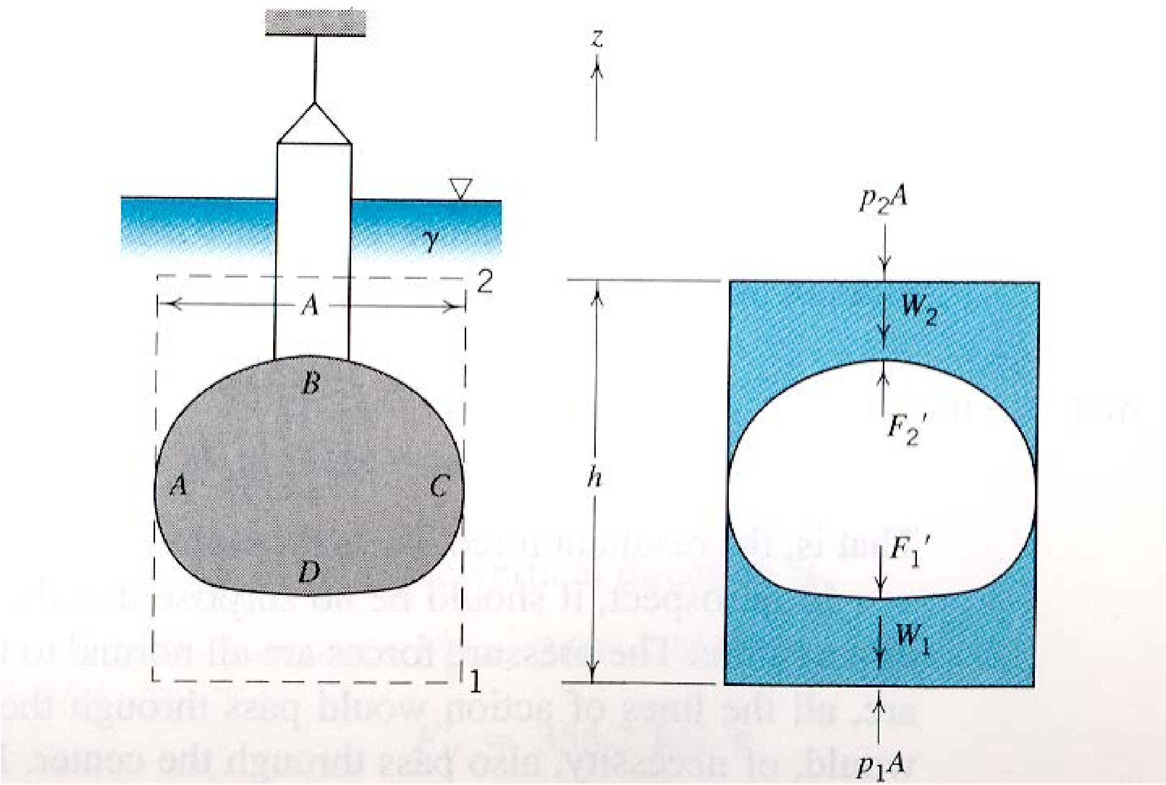
→ Calculation of draft of surface vessels, lift of airships and balloons

(i) Immersed body

Isolate a free body of fluid with vertical sides tangent to the body

→ F_1' = vertical force exerted by the lower surface (ADC) on the surrounding fluid

2.6 Buoyancy and Floatation



2.6 Buoyancy and Floatation

F_2' = vertical force exerted by the upper surface (ABC) on the surrounding fluid

$$F_1' - F_2' = F_B$$

F_B = buoyancy of fluid; act vertically upward.

For upper portion of free body

$$\sum F_z = F_2' - W_2 - P_2 A = 0 \qquad \text{(a)}$$

For lower portion

$$\sum F_z = F_1' - W_1 + P_1 A = 0 \qquad \text{(b)}$$

2.6 Buoyancy and Floatation

Combine (a) and (b)

$$F_B = F_1^{'} - F_2^{'} = (\overbrace{P_1 - P_2}^{\gamma h})A - (W_1 + W_2)$$

$(P_1 - P_2)A = \gamma h A =$ weight of free body

$W_1 + W_2 =$ weight of dashed portion of fluid

$\therefore (P_1 - P_2)A - (W_1 + W_2) =$ weight of a volume of fluid equal to that of the body

$ABCD$

$\therefore F_B = \gamma_{fluid}$ (volume of submerged object) (2.16)

2.6 Buoyancy and Floatation

(ii) Floating body

For floating object

$$F_B = \gamma_f \ \text{(volume displaced, } ABCD\text{)} \qquad F_B = \gamma_f ABCD$$

$$W_{ABCDE} = \gamma_s V_{ABCDE} \qquad W = \gamma_s ABCDE$$

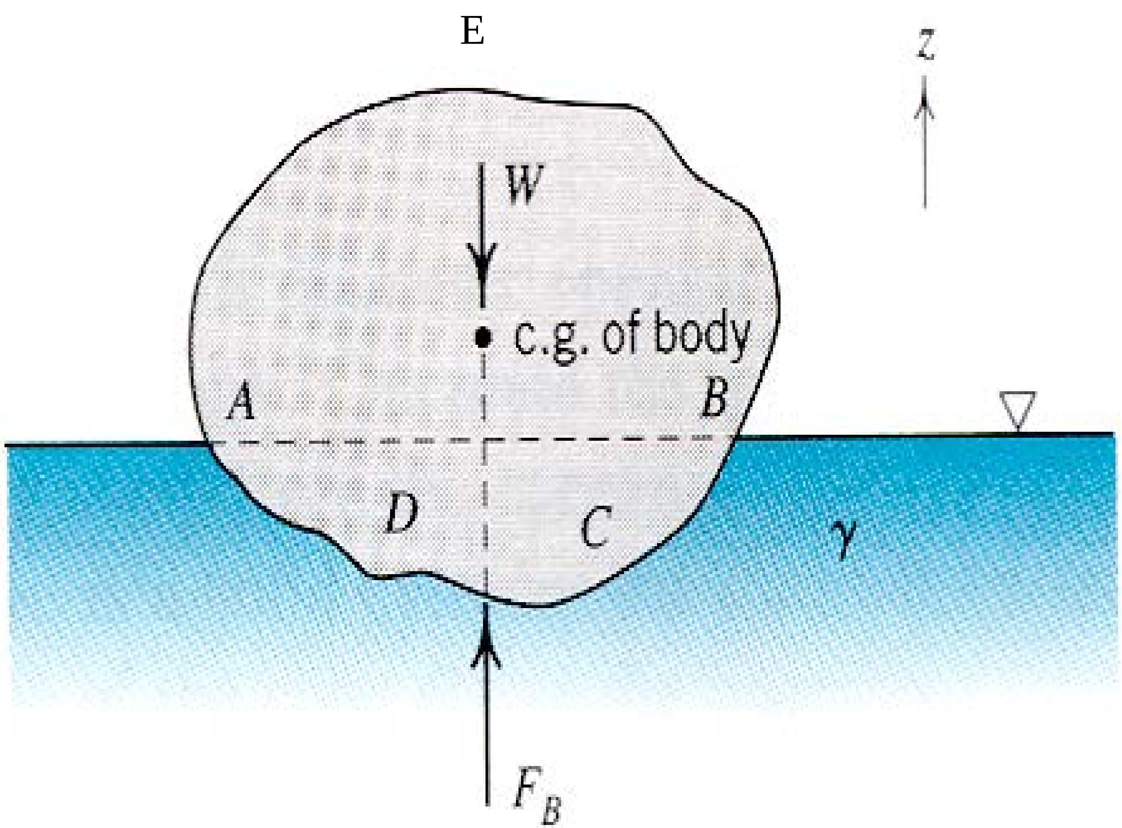
where γ_s = specific weight of body

From static equilibrium: $F_B = W_{ABCDE}$

$$\gamma_f V_{ABCD} = \gamma_s V_{ABCDE}$$

$$\boxed{\therefore \quad V_{ABCD} = \frac{\gamma_s}{\gamma_f} V_{ABCDE}}$$

2.6 Buoyancy and Floatation



2.6 Buoyancy and Floatation

[Ex] Iceberg in the sea

Ice s.g.= 0.9

Sea water s.g.= 1.03

$$V_{sub} = \frac{0.9(9800)}{1.03(9800)} V_{total} = 0.97 V_{total}$$

- Stability of submerged or floating bodies

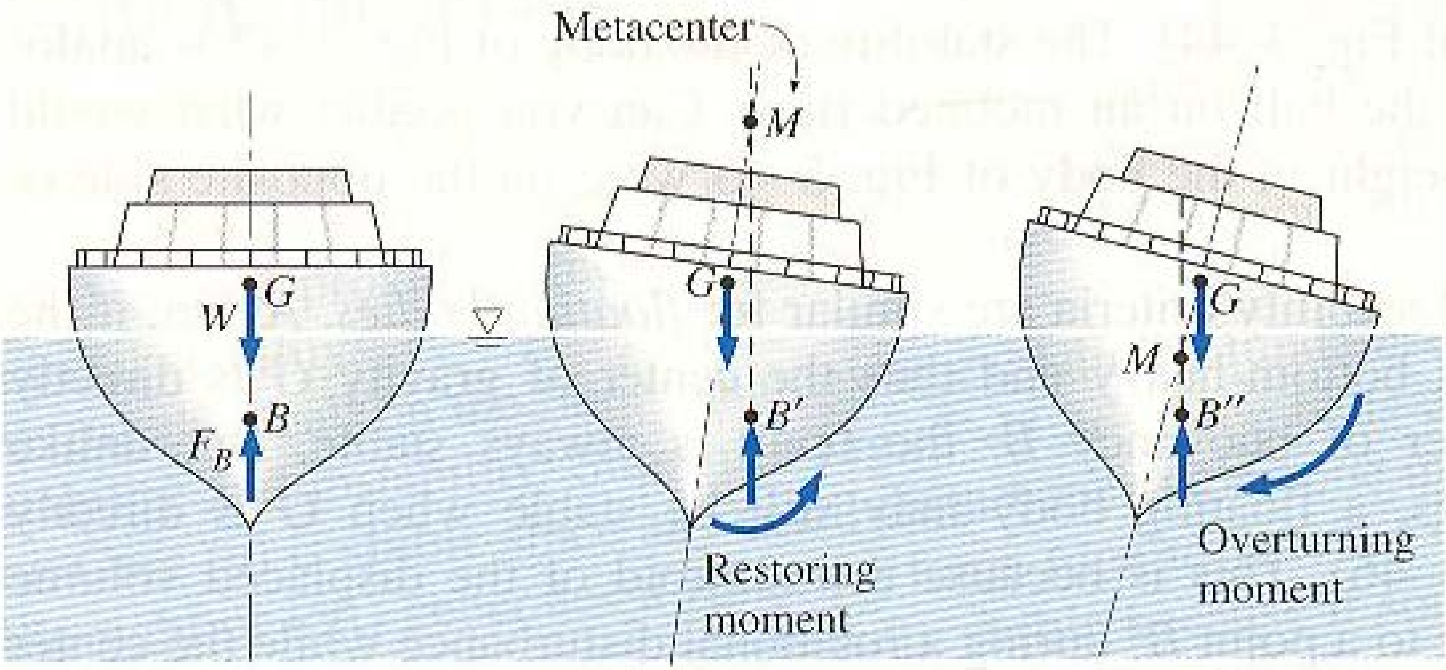
$G_1 < M$ → stable, righting moment

$G_2 > M$ → unstable, overturning moment

G_1, G_2 = center of gravity

M = metacenter

2.6 Buoyancy and Floatation



(a) Stable (b) Stable (c) Unstable

2.7 Fluid Masses Subjected to Acceleration

- Fluid masses can be subjected to various types of acceleration without the occurrence of relative motion between fluid particles or between fluid particles and boundaries.

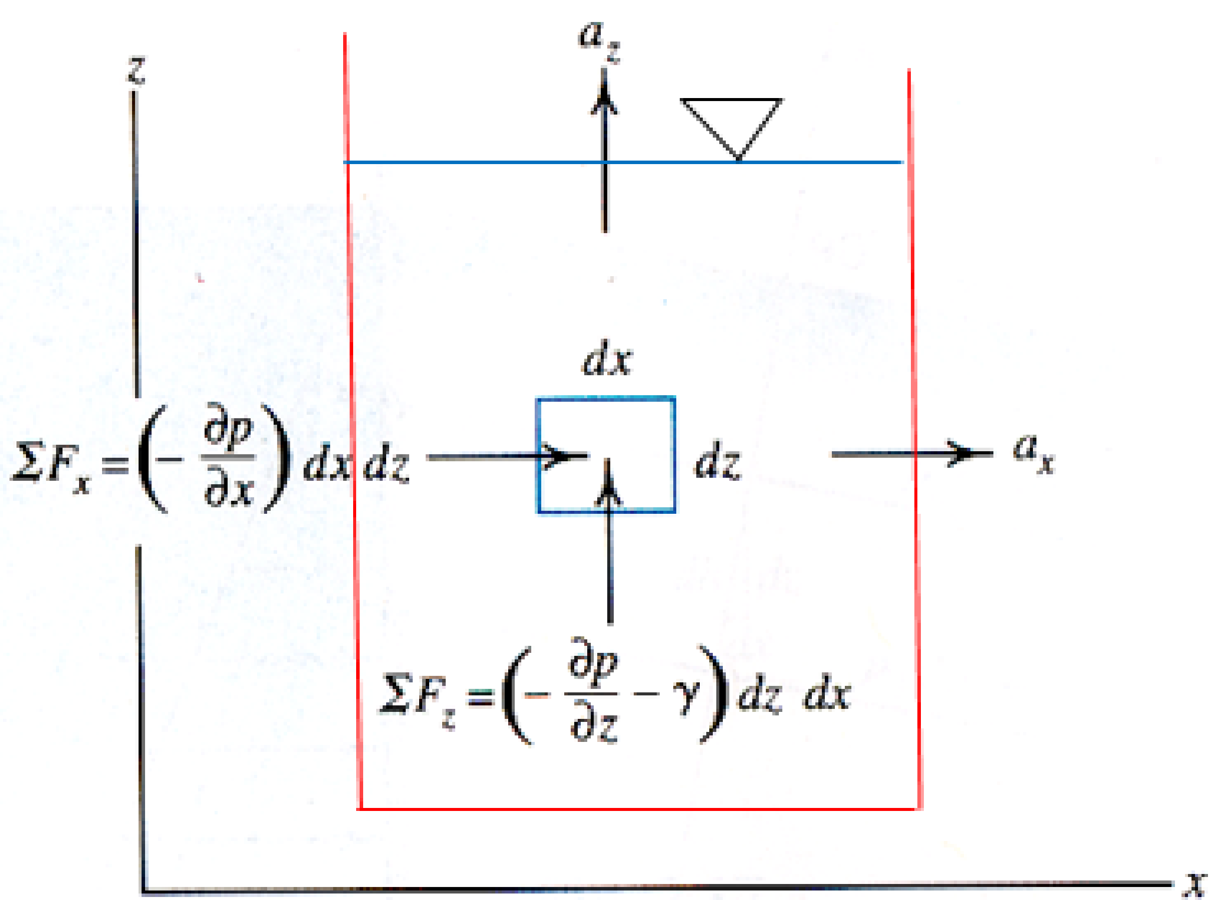
→ laws of fluid statics modified to allow for the effects of acceleration

- A whole tank containing fluid system is accelerated.

- Newton's 2nd law of motion (Sec. 2.1)

$$\sum F = Ma \qquad (2.17)$$

2.7 Fluid Masses Subjected to Acceleration



2.7 Fluid Masses Subjected to Acceleration

First, consider force

$$\sum F_x = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2} \right) dz - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2} \right) dz = \left(-\frac{\partial p}{\partial x} \right) dxdz \qquad (2.18a)$$

$$\sum F_z = \left(-\frac{\partial p}{\partial z} - \gamma \right) dxdz \qquad (2.18b)$$

Then, consider acceleration

mass

$$x: \left(-\frac{\partial p}{\partial x} \right) dxdz = \left(\frac{\gamma}{g} dxdz \right) a_x$$

$$z: \left(-\frac{\partial p}{\partial z} - \gamma \right) dxdz = \left(\frac{\gamma}{g} dxdz \right) a_z$$

2.7 Fluid Masses Subjected to Acceleration

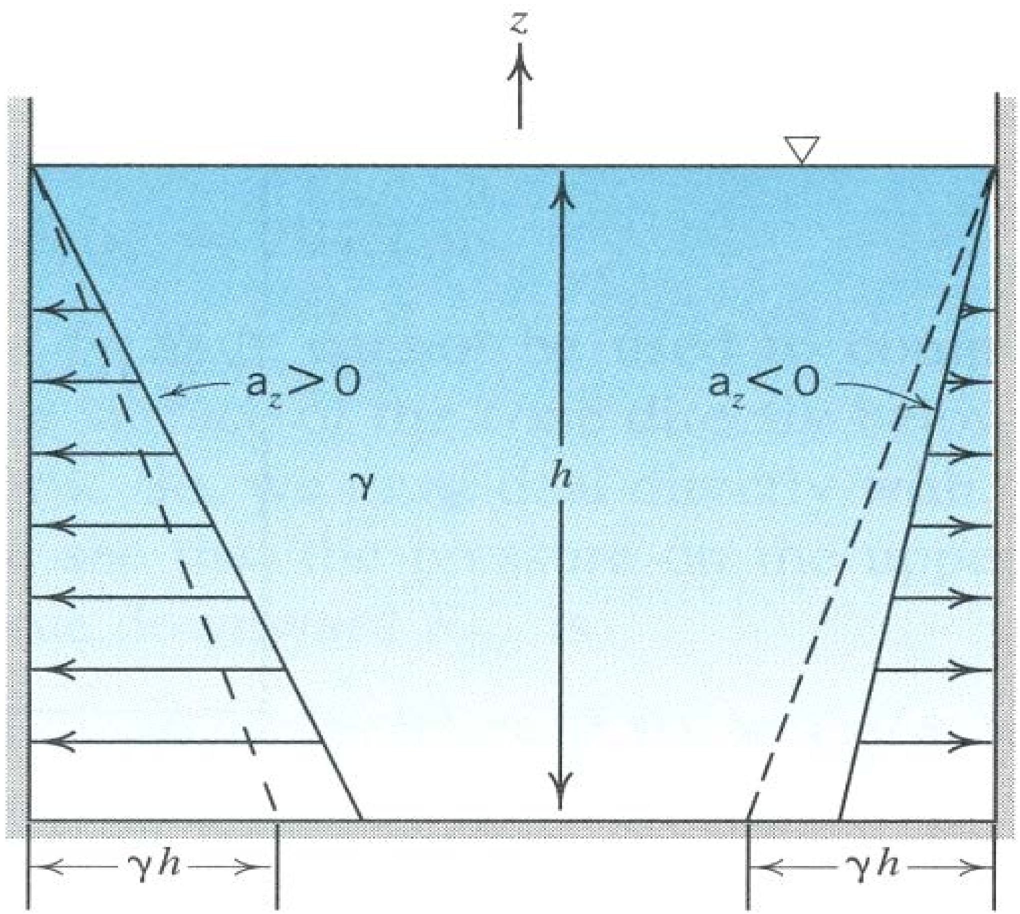
where mass = $\rho vol. = \frac{\gamma}{g} dxdz \times 1$

$$\frac{\partial p}{\partial x} = -\frac{\gamma}{g} a_x \tag{2.19}$$

$$\frac{\partial p}{\partial z} = -\frac{\gamma}{g} (g + a_z) \tag{2.20}$$

→ pressure variation through an accelerated mass of fluid

2.7 Fluid Masses Subjected to Acceleration



2.7 Fluid Masses Subjected to Acceleration

[Cf] For fluid at rest,

$$\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial z} = -\gamma$$

• Chain rule for the total differential for dp (App. 5)

$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial z} dz \qquad \text{(a)}$$

Combine (2.19) , (2.20), and (a)

$$dp = -\frac{\gamma}{g} a_x dx - \frac{\gamma}{g}(g + a_z) dz \qquad (2.21)$$

2.7 Fluid Masses Subjected to Acceleration

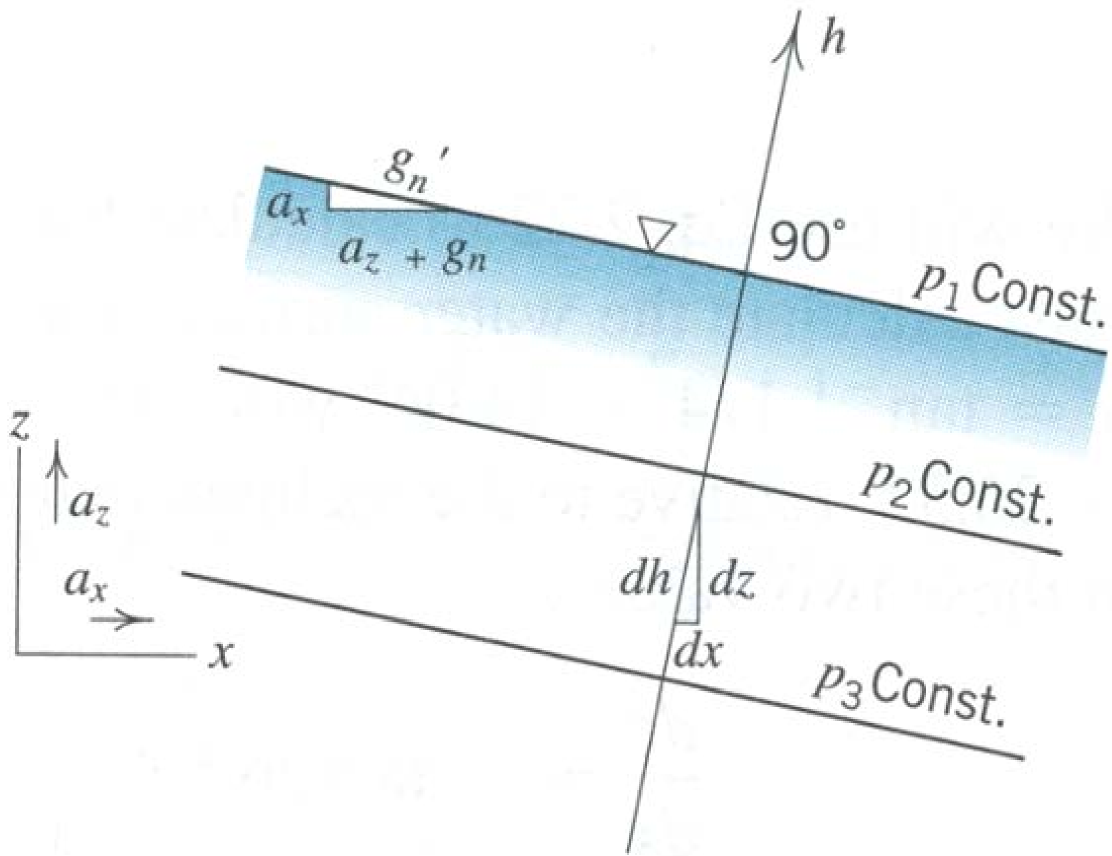
- Line of constant pressure $dp = 0$

$$-\frac{\gamma}{g}a_x dx - \frac{\gamma}{g}(g + a_z)dz = 0$$

$$\therefore \frac{dz}{dx} = -\left(\frac{a_x}{g + a_x}\right) \qquad (2.22)$$

→ slope of a line of constant pressure

2.7 Fluid Masses Subjected to Acceleration



2.7 Fluid Masses Subjected to Acceleration

1) No horizontal acceleration: $a_x = 0$

$$\frac{\partial p}{\partial x} = 0$$

$$\therefore \frac{dp}{dz} = -\gamma\left(\frac{g + a_z}{g}\right)$$

- For free falling fluid, $a_z = -g$

$$\frac{dp}{dz} = 0$$

2) Constant linear acceleration

Divide (2.21) by dh

$$\frac{dp}{dh} = -\gamma\left(\frac{a_x}{g}\frac{dx}{dh} + \frac{g + a_z}{g}\frac{dz}{dh}\right) \qquad \text{(a)}$$

2.7 Fluid Masses Subjected to Acceleration

Use similar triangles

$$\frac{dx}{dh} = \frac{a_x}{g^{'}} \quad \text{(b.1)}$$

$$\frac{dz}{dh} = \frac{a_z + g}{g^{'}} \quad \text{(b.2)}$$

$$g^{'} = \left[a_x^2 + (a_z + g)^2 \right]^{1/2}$$

Substitute (b) into (a)

$$\frac{dp}{dh} = -\gamma \frac{g^{'}}{g}$$

→ pressure variation along h is linear.

2.7 Fluid Masses Subjected to Acceleration

[IP 2.13] p. 70

An open tank of water is accelerated vertically upward at 4.5 m/s^2. Calculate the pressure at a depth of 1.5 m.

[Sol]

$$\frac{dp}{dz} = -\gamma\left(\frac{g + a_z}{g}\right) = (-9{,}800\ \mathrm{N/m^3})\left(\frac{9.81 + 4.5}{9.81}\right) = -14{,}300\ \mathrm{N/m^3}$$

$$dp = -14{,}300 dz$$

integrate

$$\int_0^p dp = \int_0^{-1.5} -14{,}300 dz$$

$$p = -14{,}300[z]_0^{-1.5} = 14{,}300(-1.5 - 0) = 21{,}450\,\mathrm{N/m^2} = 21.45\ \mathrm{kPa}$$

2.7 Fluid Masses Subjected to Acceleration

[Cf] For $a_z = 0$

$$p = \gamma h = 9800(1.5) = 14.7\ \text{kPa}$$

2.7 Fluid Masses Subjected to Acceleration

Homework Assignment # 2

Due: 1 week from today

Prob. 2.4

Prob. 2.6

Prob. 2.11

Prob. 2.26

Prob. 2.31

Prob. 2.39

Prob. 2.52

Prob. 2.59

Prob. 2.63

Prob. 2.76

Prob. 2.91

Prob. 2.98

Prob. 2.129