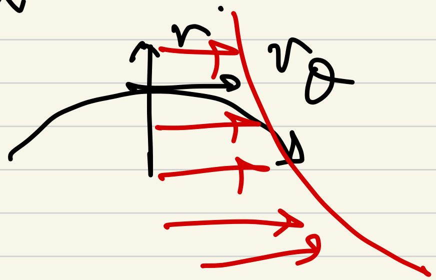
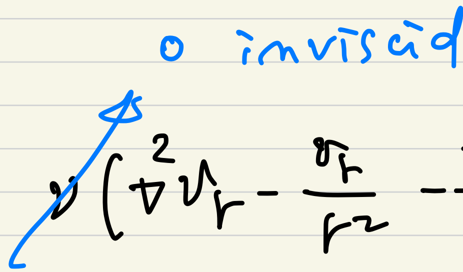
- Line irrotational vortex (free vortex)

$$\rightarrow \quad v_\theta = \frac{k}{r}$$
$$v_r = 0$$



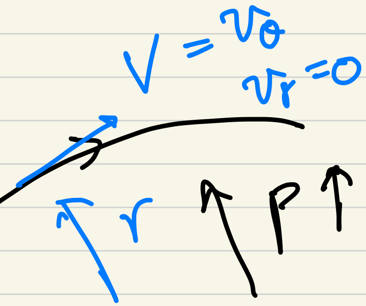
<u>r- momentum eq.</u>



$$\cancel{\frac{\partial v_r}{\partial t}}^{\,0} + (\underline{V}\cdot\nabla)\cancel{v_r}^{\,0} - \frac{1}{r}v_\theta^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \cancel{\nu}^{\,0}\left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)$$

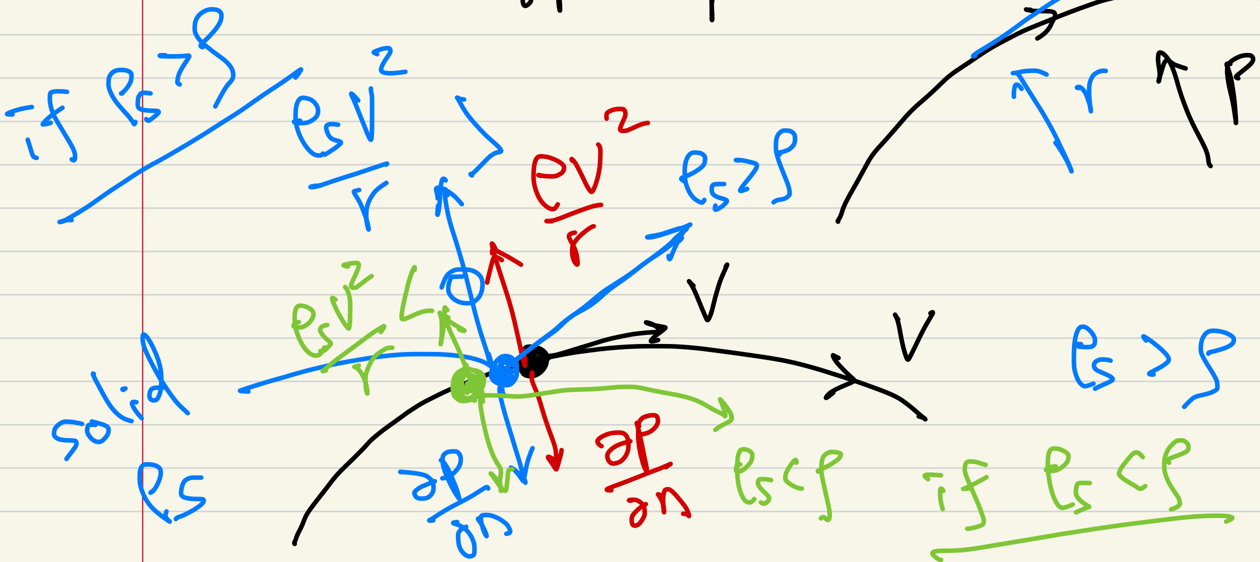
$$\Rightarrow \quad \frac{\partial p}{\partial r} = \frac{\rho}{r}v_\theta^2$$

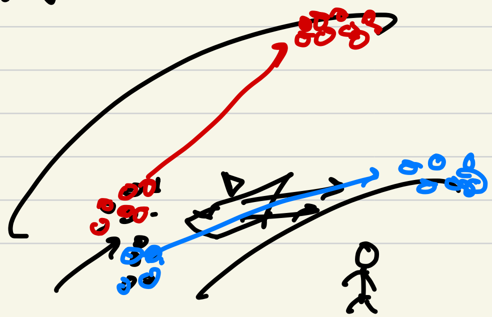
$$\frac{\partial p}{\partial r} = \frac{\rho v_\theta^2}{r} = \frac{\rho V^2}{r} > 0$$



$$\boxed{\frac{\partial p}{\partial n} = \frac{\rho V^2}{r}_{>0}}$$ Euler-n eq.

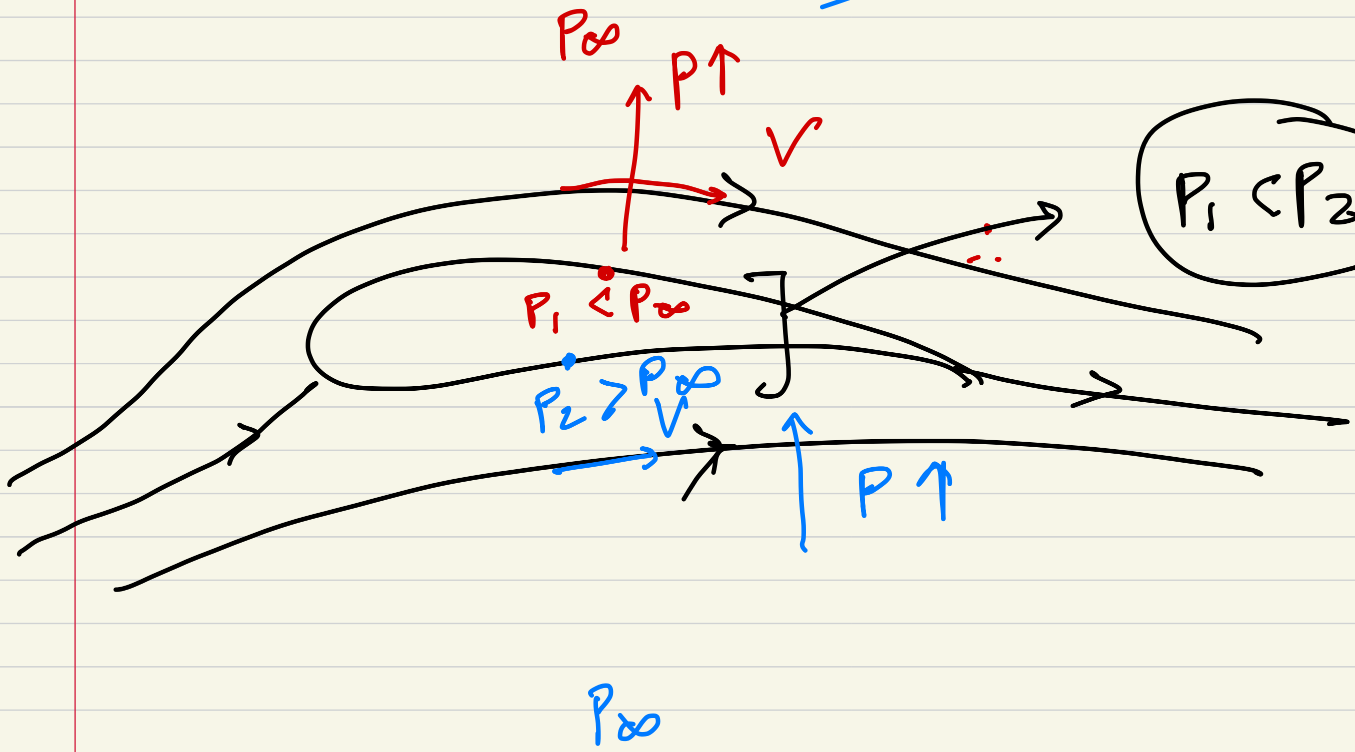
↑ normal





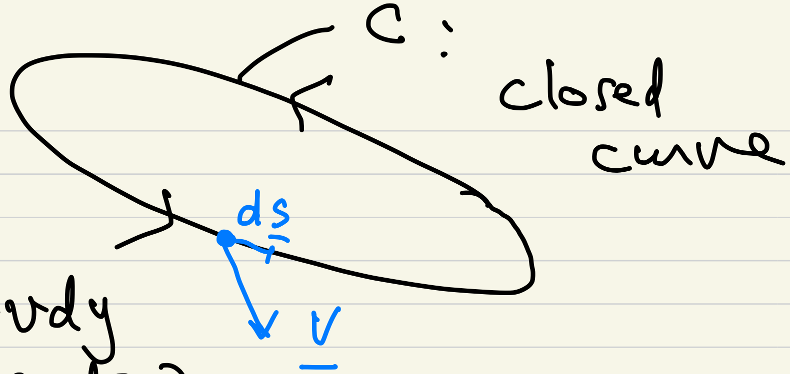
flow visualization

$\rho_s \simeq \rho$



$P_1 < P_2 \rightarrow$ Lift force

• Circulation (순환) Γ



invisçid ~~viscous~~ → $\boxed{\Gamma \equiv \oint_C \underline{V} \cdot d\underline{s}}$ $= \int_C (u\,dx + v\,dy + w\,dz)$

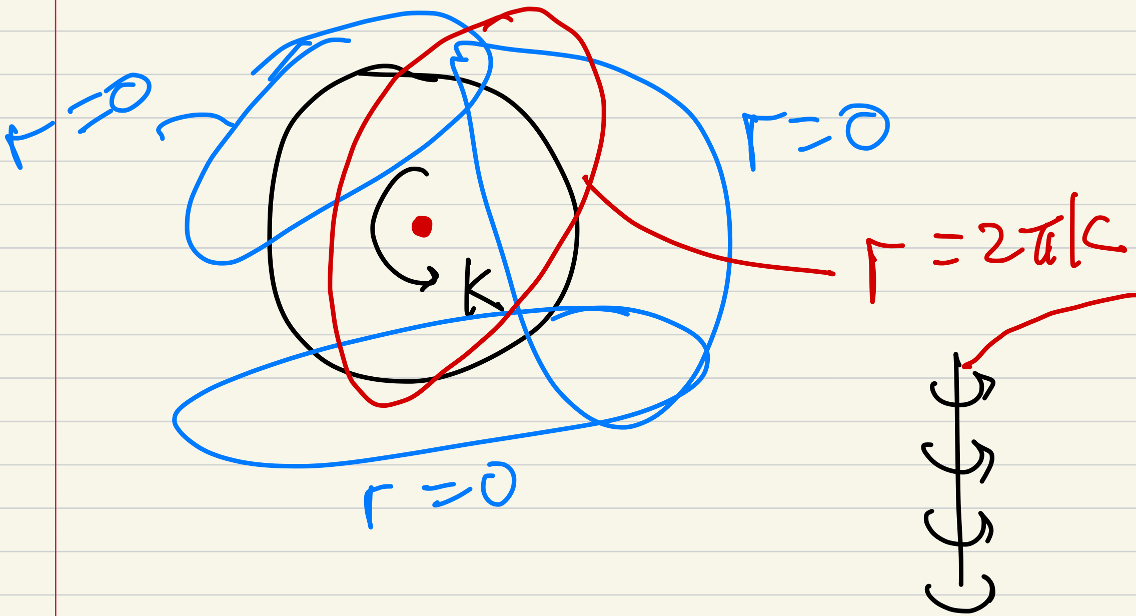
irrotational → $= \oint_C \nabla\phi \cdot d\underline{s} = \oint_0 d\phi$

$\underline{\omega} = \nabla \times \underline{V} = 0$

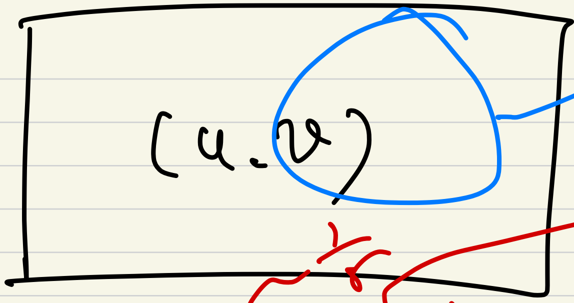
$\quad \hookrightarrow \underline{V} = \nabla\phi$

$\Gamma = 0$ for irrotational flow on the whole domain

Free vortex $\phi = k\theta \rightarrow \Gamma = \oint_C d\phi = 2\pi k \neq 0$

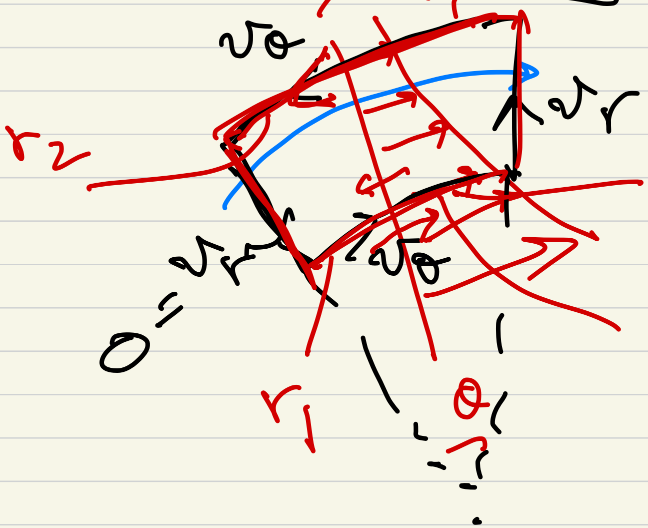


Γ denotes the net algebraic strength of all the vortex filaments contained within the closed curve.



$\Gamma \neq 0 \leftarrow \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \neq 0$ rotational flow

$\omega_z = 0$ irrotational flow



$v_r = 0$

$v_\theta = \frac{k}{r}$

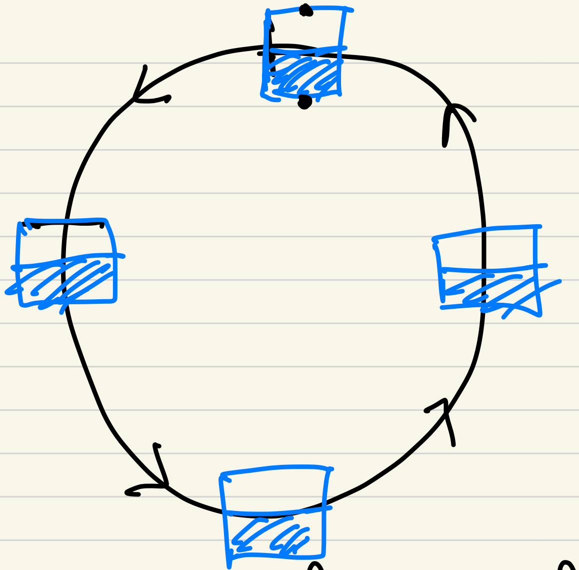
$$\oint \underline{v}\cdot d\underline{s} = -\frac{k}{r_1}\cdot r_1\theta + \frac{k}{r_2}\cdot r_2\theta = 0$$

$v_\theta = k\cdot r$

$$\oint \underline{v}\cdot d\underline{s} = -kr_1\cdot r_1\theta + kr_2\cdot r_2\theta \neq 0$$

$\underline{\omega} = 0$

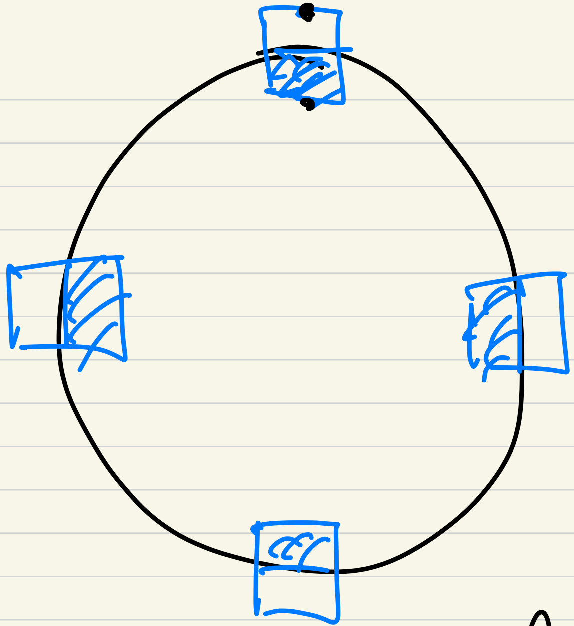
$v_\theta \sim \frac{1}{r}$



irrotational circular flow

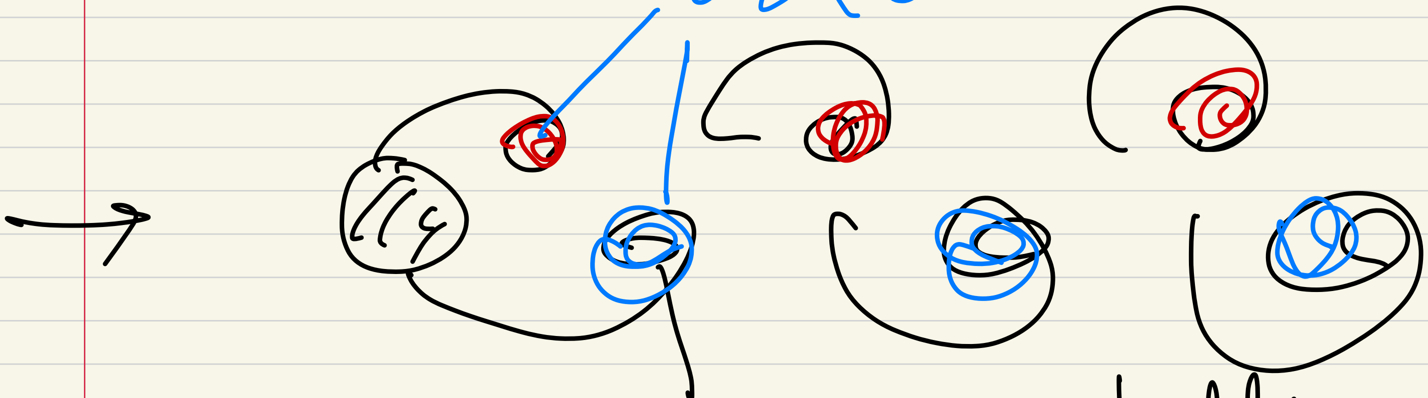
$\underline{\omega} \neq 0$

$v_\theta \sim r$



rotational circular flow

$\omega_z \neq 0$



Karman vortex shedding

8.3 Superposition of plane-flow sols.

$$\left(\begin{array}{l}\nabla^2\phi=0\\ \nabla^2\psi=0\end{array}\right] \rightarrow \text{linear eq.} \rightarrow \text{superposition is possible.}$$

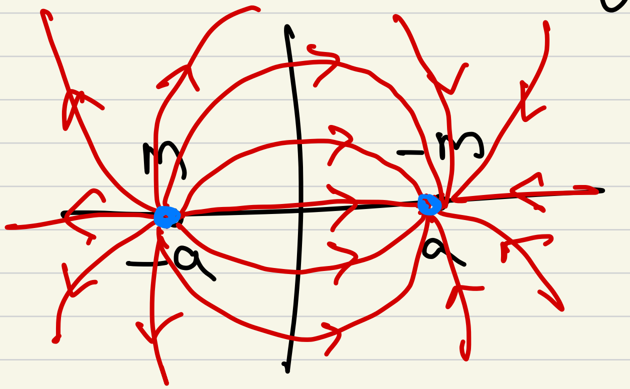
$$\phi_1,\ \phi_2 \longrightarrow c_1\phi_1+c_2\phi_2$$

$$P_1 \quad P_2 \rightarrow c_1P_1+c_2P_2 \times$$

Bernoulli eq

$$\frac{V^2}{2}+\frac{P}{\rho}=\text{const}$$

① source + an equal sink



$$\left(\begin{array}{l}\psi=m\theta \ \text{"}m\tan^{-1}\frac{y}{x}\text{"}\\ \phi=m\ln r\end{array}\right)$$

$$\psi=\psi_{source}+\psi_{sink}$$

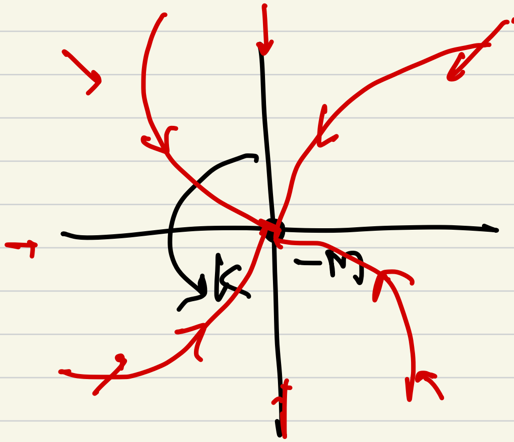
$$=m\tan^{-1}\frac{y}{x+a}-m\tan^{-1}\frac{y}{x-a}$$

$$=-m\tan^{-1}\frac{2ay}{x^2+y^2-a^2}$$

$$\phi=\phi_{source}+\phi_{sink}=\frac{1}{2}m\ln\left[(x+a)^2+y^2\right]-\frac{1}{2}m\ln\left[(x-a)^2+y^2\right]$$

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \rightarrow P \text{ from Bernoulli eq.}$$

② sink + vortex at the origin



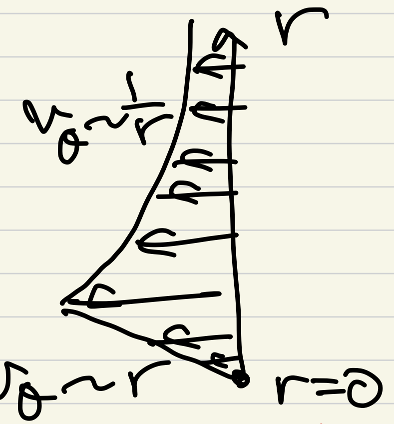
$$\psi = -m\theta - K\ln r$$

$$\phi = -m\ln r + K\theta$$

tornado, rapidly draining bathtub

@ $r=0$, $v_\theta \to \infty$

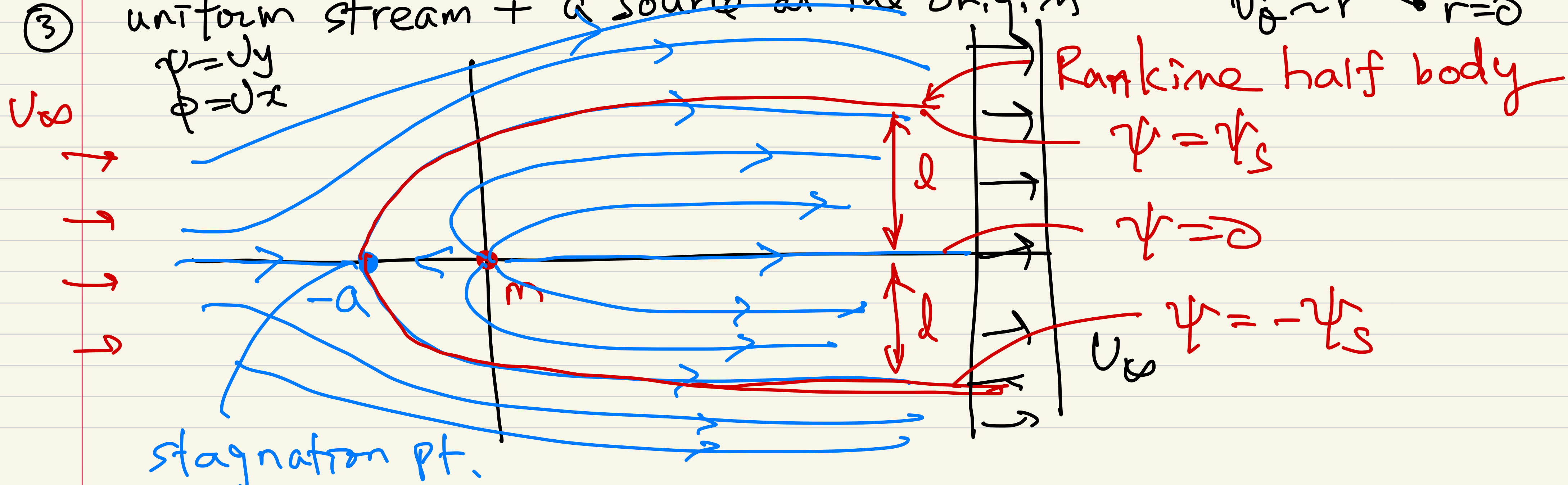
In reality, near $r=0$, solid-body rotation



③ uniform stream + a source at the origin

$\psi = Uy$

$\phi = Ux$



$\psi = u_\infty r \sin\theta + m\theta$ $\rightarrow$ $u = \frac{\partial\psi}{\partial y} = u_\infty + \frac{m}{r}\cos\theta$

$\phi = U_\infty r \cos\theta + m \ln r$ $\quad v = -\frac{\partial\psi}{\partial x} = \frac{m}{r}\sin\theta$

stag. pt: $u = v = 0$ $\quad \theta = \pi \rightarrow u = u_\infty - \frac{m}{a} = 0 \Rightarrow a = \frac{m}{u_\infty}$

$m = \frac{Q}{2\pi b} \rightarrow \frac{Q}{b} = 2\pi m$: flow rate from source

$= 2l u_\infty \rightarrow l = \frac{\pi m}{u_\infty} = \pi a$

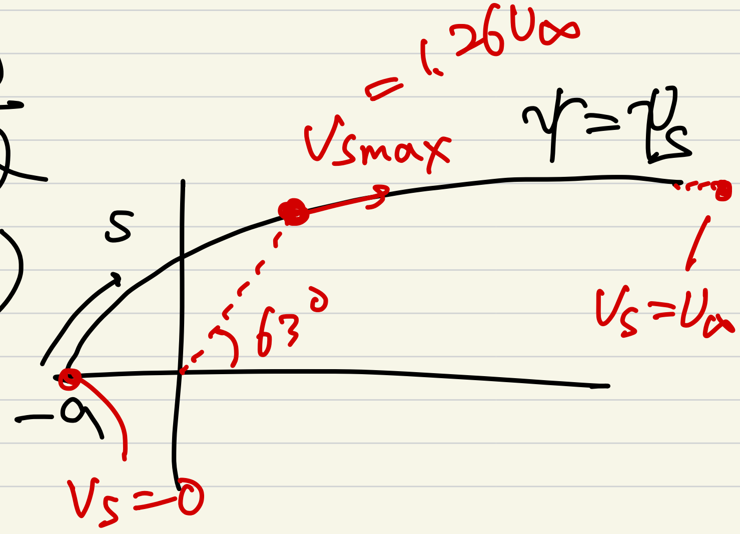
$\frac{Q}{b} = \int d\psi = 2\psi_s = 2\pi m \rightarrow \psi_s = \pi m$

upper surface : $\psi_s = \pi m = u_\infty r \sin\theta + m\theta$

$\rightarrow r = \frac{m(\pi - \theta)}{U_\infty \sin\theta}$

$V_s^2 = u^2 + v^2 = u_\infty^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$

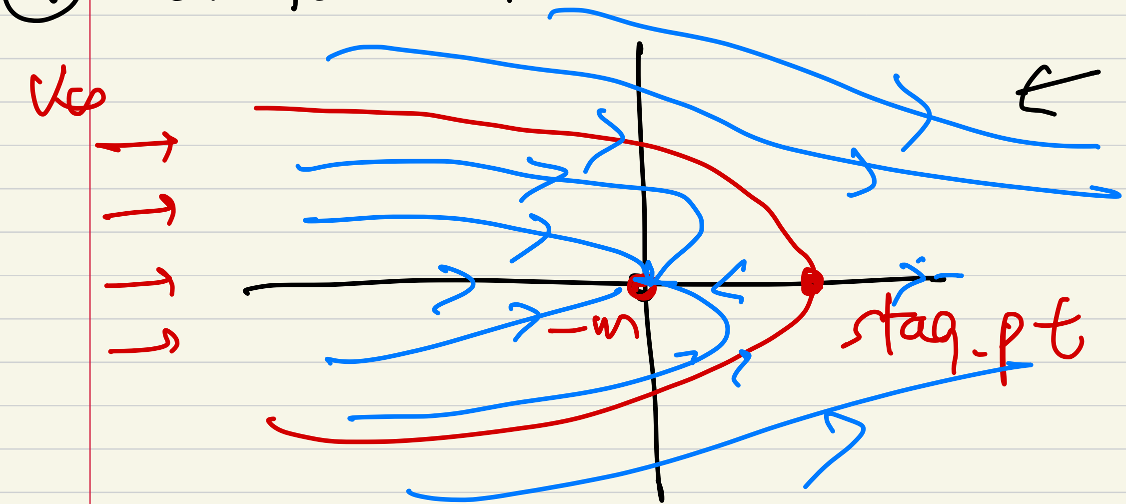
$\frac{\partial V_s^2}{\partial\theta} = 0$: $V_{s\,max} = 1.26\,u_\infty$ @ $\theta = 63^\circ$



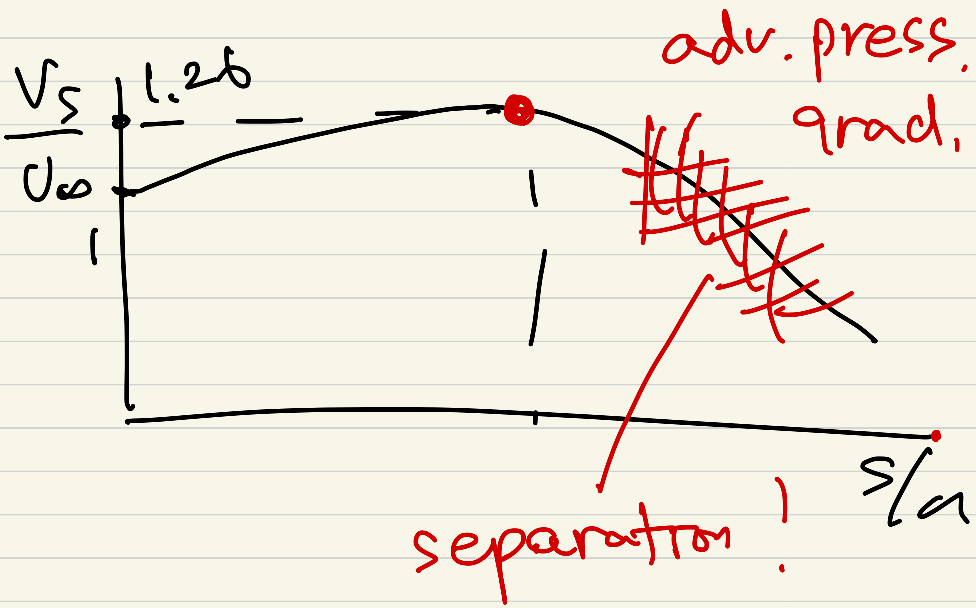
→ possibility of flow sep.

→ in reality, no sep.

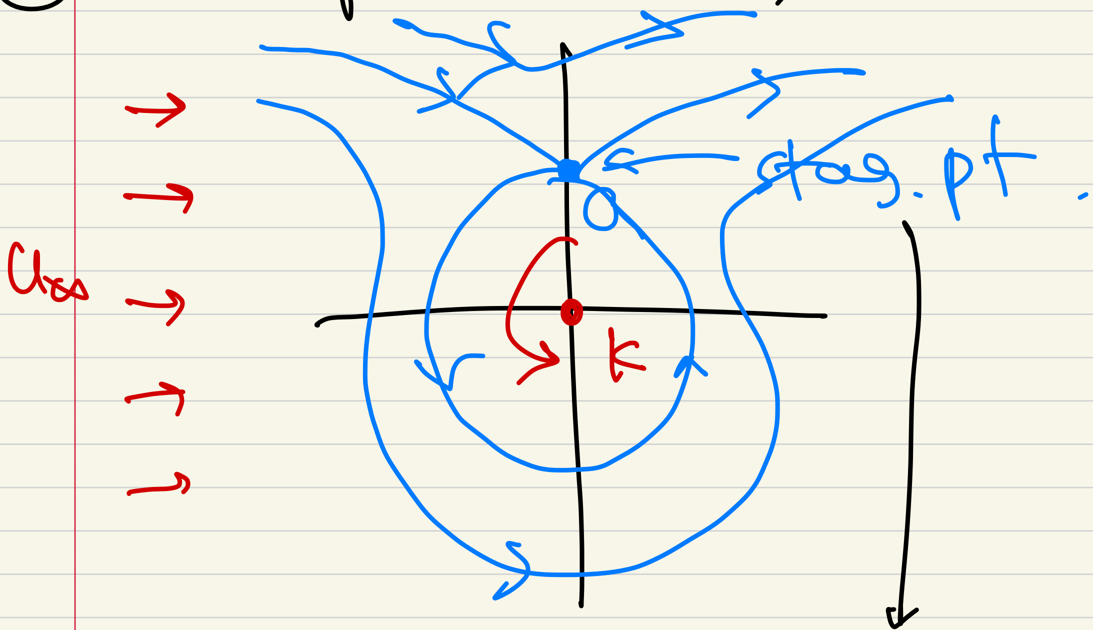
④ uniform stream + a sink



← has a mirror image of U_∞ + source



⑤ Flow past a vortex = unif. stream + a vortex



$$\psi = U_\infty r \sin\theta - k \ln r$$

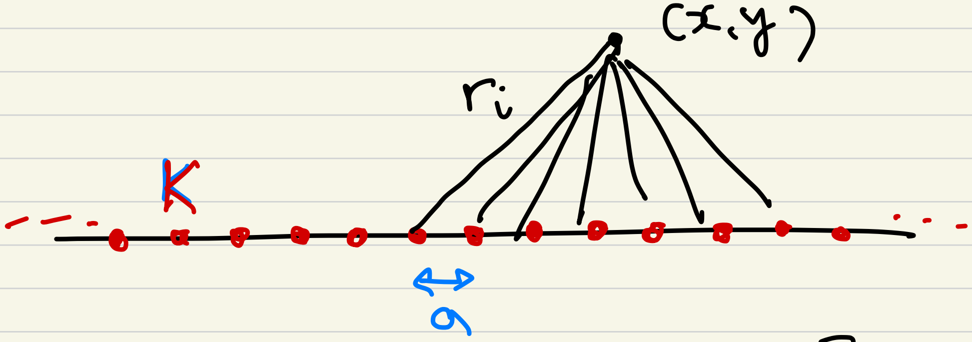
$$\phi = - - .$$

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = u_\infty \cos\theta$$

$$v_\theta = -\frac{\partial \psi}{\partial r} = -u_\infty \sin\theta + \frac{k}{r}$$

$$v_r = v_\theta = 0 \;:\; \theta = \frac{\pi}{2}, \quad r = a = \frac{k}{u_\infty}$$

⑥ Infinite row of vortices

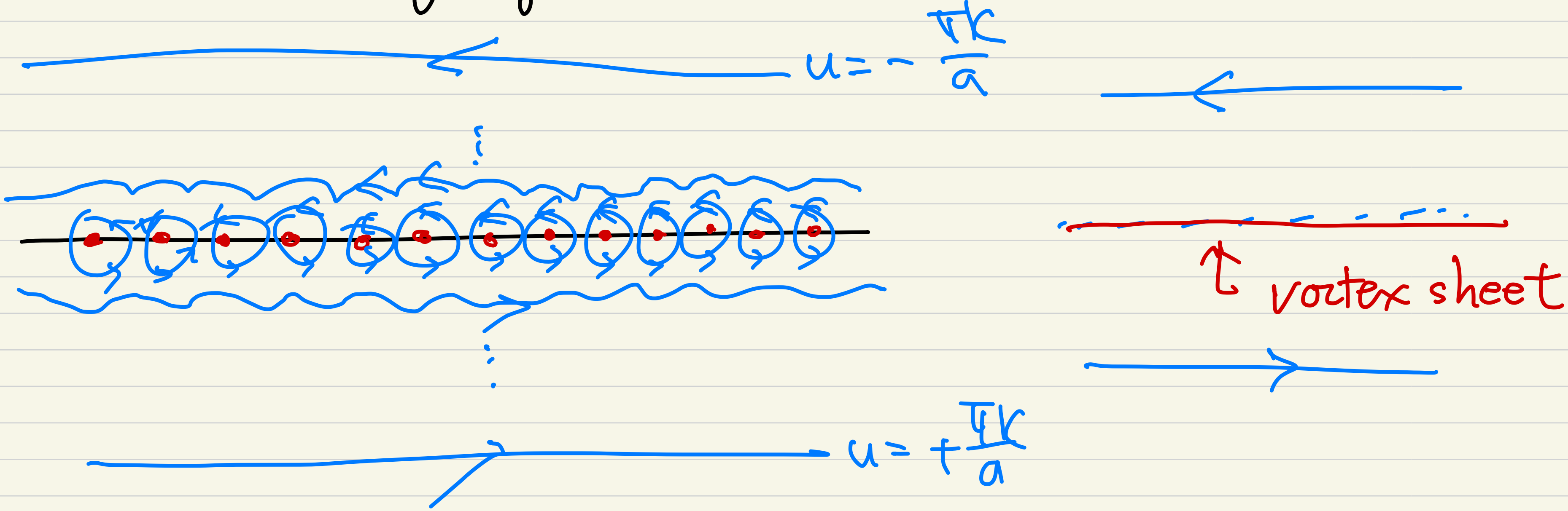


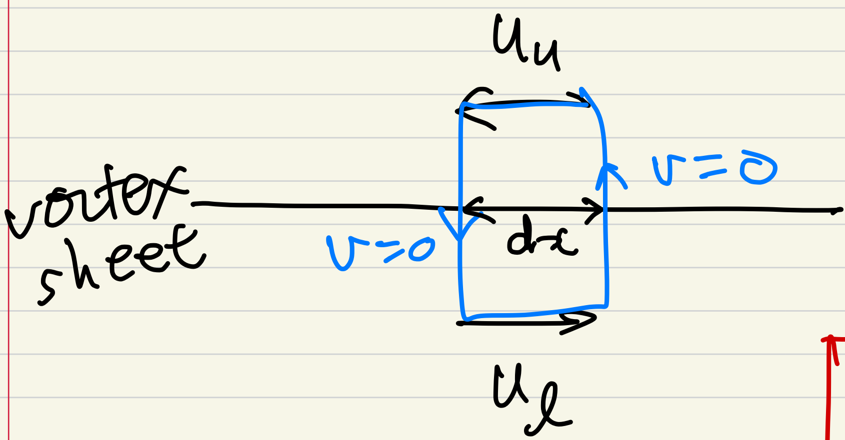
$$\psi = -k \sum_{i=1}^{\infty} \ln r_i$$

(complex variable)

$$\rightarrow \quad \psi = -\frac{1}{2} k \ln\left[\frac{1}{2}\left(\cosh\frac{2\pi y}{a} - \cos\frac{2\pi x}{a}\right)\right]$$

$$u = \frac{\partial \psi}{\partial y}\Big|_{|y| \gg a} = \pm \frac{\pi k}{a}$$





$$d\Gamma = \oint \underline{u}\cdot d\underline{l} = -\frac{\pi k}{a}\cdot \underset{\overset{\wedge}{-}}{dx} + \frac{\pi k}{a}\cdot dx$$

$$= \frac{2\pi k}{a}\,dx$$

$$\boxed{\frac{d\Gamma}{dx} = \frac{2\pi k}{a} = \gamma}$$: strength of vortex sheet.

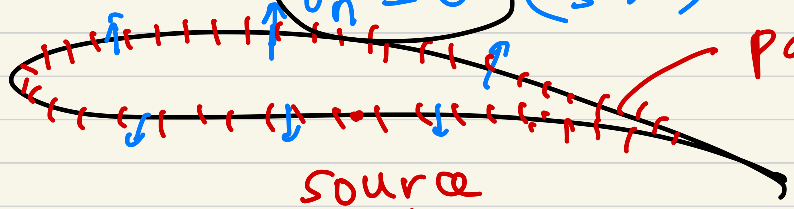
circulation per unit length of the vortex sheet

→ This is used to simulate a <u>thin-body shape</u>.

airfoil, flat plate

u_∞

$v_n = 0$ $(v_s \neq 0)$



panel

source

sink

vortex

<u>panel method</u>

$\Downarrow$

u. v. p