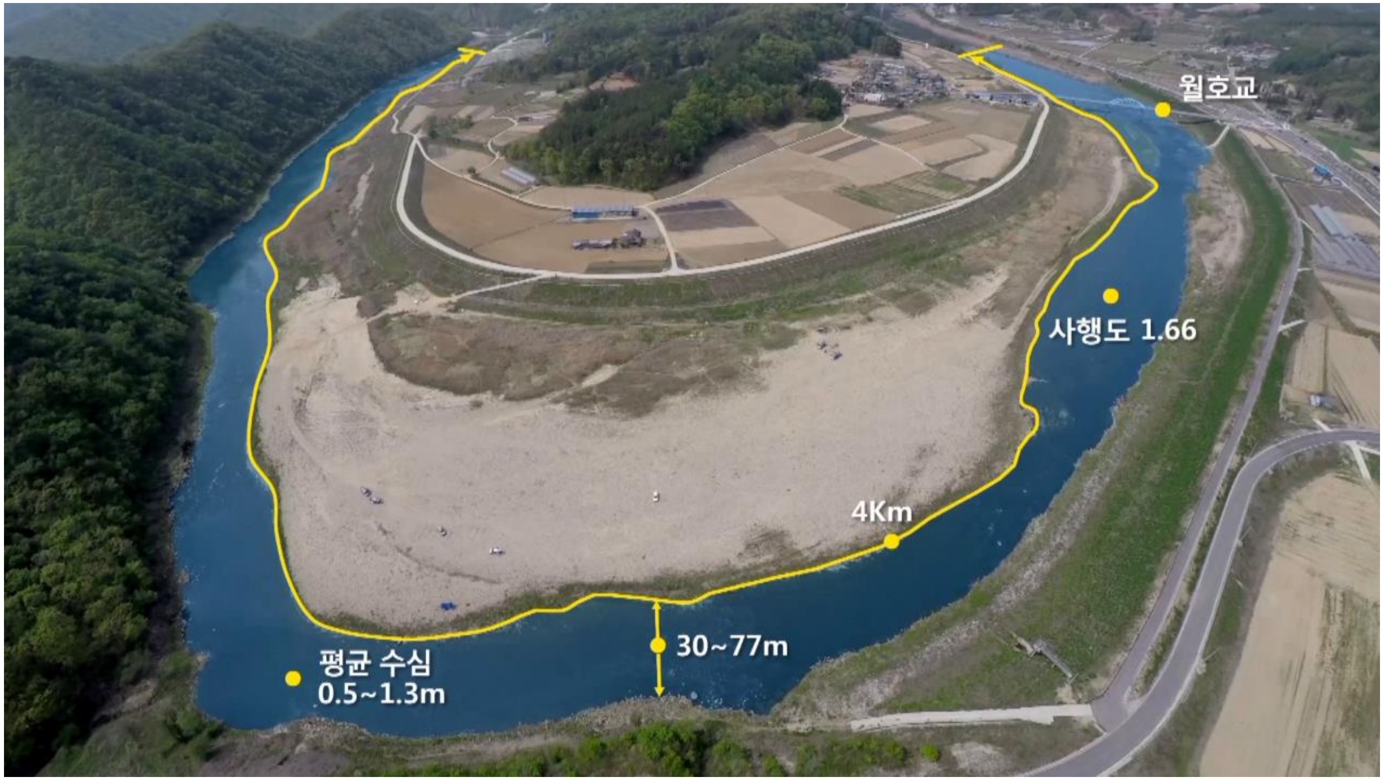
Chapter 5

Mixing in Rivers



Chapter 5 Mixing in Rivers

Contents

Objectives

- Discuss turbulent diffusion
- Study transverse mixing in the mid-field
- Discuss process of longitudinal dispersion for the analysis of final stage
- Study prediction methods for dispersion coefficients

5.1 Mixing Process of Pollutants in Rivers

Consider a stream of pollutant or effluent discharged into a river.

What happens can be divided into three stages:

Stage I: Three-dimensional mixing

→ vertical + lateral + longitudinal mixing

Stage II: Two-dimensional mixing

→ lateral + longitudinal mixing

Stage III: One-dimensional mixing

→ longitudinal mixing

5.1 Mixing Process of Pollutants in Rivers

- Two types of contaminant source

1) Effluent discharge through outfall structure
2) Accidental spill of slug of contaminants

1) Effluent discharge

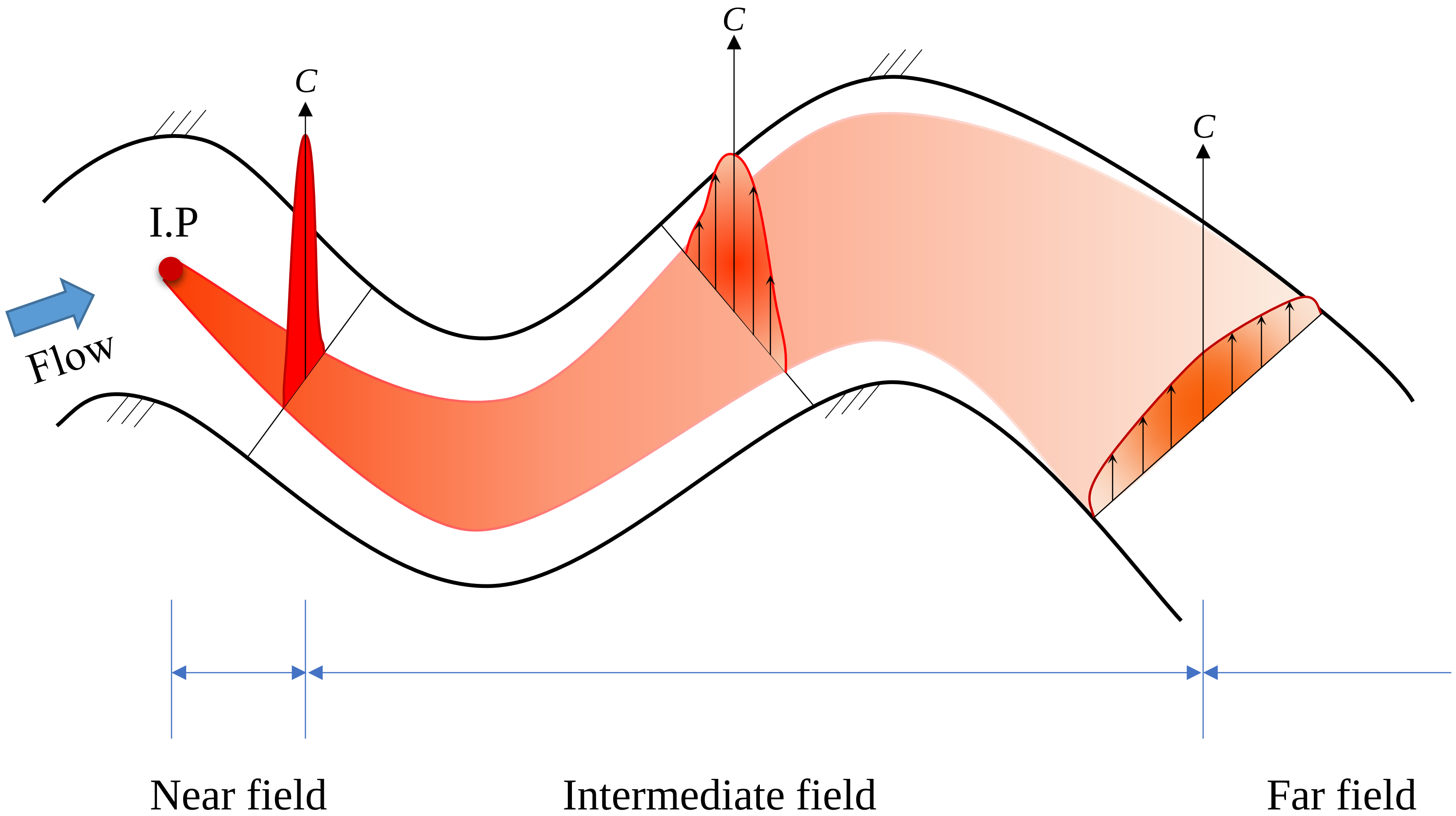
~ Effluents are discharged continuously with initial momentum and buoyancy which determine mixing near the outlet → active mixing

2) Accidental spill of slug of contaminant

~ contaminants discharged instantaneously without any initial momentum and buoyancy → passive mixing

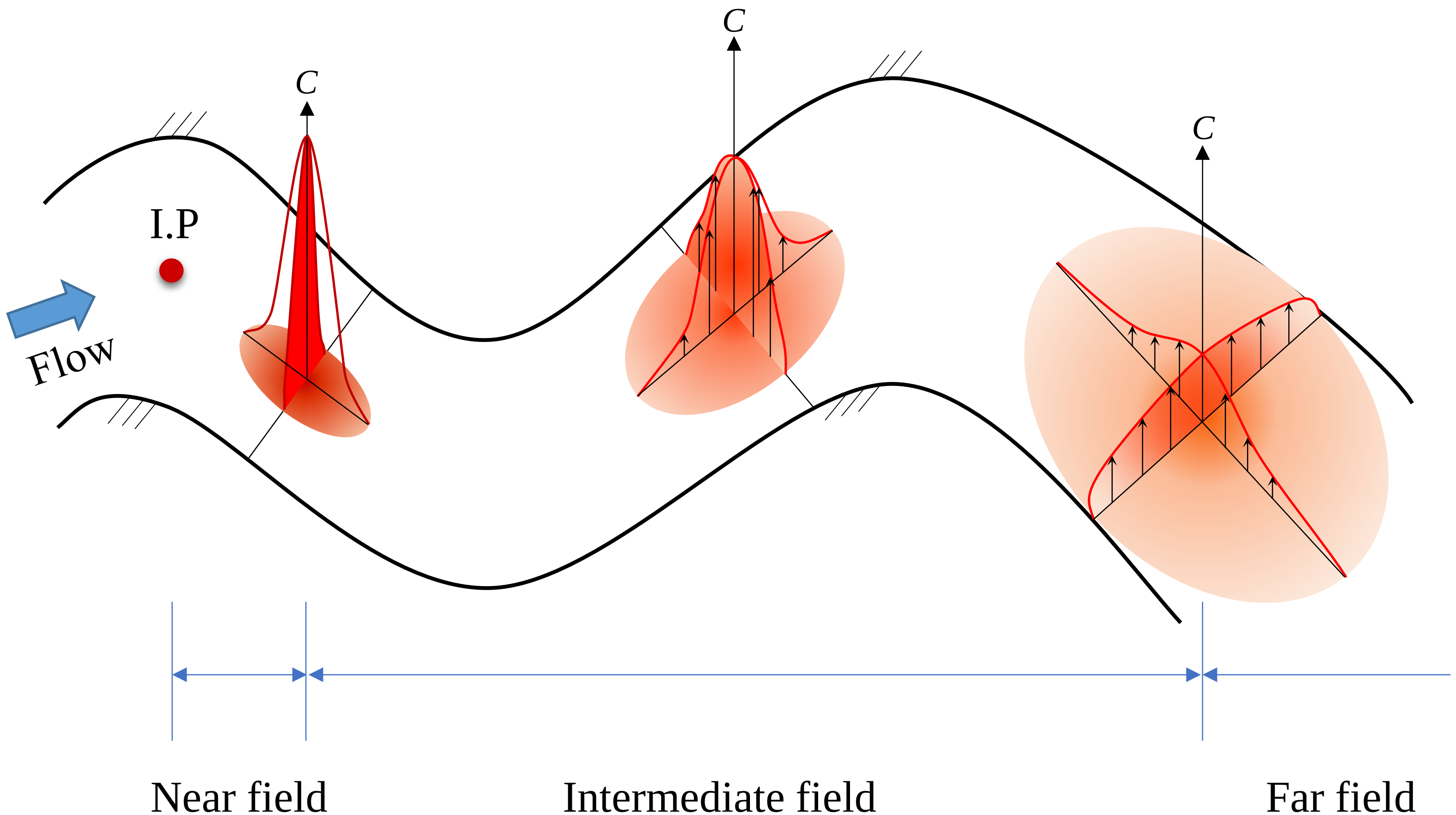
5.1 Mixing Process of Pollutants in Rivers

a) Continuous Source



5.1 Mixing Process of Pollutants in Rivers

b) Instantaneous Source



5.1 Mixing Process of Pollutants in Rivers

5.1.1 Near Field Mixing

Three-dimensional mixing at **Stage I**

~ Vertical mixing is usually completed at the end of this region.

1) Effluent discharge

i) Jet Integral Model

- CORMIX (Cornell Mixing Zone Expert System)
- VISJET

ii) 3D Hydrodynamic Model

- FLOW3D / FLUENT
- OpenFoam

5.1 Mixing Process of Pollutants in Rivers

2) Accidental spill of slug of contaminant

~ apply 3D advection-diffusion equation for turbulent mixing in rivers

$$\frac{\partial c}{\partial t} + u_x \frac{\partial c}{\partial x} + u_y \frac{\partial c}{\partial y} + u_z \frac{\partial c}{\partial z} = \frac{\partial}{\partial x}(\varepsilon_l \frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(\varepsilon_t \frac{\partial c}{\partial y}) + \frac{\partial}{\partial z}(\varepsilon_v \frac{\partial c}{\partial z})$$

where c = time-averaged concentration; t = time; u_x, u_y, u_z = velocity components; ε_l = longitudinal turbulent mixing coefficient; ε_t = transverse turbulent mixing coefficient; ε_v = vertical turbulent mixing coefficient

5.1 Mixing Process of Pollutants in Rivers

5.1.2 Intermediate field mixing

Two-dimensional mixing (longitudinal + lateral mixing) at **Stage II**

~ Contaminant is mixed across the channel primarily by turbulent dispersion and spread longitudinally in the receiving stream.

5.1 Mixing Process of Pollutants in Rivers

→ apply 2D depth-averaged advection-dispersion equation for mixing in rivers

$$\frac{\partial \overline{c}}{\partial t} + u\frac{\partial \overline{c}}{\partial x} + v\frac{\partial \overline{c}}{\partial y} = \frac{\partial}{\partial x}\left(D_L \frac{\partial \overline{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_T \frac{\partial \overline{c}}{\partial y}\right)$$

where $\overline{c}$ = depth-averaged concentration; u = depth-averaged longitudinal velocity; v = depth-averaged transverse velocity; D_L = 2D longitudinal mixing coefficient; D_T = transverse mixing coefficient.

5.1 Mixing Process of Pollutants in Rivers

5.1.3 Far field mixing

~ Longitudinal dispersion at **Stage III**

~ Process of longitudinal shear flow dispersion erases any longitudinal concentration variations.

~ Apply 1D longitudinal dispersion model proposed by Taylor (1954)

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(K\frac{\partial C}{\partial x}\right)$$

where C = cross-sectional-averaged concentration; U = cross-sectional-averaged longitudinal velocity; K = 1D longitudinal mixing coefficient.

5.2 Near-field Mixing

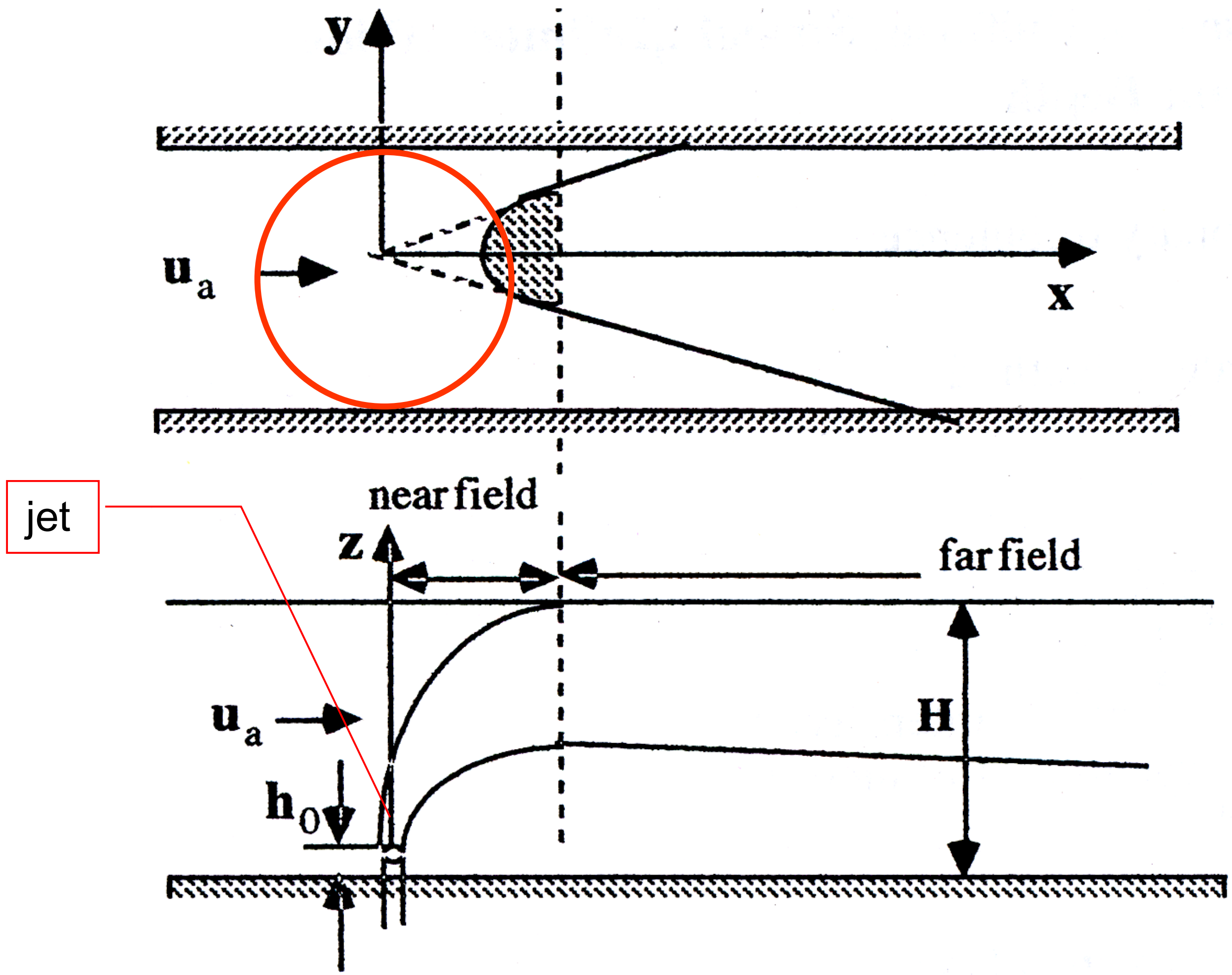
5.2.1 Analysis of Active Mixing

Effluents are discharged continuously with initial momentum and buoyancy by means of diffusers

Analyze jet mixing based on three groups of parameters

1) Pollutant discharge characteristics: discharge velocity (momentum), flow rate, density of pollutant (buoyancy)
2) Diffuser characteristics: single/multi ports, submerged/surface discharge, alignment of port
3) Receiving water flow patterns: ambient water depth, velocity, density stratification

5.2 Near-field Mixing



5.2 Near-field Mixing

- Jet analysis model:

1) CORMIX: expert system

2) VISJET: Lagrangian jet integral model

- Multiport diffuser

~ linear structure consisting of many closely spaced ports, or nozzles, through which wastewater effluent is discharged at high velocity into the receiving water body

~ attractive engineering solution to the problem of managing wastewater discharge in an environmentally sound way

→ offer high degree of <u>initial dilution</u>

5.2 Near-field Mixing

1) Thermal diffuser: heated water discharge from the once-through cooling systems of nuclear power plant and fossil fuel power plant
2) Wastewater diffuser: wastewater discharge from the sewage treatment plants

[Cf] Classification of discharges

- Positive buoyant jets: heated water discharge, wastewater discharge
- Negative buoyant jets: cooled water discharge (LNG terminal), brine-water discharge (desalination plant)

5.2 Near-field Mixing

▪ Water quality policy in USA

"Technical support document for water quality-based toxics control," Office of Water (1991)

~ regulations on toxic control with higher initial mixing requirements by U.S. EPA

・ Regulatory Mixing Zone (RMZ): limited area or volume of water where initial dilution of an aqueous pollutant discharge occurs

→ should predict the initial dilution of a discharge and extent of its mixing zone

5.2 Near-field Mixing

	streams, rivers	lakes, estuaries
Florida	RMZ ≤ 800m and ≤ 10% total length	≤ 125,600 m^2 and ≤ 10% surface area
Michigan	RMZ ≤ 1/4 cross-sectional area	≤ 1,000 ft radius
West Virginia	RMZ ≤ 20~33% cross-sectional area and ≤ 5~10 times width	≤ 300 ft any direction

5.2 Near-field Mixing

5.2.2 Transport Equation for Passive Mixing in the Near-field

Consider advection and turbulent diffusion coefficient for 3-D flow

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=\frac{\partial}{\partial x}(\varepsilon_l\frac{\partial c}{\partial x})+\frac{\partial}{\partial y}(\varepsilon_t\frac{\partial c}{\partial y})+\frac{\partial}{\partial z}(\varepsilon_v\frac{\partial c}{\partial z})$$

Consider shear stress tensor for turbulent diffusion coefficients in 3-D flow

$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

5.2 Near-field Mixing

Now, consider velocity gradients for each turbulent diffusion coefficient

$$\tau_{xz} = \rho\varepsilon_v \frac{du}{dz} \qquad \tau_{yz} = \rho\varepsilon_v \frac{dv}{dz} \qquad \cancel{\sigma_{zz}} = \rho\varepsilon_v \frac{dw}{dz}$$

$$\tau_{xy} = \rho\varepsilon_t \frac{du}{dy} \qquad \cancel{\sigma_{yy}} = \rho\varepsilon_t \frac{dv}{dy} \qquad \tau_{zy} = \rho\varepsilon_t \frac{dw}{dy}$$

$$\cancel{\sigma_{xx}} = \rho\varepsilon_l \frac{du}{dx} \qquad \tau_{yx} = \rho\varepsilon_l \frac{dv}{dx} \qquad \tau_{zx} = \rho\varepsilon_l \frac{dw}{dx}$$

5.2 Near-field Mixing

1) vertical mixing

- vertical profile of u-velocity ~ logarithmic
- vertical profile of v-velocity ~ linear/cubic → might be neglected because v-velocity is relatively small compared to u-velocity

2) transverse mixing

- transverse profile of u-velocity ~ parabolic/beta function
- transverse profile of w-velocity → might be neglected because w-velocity is usually very small

5.2 Near-field Mixing

3) longitudinal mixing

- longitudinal profile of v-velocity ~ linear/cubic
- longitudinal profile of w-velocity → might be neglected because w-velocity is usually very small

5.2 Near-field Mixing

5.2.3 Vertical Mixing Coefficient

Consider mixing of source of tracer without its own momentum or buoyancy in a straight channel of constant depth and great width

The turbulence is homogeneous, stationary because the channel is uniform.

If the sidewalls are very far apart the width of the flow should play no role.

→ The important length scale is depth.

From Eq. (3.40), turbulent mixing coefficient is given as

$$\varepsilon = \ell_L \left[\overline{u'^2} \right]^{\frac{1}{2}} \qquad (1)$$

5.2 Near-field Mixing

where ε = turbulent mixing coefficient

ℓ_L = Lagrangian length scale ≈ d (a)

$\left[\overline{u'^2}\right]^{\frac{1}{2}}$ = <u>intensity of turbulence</u>

$$\overline{u'^2} = \frac{1}{T}\int u'^2\,dt = \frac{1}{T}\int (u-\overline{u})^2\,dt$$

5.2 Near-field Mixing

- Experiments (Lauffer, 1950) show that in any wall shear flow

$$\left[\overline{u'^2}\right]^{\frac{1}{2}} \propto \sqrt{\tau_0} \qquad \text{(b)}$$

$$\boxed{\tau = \tau_0 = -\rho\overline{u'v'} = \frac{1}{T}\int (u-\overline{u})(v-\overline{v})dt}$$

For dimensional reasons use shear velocity

$$u^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{gdS} \qquad (5.1)$$

where τ_0 = shear stress on the channel bottom

5.2 Near-field Mixing

[Re] shear stress (Henderson, 1966)

~ bottom shear stress is evaluated by a force balance

$$\tau_0 = \rho g d S$$

where S = slope of the channel

Substitute (a) & (b) into (1)

$$\varepsilon \propto d\, u^*$$

$$\boxed{\varepsilon = \alpha\, d\, u^*}$$

→ turbulence will <u>not be isotropic</u>

5.2 Near-field Mixing

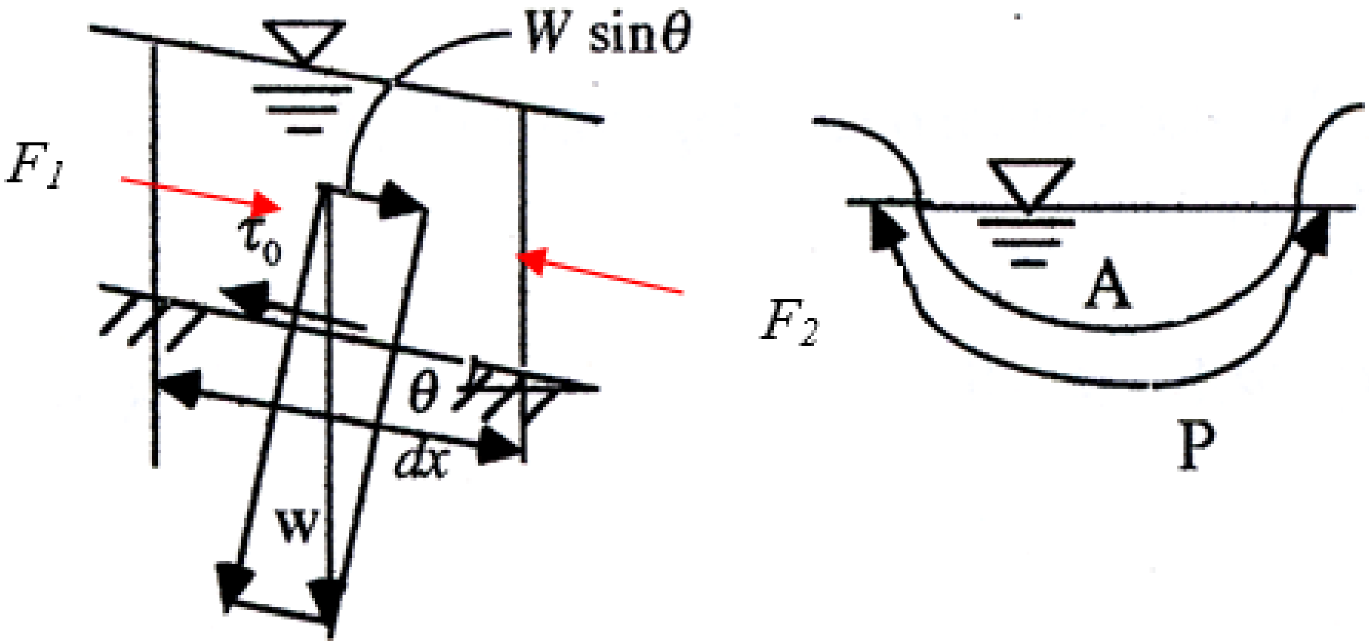
i) vertical mixing, ε_v

~ influence of surface and bottom boundaries

ii) transverse and longitudinal mixing, $\varepsilon_t, \varepsilon_l$

~ no boundaries to influence flow

5.2 Near-field Mixing



5.2 Near-field Mixing

Apply Newton's 2nd law of motion to uniform flow

$$\Sigma\vec{F} = m\vec{a} \qquad \boxed{\vec{a} = 0}$$

$$F_1 - bottom\,shear + W\sin\theta - F_2 = 0 \qquad \boxed{F_1 = F_2}$$

$$-\tau_0 P dx + \rho g A dx \sin\theta = 0$$

$$\tau_0 = \rho g \frac{A}{P} \sin\theta$$

where P = wetted perimeter

Set $S = \tan\theta \approx \sin\theta$

R = hydraulic radius $= \dfrac{A}{P}$

5.2 Near-field Mixing

Then $\tau_0 = \gamma RS$

For very wide channel (b>>d)

$$R = \frac{bd}{b+2d} = \frac{d}{1+2\dfrac{d}{b}} \approx d$$

$$\boxed{\tau_0 = \gamma dS}$$

Vertical mixing coefficient is needed for 3D model

→ there is no dispersion effect by shear flow

5.2 Near-field Mixing

1) The vertically varying coefficient

The vertical mixing coefficient for momentum (eddy viscosity) can be derived from logarithmic law velocity profile (Eq. 4.43).

$$\boxed{\varepsilon_v = \kappa d u^* \frac{z}{d}\left(1 - \frac{z}{d}\right)} \tag{5.2}$$

[Re] Derivation of (5.2)

$$u(z) = \bar{u} + \frac{u^*}{\kappa}(1 + \ln\frac{z}{d}) = \bar{u} + \frac{u^*}{\kappa}(1 + \ln z') \tag{1}$$

5.2 Near-field Mixing

$$\frac{du}{dz} = \frac{u^*}{\kappa}\frac{1}{z'}\frac{1}{d} \tag{2}$$

Reynolds analogy

$$\tau = \tau_0\left(1 - \frac{z}{d}\right) = \rho\varepsilon_v \frac{du}{dz} \tag{3}$$

Substitute (2) into (3)

$$\tau_0\left(1 - z'\right) = \rho\varepsilon_v \frac{u^*}{\kappa}\frac{1}{z'}\frac{1}{d} \tag{4}$$

Rearrange (4)

$$\varepsilon_v = \kappa d\frac{\tau_0}{\rho} z'\left(1 - z'\right) = \kappa d u^* z'\left(1 - z'\right) \tag{5}$$

→ parabolic distribution

5.2 Near-field Mixing

The Reynolds analogy states that the same coefficient can be used for transports of mass and momentum.

→ verified by Jobson and Sayre (1970)

[Re] Relation between eddy viscosity (ν_t) and turbulent diffusion coefficient (ε_t)

→ use turbulent Prandtl (heat) or Schmidt number (mass), σ_t

$$\varepsilon_t = \frac{\nu_t}{\sigma_t}$$

where σ_t~ is assumed to be constant, and usually less than unity

5.2 Near-field Mixing

[Re] Velocity profiles:

- vertical profile of u-velocity ~ logarithmic
- vertical profile of v-velocity ~ linear/cubic → might be neglected because v-velocity is relatively small compared to u-velocity

2) The depth-averaged coefficient

Average Eq. (5.2) over the depth, taking $\kappa = 0.4$

$$\boxed{\overline{\varepsilon}_v = \frac{1}{d}\int_0^d \kappa du^* \left(\frac{z}{d}\right)\left[1-\left(\frac{z}{d}\right)\right]dz = \frac{\kappa}{6}du^* = 0.067du^*} \qquad (5.3)$$

5.2 Near-field Mixing

[Cf] For atmospheric boundary layer: $\overline{\varepsilon_v} = 0.05 d u^*$

where d = depth of boundary layer; u^* = shear velocity at the earth surface

5.2 Near-field Mixing

5.2.4 Longitudinal and Transverse Mixing Coefficients

(1) **Transverse Mixing Coefficient**

Transverse mixing coefficient in 3D model

ε_t ~ no dispersion effect by shear flow, turbulence effect only

For infinitely wide uniform channel, there is no transverse profile of velocity.

~ not possible to establish a transverse analogy of Eq. (5.2)

→ need to know velocity profiles:

5.2 Near-field Mixing

- Depth-averaged coefficient for rectangular open channels

→ rely on experiments (Table 5.1 for results of 75 separate experiments)

$$\varepsilon_t \cong 0.15du^* \tag{5.4}$$

(2) **Longitudinal Mixing Coefficient**

Longitudinal mixing coefficient in 3D model

~longitudinal turbulent mixing is the same rate as transverse mixing because there is an equal lack of boundaries to inhibit motion

$$\varepsilon_l \cong 0.15du^*$$

5.3 Intermediate-field Mixing

5.3.1 Transport Equation for Intermediate-field Mixing

The 2D depth-averaged advection-dispersion equation can be obtained by averaging 3D advection-turbulent diffusion equation.

$$\frac{\partial \overline{c}}{\partial t} + \overline{u}\frac{\partial \overline{c}}{\partial x} + \overline{v}\frac{\partial \overline{c}}{\partial z} = D_L \frac{\partial^2 \overline{c}}{\partial x^2} + D_T \frac{\partial^2 \overline{c}}{\partial z^2}$$

1) D_L : longitudinal mixing coefficient in 2D model

~ Longitudinal mixing by turbulent motion is unimportant because shear flow dispersion coefficient caused by the velocity gradient (vertical variation of u-velocity) is much bigger than mixing coefficient caused by turbulence alone

5.3 Intermediate-field Mixing

Aris (1956) showed that coefficients due to turbulent mixing and shear flow are additive.

$$D_L = D_l + \varepsilon_\ell$$

Elder's result for depth-averaged longitudinal dispersion coefficient

$$D_L = 5.93HU^* \approx 40\varepsilon_t$$

Field data from tracer tests in natural rivers shows that (Seo et al. 2014)

$$\frac{D_L}{HU^*} \approx 10 \sim 100$$

5.3 Intermediate-field Mixing

	$\frac{W}{H}$	$\frac{D_L}{HU^*}$
Laboratory meandering flume (SNU)	4.80~14.3	5.70~22.6
Hong-cheon River (Seo et al., 2006)	69.1~167.4	9.80~87.7
Dae-gok Creek (Seo et al., 2013)	29.0	20.5
Han Creek (Seo et al., 2013)	41.0	22.8
Gam Creek (Seo et al., 2013)	34.0~58.0	12.2~26.5
Mi-ho Creek (Seo et al., 2013)	63.0	15.9~35.9

5.3 Intermediate-field Mixing

2) D_T : transverse mixing coefficient in 2D model

Include dispersion effect by shear flow due to vertical variation of v-velocity

$$v = v(z) = \overline{v} + v'$$

Decompose mixing coefficient

$$D_T = D_t + \varepsilon_t$$

where D_t = transverse dispersion coefficient due to vertical profile of v-velocity

ε_t = transverse turbulent mixing coefficient due to transverse profile of u-velocity

5.3 Intermediate-field Mixing

Researchers (Okoye, 1970; Lau and Krishnappan, 1977) proposed that

$$\frac{D_T}{HU^*} = f\left(\frac{W}{H}, \frac{U}{U^*}, S_n\right)$$

5.3.2 Transverse Mixing in Natural Streams

Natural streams differ from uniform rectangular channels:

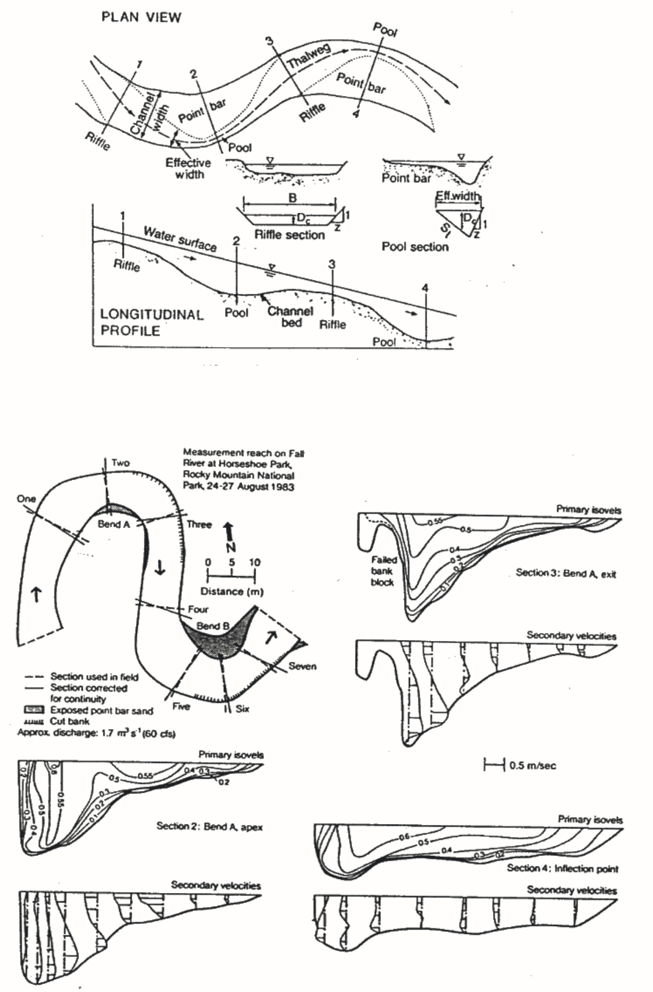
- depth may vary irregularly → <u>pool and riffle</u> sequences

- the channel is likely to curve → <u>meandering</u> rivers

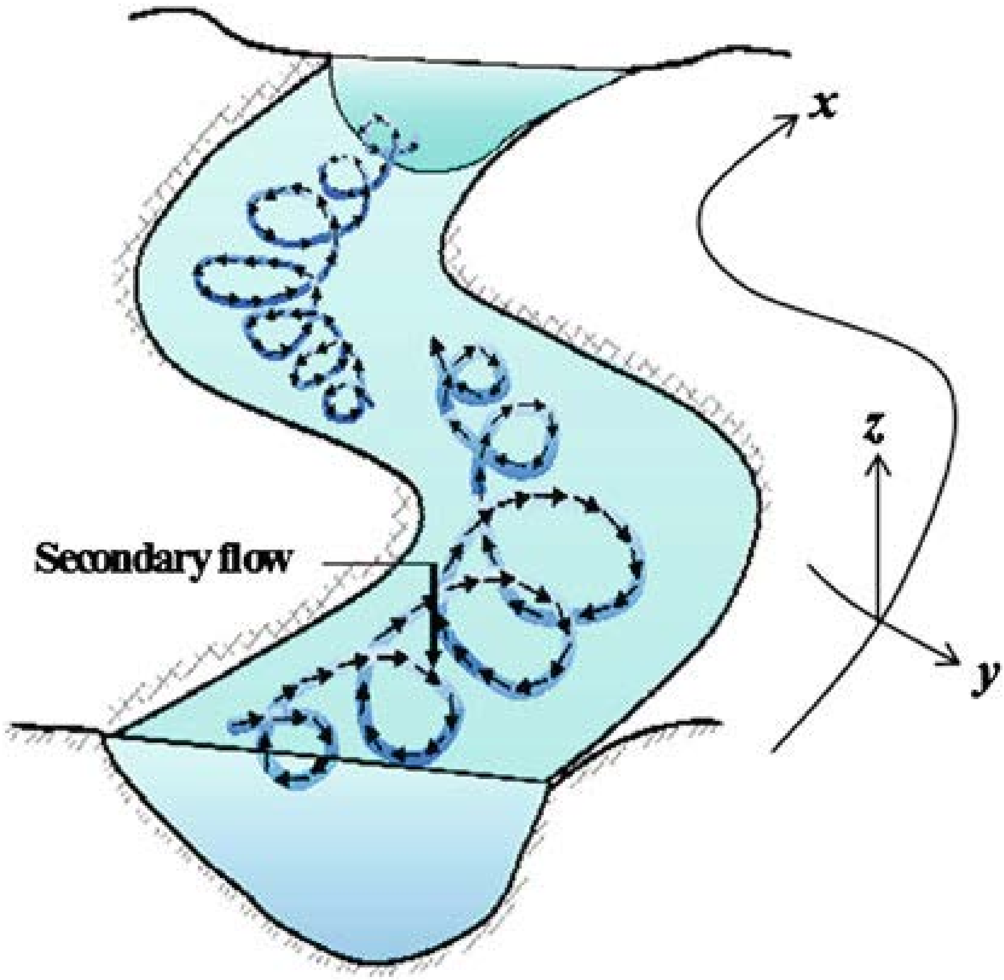
- there may be large sidewall irregularities → groins, dikes

5.3 Intermediate-field Mixing

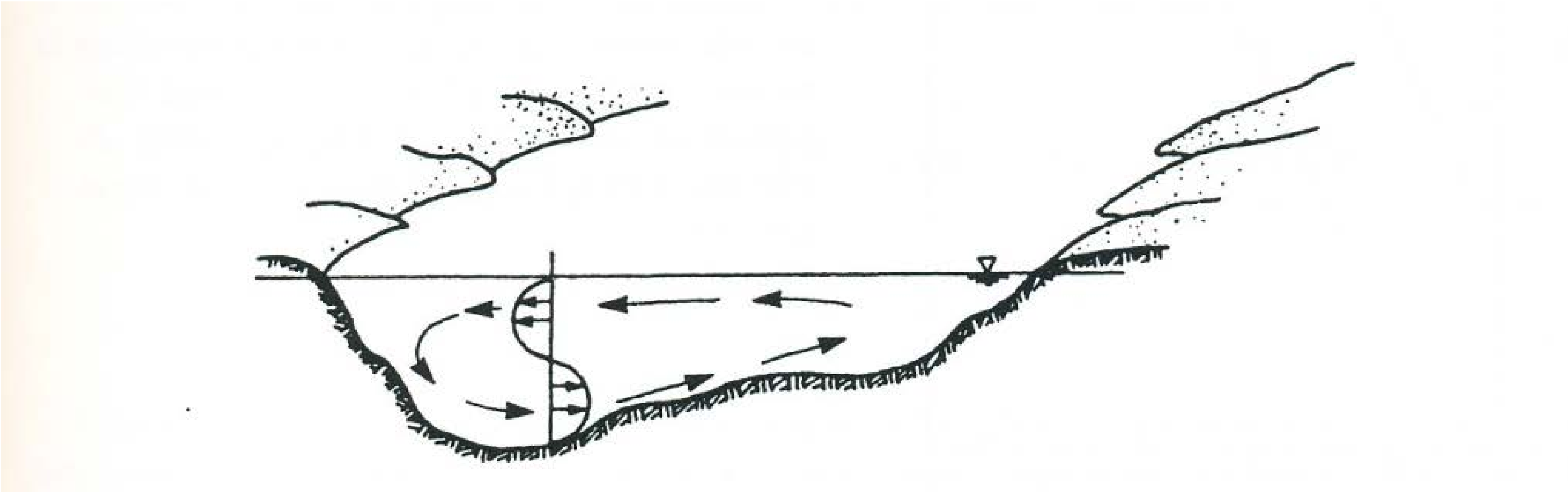
5.3 Intermediate-field Mixing



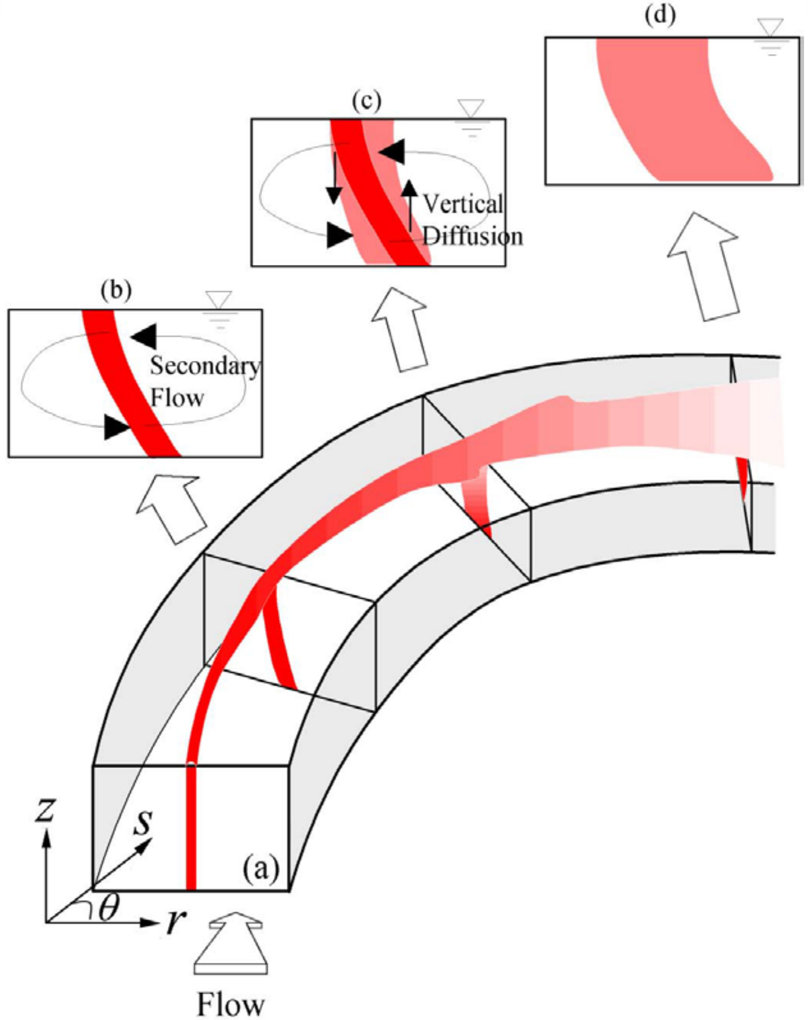
5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

1) Effect of depth variation

Transverse mixing is strongly affected by the channel irregularities because they are capable of generating a wide variety of transverse motions.

2) Effect of channel irregularity

~ major effect on transverse mixing

~ the bigger the irregularity, the faster the transverse mixing

$$\rightarrow 0.3 < \frac{D_T}{HU^*} < 0.7$$

5.3 Intermediate-field Mixing

3) Effect of channel curvature

~ when a flow rounds a bend, the centrifugal forces induce a flow towards the outside bank at the surface, and a compensating reverse flow near the bottom.

→ secondary flow generates

→ secondary flow causes transverse dispersion due to shear flow

→ transverse dispersion enhanced by vertical variation of v-velocity

5.3 Intermediate-field Mixing

For straight, uniform channels, $\frac{D_T}{HU^*} = 0.15$

For natural channels with side irregularities, $\frac{D_T}{HU^*} = 0.4$

For meandering channels with side irregularities, $\frac{D_T}{HU^*} = 0.3 \sim 0.9$

Fischer (1969) predict a transverse dispersion coefficient based on the transverse shear flow

~ used velocity profile given by Rozovskii (1959)

$$\frac{D_T}{HU^*} = 25\left(\frac{U}{U^*}\right)^2\left(\frac{H}{R_c}\right)^2 \qquad (5.5)$$

where R_c = radius of curvature

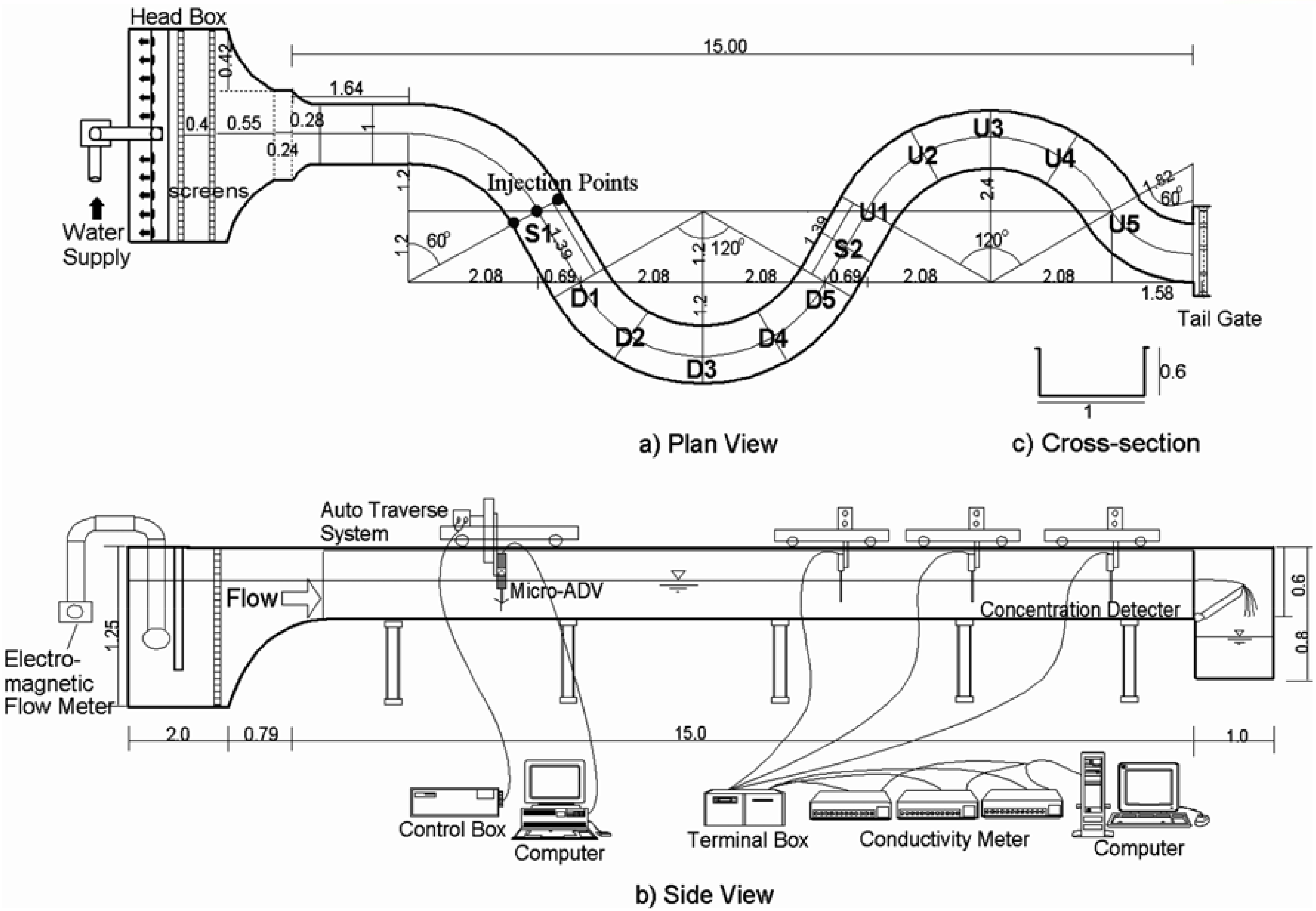
5.3 Intermediate-field Mixing

Yotsukura and Sayre (1976) revised Eq. 5.5) (Fig. 5.3)

$$\frac{D_T}{HU^*} = 0.4\left(\frac{U}{U^*}\right)^2\left(\frac{W}{R_c}\right)^2$$

where W = channel width

5.3 Intermediate-field Mixing

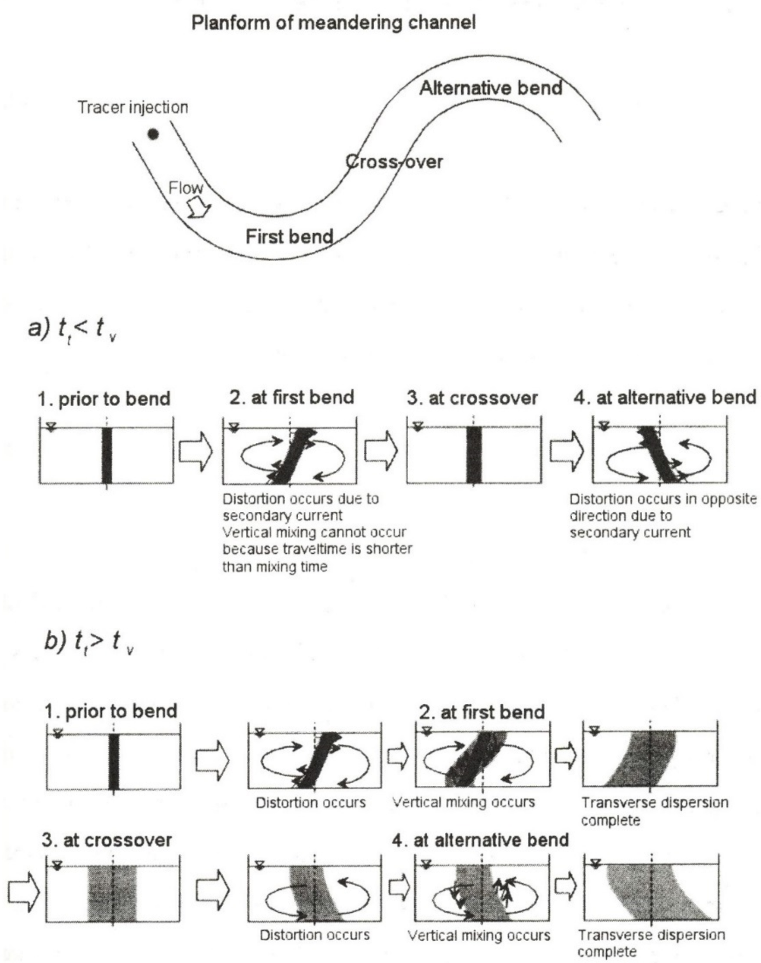


a) Plan View

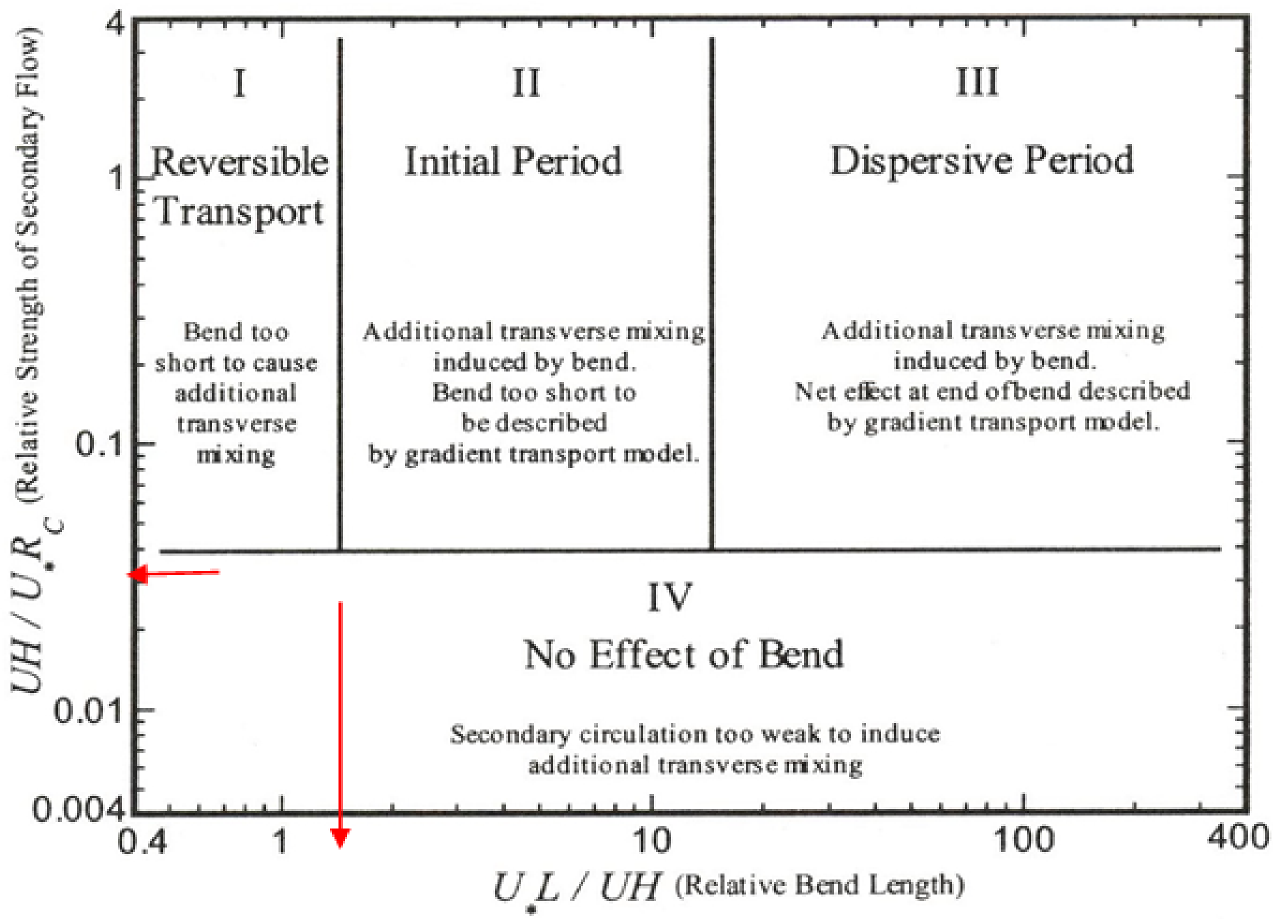
c) Cross-section

b) Side View

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

After initial period, the additional transverse mixing coefficient, $\Delta\alpha$ is given as

$$\Delta\alpha = 25\left(\frac{U}{U^*}\right)^2\left(\frac{H}{R_c}\right)^2$$

Dispersive period

$$\frac{t_t}{t_v} = \frac{L/U}{H^2/\varepsilon_v} > 1$$

$$\frac{U^*L}{UH} > 14$$

$$\boxed{\varepsilon_v = 0.067HU^*}$$

5.3 Intermediate-field Mixing

Rutherford (1979) suggested that

For straight channels, $\dfrac{D_T}{HU^*} = 0.15 \sim 0.30$

For meandering channels, $\dfrac{D_T}{HU^*} = 0.30 \sim 0.90$

For sharp meandering channels, $\dfrac{D_T}{HU^*} = 1.0 \sim 3.0$

- Transverse dispersion coefficient in <u>meandering channels</u>

  - Baek et al. (2006) - observation

  - Baek and Seo (2008), Baek and Seo (2011) – prediction

- Transverse dispersion coefficient in <u>natural streams</u>

  - Seo et al. (2006), Baek and Seo (2010) - observation

  - Jeon et al. (2007), Baek and Seo (2013) - prediction

5.3 Intermediate-field Mixing

- Jeon et al. (2007)

$$\frac{D_T}{HU^*} = a\left(\frac{U}{U^*}\right)^b \left(\frac{W}{H}\right)^c \left(\frac{H}{R_C}\right)^d S_n{}^e$$

a=0.029; b=0.463; c=0.299; d=0; e=0.733

- Baek and Seo (2008)

$$\frac{D_T}{HU^*} = 0.04\left(\frac{U}{U^*}\right)^2 \left(\frac{W}{R_c}\right)^2 \left(\left|\frac{x}{2L_C}\sin(2\pi\frac{x}{L_C})\right| + \frac{1}{2}\right)^2 I$$

5.3 Intermediate-field Mixing

- Baek and Seo (2011)

$$\frac{D_T}{HU^*} = \frac{1}{24\kappa^7}\left(2\kappa\frac{U}{U^*}+1\right)^2\left(\frac{H}{R_c}\right)^2\left(1-\exp\left(-\frac{2\kappa^2}{\left(\kappa\frac{U}{U^*}+1\right)}\frac{x}{H}\right)\right)^2$$

- Baek and Seo (2013)

$$\frac{D_T}{HU^*} = \left(88.66\frac{U}{U^*}\frac{H}{R_c}\right)^2\left(1-\exp\left(-\frac{1}{94.02\frac{U}{U^*}\frac{H}{R_c}}\right)\right)^2$$

5.3 Intermediate-field Mixing

[Re] Determination of dispersion coefficients for 2D numerical models

1) Observation – calculation of observed concentration curves from field data
2) Prediction – estimation of dispersion coefficient using theoretical or empirical equations

5.3 Intermediate-field Mixing

Observation Method	
Moment method	Simple moment method
	Stream-tube moment method
Routing procedure	2-D routing method
	2-D stream-tube routing method

5.3 Intermediate-field Mixing

Prediction Method	
Theoretical equation for D_T	Use vertical profile of v-velocity
	Baek and Seo (2008), Baek and Seo (2011), Baek and Seo (2013)
Empirical equation for D_T	Use mean hydraulic data
	Fischer (1969) Yotsukura & Sayre (1976) Jeon et al. (2007)

5.3 Intermediate-field Mixing

- Numerical model

· In numerical calculations of large water bodies, additional processes are represented by the diffusivity.

1) Sub-grid advection

Owing to computer limitations, the numerical grid of the numerical calculations cannot be made so fine as to obtain grid-independent solutions.

→ All advective motions smaller than the mesh size, such as in small recirculation zones, cannot be resolved. Thus, their contribution to the transport must be accounted for by the diffusivity.

5.3 Intermediate-field Mixing

2) Numerical diffusion

The approximation of the differential equations by difference equations introduces errors which act to smooth out variations of the dependent variables and thus effectively increase the diffusivity.

→ This numerical diffusion is larger for coarser grids.

· An effective diffusivity accounts for turbulent transport, numerical diffusion, sub-grid scale motions, and dispersion (in the case of depth-average calculations).

→ The choice of a suitable mixing coefficient (D_{MT}) is usually not a turbulence model problem but a matter of numerical model calibration.

5.3 Intermediate-field Mixing

For 2D model,

$$D_{MT} = D_t + \varepsilon_t + \varepsilon_{sgm} - \varepsilon_{nd}$$

5.3 Intermediate-field Mixing

5.3.3 2D Concentration Distributions

Compute the distribution of concentration downstream from a continuous effluent discharge in a flowing stream

In most of the natural streams the flow is much wider than it is deep; a typical channel dimension might be 30 m wide by 1 m deep, for example.

Recall that the mixing time is proportional to the square of the length divided by the mixing coefficient,

5.3 Intermediate-field Mixing

$$T \propto \frac{(length)^2}{\varepsilon}$$

$$\frac{W}{d} \cong \frac{30}{1} = 30$$

$$\frac{\varepsilon_t}{\varepsilon_v} = \frac{0.6du^*}{0.067du^*} \approx 10$$

$$\therefore \frac{T_t}{T_v} = \frac{(W)^2}{\varepsilon_t} / \frac{(d)^2}{\varepsilon_v} = \left(\frac{W}{d}\right)^2 \frac{\varepsilon_v}{\varepsilon_t} = \left(\frac{30}{1}\right)^2 \left(\frac{1}{10}\right) = 90 \approx 10^2$$

$$\therefore T_t \approx 10^2 T_v \qquad (5.6)$$

5.3 Intermediate-field Mixing

→ vertical mixing is instantaneous compared to transverse mixing

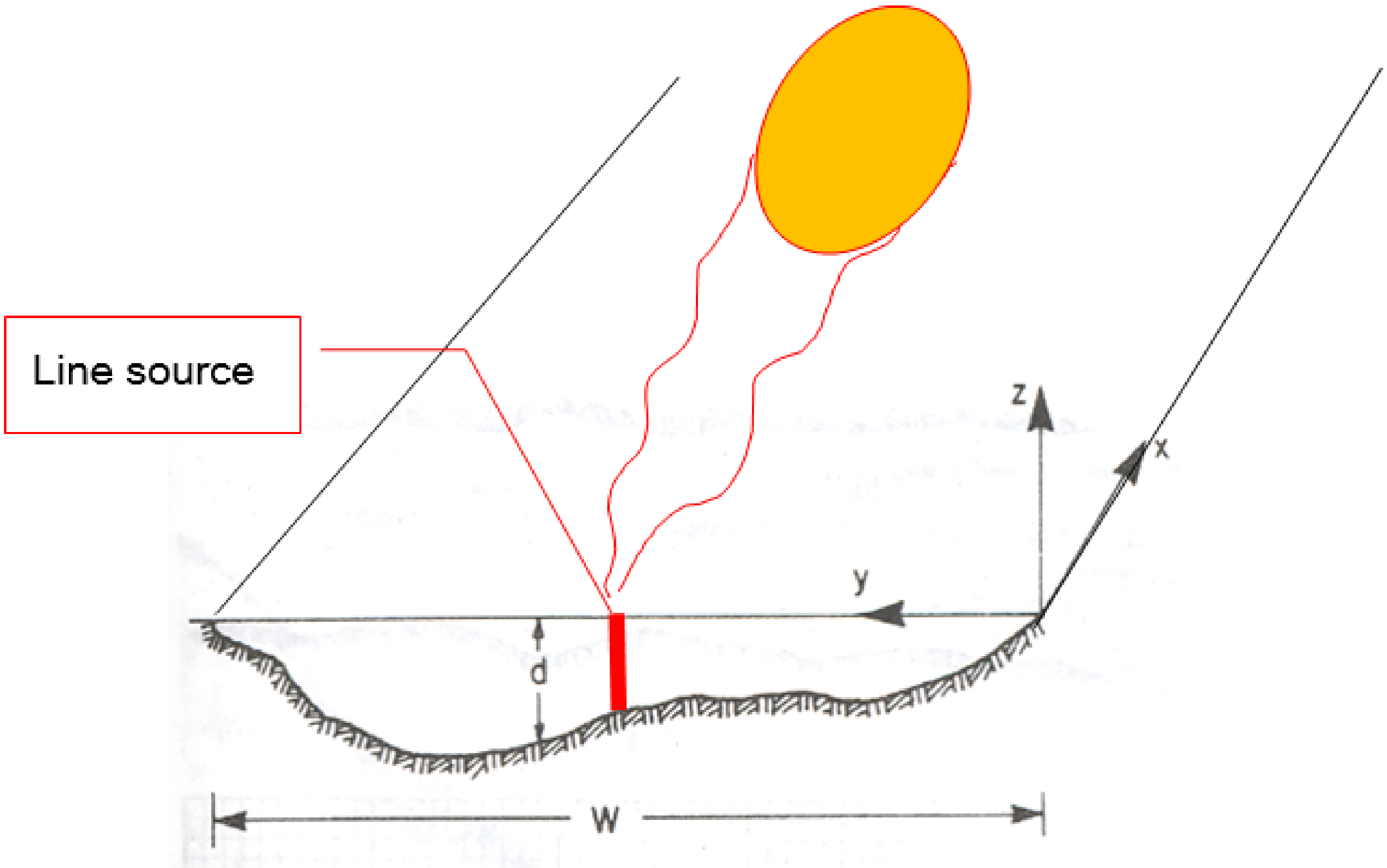
Thus, in most practical problems, we can start assuming that the effluent is uniformly distributed over the vertical.

→ analyze the two-dimensional spread from a uniform line source

Now consider the case of a rectangular channel of depth d into which is discharged $\dot{M}$ units of mass (per time) in the form of line source.

~ is equivalent to a point source of strength $\dot{M}/d$ in a two-dimensional flow → maintained source in 2D

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

Recall Eq. (2.68)

$$C = \frac{\dot{M}/d}{\overline{u}\sqrt{4\pi\varepsilon_t \frac{x}{\overline{u}}}} \exp\left(-\frac{y^2 \overline{u}}{4\varepsilon_t x}\right) \tag{5.7}$$

i) For very wide channel, when $t >> 2\varepsilon_t / \overline{u}^2$

→ use Eq. (5.7)

ii) For narrow channel, consider effect of boundaries

$$\frac{\partial C}{\partial y} = 0 \; at \; y = 0 \; and \; y = W$$

→ method of superposition

5.3 Intermediate-field Mixing

Define dimensionless quantities by setting

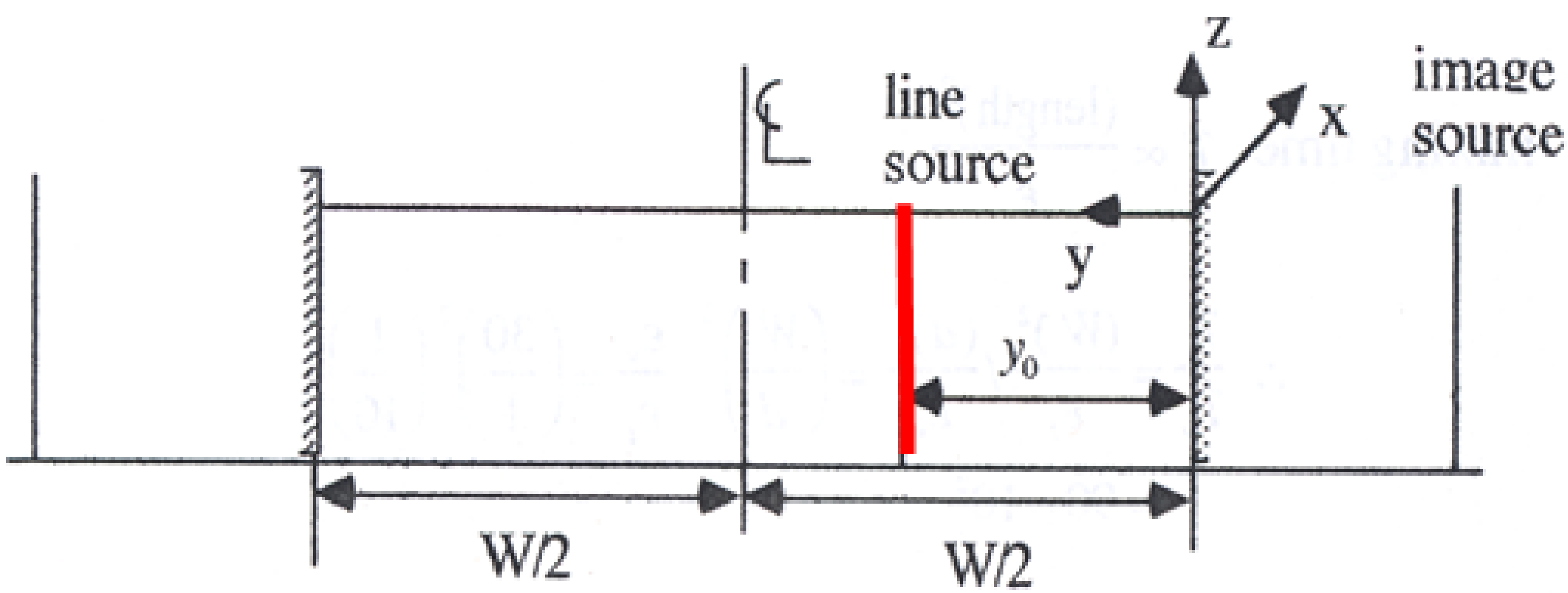
$C_0 = \frac{\dot{M}}{\bar{u}dW}$ = mass rate / volume of ambient water

~ concentration after cross-sectional mixing is completed

$$x' = \frac{x\varepsilon_t}{\bar{u}W^2}$$

$$y' = y/W$$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

Then Eq. (5.7) becomes

$$C = \frac{\dfrac{\dot{M}}{\bar{u} d W}}{\sqrt{\dfrac{4\pi \varepsilon_t x}{\bar{u} W^2}}} \exp\left(-\frac{(\dfrac{y}{W})^2}{\dfrac{4\varepsilon_t x}{\bar{u} W^2}} \right)$$

$$= \frac{C_0}{\sqrt{4\pi x^{'}}} \exp\left(-\frac{y^{'2}}{4x^{'}} \right)$$

$$\frac{C}{C_0} = \frac{1}{(4\pi x^{'})^{1/2}} \exp\left(-\frac{y^{'2}}{4x^{'}} \right)$$

5.3 Intermediate-field Mixing

If the source is located at $y = y_0 (y' = y'_0)$

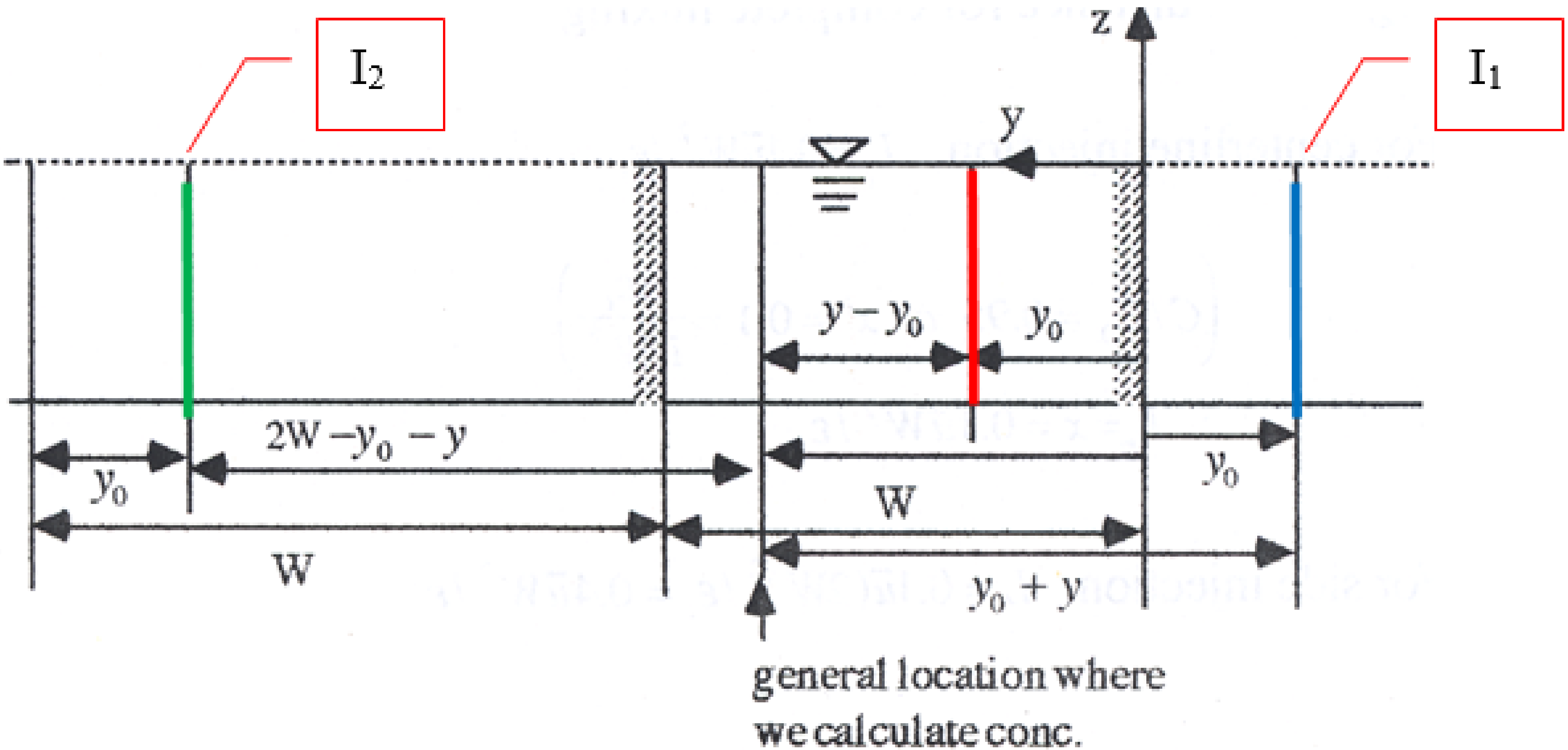
Consider real and image sources, then superposition gives the downstream concentration distribution as

$$\frac{C}{C_0} = \frac{1}{(4\pi x')^{\frac{1}{2}}}\left[\overbrace{\exp\left(-\left\{\frac{(y' - y'_0)^2}{4x'}\right\}\right)}^{\text{real}} + \overbrace{\exp\left(-\left\{\frac{(y' + y'_0)^2}{4x'}\right\}\right)}^{\mathrm{I}_1} + \overbrace{\exp\left(-\left\{\frac{(y' - 2 + y'_0)^2}{4x'}\right\}\right)}^{\mathrm{I}_2} + \bullet\bullet\bullet\right]$$

$$= \frac{1}{(4\pi x')^{\frac{1}{2}}} \sum_{n=-\infty}^{\infty} \left\{\exp\left[-(y' - 2n + y'_0)^2 / 4x'\right] + \exp\left[-\left(y' - 2n + y'_0\right)^2 / 4x'\right]\right\}$$

Sum for $n = 0, \pm 1, \pm 2$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

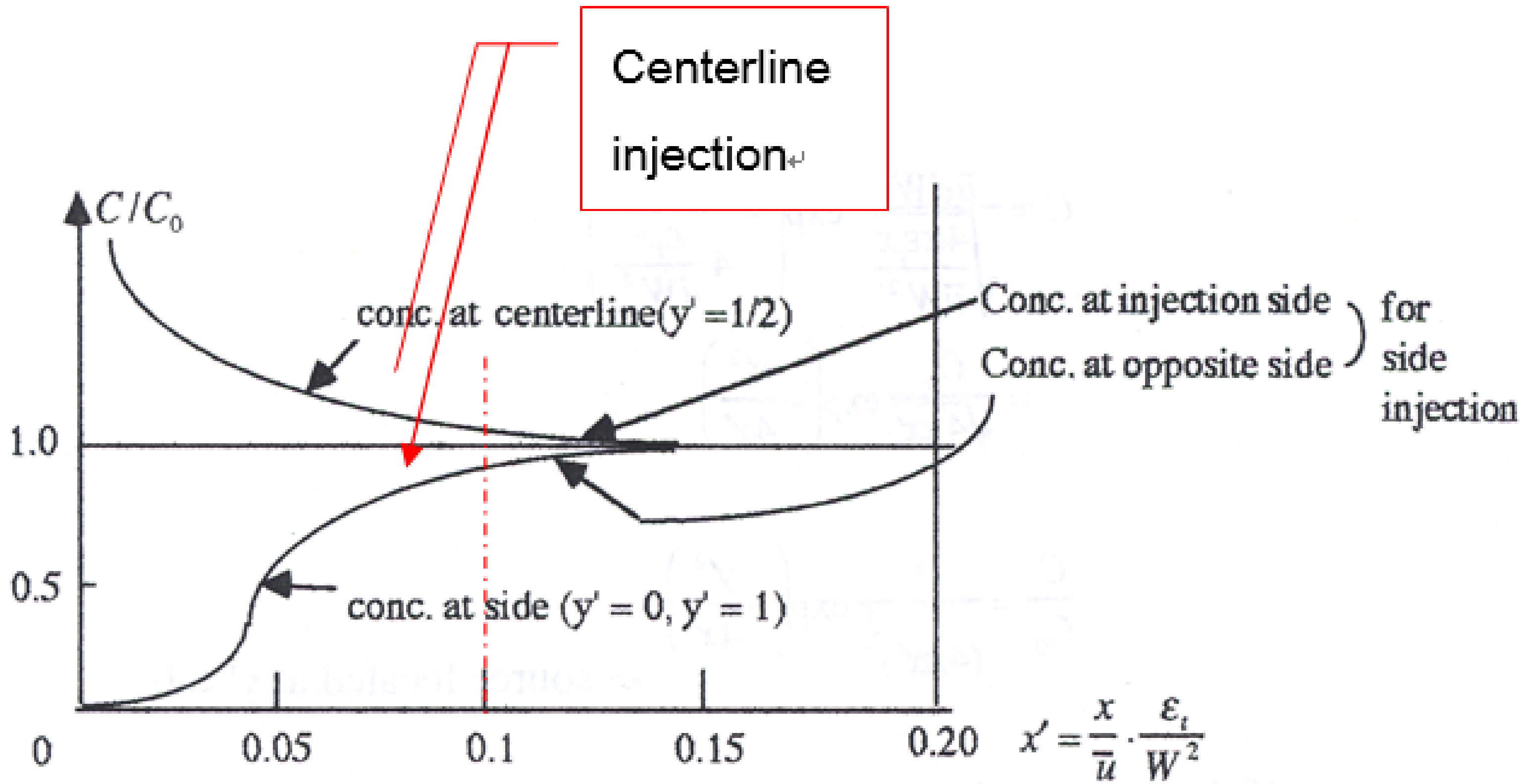
Continuous centerline discharge: $y'_0 = 1/2$

From this figure, for *x'* greater than about 0.1 the concentration is within 5 % of its mean value everywhere on the cross section.

Thus, the longitudinal distance for complete transverse mixing for centerline injection is

$$\boxed{L_c = 0.1\bar{u}W^2 / \varepsilon_t} \qquad (5.8)$$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

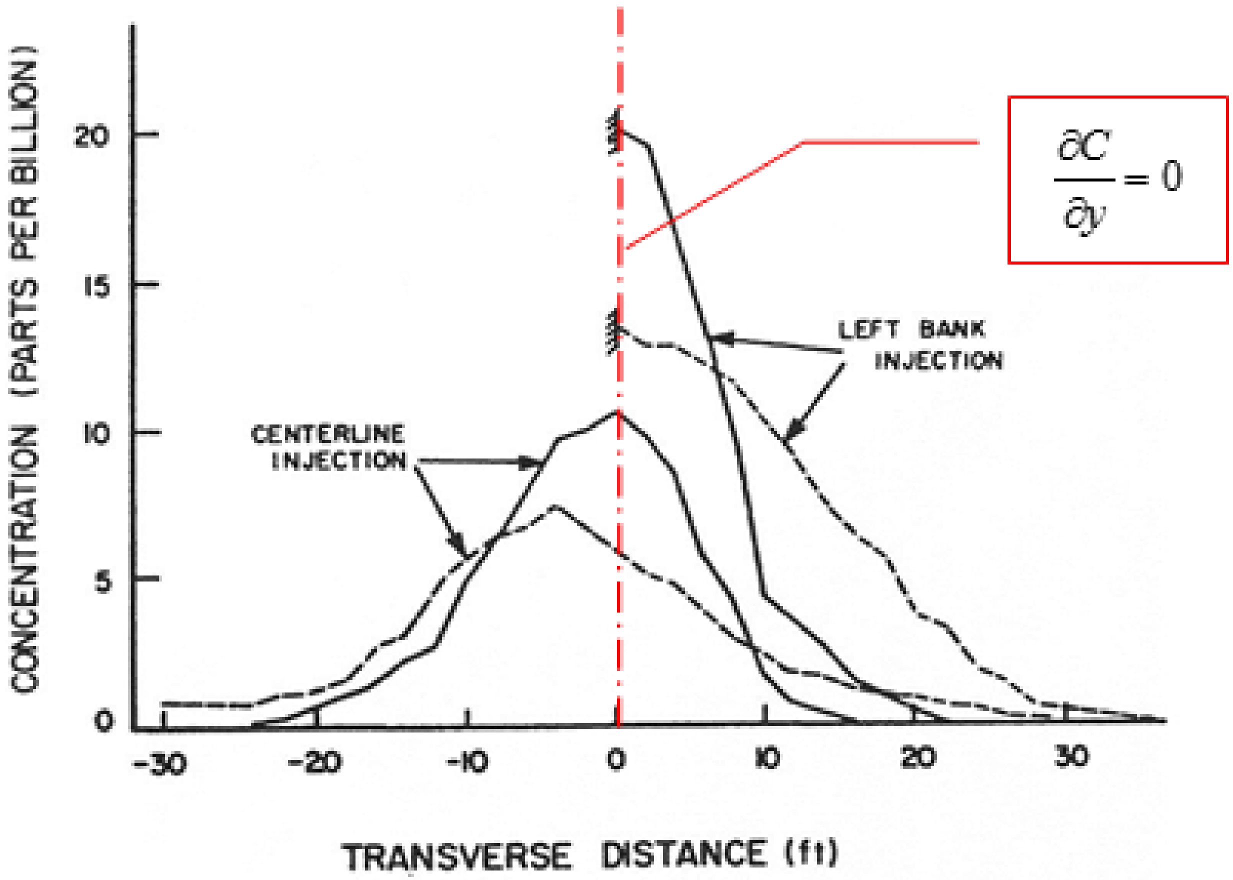
[Re] $$\frac{C}{C_0} = 0.95\, at\; x' = 0.1 = \frac{x\varepsilon_t}{\overline{u}W^2}$$

$$L_c = x = 0.1\overline{u}W^2 / \varepsilon_t$$

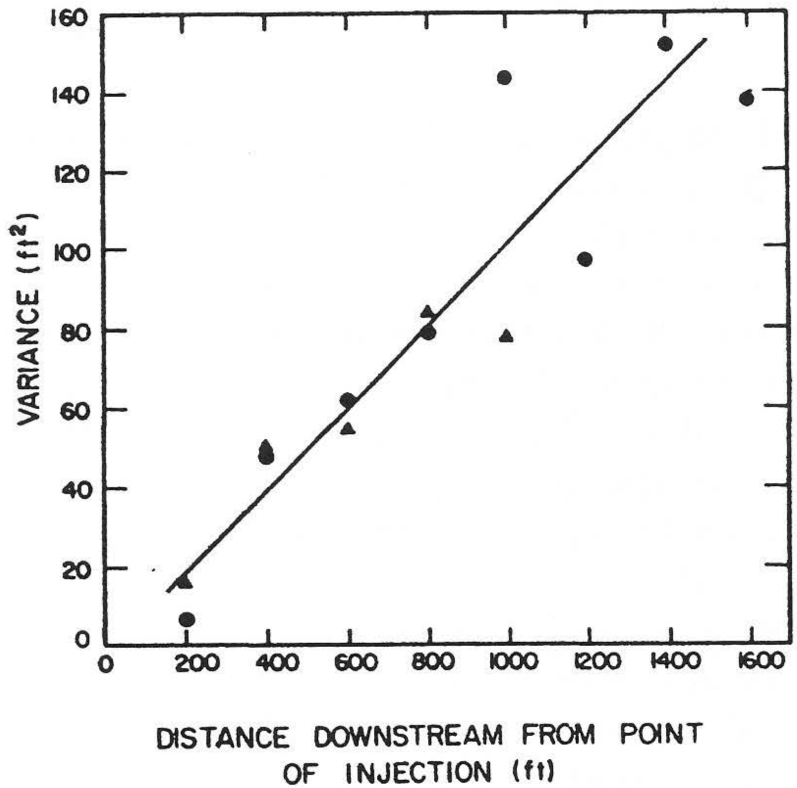
For side injection, the width over which mixing must take place is twice that for a centerline injection

$$\boxed{L = 0.1\overline{u}(2W)^2 / \varepsilon_t = 0.4\overline{u}W^2 / \varepsilon_t}$$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

[Ex 5.1] Spread of a plume from a point source

An industry discharges effluent;

Continuous injection

C = 200ppm

$$Q = 3MG / day = 11,356.2 m^3 / day = 0.13 m^3 / s$$

Thus, rate of mass input is

$$\dot{M} = QC = 0.13(200ppm) = 26\, m^3 / s \cdot ppm$$

5.3 Intermediate-field Mixing

Centerline injection in very wide, slowly meandering stream

$$d = 9.14m; \quad \bar{u} = 0.61m/s; \quad u^* = 0.061m/s$$

Determine the width of the plume, and maximum concentration 1000 ft downstream from discharge assuming that the effluent is completely mixed over the vertical.

[Sol]

For meandering stream,

$$\varepsilon_t = 0.6du^* = 0.6(9.14)(0.061) = 0.33m^2/s$$

5.3 Intermediate-field Mixing

Use Eq.(5.7) for line source

Peak concentration

Exponential decay

$$C(x,y)=\frac{\dot{M}}{\bar{u}d\left(\frac{4\pi\varepsilon_t x}{\bar{u}}\right)^{\frac{1}{2}}}\exp\left(-\frac{y^2\bar{u}}{4\varepsilon_t x}\right) \qquad (5.7)$$

Compare with normal distribution; $C=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{y^2}{2\sigma^2}\right)$

$$\exp\left(-\frac{y^2}{\frac{4\varepsilon_t x}{\bar{u}}}\right)=\exp\left(-\frac{y^2}{2\sigma^2}\right)$$

5.3 Intermediate-field Mixing

$$\sigma^2 = \frac{2\varepsilon_t x}{\bar{u}}$$

$$\sigma = \sqrt{\frac{2\varepsilon_t x}{\bar{u}}}$$

a) width of plume can be approximate by 4σ (includes 95% of total mass)

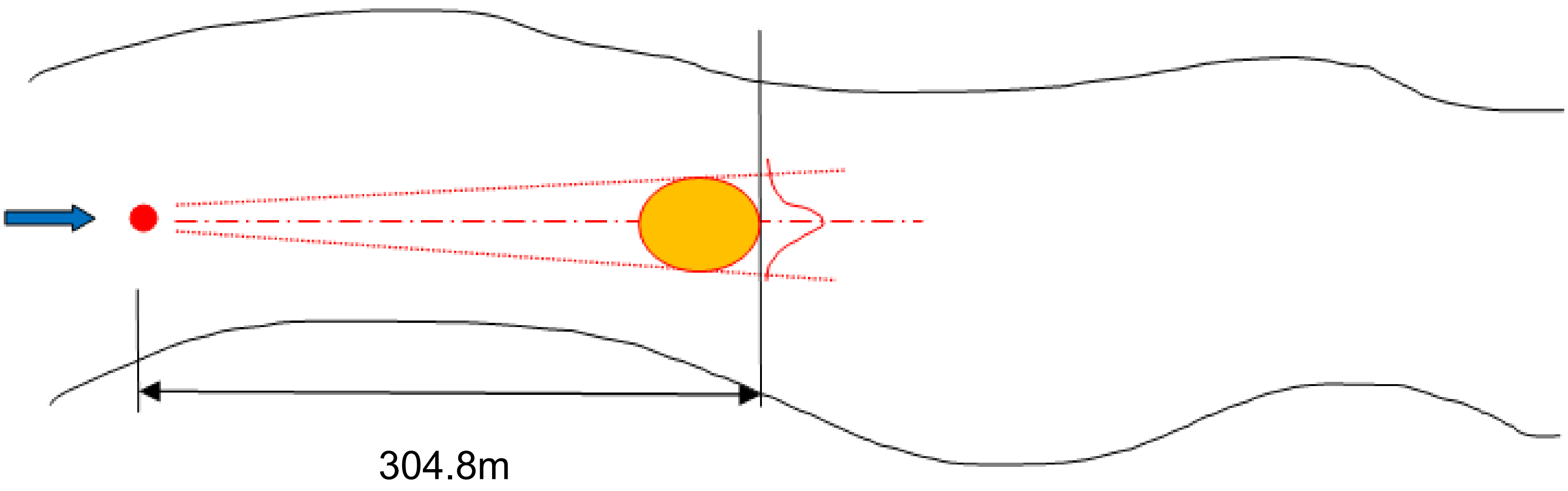
$$b = 4\sigma = 4\sqrt{\frac{2\varepsilon_t x}{\bar{u}}} = 4\sqrt{\frac{2(0.33)(304.8)}{0.61}} = 72.6m$$

b) maximum concentration

$$C_{\max} = \frac{M}{\bar{u}d\left(\frac{4\pi\varepsilon_t x}{\bar{u}}\right)^{\frac{1}{2}}} = \frac{26m^3/s\cdot ppm}{(0.61m/s)(9.14m)\left(\frac{4\pi\times 0.33m^2/s\times 304.8m}{0.61m/s}\right)^{\frac{1}{2}}}$$

$$= 0.102\,ppm$$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

[Ex 5.2] Mixing across a stream

→ consider boundary effect

Given:

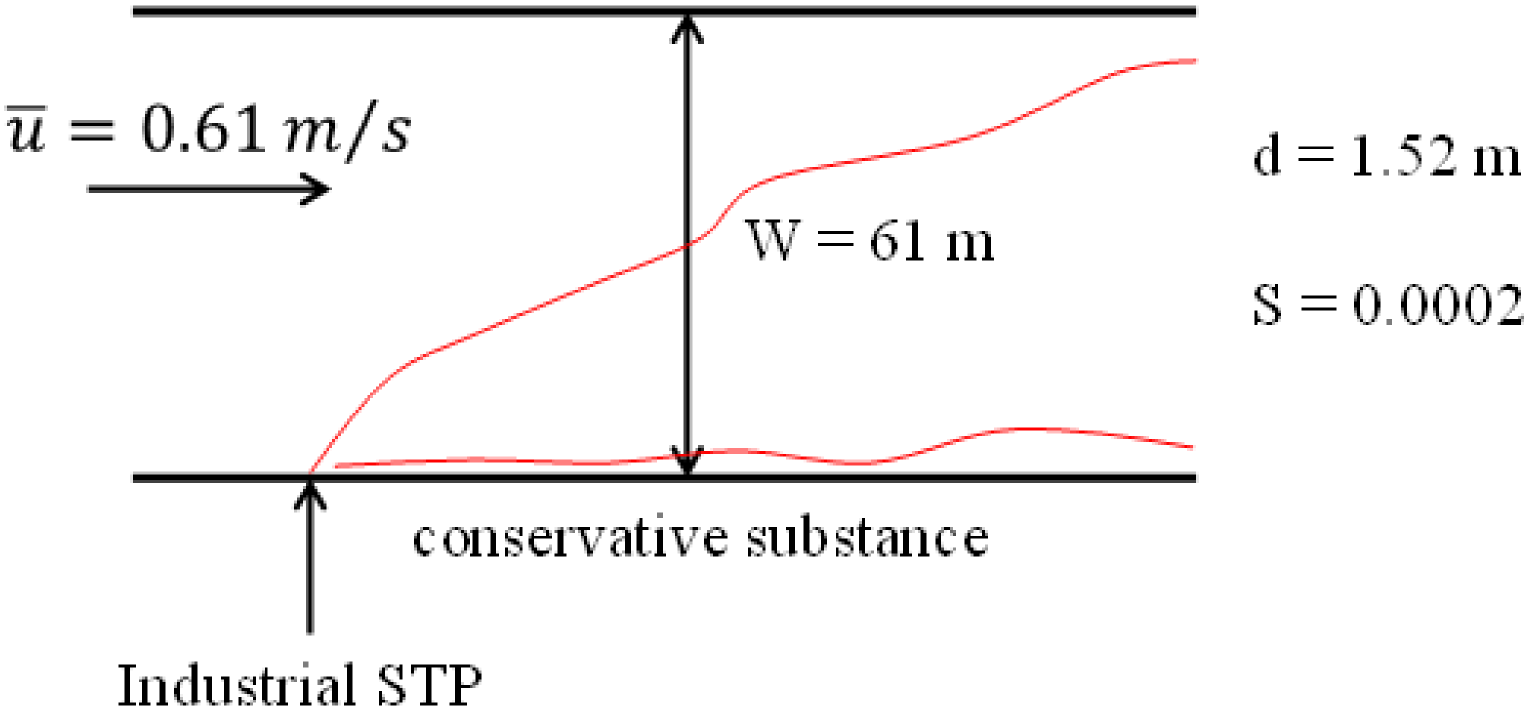
Find: length of channel required for "complete mixing" as defined to mean that the concentration of the substance varies by no more than 5% over the cross section

[Sol]

Shear velocity

$$u^* = \sqrt{gdS} = \sqrt{9.81(1.52)(0.0002)} = 0.055 m/s$$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

For uniform, straight channel

$$\varepsilon_t = 0.15du^* = 0.15(1.52)(0.055) = 0.125\, ft^3 / s$$

For complete mixing from a side discharge

$$L = 0.4\bar{u}W^2 / \varepsilon_t$$

$$L = 0.4(0.61)(61)^2 / 0.0125 = 72,634m \approx 73km$$

Very long distance for a real channel

5.3 Intermediate-field Mixing

[Ex 5.3] Blending of two streams

Compute the mixing of two streams which flow together at a smooth junction so that the streams flow side by side until turbulence accomplishes the mixing.

Given:

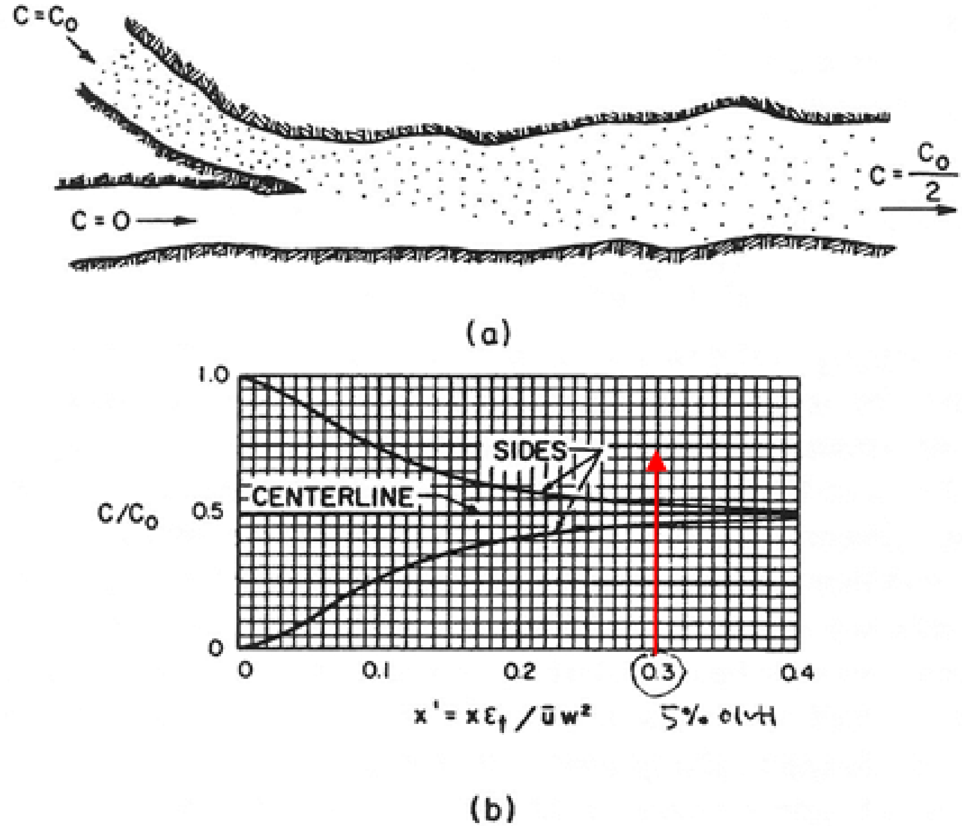
$$Q = 1.42 m^3 / s;\ W = 6.1 m;\ S = 0.001;\ n = 0.030$$

Find:

a) length of channel required for complete mixing for uniform straight channel

b) length of channel required for complete mixing for curved channel with a radius of 30.5 ft

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

[Sol]

The velocity and depth of flow can be found by solving Manning's formula

$$\bar{u} = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

R = hydraulic radius = A/P

$$Q = A\bar{u} = \frac{1}{n} A R^{2/3} S^{1/2} = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

$$2.84 = \frac{1}{0.030} \frac{(6.1d)^{5/3}}{(6.1+2d)^{2/3}} (0.001)^{1/2} = 21.5 \frac{d^{5/3}}{(6.1+2d)^{2/3}}$$

$$d^{5/3} = 0.132(6.1+2d)^{2/3}$$

$$d = 0.297(6.1+2d)^{2/5}$$

5.3 Intermediate-field Mixing

By trial-error method, $d = 0.66m$

$$R = \frac{0.66(6.1)}{(6.1+1.32)} = 0.54m$$

$$\overline{u} = \frac{1}{0.030}\left(\frac{0.66\times 6.1}{6.1+1.32}\right)^{2/3}(0.001)^{1/2} = 0.70m/s$$

$$\therefore u^{*} = \sqrt{gRS} = \sqrt{9.81(0.54)(0.001)} = 0.073m/s$$

$$\varepsilon_t = 0.15du^{*} = 0.15(0.66)(0.073) = 0.0072\ m2/s$$

5.3 Intermediate-field Mixing

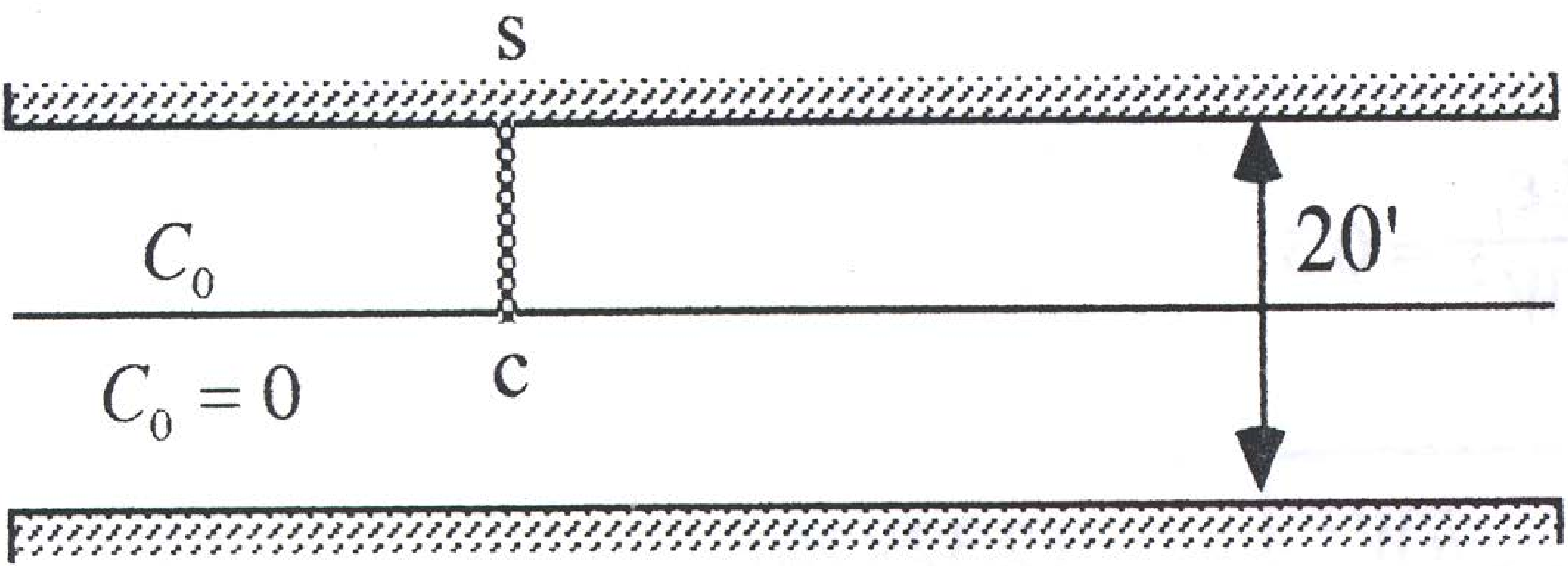
For the case of blending of two streams, there is a tracer whose concentration is C_0 in one stream and zero in the other.

If the steams were mixed completely the concentration would be 1/2 C_0 everywhere on the cross section.

The initial condition may be considered to consist of a uniform distribution of unit inputs in one-half of the channel.

→ The exact solution can be obtained by superposition of solutions for the step function in an unbounded system [Eq. (2.33)].

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

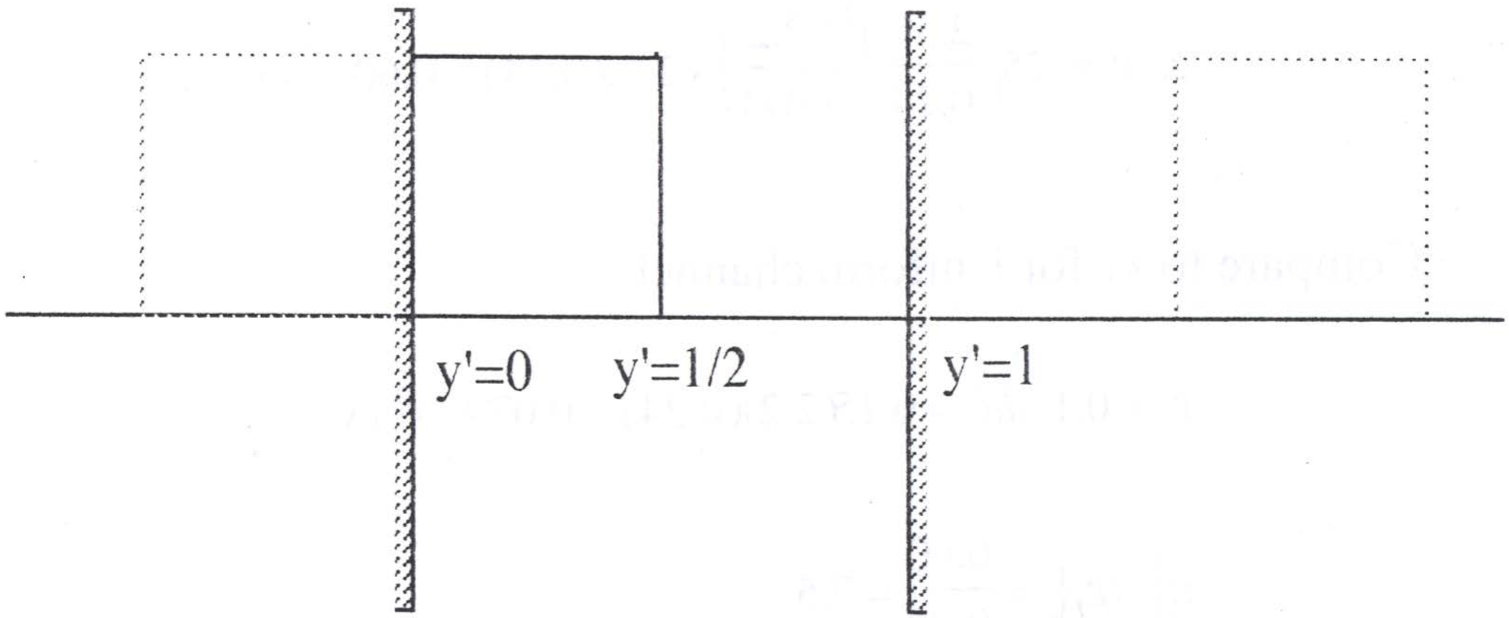
Consider sources ranging $y'_0 = 0 \sim 1/2$

Method of images gives

$$\frac{C}{C_0} = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(erf \frac{y' + 1/2 + 2n}{\sqrt{4x'}} - erf \frac{y' - 1/2 + 2n}{\sqrt{4x'}} \right)$$

where $y' = y/W; x' = \dfrac{x\varepsilon_t}{\bar{u}W^2}$

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

From Fig. 5.9, maximum deviation in concentration is 5% of the mean when $x' \approx 0.3$.

$$x' = \frac{L\varepsilon_t}{\overline{u}W^2} = 0.3$$

$$L = 0.3\frac{\overline{u}W^2}{\varepsilon_t} = 0.3\frac{(0.70)(6.1)^2}{0.0072} = 1{,}085m < 1{,}447m$$

[Re] For side injection only

$$L = 0.4\frac{\overline{u}W^2}{\varepsilon_t} = 0.4\frac{(0.70)(6.1)^2}{0.0072} = 1{,}447m$$

5.3 Intermediate-field Mixing

For curved channel

$$\frac{\varepsilon_t}{du^*} = 25\left(\frac{\overline{u}}{u^*}\right)^2\left(\frac{d}{R}\right)^2$$

$$\therefore \varepsilon_t = 25\left(\frac{0.7}{0.073}\right)^2\left(\frac{0.66}{30.5}\right)^2 du^*$$

$$= 1.079(0.66)(0.073) = 0.052m^2/s > 0.0072m^2/s$$

$$L = 0.3\frac{\overline{u}W^2}{\varepsilon_t} = \frac{0.3(0.70)(6.1)^2}{0.052} = 150.3m$$

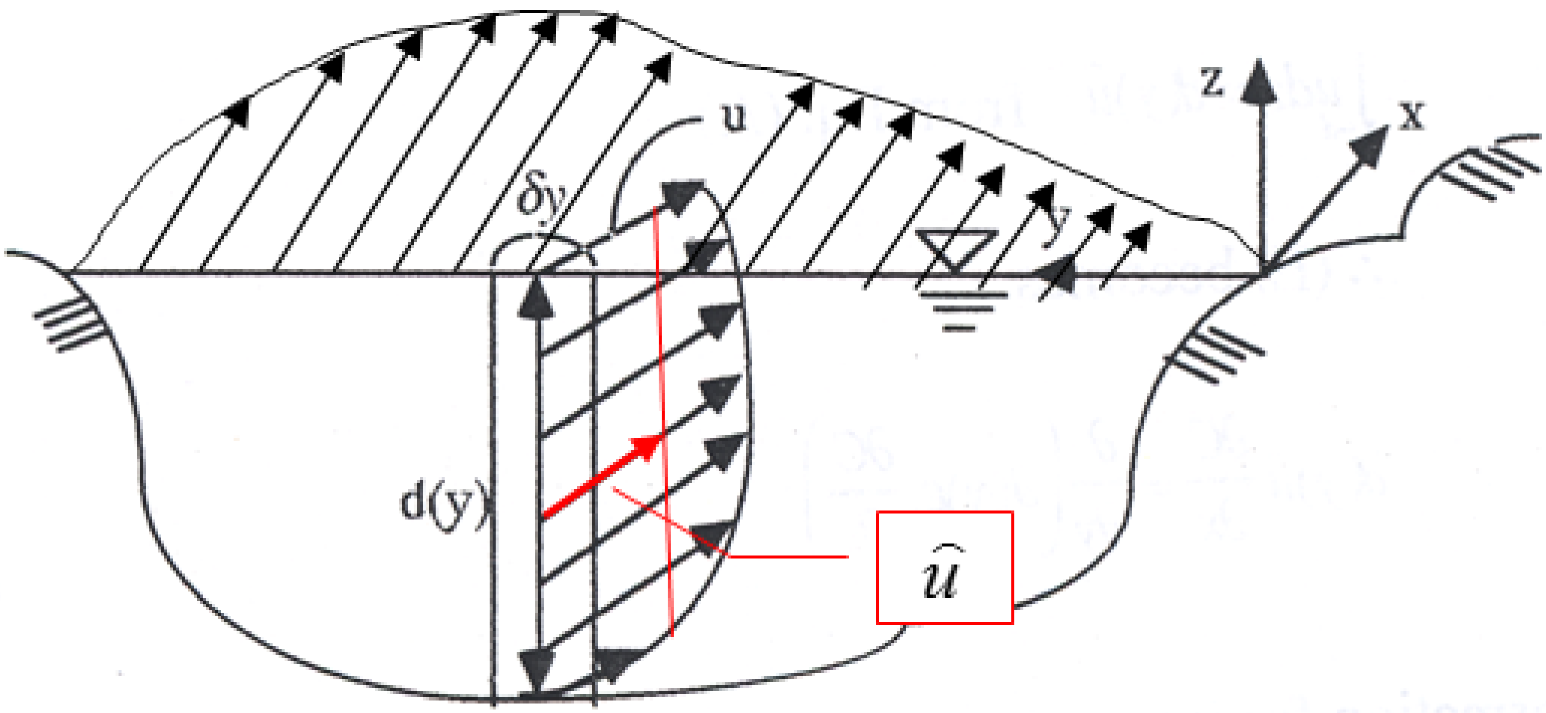
5.3 Intermediate-field Mixing

5.3.4 Cumulative Discharge Method for 2D Mixing

Previous analysis was presented assuming a uniform flow of constant velocity everywhere in the channel.

However, in real rivers, the downstream velocity varies across the cross section, and there are irregularities along the channel.

5.3 Intermediate-field Mixing



5.3 Intermediate-field Mixing

Use cumulative discharge method (Stream-tube method) by Yotsukura and Sayre (1976)

Define velocity averaged over depth at some value of y as

$$\hat{u} = \frac{1}{d(y)} \int_{-d(y)}^{0} u dz \qquad \text{(a)}$$

Then, cumulative discharge is given as

$$\boxed{q(y) = \int_0^y dq = \int_0^y d(y) \hat{u} dy} \qquad \text{(b)}$$

$$q(y) = 0 \quad at \quad y = 0 \qquad \text{(c)}$$

$$q(y) = Q \quad at \quad y = W$$

5.3 Intermediate-field Mixing

[Cf] $\bar{u}$ = cross-sectional average velocity

Now, derive depth-averaged 2D equation for transverse diffusion assuming steady-state concentration distribution and neglecting longitudinal mixing and v-velocity

$$\cancel{\frac{\partial C}{\partial t}} + u\frac{\partial C}{\partial x} + \cancel{v\frac{\partial C}{\partial y}} = \cancel{\frac{\partial}{\partial x}\left(\varepsilon_l \frac{\partial C}{\partial x}\right)} + \frac{\partial}{\partial y}\left(\varepsilon_t \frac{\partial C}{\partial y}\right) \qquad \text{(d)}$$

Integrate (d) over depth

$$\int_{-d}^{0} u\frac{\partial C}{\partial x}dz = \int_{-d}^{0} \frac{\partial}{\partial y}\left(\varepsilon_t \frac{\partial C}{\partial y}\right)dz \qquad \text{(e)}$$

5.3 Intermediate-field Mixing

From Eq.(a)

$$\int_{-d}^{0} u dz = d(y)\hat{u}$$

Eq. (e) becomes

$$d(y)\hat{u}\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(d(y)\varepsilon_t \frac{\partial C}{\partial y} \right)$$

$$\frac{\partial C}{\partial x} = \frac{1}{d(y)\hat{u}}\frac{\partial}{\partial y}\left(d(y)\varepsilon_t \frac{\partial C}{\partial y} \right) \qquad \text{(f)}$$

5.3 Intermediate-field Mixing

Transformation from y to q gives

$$\frac{\partial}{\partial y}=\frac{\partial q}{\partial y}\frac{\partial}{\partial q}=d(y)\hat{u}\frac{\partial}{\partial q}$$

$$\boxed{\frac{\partial q}{\partial y}=\frac{\partial}{\partial y}\left[\int_0^y d(y)\hat{u}dy\right]=d(y)\hat{u}} \quad \text{(g)}$$

Substituting Eq. (g) into Eq.(f) yields

$$\frac{\partial C}{\partial x}=\frac{1}{\cancel{d(y)\hat{u}}}\cancel{d(y)\hat{u}}\frac{\partial}{\partial q}\left(d(y)\varepsilon_t\left(d(y)\hat{u}\frac{\partial C}{\partial q}\right)\right)=\frac{\partial}{\partial q}\left(d^2(y)\varepsilon_t\hat{u}\frac{\partial C}{\partial q}\right)$$

If we set $\varepsilon_q=d^2\varepsilon_t\hat{u}\cong$ constant diffusivity, then equation becomes

$$\boxed{\frac{\partial C}{\partial x}=\varepsilon_q\frac{\partial^2 C}{\partial q^2}}$$

→ Fickian Diffusion equation; Gaussian solution in the x-q coordinate system

5.3 Intermediate-field Mixing

- Advantage of x-q coordinate system

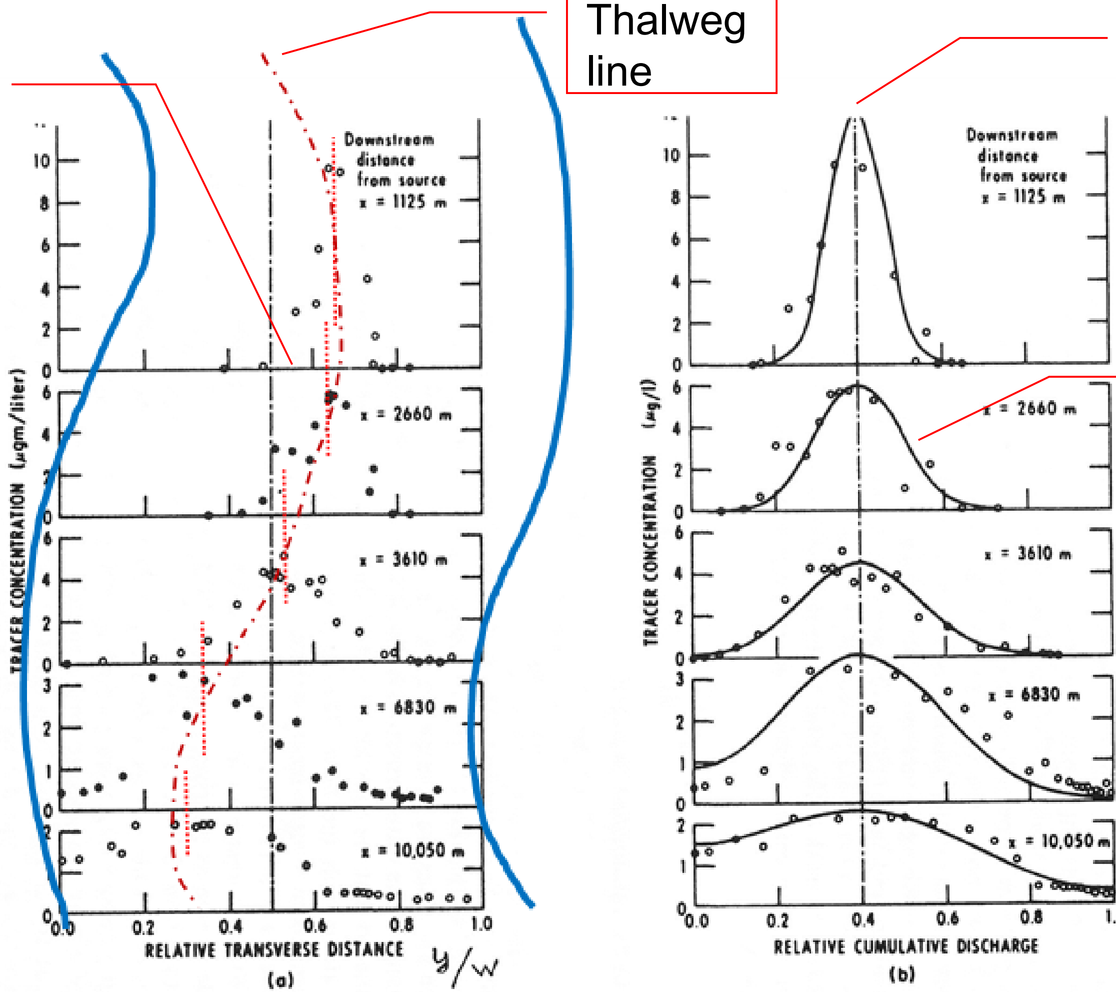
- A fixed value of q is attached to a fixed streamline, so that the coordinate system shifts back and forth within the cross section along with the flow.

→ simplifies interpretation of tracer measurements in meandering streams

→ Transformation from transverse distance to cumulative discharge as the independent variable essentially transforms meandering river into an equivalent straight river.

5.3 Intermediate-field Mixing

The peak of the concentration curves moves from side to side as the river meanders.



The peak remains at the injection location.

Gaussian distribution

5.4 Far-field Mixing

5.4.1 Transport Equation for Far-field Mixing

The 1D cross-sectional-averaged advection-dispersion equation can be obtained by averaging 2D advection-dispersion equation.

$$\frac{\partial \overline{C}}{\partial t} + U\frac{\partial \overline{C}}{\partial x} = K\frac{\partial^2 \overline{C}}{\partial x^2}$$

Apply shear flow dispersion theory to evaluate the longitudinal dispersion coefficient K

$$\rightarrow K = K1_l + K2_l + \varepsilon_\ell$$

where $K1_l$ ~ due to lateral variation of u-velocity;

$K2_l$ ~ due to vertical variation of u-velocity

5.4 Far-field Mixing

After a tracer has mixed across the cross section, the final stage in the mixing process is the reduction of longitudinal gradients by longitudinal dispersion.

Practical cases where longitudinal dispersion is important are accidental spill of a quantity of pollutant; output from a STP which has a daily cyclic variation

The longitudinal dispersion may be neglected when effluent is discharged at a constant rate

5.4 Far-field Mixing

5.4.2 Theoretical Derivation of Longitudinal Dispersion Coefficient

Elder's analysis

- dispersion due to vertical variation of *u*-velocity (logarithmic profile)

$$u(z) = \overline{u} + \frac{u^*}{\kappa}\left\{1 + \ln\left[z + d / d\right]\right\}$$

$$\boxed{D_{le} = 5.93du^*}$$

Elder's equation does not describe longitudinal dispersion in real streams (1D model).

Experimental results shows $K >> 5.93du^* \rightarrow$ Table 5.3

5.4 Far-field Mixing

1) Fischer (1967) - Laboratory channel

$$\frac{K}{du^*} = 150 \sim 392$$

2) Fischer (1968) - Green-Duwamish River

$$\frac{K}{du^*} = 120 \sim 160$$

3) Godfrey and Frederick (1970)

– natural streams in which radioactive tracer Gold-198 was used

$$\frac{K}{du^*} = 140 \sim 500$$

5.4 Far-field Mixing

4) Yotsukura et al. (1970) - Missouri River

$$\frac{K}{du^*} = 7500$$

- Fischer's model (1966, 1967)

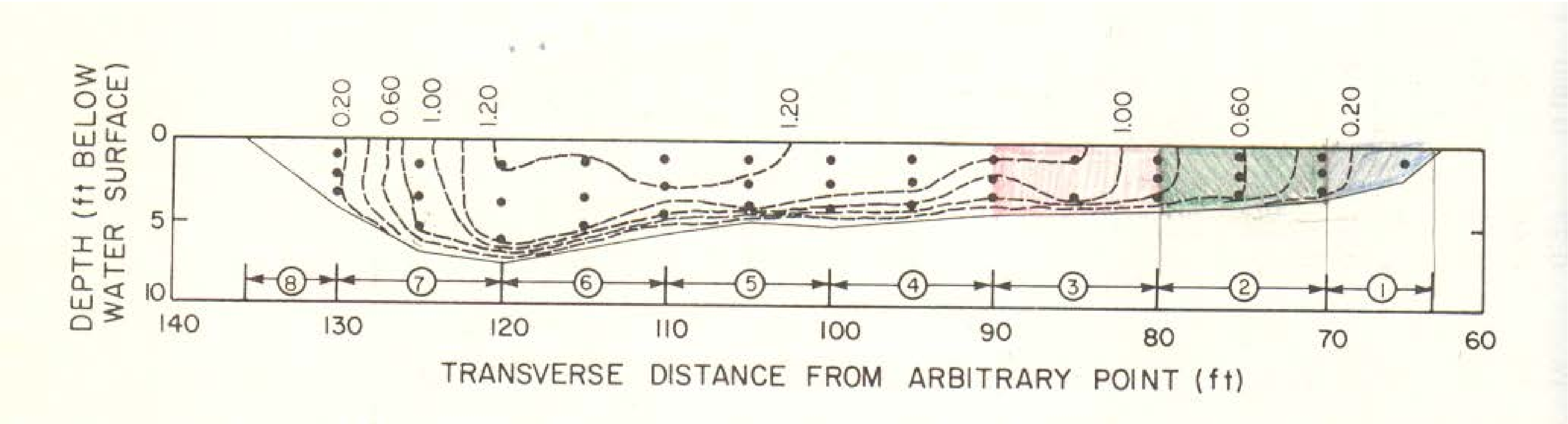
He showed that the reason that Elder's result does not apply to 1D model is because of <u>transverse variation</u> of across the stream.

Vertical velocity profile, $u(z)$ is approximately logarithmic.

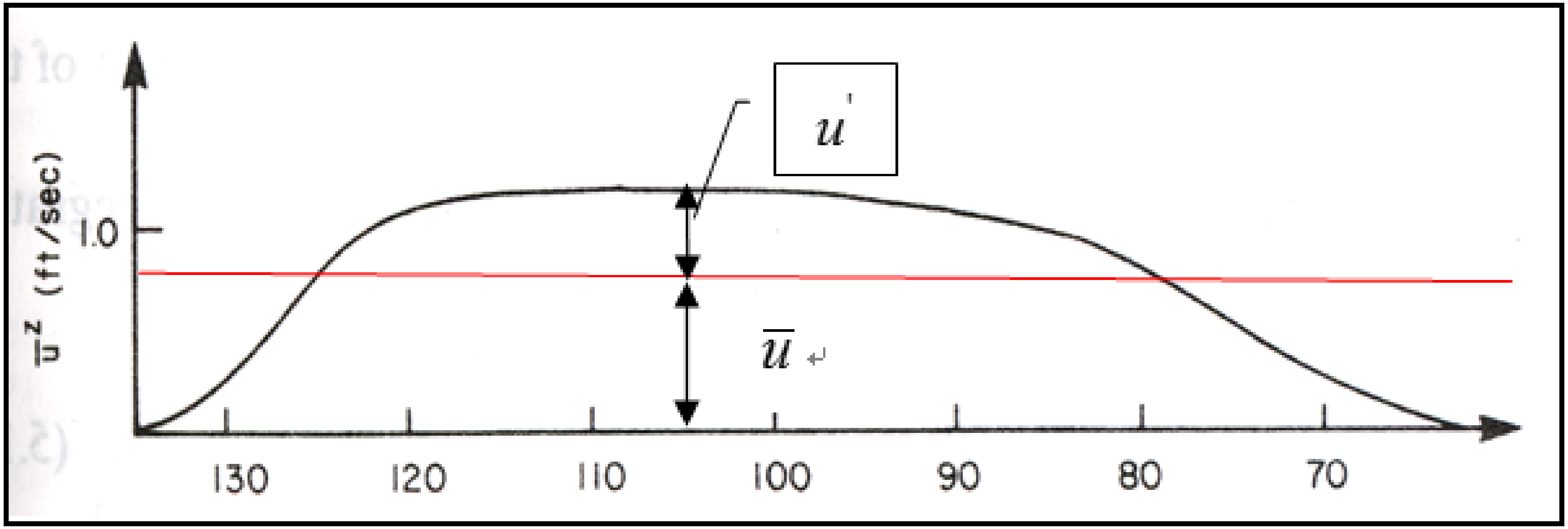
Now, consider transverse variation of depth-averaged velocity

$$\hat{u}(y) = \frac{1}{d(y)} \int_{-d(y)}^{0} u(y,z)\,dz$$

5.4 Far-field Mixing



5.4 Far-field Mixing



5.4 Far-field Mixing

Transverse velocity profile would be approximated by parabolic, polynomial, or beta function.

5.4 Far-field Mixing

3. Shear Flow Dispersion

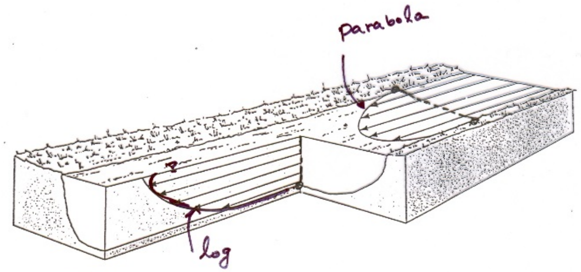
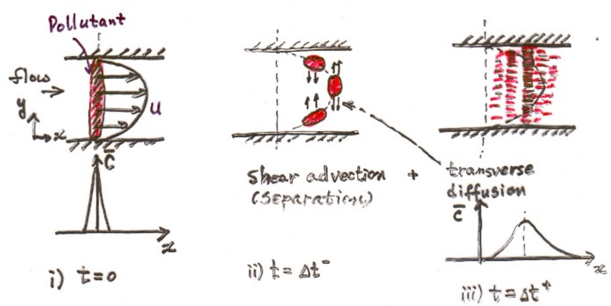


Figure 10.5
Variations in the velocity of flow in natural stream channels occur both horizontally and vertically. Friction reduces the velocity along the floor and sides of the channels. The maximum velocity in a straight channel is near the top and center of the channel.



5.4 Far-field Mixing

$\hat{u}(y)$ is a shear flow velocity profile extending over the stream width W, whereas $u(z)$, the profile used in Elder's analysis, extends only over the depth of flow d.

Remember that longitudinal dispersion coefficient is proportional to the square of the distance over which the shear flow profile extends.

Eq. (5.11): $$K = \frac{h^2 \overline{u'^2}}{E} I$$

$$K \propto h^2$$

where h = characteristic length, W or d

5.4 Far-field Mixing

Say that $W/d \approx 10$

Therefore,

$$\boxed{K_W \approx 100 K_d}$$

→ Transverse profile $u(y)$ is 100 or more times as important in producing longitudinal dispersion as the vertical profile.

→ The dispersion coefficient in a real stream (1D model) should be obtained by neglecting the vertical profile entirely and applying Taylor's analysis to the transverse velocity profile.

5.4 Far-field Mixing

Consider balance of diffusion and advection

Let $u'(y) = \hat{u}(y) - \bar{u}$

$C'(y) = \hat{C}(y) - \bar{C}$

$\bar{u}$ = cross-sectional average velocity = U

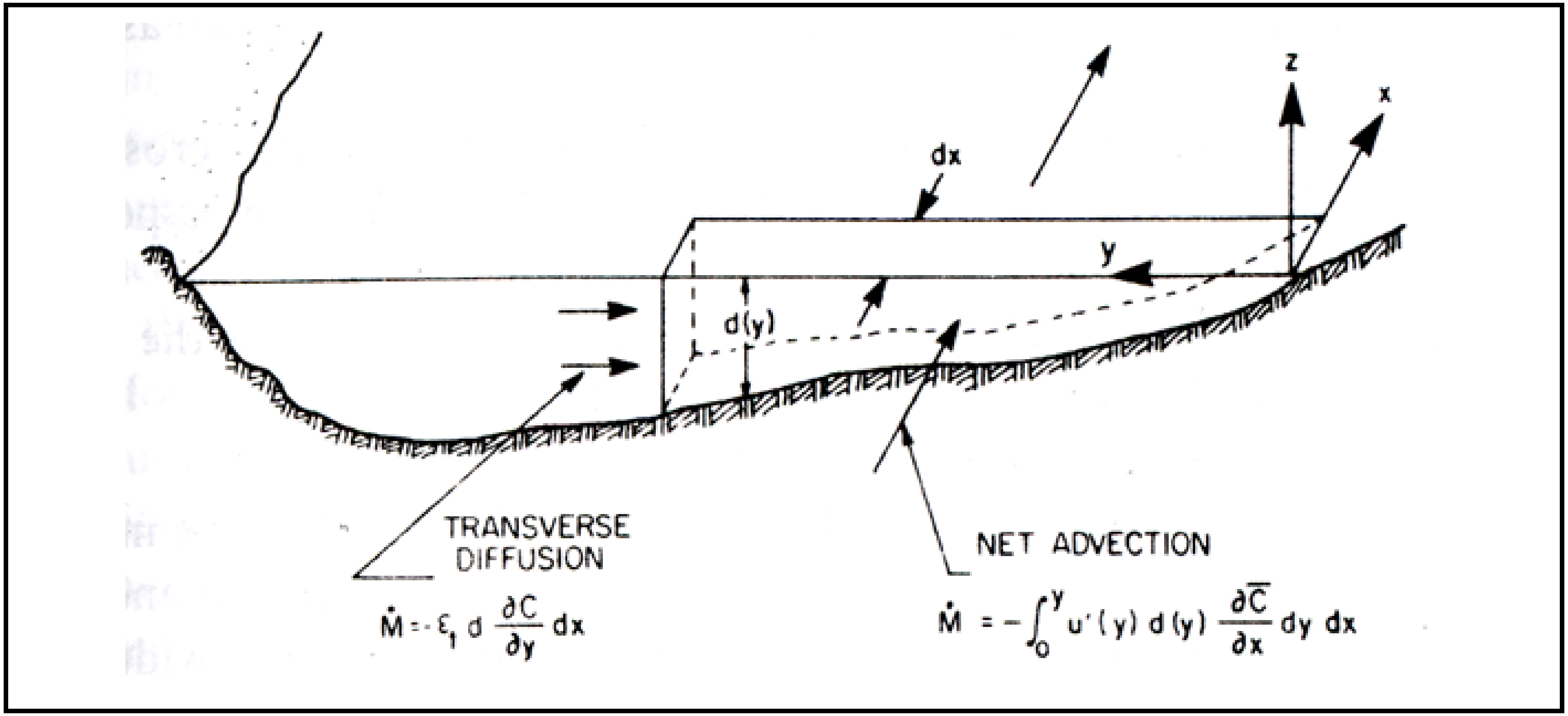
Equivalent of Eq. (4.35) is

$$u'(y)\frac{\partial \bar{C}}{\partial x} = \frac{\partial}{\partial y}\varepsilon_t \frac{\partial C'}{\partial y} \qquad \text{(a)}$$

Shear advection

Transverse diffusion

5.4 Far-field Mixing



5.4 Far-field Mixing

Integrate Eq. (a) over the depth

$$\int_{-d}^{0} u^{'}(y)\frac{\partial \overline{C}}{\partial x}dz = \int_{-d}^{0}\frac{\partial}{\partial y}\varepsilon_t\frac{\partial C^{'}}{\partial y}dz \qquad \text{(b)}$$

$$u^{'}(y)d(y)\frac{\partial \overline{C}}{\partial x} = \frac{\partial}{\partial y}d(y)\varepsilon_t\frac{\partial C^{'}}{\partial y} \qquad \text{(c)}$$

Integrate Eq. (c) w.r.t. y (in the transverse direction)

$$\int_{0}^{y} u^{'}(y)d(y)\frac{\partial \overline{C}}{\partial x}dy = d\varepsilon_t\frac{\partial C^{'}}{\partial y} \qquad \text{(5.9)}$$

$$\frac{\partial C^{'}}{\partial y} = \frac{1}{d\varepsilon_t}\int_{0}^{y} u^{'}(y)d(y)\frac{\partial \overline{C}}{\partial x}dy \qquad \text{(d)}$$

5.4 Far-field Mixing

Integrate again Eq. (d) w.r.t. y (in the transverse direction)

$$C' = \int_0^y \frac{1}{d\varepsilon_t} \int_0^y u'(y) d(y) \frac{\partial \overline{C}}{\partial x} dy dy \qquad \text{(e)}$$

Eq. (4.27)

$$K = -\frac{1}{A \frac{\partial \overline{C}}{\partial x}} \int_A u' C' dA \qquad \text{(f)}$$

5.4 Far-field Mixing

Substitute Eq. (e) into Eq. (f)

$$K = -\frac{1}{A}\frac{1}{\cancel{\frac{\partial \bar{C}}{\partial x}}}\cancel{\frac{\partial \bar{C}}{\partial x}}\int_A u' \int \frac{1}{d\varepsilon_t}\int du' \, dy dy dA$$

Substitute $dA = dy\, d$

$$\boxed{K = -\frac{1}{A}\int_0^W u' d \int_0^y \frac{1}{\varepsilon_t d}\int_0^y u' d \; dy dy dy} \qquad (5.10)$$

5.4 Far-field Mixing

This result is only an estimate because it is based on the concept of a <u>uniform flow in a constant cross section</u>.

[Re] $K = K1_l + K2_l + K_{li} + \varepsilon_\ell$

where $K1_l$ ~ due to lateral variation of u-velocity;

$K2_l$ ~ due to vertical variation of u-velocity

- Simplified equation

Let $d^{'} = d / \overline{d}$; $u^{''} = \dfrac{u^{'}}{\sqrt{\overline{u^{'2}}}}$; $\varepsilon^{'}_{t} = \dfrac{\varepsilon_t}{\overline{\overline{\varepsilon_t}}}$; $y^{'} = \dfrac{y}{W}$

5.4 Far-field Mixing

Overbars mean cross-sectional average; $\bar{d}$ = cross-sectional average depth

Then

$$\boxed{K = \frac{W^2 \overline{u'^2}}{\bar{\varepsilon}_t} I} \qquad (5.11)$$

where I is dimensionless integral given as

$$I = -\int_0^1 u'' d' \int_0^{y'} \frac{1}{\varepsilon_t' d'} \int_0^{y'} u'' dy' dy' dy'$$

Compare with Eq. (5.11)

$$K = \frac{h^2 \overline{u'^2}}{E} I$$

5.4 Far-field Mixing

[Example 5.4] cross-sectional distribution of velocity (Fig. 5.11) of Green-Duwamish at Renton Junction

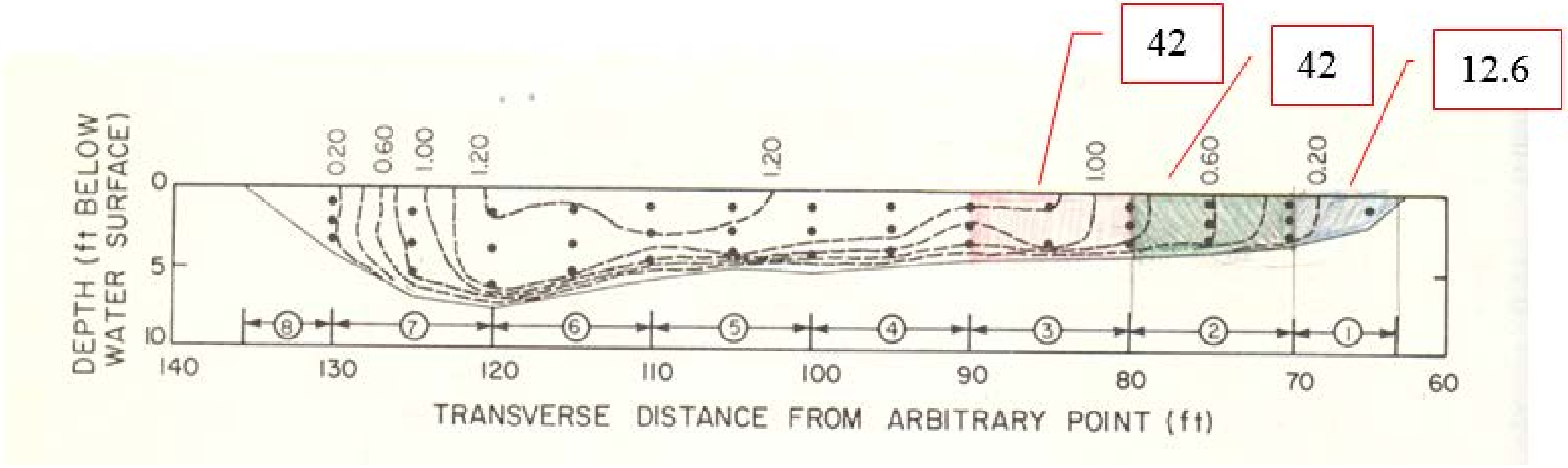
$$\varepsilon_t = 0.133\, ft^2 / \sec$$

Estimate longitudinal dispersion coefficient

Solution: divide whole cross section into 8 subarea

$$K = -\frac{1}{A}\int_0^W u' d \int_0^y \frac{1}{\varepsilon_t d}\int_0^y du' dy dy dy$$

5.4 Far-field Mixing



5.4 Far-field Mixing

→ perform inner integral first

Column 2: transverse distance to the end of subarea

Column 4: $\Delta A = d\,\Delta y$

Column 46: $\Delta Q = \hat{u}\,\Delta A$

Column 8: $Relative\,\Delta Q = u'\,\Delta A$

Column 9: Cumulative of $Relative\,\Delta Q = u'\,\Delta A$

Column 11: $\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dydy = \sum Col(10)\frac{\Delta y}{\varepsilon_t d}$

Column 13: $\int_0^W u'd\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dydydy = Col(8)\times Col(12)$

$$K = -\frac{1}{A}Cumulative\,of\;Col(13)$$

5.4 Far-field Mixing

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	(1)	(2)	(3)	(4)	(5) $\hat{u}$ ($\hat{u}$ (측정치))	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
2	(1)						(7)	Rel. ΔQ =						= (13)의 누가치
3			측정치	ΔA =	Subarea mean	ΔQ=	u' =	u' * ΔA,	$\int_0^y u' dA$	Average	$\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dy\,dy$	Average		
4		y,	d,	d*Δy,	Velocity,	$\hat{u}$ΔA,	$\hat{u}$ - ū,	CFS	(8)을	of		of		
5	Subareas	ft	ft	ft2	ft/s	CFS	fps	(4)*(7)	누가한 값	(9)	(누적치)	(11)	(8)*(12)	Σ(13)
6		63							0		0			0
7	1		1.8	12.6	0.105	1.323	-0.796	-10.026		-5.013		-73	735	
8		70		= 1.8(7)		= 0.105(12.6)			-10.026		-147			735
9	2		4.2	42	0.526	22.092	-0.375	-15.738		-17.895		-307	4828	
10		80							-25.764		-467			5563
11	3		4.2	42	0.986	41.412	0.085	3.582		-23.973		-682	-2441	
12		90							-22.182		-896			3121
13	4		4.8	48	1.091	52.368	0.190	9.134		-17.616		-1034	-9445	
14		100							-13.049		-1172			-6323
15	5		5.2	52	1.196	62.192	0.295	15.355		-5.371		-1211	-18593	
16		110							2.306		-1250			-24916
17	6		6.6	66	1.148	75.768	0.247	16.321		10.466		-1190	-19423	
18		120							18.627		-1130			-44339
19	7		6.4	64	0.766	49.024	-0.135	-8.622		14.316		-1046	9022	
20		130							10.005		-962			-35317
21	8		2	12	0.067	0.804	-0.834	-10.005		5.002		-906	9063	
22		136							0.000		-849			-26254
23														
24	Sum=		A =	338.6	Q =	304.983	0	0.000						
25														
26	(6-1)	ū =	Q/A=	0.90	fps			K =	- (-26254)/A =	77.54	ft²/s			
27														
28		ε_t =	0.133	ft²/s										

(5) given in P.128

$$K = -\frac{1}{A}\int_0^W u'd \underbrace{\int_0^y \frac{1}{\varepsilon_t d} \underbrace{\int_0^y du'\,dy}_{(9)}\,dy}_{(11)}\,dy \quad (14)$$

(5.16) → Inner integral first

(11): $(-5.013)(7)/(0.133)(1.8) = -146.6$

$-146.6 + (-17.895 \times 10)/(0.133 \times 4.2) = -467.0$

(9) $\int_0^y du'dy = \Sigma\, du'\Delta y = \Sigma\, u'\Delta A$ (∵ $d\Delta y = \Delta A$)

(11) $\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dy\,dy = \Sigma \int_0^y du'dy\, \frac{\Delta y}{\varepsilon_t d}$ $= \Sigma(10) \times \Delta y / \varepsilon_t d$

(14) $\Sigma\, \underbrace{u'd\Delta y}_{\Delta A} \underbrace{\left[\int \frac{1}{\varepsilon_t d}\int du'\,dy\,dy\right]}_{(12)}$ $= \Sigma(8) \times (12)$

Rel. ΔQ = (8)

5.4 Far-field Mixing

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
subarea	y (ft)	d (측정치) (ft)	$\Delta A = d \times \Delta y$ (ft^2)	$\hat{u}$ Stream mean velocity (측정치) (ft/s)	$\Delta Q = \hat{u} \times \Delta A$ (CFS)	$u' = \hat{u} - \bar{u}$ (fps)	Rel. $= \hat{u} \times \Delta A$ (CFS) (4)*(7)	$\int_0^y u' dA$ (8)을 누가한 값	Average of (9)	$\int_0^y \frac{1}{\varepsilon_t d} \int_0^y du' dy dy$	Average of (11)	(8) (12)	$\sum(13)$
	63							0		0			0
1		1.8	12.6	0.105	1.323	-0.796	-10.026		-5.013		-73	735	
	70		=1.8(7)		=0.105 (12.6)			-10.026		-147			735
2		4.2	42	0.526	22.092	-0.375	-15.738		-17.895		-307	4828	
	80							-25.764		-467			5563
3		4.2	42	0.986	41.412	0.085	3.582		-23.973		-682	-2441	
	90							-22.182		-896			3121
4		4.8	48	1.091	52.368	0.190	9.134		-17.616		-1034	-9445	
	100							-13.049		-1172			-6323
5		5.2	52	1.196	62.192	0.295	15.355		-5.371		-1211	-18593	
	110							2.306		-1250			-24916
6		6.6	66	1.148	75.768	0.247	16.321		10.466		-1190	-19423	
	120							18.627		-1130			-44339
7		6.4	64	0.766	49.024	-0.135	-8.622		14.316		-1046	9022	
	130							10.005		-962			-35317
8		2	12	0.067	0.804	-0.834	-10.005		5.002		-906	9063	
	136							0.000		-849			-26254
Sum		$A=$	338.6	$Q=$	304.98		0.000						
		$\varepsilon_t=$	0.133 ft^2/s	$\bar{u} = Q/A =$	0.90 fps		$K=$	-(-26254)/A = 77.54 ft^2/s					

5.4 Far-field Mixing

(5) given in p.128

(9) $\int_0^y du'\,dy = \sum du'\Delta y = \sum u'\Delta A \;\; (\because d\Delta y = \Delta A)$

(5.16) : Inner integral first

(11) $\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dy\,dy = \sum \int_0^y du'\,dy \frac{\Delta y}{\varepsilon_t d} = \sum (10) \times \Delta y / \varepsilon_t d$

(11): $(-5.013)(7)/(0.133)(1.8) = -146.6$

5.4 Far-field Mixing

$$K = -\frac{1}{A}\underbrace{\int_0^W u'd \underbrace{\int_0^y \frac{1}{\varepsilon_t d}\underbrace{\int_0^y du'\,dy}_{(9)}\,dy}_{(11)}\,dy}_{(14)}$$

(14) $$\sum \underbrace{u'\,\underbrace{d\,\Delta y}_{\Delta A}}_{\mathrm{Rel.}\ \Delta Q=(8)}\underbrace{\left[\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dy\,dy\right]}_{(12)} = \sum (8)\times(12)$$

$$-146.6+(-17.895)(7)/(0.133\times 4.2) = -467.0$$

5.4 Far-field Mixing

Homework Assignment #5-1

Due: Two weeks from today

1. Estimate the longitudinal dispersion coefficient using the cross-sectional distribution of velocity measured in the field using Eq. (5.10). Take S (channel slope) = 0.00025 for natural streams.

2. Compare this result with Elder's analysis and Fischer's approximate formula, Eq. (5.12).

5.4 Far-field Mixing

Station	Y from left bank (ft)	Depth, d (ft)	Mean Velocity (ft/sec)
1	0.00	0.0	0.00
2	4.17	1.4	0.45
3	7.83	3.0	0.68
4	11.50	3.7	1.05
5	15.70	4.7	0.98
6	22.50	5.3	1.50
7	29.83	6.2	1.65
8	40.83	6.7	2.10
9	55.50	7.0	1.80
10	70.17	6.5	2.40
11	84.83	6.3	2.55
12	99.50	6.8	2.45
13	114.17	7.4	2.20
14	132.50	7.3	2.65
15	150.83	7.1	2.70

5.4 Far-field Mixing

16	169.16	7.4	2.35
17	187.49	7.8	2.65
18	205.82	7.8	2.80
19	224.15	7.8	2.60
20	242.48	6.6	2.50
21	260.81	6.3	2.30
22	279.14	6.2	2.35
23	297.47	6.6	2.30
24	315.80	6.0	2.65
25	334.13	5.5	2.50
26	352.46	5.4	2.10
27	370.79	5.2	2.25
28	389.12	5.5	2.30
29	407.45	5.7	1.50
30	416.62	3.2	1.30
31	422.00	0.0	0.00

5.4 Far-field Mixing

5.4.3 Estimation of Longitudinal Dispersion Coefficients

1) *Theoretical equation*

2) $$K = -\frac{1}{A}\int_0^W u' d \int_0^y \frac{1}{\varepsilon_t d}\int_0^y d u' dydydy \qquad (5.10)$$

• Elder (1959) use vertical profile

• Seo and Baek (2004)

~ use beta function for transverse profile of u-velocity

5.4 Far-field Mixing

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx,\ \alpha > 0$$

$$\frac{u}{U} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{y}{W}\right)^{\alpha-1} (1-\frac{y}{W})^{\beta-1}$$

$$K = \gamma \frac{U^2 W^2}{du^*}$$

2) Empirical equation

- Fischer (1975)

$$K' = \frac{\overline{Iu'^2 h^2}}{E} \quad (5.11)$$

5.4 Far-field Mixing

Select $I = 0.07(0.054 \sim 0.10)$

$$h = 0.7W(0.5 \sim 1.0W)$$

$$\overline{u'^2} = 0.2\overline{u}^2(0.17 \sim 0.25)$$

$$E = \varepsilon_t = 0.6du^*$$

Then (5.11) becomes

$$K = 0.011\frac{U^2W^2}{du^*} \qquad (5.12)$$

5.4 Far-field Mixing

- Seo and Cheong (1998)

Use dimensional analysis to find significant factors

Include dispersion by shear flow and mixing by storage effects

$$\frac{K}{du^*} = a\left(\frac{U}{u^*}\right)^b \left(\frac{W}{d}\right)^c$$

Fischer (1975):	a=0.011; b=2.0; c=2.0
Liu (1979):	a=0.18; b=0.5; c=2.0
Iwasa and Aya (1991):	a=2.0; b=0; c=1.5
Koussis and Rodrguez-Mirasol (1998):	a=0.6; b=0; c=2.0
Seo and Cheong (1998):	a=5.92; b=1.43; c=0.62

5.4 Far-field Mixing

[Re] Empirical methods

1) Data driven methods

Dimensional analysis → regression method

2) Soft computing methods

Artificial Neural Network (ANN)

Adaptive Neuro-Fuzzy Inference System technique

Expert System

5.4 Far-field Mixing

Fuzzy Logic

Genetic Algorithm (GA)

Machine Learning Approach

Model Tree: M5 vs M5'

Neural Networks

Support Vector Machine

5.4 Far-field Mixing

[Ex 5.5] Dispersion of slug (instantaneous input)

Given:

$M = 10 lb$ (Rhodamine WT dye); $\bar{u} = 0.90\, ft/s$; $W = 73\, ft$; $A = 338.6$

$\bar{d} = 4.46\, ft,$ (weighted average)

$\varepsilon_t = 0.133\, ft^2/s$

$$u^* = \frac{\varepsilon_t}{0.4d} = \frac{0.133}{0.4(4.64)} = 0.072\, ft/s$$

5.4 Far-field Mixing

Find:

(a) K by Eq. (5.12)

(b) length of initial zone in which Taylor's analysis does not apply

(c) length of dye cloud at the time that peak passes =20,000 ft

(d) C_{peak} at x =20,000ft

[Solution]

(a) Eq. (5.12)

$$K = 0.011\bar{u}^2 W^2 / du^*$$
$$= 0.011(0.90)^2 (73)^2 / (4.46)(0.072)$$
$$= 142.1\, ft^2 / s$$

$$K(5.19) / K(5.16) = 142.1 / 77.5 = 1.83$$

5.4 Far-field Mixing

[Cf] K by Seo & Cheong (1998)

$$\frac{K}{du^*} = 5.92\left(\frac{U}{u^*}\right)^{1.43}\left(\frac{W}{d}\right)^{0.62} = 294\ ft^2/s$$

→ include dispersion by shear flow and <u>storage effects</u>

(b) initial period

$$x = 0.4\bar{u}W^2/\varepsilon_t = 0.4(0.90)(73)^2/(0.133) = 14,424\ ft$$

(c) length of cloud

$$x' = x\varepsilon_t/\bar{u}W^2 = \frac{(20,000)(0.133)}{(0.90)(73)^2} = 0.55$$

5.4 Far-field Mixing

- decay of skewed concentration distribution

→ assume Gaussian distribution

$$\frac{d\sigma^2}{dt} = 2K$$

From Fig. 5.14

$$\frac{\sigma^2 \varepsilon_t}{2KW^2} = \left(x' - 0.07\right)$$

$$\sigma^2 = 2K(W^2/\varepsilon_t)(x' - 0.07)$$

$$= 2\left(142\right)\left(73\right)^2/0.133\left(0.55 - 0.07\right) = 5.46 \times 10^{-6}\ ft^2$$

$$\therefore \sigma = 2.337$$

5.4 Far-field Mixing

length of cloud $= 4\sigma = 4(2,337) = 9,348\,ft$

(d) peak concentration

$$C_{\max} = \frac{M}{A\sqrt{4\pi Kx/\overline{u}}} = \frac{10}{(338.6)\sqrt{4\pi(142)(20,000)/(0.90)}} = 4.69\times10^{-6}\,lb/\,ft^3$$

$$= 4.69\times10^{-6}\times\frac{453.6g}{0.0283m^3} = 75.1\times10^{-3}\,g/m^3\,(= mg/l = ppm)$$

$$= 75.1ppb$$

5.4 Far-field Mixing

Homework Assignment #5-2

Due: Two weeks from today

Concentration-time data listed in Table 2 are obtained from dispersion study by Godfrey and Fredrick (1970).

1) Plot concentration vs. time

2) Calculate time to centroid, variance, skew coefficient.

3) Calculate dispersion coefficient using the change of moment method and routing procedure.

4) Compare and discuss the results.

5.4 Far-field Mixing

Test reach of the stream is straight and necessary data for the calculation of dispersion coefficient are

$$\bar{u} = 1.70\ ft/s; \qquad W = 60\ ft;$$

$$d = 2.77\ ft; \qquad u^{*} = 0.33\ ft/s$$

5.4 Far-field Mixing

Section 1 x=630ft		Section 2 x=3310ft		Section 3 x=5670ft		Section 4 x=7870ft		Section 5 x=11000ft		Section 6 x=13550ft	
T(hr)	C/C_0	T(hr)	C/C_0	T(hr)	C/C_0	T(hr)	C/C_0	T(hr)	C/C_0	T(hr)	C/C_0
1111.5	0.00	1125.0	0.00	1138.0	0.00	1149.0	0.00	1210.0	0.00	1226.0	0.00
1112.5	2.00	1126.0	0.15	1139.0	0.12	1152.0	0.26	1215.0	0.05	1231.0	0.07
1112.5	16.50	1127.0	1.13	1140.0	0.30	1155.0	0.67	1220.0	0.25	1236.0	0.22
1113.0	13.45	1128.0	2.30	1143.0	1.21	1158.0	0.95	1225.0	0.52	1241.0	0.40
1113.5	7.26	1128.5	2.74	1145.0	1.61	1200.0	1.09	1228.0	0.64	1245.0	0.50
1114.0	5.29	1129.0	2.91	1147.0	1.64	1202.0	1.13	1231.0	0.70	1249.0	0.58
1115.0	3.37	1129.5	2.91	1149.0	1.56	1204.0	1.10	1234.0	0.72	1251.0	0.59
1116.0	2.29	1130.0	2.80	1153.0	1.26	1206.0	1.04	1237.0	0.71	1253.0	0.59

5.4 Far-field Mixing

1117.0	1.54	1131.0	2.59	1158.0	0.86	1208.0	0.95	1240.0	0.65	1257.0	0.54
1118.0	1.03	1133.0	2.18	1203.0	0.53	1213.0	0.72	1244.0	0.55	1304.0	0.44
1120.0	0.40	1137.0	1.34	1208.0	0.30	1218.0	0.50	1248.0	0.45	1313.0	0.27
1124.0	0.10	1143.0	0.60	1213.0	0.17	1223.0	0.31	1258.0	0.24	1323.0	0.14
1128.0	0.04	1149.0	0.23	1218.0	0.10	1228.0	0.21	1308.0	0.12	1333.0	0.06
1133.0	0.02	1158.0	0.08	1228.0	0.04	1238.0	0.08	1318.0	0.06	1343.0	0.03
1138.0	0.00	1208.0	0.03	1238.0	0.01	1248.0	0.02	1333.0	0.03	1403.0	0.02
-	-	1218.0	0.00	1248.0	0.00	1300.0	0.00	1353.0	0.00	1423.0	0.00

5.4 Far-field Mixing

5.4.4 Non-Fickian Dispersion in Real Streams

So far the analyses have been limited to uniform channels because Taylor's analysis assumes that everywhere along the stream the cross section is the same.

Real streams have <u>bends</u>, <u>sandbars</u>, <u>side pockets</u>, <u>pools and riffles</u>, <u>bridge piers</u>, <u>man-made revetments</u>.

→ Every <u>irregularities</u> contribute to dispersion.

→ It is not suitable to apply Taylor's analysis to real streams with these irregularities.

5.4 Far-field Mixing

Limitation of Taylor's analysis

Taylor's analysis cannot be applied until after the initial period.

Numerical experiments showed that in a uniform channel the variance of dispersing cloud behaves as a line as shown in Fig. 5.14.

A) generation of skewed distribution: $x' \left(= \dfrac{x}{\bar{u}W^2/\varepsilon_t}\right) < 0.4$ (initial period)

B) decay of the skewed distribution: $0.4 < x' < 1.0$

C) approach to Gaussian distribution: $1.0 < x'$

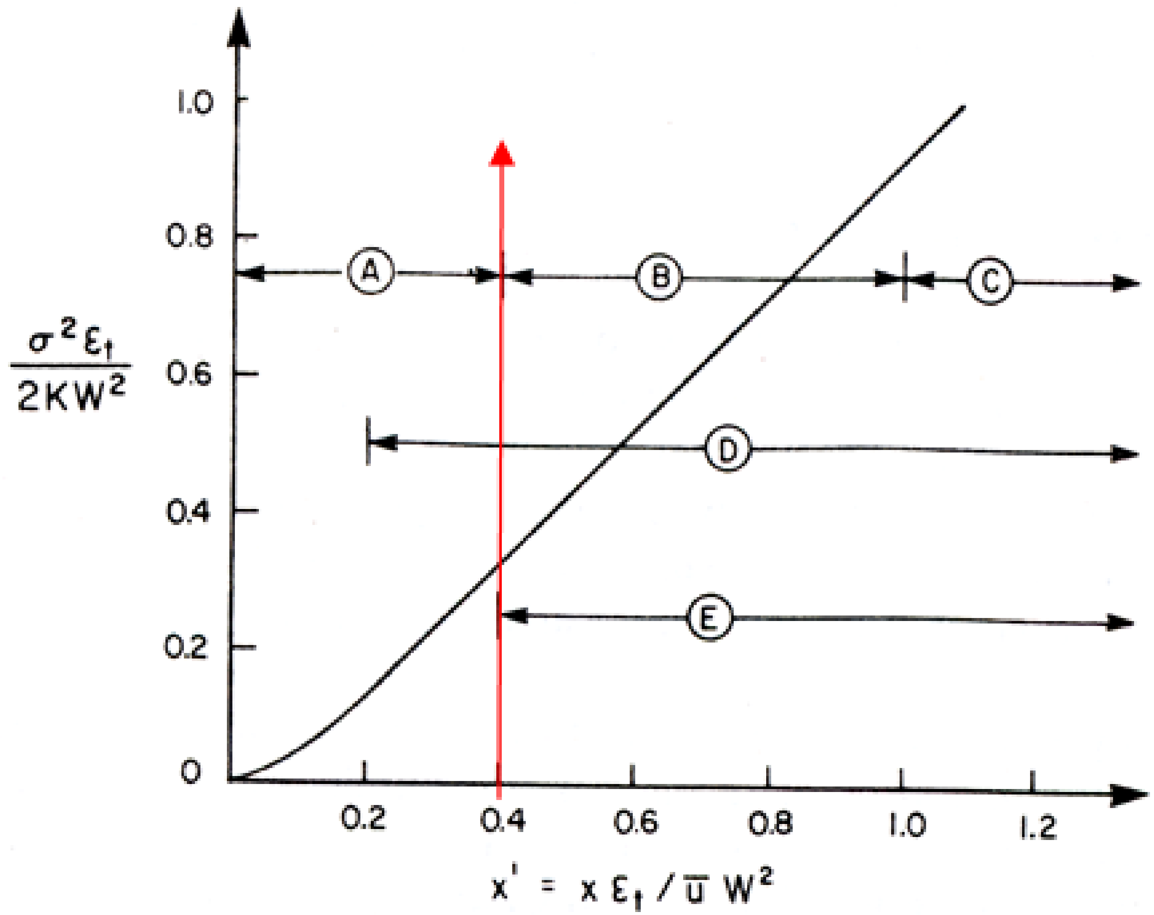
5.4 Far-field Mixing

D) zone of linear growth of the variance: $0.2 < x^{'}\ ;\ \dfrac{\partial \sigma^2}{\partial t} = 2D$

E) zone where use of the routing procedure is acceptable: $0.4 < x^{'}$

Analytical solution of 1D advection-dispersion model

5.4 Far-field Mixing



5.4 Far-field Mixing

5.4.5 Two-zone Models

Irregularities in real streams <u>increase the length of the initial period</u>, and <u>produce long tail</u> on the observed concentration distribution due to <u>detention of small amounts</u> of effluent cloud and release slowly after the main cloud has passed.

Pockets of dye are retained in small irregularities along the side of the channel. The dye is released slowly from these pockets, and causes measurable concentrations of dye to be observed after the main portion of the cloud has passed.

5.4 Far-field Mixing

• Field studies

Godfray and Frederick (1974); Nordin and Savol (1974); Day (1975); Legrand-Marcq and Laudelot (1985) showed <u>nonlinear behavior of variance</u> for times beyond the initial period. (increased faster than linearly with time)

$$\sigma^2 = f\left(t^{1.4}\right)$$

→ skewed concentration distribution

→ cannot apply Taylor's analysis

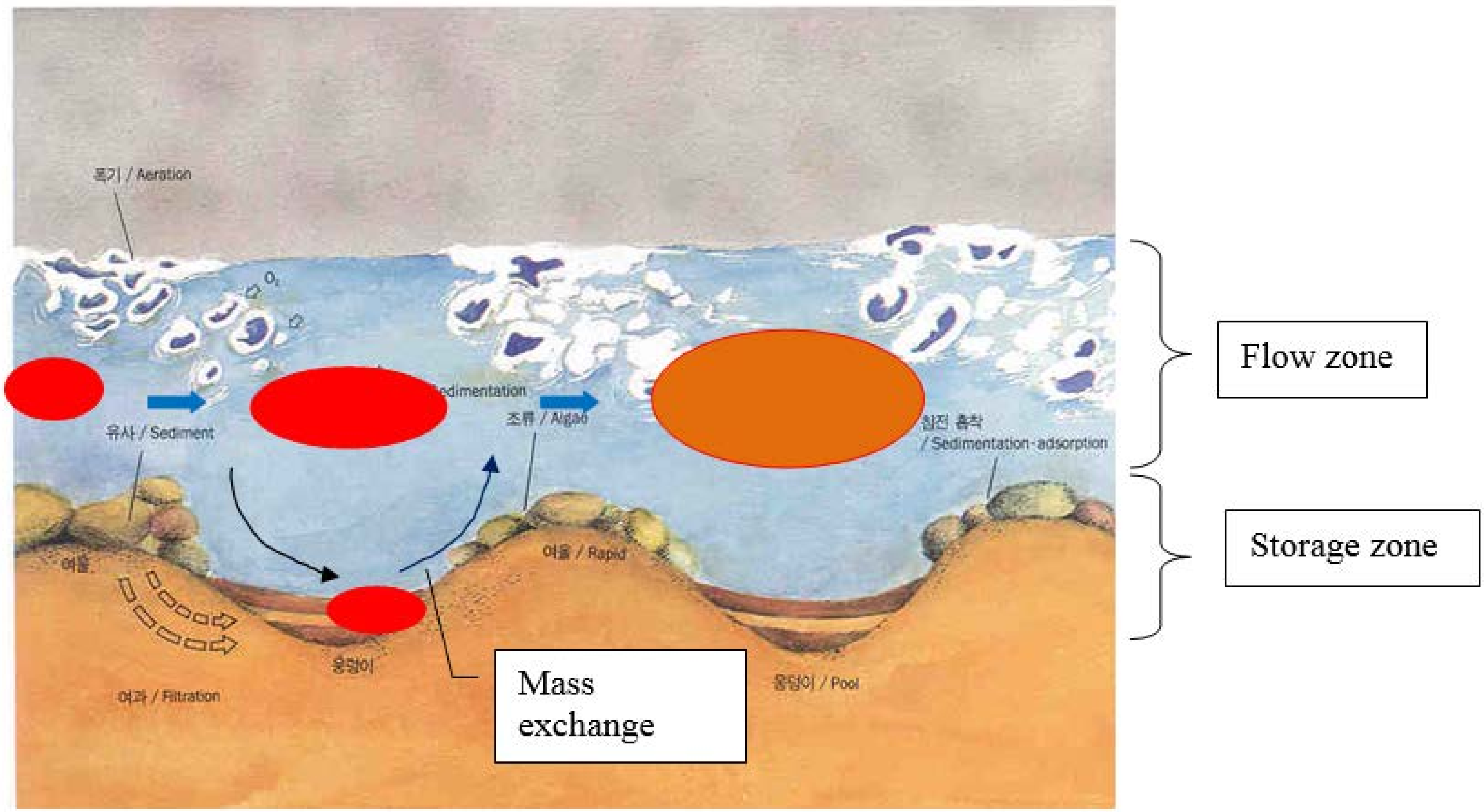
5.4 Far-field Mixing

- Effect of storage zones (dead zones)

1) increases the length of the initial period

2) increases the magnitude of the longitudinal dispersion coefficient

5.4 Far-field Mixing



5.4 Far-field Mixing

- Two zone models

~ divide stream area into two zones

Flow zone: advection, dispersion, reaction, mass exchange

$$A_F \frac{\partial C_F}{\partial t} + U_F A_F \frac{\partial C_F}{\partial x} = \frac{\partial}{\partial x}\left(K A_F \frac{\partial C_F}{\partial y} \right) + F$$

Storage zone: vortex, dispersion, reaction, mass exchange

$$A_S \frac{\partial C_S}{\partial t} = -F$$

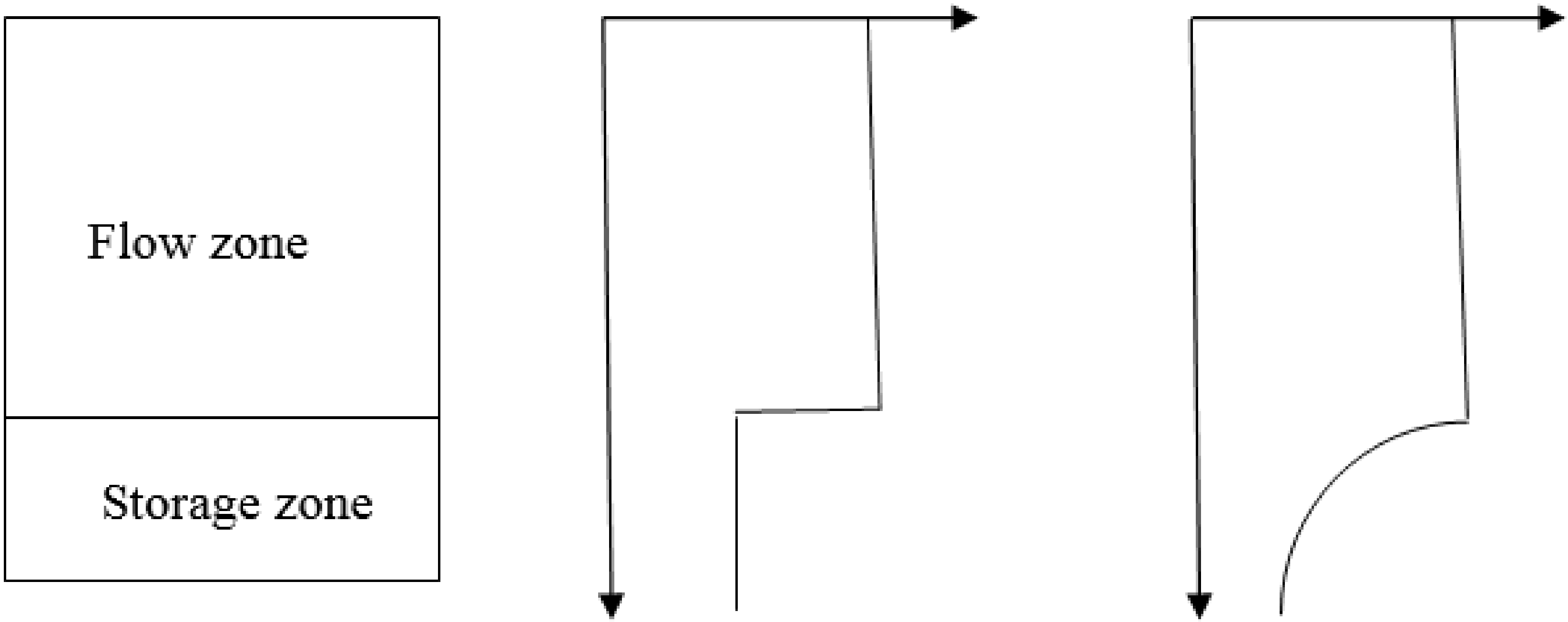
5.4 Far-field Mixing

Introduce auxiliary equation for mass exchange term F

Exchange model: $$F = k(C_F - C_S)P$$

Diffusion model: $$F = -\varepsilon_y \left.\frac{\partial C_S}{\partial y}\right|_{y=0}$$

5.4 Far-field Mixing



5.4 Far-field Mixing

- Dead zone model

Hays et al (1967)

Valentine and Wood (1977, 1979), Valentine (1978)

Tsai and Holley (1979)

Bencala and Waters (1983), Jackman et al (1984)

- Storage zone model

Seo (1990), Seo and Maxwell (1991, 1992)

Seo and Yu (1993)

Seo & Cheong (2001), Cheong & Seo (2003)

5.4 Far-field Mixing

- Effect of bends

1) Bends increase the rate of transverse mixing.

2) Transverse velocity profile induced by meandering flow increase longitudinal dispersion coefficient significantly because the velocity differences across the stream are accentuated.

(3) Effect of alternating series of bends depends on the ratio of the cross-sectional diffusion time to the time required for flow round the bend.

$$\gamma = \frac{W^2 / \varepsilon_t}{L / \bar{u}} \qquad (5.13)$$

5.4 Far-field Mixing

where L= length of the curve

$$\gamma \le 25 = \gamma_0 \rightarrow K = K_0$$ → no effect due to alternating direction

$$\gamma > 25 \rightarrow K = K_0 \frac{\gamma_0}{\gamma}$$

K_0 = dispersion coefficient for the steady-state concentration profile, Eq. (5.10)