Advanced Bridge Engineering

Operational Modal Analysis
: NExT-ERA

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Analysis

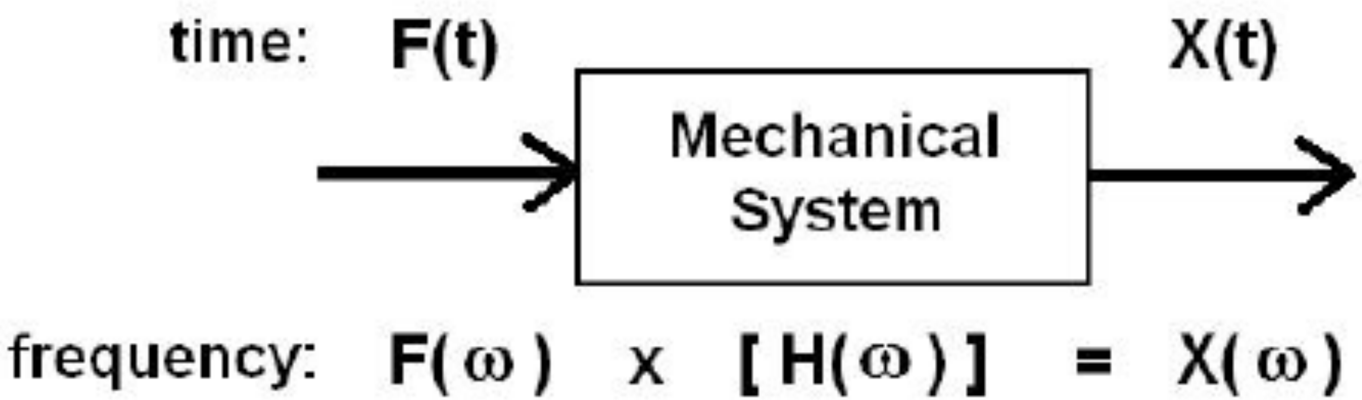
$$\underline{\textit{Force}} \qquad\qquad\qquad \underline{\textit{System}}\ \underline{\textit{Response}}$$

$$F = K\,x$$

☐ Known

☐ Unknown

Inverse analysis



Ratio of the Fourier transform of an output response and input force

Known

Unknown

Fourier Transform

- From Fourier series of non-periodic functions
- Allow period to go to infinity
- Similar to Laplace Transform
- Useful for random inputs

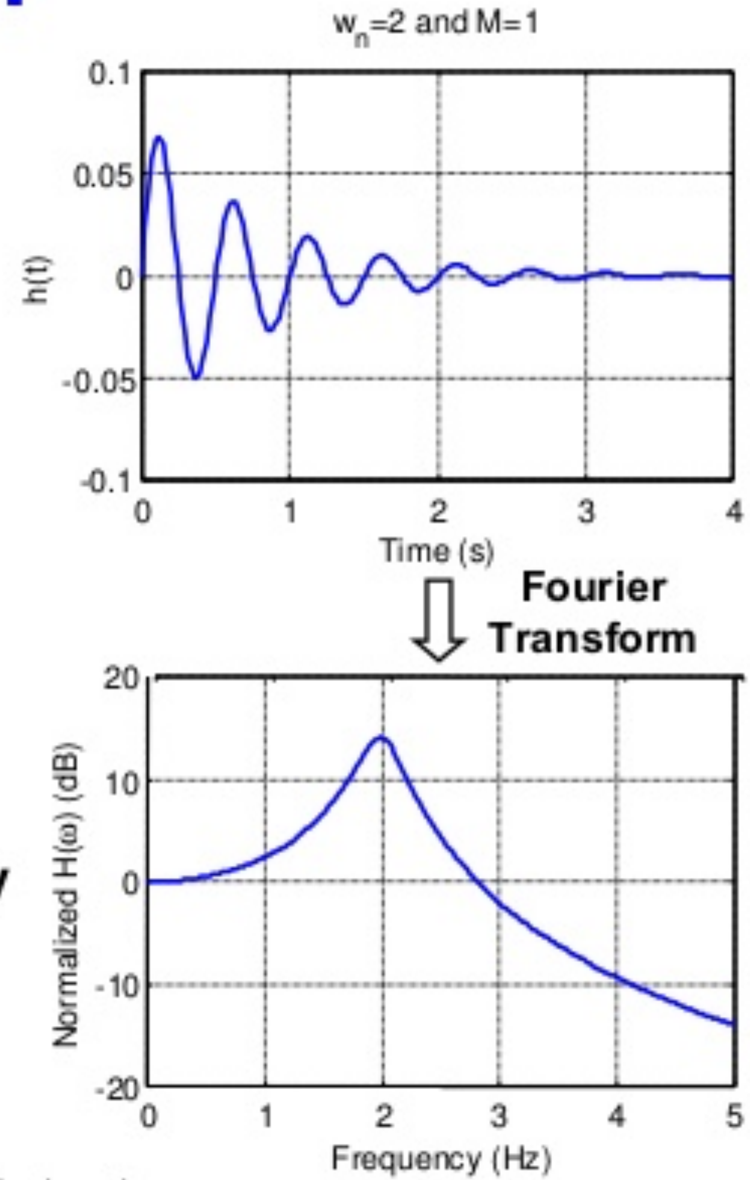
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

- Corresponding inverse transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

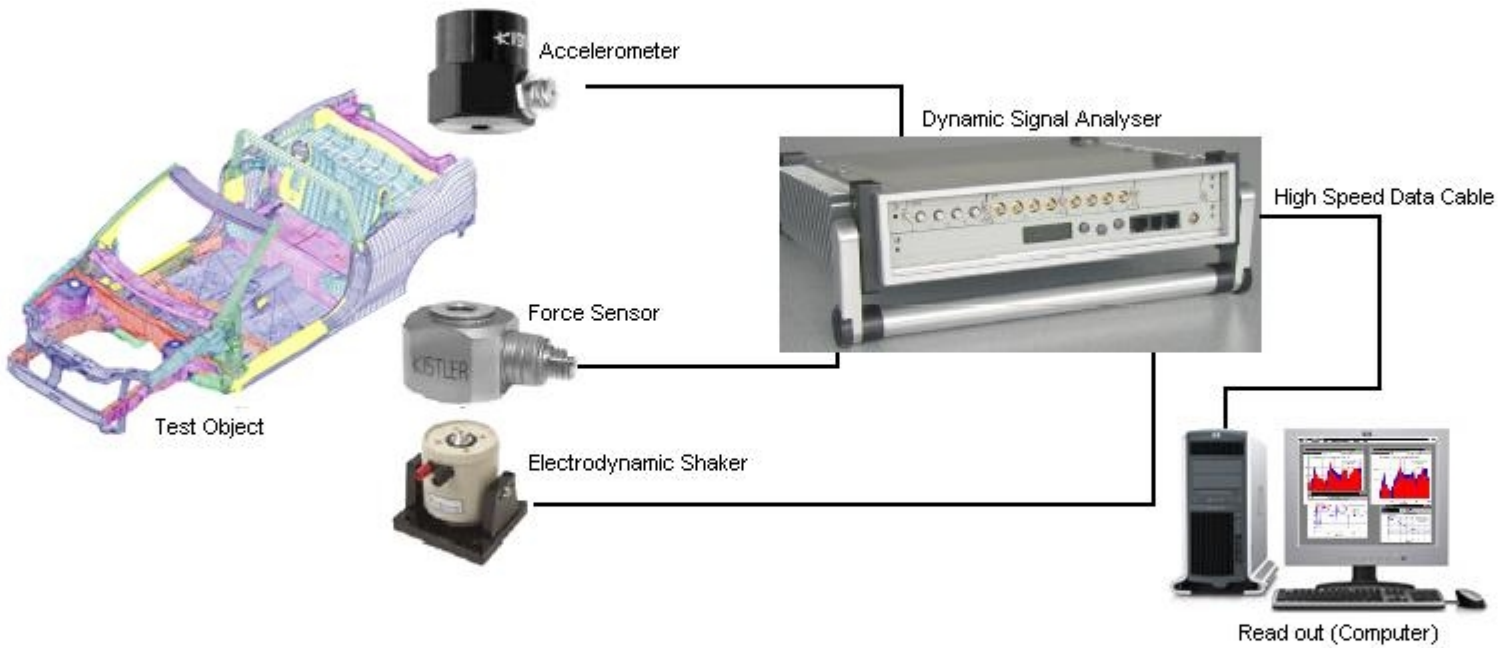
- Fourier transform of the unit impulse response is the frequency response function

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$



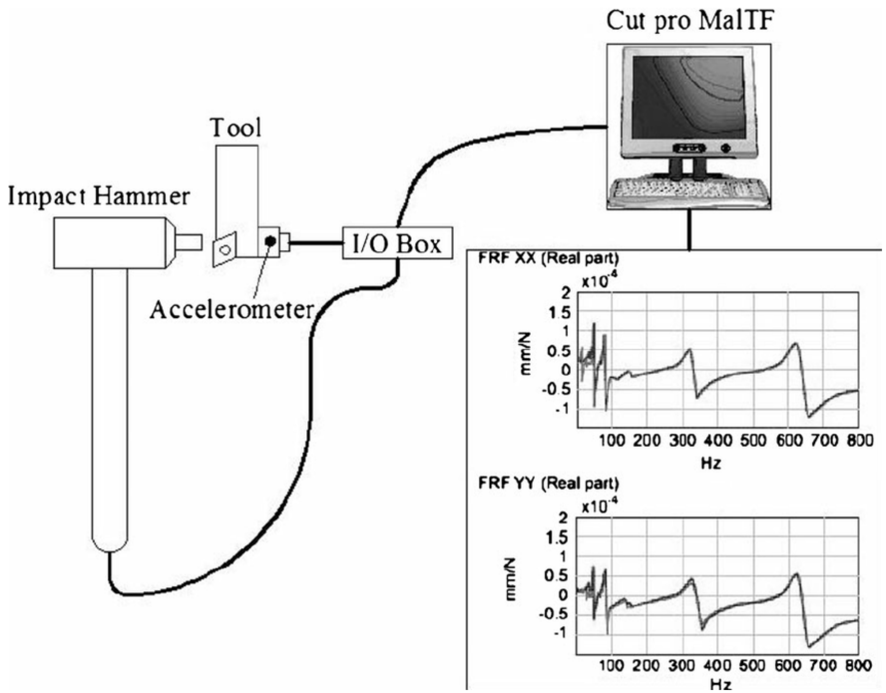
Impact test

- **Modal testing?**
  - A form of vibration testing of an object whereby the natural frequencies, modal masses, modal damping ratio and mode shape
  - Contents: Impact Hammer test, Shaker Modal Testing



Impact test

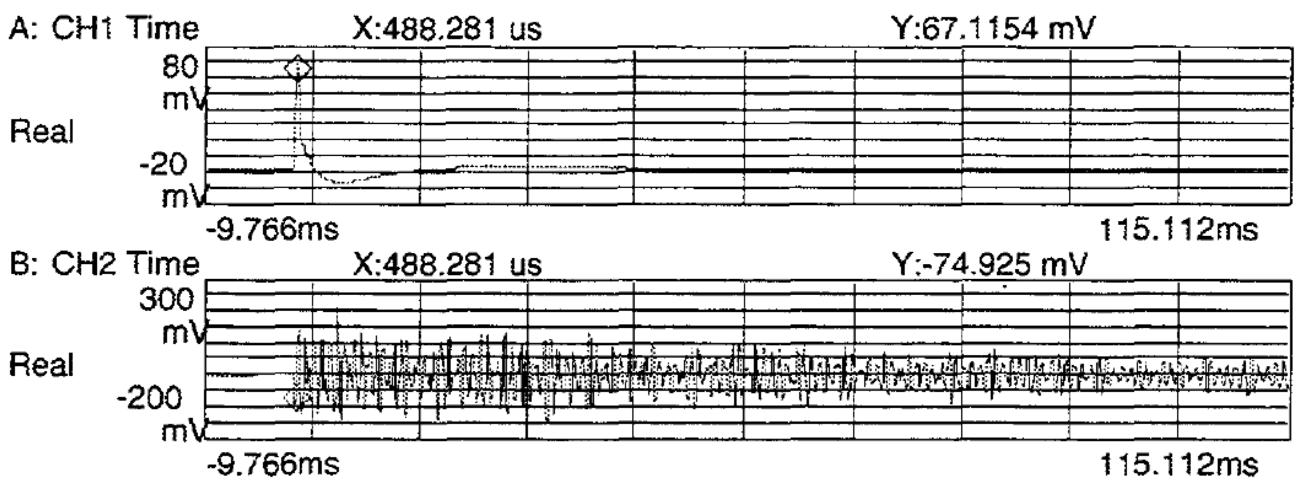
- **Impact Hammer Modal Testing**
  - Strike the test object with the force-instrumented hammer
  - Measure the resultant motion with an accelerometer
  - Analyze the signal with FFT analyzer



Impact test

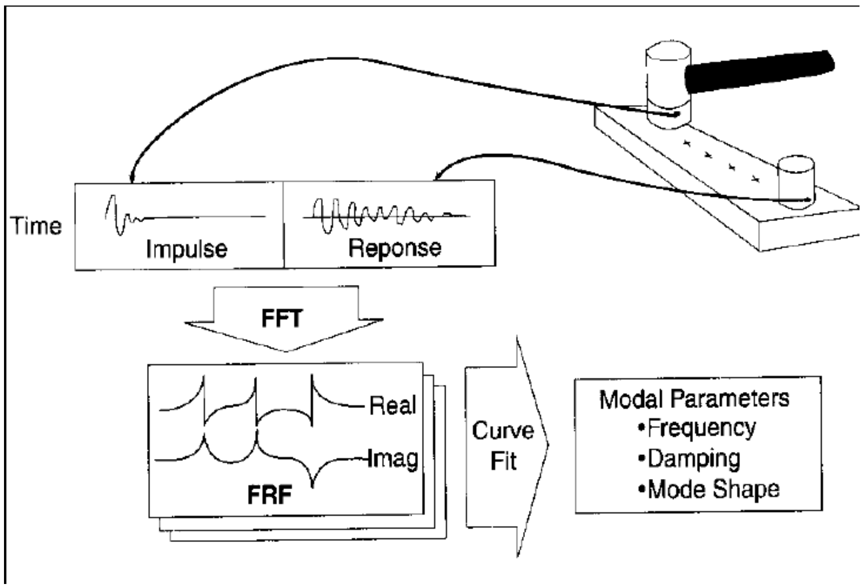
▶ **Response function of structure**

- Measured data: Input (Force) and Output (Accelerometer)
- Calculate the PSD of each signal
- Get the "Frequency Response Function" of the structure



Impact test

- **Steps of Impact hammer test**
  - Measure the excitation and response
  - FFT both signals
  - Calculate Auto Power Spectrum and Cross Power Spectrum
  - Calculate Average Spectrum

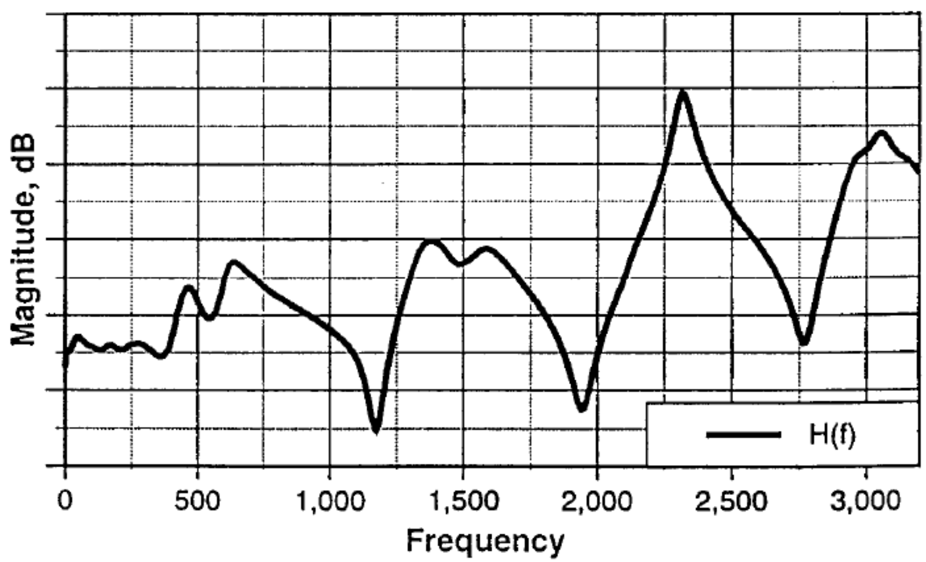


Impact test

▶ **Modal properties from FRF**

- Natural frequencies: peak value of FRF
- Modal damping for each mode: Half-band width theory

$$\frac{\omega_b - \omega_a}{\omega_n} = 2\xi$$



In civil structures

In civil structures

- Shaker and force-balance accelerometers

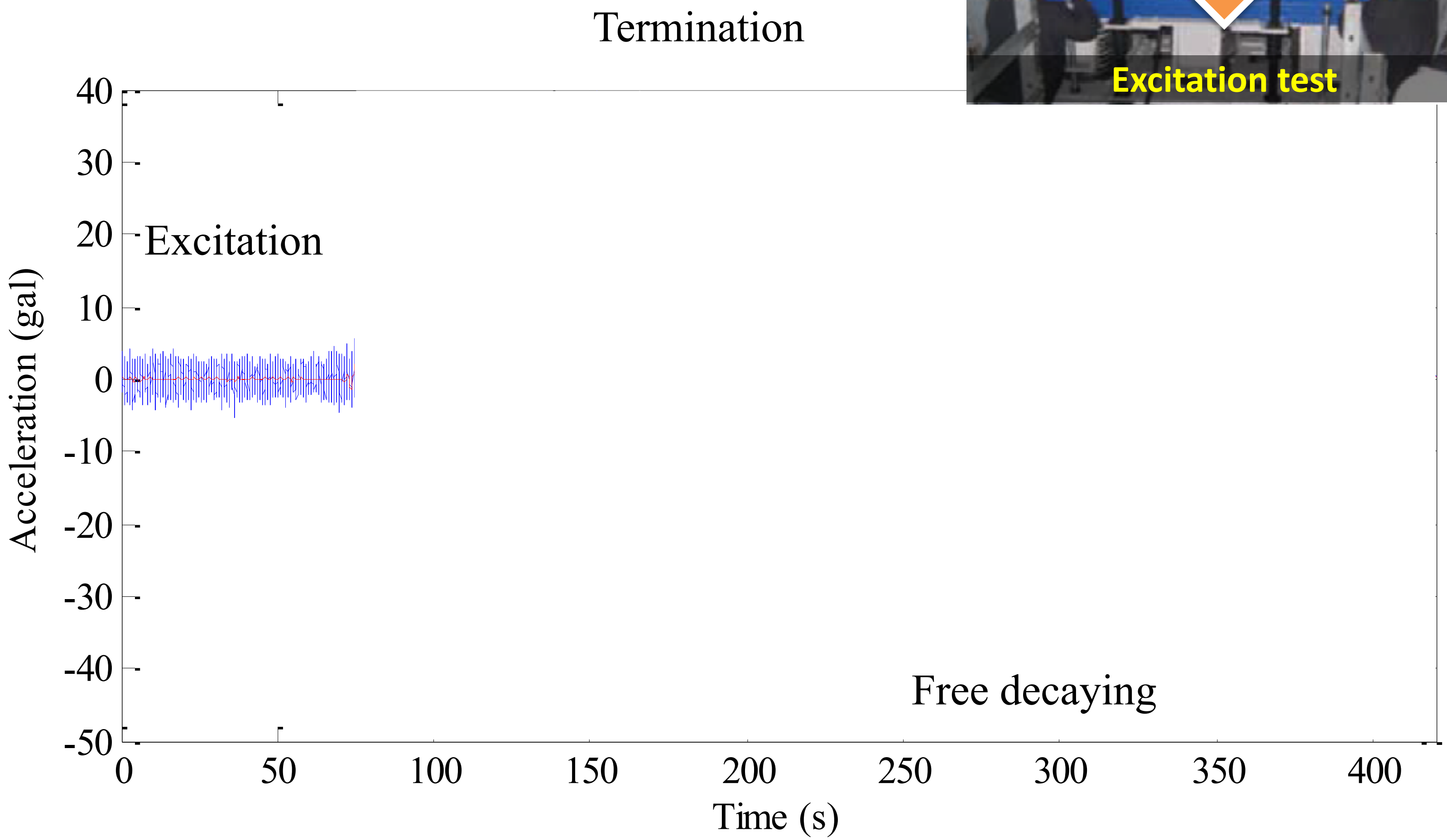
In civil structures

- Free vibration test



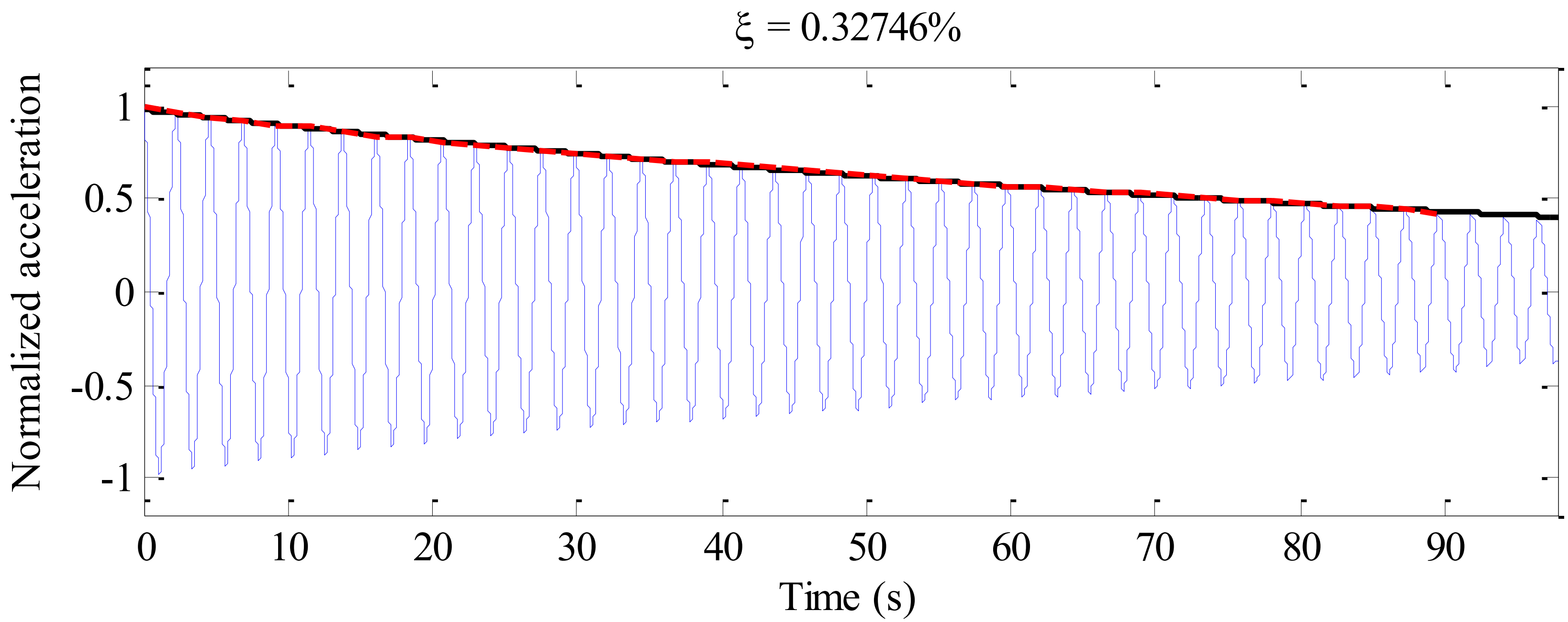
Damping estimation using excitation test

▶ **Excitation test using TMD**



Damping estimation using excitation test

▶ **Logarithmic decrement**



- Red line: utilized peaks
- Black line: theoretical envelope function

Output-only system identification

$$\underset{\text{Force}}{F} = \underset{\text{System}}{K}\ \underset{\text{Response}}{x}$$

□ Known

□ Unknown

NExT-ERA

▶ Impulse response function in modal coordinate (1)

$$\mathbf{Ma}(\mathrm{t}) + \mathbf{Cv}(\mathrm{t}) + \mathbf{Kx}(\mathrm{t}) = \mathbf{f}(\mathrm{t})$$

- For the modal coordinates:

$$\{x(t)\} = [\Phi]\{q(t)\} = \sum_{r=1}^{n} \{\phi^r\} q^r(t)$$

- Since real normal modes are assumed, a premultiplication by transpose of modal matrix makes a simultaneously diagonalized matrix. Therefore, each motion of equation can be expressed as a set of scalar equations in the modal coordinates:

$$\ddot{q}^r(t) + 2\zeta^r \omega_n^r \dot{q}^r(t) + \omega_n^{r2} q^r(t) = \frac{1}{m^r}\{\phi^r\}^T\{f(t)\}$$

NExT-ERA

▶ Basic procedure

- The output of structure *q(t)* can be obtained from convolution integral between input *f(t)* and impulse response function *g(t)*.

$$q^r(t) = \int_{-\infty}^{t} \{f(\tau)\} g^r(t-\tau)\, d\tau$$

$$g^r(t) = \frac{1}{m^r \omega_d^r} \exp(-\zeta^r \omega_n^r t) \sin(\omega_d^r t)$$

NExT-ERA

▶ **Basic procedure**

- Correlation function of output signal can be expressed as:

$$R(T)=\int_{-\infty}^{t}\int_{-\infty}^{t} g^{r}(t+T-\sigma)g^{s}(t-\tau)E\left[f(\sigma)\quad f(\tau)\right]d\sigma d\tau$$

- The expectation between white noise can be truncated to Dirac delta function.

$$E\left[f(\sigma)\quad f(\tau)\right]=\alpha\delta(\tau-\sigma)$$

- Finally, the correlation function of output is formulated as a same form of IRF.

$$R(T)=\alpha\cdot\int_{-\infty}^{t} g^{r}(t+T-\tau)g^{s}(t-\tau)d\tau$$

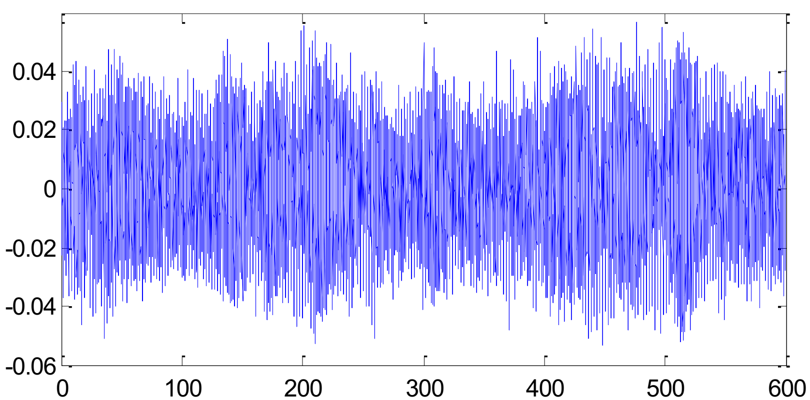
$$R(T)=\alpha\cdot\int_{0}^{\infty} g^{r}(\lambda+T)g^{s}(\lambda)d\lambda$$

Main procedure of NExT in MATLAB

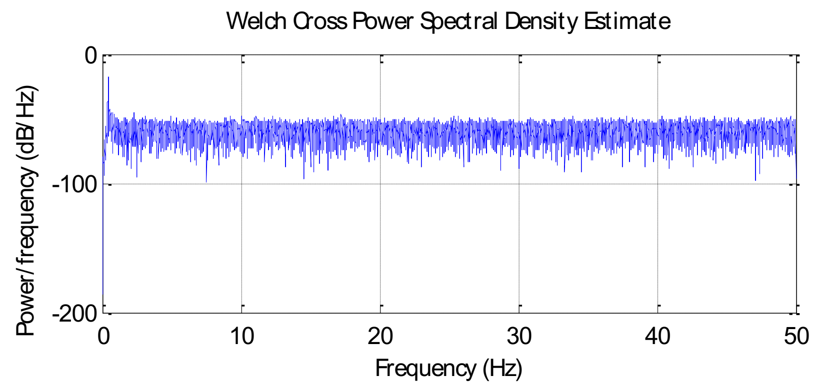
▶ **Calculation the power spectral density function of output.**

Output matrix **Y(t)** $\xrightarrow{\text{FFT}}$ Spectral density function $\boldsymbol{S}_{\mathbf{yy}}(\boldsymbol{\omega})$ $\xrightarrow{\text{IFFT}}$ Correlation function $\boldsymbol{R}_{\mathbf{yy}}(\boldsymbol{t})$

```
% Calculate the auto power spectral density
[CPSxy, Fxy] = cpsd(acc, acc, hanning(Nfft), Nfft/2, Nfft, fs);
```



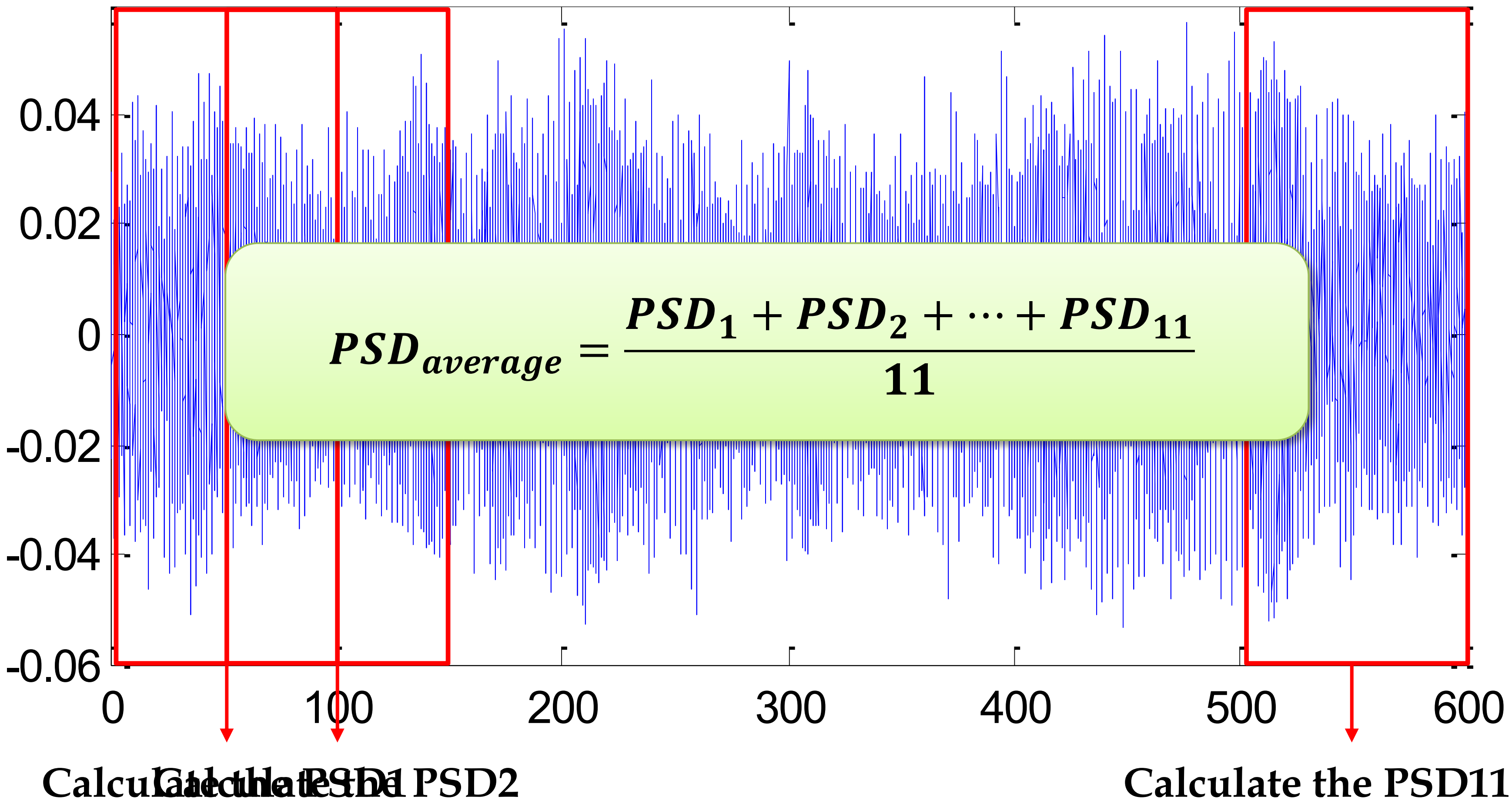
Output signal



PSD

What is Nfft!?

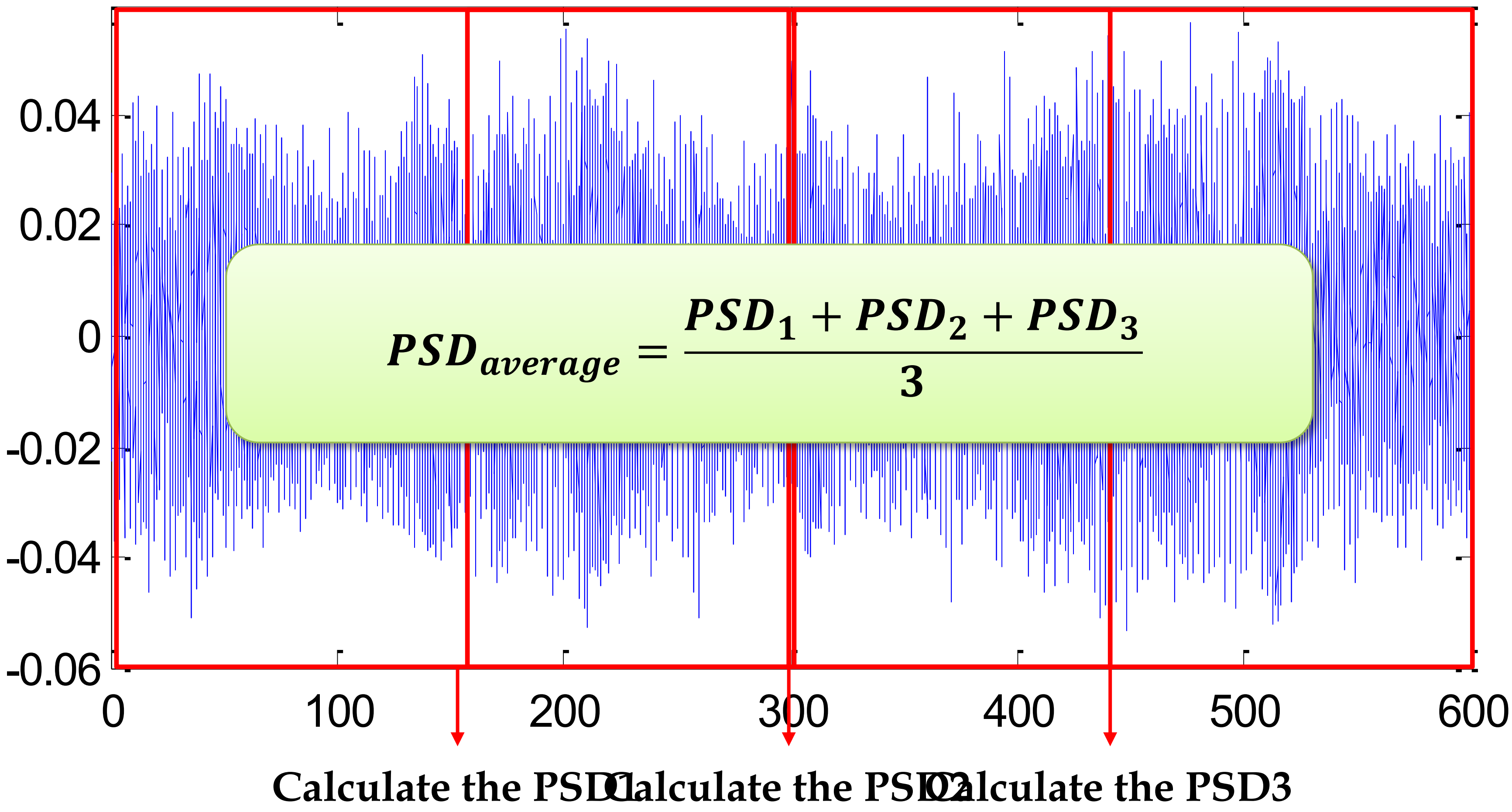
- **Example (1): Nfft: 100s / Overlap: 50s**



$$\boldsymbol{PSD_{average} = \frac{PSD_1 + PSD_2 + \cdots + PSD_{11}}{11}}$$

What is Nfft!?

- **Example (1): Nfft: 300s / Overlap: 150s**



$$PSD_{average} = \frac{PSD_1 + PSD_2 + PSD_3}{3}$$

Eigensystem Realization Algorithm

- **Objective**
  - Get the modal parameter from impulse response function

- **Methodology**
  - State-space model (S-S model)
  - Markov parameter from impulse response function
  - Hankel matrix composed of Markov parameter

- **Expected result**
  - Natural frequency
  - Damping ratio
  - Mode shape (not yet!)

State-space model for dynamic system

Derivation (2)

- Continuous-time state-space model

$$\ddot{\mathbf{w}} = -\mathbf{M}_{st}^{-1}\mathbf{C}_{st}\dot{\mathbf{w}} - \mathbf{M}_{st}^{-1}\mathbf{K}_{st}\mathbf{w} + \mathbf{M}_{st}^{-1}\mathbf{B}_{f}\mathbf{u}$$

$$\rightarrow \begin{bmatrix} \dot{\mathbf{w}}(t) \\ \ddot{\mathbf{w}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{st}^{-1}\mathbf{K}_{st} & -\mathbf{M}_{st}^{-1}\mathbf{C}_{st} \end{bmatrix} \begin{bmatrix} \mathbf{w}(t) \\ \dot{\mathbf{w}}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{st}^{-1}\mathbf{B}_{f} \end{bmatrix} \mathbf{u}(t)$$

$$\therefore \dot{\mathbf{x}} = \mathbf{A}_c\mathbf{x} + \mathbf{B}_c\mathbf{u}$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x}(t) + \mathbf{B}_c\mathbf{u}(t)$$

$$\text{where } \mathbf{x}(t) = \begin{bmatrix} \mathbf{w}(t) \\ \dot{\mathbf{w}}(t) \end{bmatrix} \quad \mathbf{A}_c = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{st}^{-1}\mathbf{K}_{st} & -\mathbf{M}_{st}^{-1}\mathbf{C}_{st} \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{st}^{-1}\mathbf{B}_{f} \end{bmatrix}$$

$$\mathbf{y}(t) = \mathbf{C}_c\mathbf{x}(t) + \mathbf{D}_c\mathbf{u}(t)$$

where $\mathbf{C}_c$ is the output influence matrix

$\mathbf{D}_c$ is the direct – transmission term

State-space model for dynamic system

Derivation (3)

- Definition of $\mathbf{C_c}$ and $\mathbf{D_c}$

$$\mathbf{M_{st}\ddot{w}} + \mathbf{C_{st}\dot{w}} + \mathbf{K_{st}w} = \mathbf{B_f u}$$

$$\Leftrightarrow \mathbf{\ddot{w}} = \mathbf{M_{st}^{-1}B_f u} - \mathbf{M_{st}^{-1}C_{st}\dot{w}} - \mathbf{M_{st}^{-1}K_{st}w}$$

substitute $\mathbf{\ddot{w}}$ to

$$\mathbf{y} = \mathbf{H_d w} + \mathbf{H_v \dot{w}} + \mathbf{H_a \ddot{w}}$$

$$\Leftrightarrow \mathbf{y} = \mathbf{H_d w} + \mathbf{H_v \dot{w}} + \mathbf{H_a}\left(\mathbf{M_{st}^{-1}B_f u} - \mathbf{M_{st}^{-1}C_{st}\dot{w}} - \mathbf{M_{st}^{-1}K_{st}w}\right)$$

$$\Leftrightarrow \mathbf{y} = \left(\mathbf{H_d} - \mathbf{H_a M_{st}^{-1}K_{st}}\right)\mathbf{w} + \left(\mathbf{H_v} - \mathbf{H_a M_{st}^{-1}C_{st}}\right)\mathbf{\dot{w}} + \left(\mathbf{H_a M_{st}^{-1}B_f}\right)\mathbf{u}$$

$$\therefore \mathbf{y} = \mathbf{C_c x} + \mathbf{D_c u}$$

$$\mathbf{C_c} = \begin{bmatrix} \mathbf{H_d} - \mathbf{H_a M_{st}^{-1}K_{st}} & \mathbf{H_v} - \mathbf{H_a M_{st}^{-1}C_{st}} \end{bmatrix}$$

$$\mathbf{D_c} = \mathbf{H_a M_{st}^{-1}B_f}$$

State-space model for dynamic system

Derivation (4)

- For displacement

$$\mathbf{H_d} = \mathbf{I},\ \mathbf{H_v} = 0,\ \mathbf{H_a} = \mathbf{0}$$
$$\therefore \mathbf{C_c} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \ \mathbf{and}\ \ \mathbf{D_c} = \mathbf{0}$$

- For acceleration

$$\mathbf{H_d} = \mathbf{0},\ \mathbf{H_v} = \mathbf{I},\ \mathbf{H_a} = \mathbf{0}$$
$$\mathbf{C_c} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \ \mathbf{and}\ \ \mathbf{D_c} = \mathbf{0}$$

- For velocity

$$\mathbf{H_d} = \mathbf{0},\ \mathbf{H_v} = \mathbf{0},\ \mathbf{H_a} = \mathbf{I}$$
$$\mathbf{C_c} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \quad \mathbf{D_c} = \mathbf{M}^{-1}\mathbf{B_f}$$

- General solution for the continuous-time dynamic system

$$\mathbf{x}(t) = \mathbf{e}^{\mathbf{A_c}(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t} \mathbf{e}^{\mathbf{A_c}(t-\tau)}\mathbf{B_c}\mathbf{u}(\tau)d\tau$$

Next work: <u>Discretization</u>!

State-space model for dynamic system

Derivation (5)

- Modal form using eigenmode change

substitute $\mathbf{w}(t) = \mathbf{\Phi\eta}(t)$

$$\mathbf{M_{st}\ddot{w}} + \mathbf{C_{st}\dot{w}} + \mathbf{K_{st}w} = \mathbf{B_f u}$$
$$\Leftrightarrow \mathbf{M_{st}\Phi\ddot{\eta}} + \mathbf{C_{st}\Phi\dot{\eta}} + \mathbf{K_{st}\Phi\eta} = \mathbf{B_f u}$$
$$\Leftrightarrow \mathbf{\Phi}^T\mathbf{M_{st}\Phi\ddot{\eta}} + \mathbf{\Phi}^T\mathbf{C_{st}\Phi\dot{\eta}} + \mathbf{\Phi}^T\mathbf{K_{st}\Phi\eta} = \mathbf{\Phi}^T\mathbf{B_f u}$$
$$\therefore \ddot{\boldsymbol{\eta}} + \mathbf{\Xi}\dot{\boldsymbol{\eta}} + \mathbf{\Omega}\boldsymbol{\eta} = \mathbf{\Phi}^T\mathbf{B_f u}$$

$$\mathbf{y} = \mathbf{H_d w} + \mathbf{H_v \dot{w}} + \mathbf{H_a \ddot{w}}$$
$$\Leftrightarrow \mathbf{y} = \mathbf{H_d\Phi\eta} + \mathbf{H_v\Phi\dot{\eta}} + \mathbf{H_a\Phi\ddot{\eta}}$$
$$\Leftrightarrow \mathbf{y} = \mathbf{H_d\Phi\eta} + \mathbf{H_v\Phi\dot{\eta}} + \mathbf{H_a\Phi}\left(\mathbf{\Phi}^T\mathbf{B_f u} - \mathbf{\Xi}\dot{\boldsymbol{\eta}} - \mathbf{\Omega}\boldsymbol{\eta}\right)$$
$$\Leftrightarrow \mathbf{y} = \left(\mathbf{H_d\Phi} - \mathbf{H_a\Phi\Omega}\right)\boldsymbol{\eta} + \left(\mathbf{H_v\Phi} - \mathbf{H_a\Phi\Xi}\right)\dot{\boldsymbol{\eta}} + \mathbf{H_a B_f u}$$

State-space model for dynamic system

Derivation (6)

- Rule of normalization of eigenvector: mass-orthonormalized

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}_{\mathbf{st}}\mathbf{\Phi} = \mathbf{I}_{\mathbf{n}}$$

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{K}_{\mathbf{st}}\mathbf{\Phi} = \mathbf{\Omega} = \mathrm{diag}(\omega_{ni}^{2},\ i = 1, \ldots, n)$$

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{C}_{\mathbf{st}}\mathbf{\Phi} = \mathbf{\Xi} = \mathrm{diag}(2\zeta_i\omega_{ni}, i = 1,\ \ldots, n)$$

where n = # of degree of freedom for the dynamic system

ω_{ni} = damped natural frequency

ζ_i = modal damping rates

State-space model for dynamic system

Derivation (7)

- S-S model for the generalized coordinates

$$\dot{\mathbf{x}}_\eta(\mathbf{t}) = \mathbf{A}_{\mathbf{mdv}}\mathbf{x}_\eta(\mathbf{t}) + \mathbf{B}_{\mathbf{mdv}}\mathbf{u}$$

$$\mathbf{y}(\mathbf{t}) = \mathbf{C}_{\mathbf{mdv}}\mathbf{x}_\eta(\mathbf{t}) + \mathbf{D}_{\mathbf{mdv}}\mathbf{u}(\mathbf{t})$$

$$\text{where } \mathbf{x}_\eta(\mathbf{t}) = \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} \mathbf{A}_{\mathbf{mdv}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega} & -\mathbf{\Xi} \end{bmatrix}, \quad \mathbf{B}_{\mathbf{mdv}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathbf{T}}\mathbf{B}_{\mathbf{f}} \end{bmatrix}$$

$$\mathbf{C}_{\mathbf{mdv}} = \begin{bmatrix} \mathbf{H}_{\mathbf{d}}\mathbf{\Phi} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{\mathbf{v}}\mathbf{\Phi} & \mathbf{0} \end{bmatrix}\mathbf{A}_{\mathbf{3}} + \begin{bmatrix} \mathbf{H}_{\mathbf{a}}\mathbf{\Phi} & \mathbf{0} \end{bmatrix}\mathbf{A}_{\mathbf{3}}^{\mathbf{2}}$$

$$\mathbf{D}_{\mathbf{mdv}} = \mathbf{H}_{\mathbf{a}}\mathbf{B}_{\mathbf{f}}$$

Basic concept of ERA (1)

State-space model for dynamic system

Derivation (8)

- Finally, obtain the response at the time argument about kΔt

$$\mathbf{x}(k+1) = \mathbf{e}^{\mathbf{A_c}\Delta t}\mathbf{x}(k) + \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{e}^{\mathbf{Ac}[(k+1)\Delta t-\tau]}\mathbf{B_c}\mathbf{u}(\tau)d\tau$$

$$\cong \mathbf{e}^{\mathbf{A_c}\Delta t}\mathbf{x}(k) + \int_0^{\Delta t} \mathbf{e}^{\mathbf{A_c}\tau'}d\tau'\mathbf{B_c}\mathbf{u}(k) \qquad \text{where } \tau' = (k+1)\Delta t - \tau$$

$$= \mathbf{A_d}\mathbf{x}(k) + \mathbf{B_d}\mathbf{u}(k)$$

$$\text{Otherwise } \mathbf{B_d} = \int_0^{\Delta t} \mathbf{e}^{\mathbf{A_c}\tau}d\tau\,\mathbf{B_c} = [\mathbf{A_c^{-1}}\mathbf{e}^{\mathbf{A_c}\tau}]_0^{\Delta t}\mathbf{B_c} = \mathbf{A_c^{-1}}(\mathbf{A_d}-\mathbf{I})\mathbf{B_c}$$

$$\boxed{\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A_d}\,\mathbf{x}(k) + \mathbf{B_d}\,\mathbf{u}(k)\\ \mathbf{y}(k) &= \mathbf{C_c}\mathbf{x}(k) + \mathbf{D_c}\mathbf{u}(k)\end{aligned}} \qquad (\mathbf{x}(k) = \mathbf{x}(k\Delta t))$$

$$\text{where } \mathbf{A_d} = \mathbf{e}^{\mathbf{A_c}(\Delta t)}, \quad \mathbf{B_d} = \int_0^{\Delta t} \mathbf{e}^{\mathbf{A_c}\tau'}d\tau'\mathbf{B_c} = \mathbf{A_c^{-1}}(\mathbf{A_d}-\mathbf{I})\mathbf{B_c}$$

State-space model for dynamic system

Example

- Continuous system matrix

$$\mathbf{M}_{st} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{C}_{st} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.1 \end{bmatrix} \qquad \mathbf{K}_{st} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{A}_c = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{st}^{-1}\mathbf{K}_{st} & -\mathbf{M}_{st}^{-1}\mathbf{C}_{st} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -0.2 & 0.1 \\ 1 & -1 & 0.1 & -0.1 \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{st}^{-1}\mathbf{B}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_c = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \ \textbf{and} \ \mathbf{D}_c = \mathbf{0} \text{ for displacement}$$

State-space model for dynamic system

Example

- Discretized form

$$\mathbf{A_d} = \mathbf{e}^{\mathbf{A_c}\Delta t} = \begin{bmatrix} 0.9610 & 0.0194 & 0.1935 & 0.0032 \\ 0.0194 & 0.9804 & 0.0032 & 0.1967 \\ -0.3837 & 0.1902 & 0.9226 & 0.0384 \\ 0.1902 & -0.1935 & 0.0384 & 0.9610 \end{bmatrix}$$

$$\mathbf{B_d} = \mathbf{A_c^{-1}}(\mathbf{A_d} - \mathbf{I_{2n}})\mathbf{B_c} = \begin{bmatrix} 0.0196 & 0.0002 \\ 0.0002 & 0.0198 \\ 0.1935 & 0.0032 \\ 0.0032 & 0.1967 \end{bmatrix}$$

Basic concept of ERA (2)

Markov Parameter

Definition

- To express the time-stepped impulse responses as a matrix form which contains modal information

Purpose

- Make a basis for identifying mathematical models for linear dynamical systems

Markov Parameter

Derivation (1)

- Concept of Markov parameters

$$\mathbf{y}(k) = \mathbf{Y}_k \, \mathbf{u}(0)$$

$$\begin{aligned} \text{where } \mathbf{Y}_k &= \text{Markov parameter for } t = k\Delta t \\ \mathbf{y}(k) &= \text{output at } t = k\Delta t \\ \mathbf{u}(k) &= \text{input at } t = k\Delta t \end{aligned}$$

- Zero initial condition: x(k) = 0 // The unit pulse: u(0) = 1, u(k) = 0 (k=1,2,..)

Markov Parameter

Derivation (2)

- From the response at the time argument about kΔt

$$\mathbf{x}(k+1) = \mathbf{A}_\mathrm{d}\,\mathbf{x}(k) + \mathbf{B}_\mathrm{d}\,\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}_\mathrm{c}\mathbf{x}(k) + \mathbf{D}_\mathrm{c}\mathbf{u}(k)$$

- Zero initial condition: x(k) = 0
- The unit pulse: u(0) = 1, u(k) = 0 (k=1,2,..)

$$\text{let } \mathbf{u}(0) = 1,\ \mathbf{u}(k) = 0\ (k = 1, 2, \cdots):$$
$$\begin{aligned}
\mathbf{x}(0) &= 0 && \Rightarrow && \mathbf{Y}_0 = \mathbf{D}_\mathrm{c} \\
\mathbf{x}(1) &= \mathbf{B}_\mathrm{d} && \Rightarrow && \mathbf{Y}_1 = \mathbf{C}_\mathrm{c}\mathbf{B}_\mathrm{d} \\
\mathbf{x}(2) &= \mathbf{A}_\mathrm{d}\mathbf{B}_\mathrm{d} && \Rightarrow && \mathbf{Y}_2 = \mathbf{C}_\mathrm{c}\mathbf{A}_\mathrm{d}\mathbf{B}_\mathrm{d} \\
&\vdots && && \vdots \\
\mathbf{x}(k) &= \mathbf{A}_\mathrm{d}^{k-1}\mathbf{B}_\mathrm{d} && \Rightarrow && \boxed{\mathbf{Y}_\mathrm{k} = \mathbf{C}_\mathrm{c}\mathbf{A}_\mathrm{d}^{\mathrm{k}-1}\mathbf{B}_\mathrm{d}}
\end{aligned}$$

It contains A, B and C matrix which contain modal information!

Basic concept of ERA (2)

Markov Parameter

Example

- From the previous example:

$$\mathbf{A_d} = \mathbf{e}^{\mathbf{A_c}\Delta t} = \begin{bmatrix} 0.9610 & 0.0194 & 0.1935 & 0.0032 \\ 0.0194 & 0.9804 & 0.0032 & 0.1967 \\ -0.3837 & 0.1902 & 0.9226 & 0.0384 \\ 0.1902 & -0.1935 & 0.0384 & 0.9610 \end{bmatrix}$$

$$\mathbf{B_d} = \mathbf{A_c^{-1}}(\mathbf{A_d} - \mathbf{I_{2n}})\mathbf{B_c} = \begin{bmatrix} 0.0196 & 0.0002 \\ 0.0002 & 0.0198 \\ 0.1935 & 0.0032 \\ 0.0032 & 0.1967 \end{bmatrix}$$

$$\mathbf{C_c} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Markov Parameter

Example

- From the previous example:

From $\mathbf{Y_k} = \mathbf{C_c A_d^{k-1} B_d}$: (In fact, it is obtained by NExT)

$$\mathbf{Y_1} = \begin{bmatrix} 0.0196 & 0.0002 \\ 0.0002 & 0.0198 \end{bmatrix}, \mathbf{Y_2} = \begin{bmatrix} 0.0563 & 0.0018 \\ 0.0018 & 0.0581 \end{bmatrix}, \mathbf{Y_3} = \begin{bmatrix} 0.0873 & 0.0063 \\ 0.0063 & 0.0935 \end{bmatrix},$$

$$\mathbf{Y_4} = \begin{bmatrix} 0.1106 & 0.0143 \\ 0.0143 & 0.1249 \end{bmatrix}, \mathbf{Y_5} = \begin{bmatrix} 0.1252 & 0.0263 \\ 0.0263 & 0.1516 \end{bmatrix}, \mathbf{Y_6} = \begin{bmatrix} 0.1308 & 0.0422 \\ 0.0422 & 0.1730 \end{bmatrix}, \cdots$$

- Markov Parameter is a matrix which has a size **<u>"# of input x # of output"</u>**

Eigensystem Realization Algorithm

Purpose

- Obtain the modal information matrix A, B, C and D

Methodology

- Make a Hankel matrix by Markov parameter
- Apply the Singular Vector Decomposition (SVD)
- Calculate the A, B, C and D

Basic concept of ERA (3)

Eigensystem Realization Algorithm

Hankel matrix (1)

- The size of Hankel matrix is $pm\times\gamma\, r$, where r is the number of inputs and m is the number of outputs. p and γ are just a integers which satisfy that condition $\gamma\, r \geq pm$

$$\mathbf{H(0)} = \begin{bmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 & \cdots & \mathbf{Y}_\gamma \\ \mathbf{Y}_2 & \mathbf{Y}_3 & \cdots & \mathbf{Y}_{\gamma+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_p & \mathbf{Y}_{p+1} & \cdots & \mathbf{Y}_{p+\gamma-1} \end{bmatrix}$$

Eigensystem Realization Algorithm

Hankel matrix (1)

- Substitute the $\mathbf{Y_k} = \mathbf{C}_c\mathbf{A}_\mathbf{d}^{\mathbf{k-1}}\mathbf{B_d}$

$$
\begin{aligned}
\mathbf{H}(0) &= \begin{bmatrix} \mathbf{C}_c\mathbf{B}_d & \mathbf{C}_c\mathbf{A}_d\mathbf{B}_d & \cdots & \mathbf{C}_c\mathbf{A}_d^{\gamma-1}\mathbf{B}_d \\ \mathbf{C}_c\mathbf{A}_d\mathbf{B}_d & \mathbf{C}_c\mathbf{A}_d^2\mathbf{B}_d & \cdots & \mathbf{C}_c\mathbf{A}_d^{\gamma}\mathbf{B}_d \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_c\mathbf{A}_d^{p-1}\mathbf{B}_d & \mathbf{C}_c\mathbf{A}_d^{p}\mathbf{B}_d & \cdots & \mathbf{C}_c\mathbf{A}_d^{p+\gamma-2}\mathbf{B}_d \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{C}_c \\ \mathbf{C}_c\mathbf{A}_d \\ \mathbf{C}_c\mathbf{A}_d^2 \\ \vdots \\ \mathbf{C}_c\mathbf{A}_d^{p-1} \end{bmatrix} \begin{bmatrix} \mathbf{B}_d & \mathbf{A}_d\mathbf{B}_d & \mathbf{A}_d^2\mathbf{B}_d & \cdots & \mathbf{A}_d^{\gamma-1}\mathbf{B}_d \end{bmatrix} \\
&= \mathbf{P}_p\mathbf{Q}_\gamma
\end{aligned}
$$

- Problem: There are infinite orders to decompose H(0) as a two of matrix

Basic concept of ERA (3)

Eigensystem Realization Algorithm

Singular Vector Decomposition (SVD)

- Purpose: Decompose the Hankel matrix, which is not a square matrix, with a desirable two of matrix containing eigen image
- Basic concept: If A is a real matrix (size: m x n), there exist two orthonormal matrices, eigenvector of **A^TA** or **AA^T**

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^T$$

$\mathbf{U}$ = eigenvector of **AA^T** (m x m)

$\mathbf{V}$ = eigenvector of **A^TA** (n x n)

$\mathbf{\Sigma}$ = square root of eigenvalue of both **A^TA** and **A^TA**

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{2n} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{\Sigma}_{2n} = \mathrm{diag}\left[\sigma_1, \quad \sigma_2, \quad \cdots, \quad \sigma_i, \quad \cdots, \quad \sigma_{2n}\right],$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_i \geq \cdots \geq \sigma_{2n} \geq 0.$$

Basic concept of ERA (3)

Eigensystem Realization Algorithm

Singular Vector Decomposition (SVD)

$$\mathbf{H}(0) \cong \mathbf{R}_{2n}\,\boldsymbol{\Sigma}_{2n}\,\mathbf{U}_{2n}^{T}$$
$$\cong \left[\mathbf{R}_{2n}\boldsymbol{\Sigma}_{2n}^{1/2}\right]\left[\boldsymbol{\Sigma}_{2n}^{1/2}\mathbf{U}_{2n}^{T}\right] \cong \mathbf{P}_{p}\mathbf{Q}_{\gamma}$$

$$\boxed{\begin{aligned}\mathbf{P}_{p} &= \mathbf{R}_{2n}\boldsymbol{\Sigma}_{2n}^{1/2}\\ \mathbf{Q}_{\gamma} &= \boldsymbol{\Sigma}_{2n}^{1/2}\mathbf{U}_{2n}^{T}\end{aligned}}$$

- **It Is not a exact matrix** P and Q because noise and truncation of nonzero small singular values for **Σ** so the approximation sign is used
- There is no mathematical supports about these formulation yet

Eigensystem Realization Algorithm

- **Calculate the A_d, B_d, C_c and D_c**
  - To compute $\mathbf{A_d}$, we form the Hankel matrix, **H(1)**

$$\mathbf{H}(1)=\begin{bmatrix}\mathbf{Y}_2 & \mathbf{Y}_3 & \cdots & \mathbf{Y}_{\gamma+1}\\ \mathbf{Y}_3 & \mathbf{Y}_4 & \cdots & \mathbf{Y}_{\gamma+2}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{Y}_{p+1} & \mathbf{Y}_{p+2} & \cdots & \mathbf{Y}_{p+\gamma}\end{bmatrix}=\begin{bmatrix}\mathbf{C_c A_d B_d} & \mathbf{C_c A_d^2 B_d} & \cdots & \mathbf{C_c A_d^{\gamma} B_d}\\ \mathbf{C_c A_d^2 B_d} & \mathbf{C_c A_d^3 B_d} & \cdots & \mathbf{C_c A_d^{\gamma+1} B_d}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{C_c A_d^{p} B_d} & \mathbf{C_c A_d^{p+1} B_d} & \cdots & \mathbf{C_c A_d^{p+\gamma-1} B_d}\end{bmatrix}$$

$$=\mathbf{P_p A_d Q_\gamma}$$

$$=\mathbf{R_{2n}\Sigma_{2n}^{1/2}\hat{A}_d\Sigma_{2n}^{1/2}U_{2n}^T}$$

$$\therefore \mathbf{\hat{A}_d}=\mathbf{\Sigma_{2n}^{-1/2}R_{2n}^T H(1) U_{2n}\Sigma_{2n}^{-1/2}}$$

$$\mathbf{\hat{B}_d}=\text{the first } r\textit{th} \text{ columns of } \mathbf{Q_\gamma}\,(=\mathbf{\Sigma_n^{1/2}U_n^T})$$

$$\mathbf{\hat{C}_c}=\text{the first } m\textit{th} \text{ rows of } \mathbf{P_p}\,(=\mathbf{R_n\Sigma_n^{1/2}})$$

$$\mathbf{D_c}=\mathbf{Y_0}$$

Eigensystem Realization Algorithm

Example

- Hankel matrix (p = γ = 3)

$$\mathbf{H}(0) = \begin{bmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 & \mathbf{Y}_3 \\ \mathbf{Y}_2 & \mathbf{Y}_3 & \mathbf{Y}_4 \\ \mathbf{Y}_3 & \mathbf{Y}_4 & \mathbf{Y}_5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0196 & 0.0002 & 0.0563 & 0.0018 & 0.0873 & 0.0063 \\ 0.0002 & 0.0198 & 0.0018 & 0.0581 & 0.0063 & 0.0935 \\ 0.0563 & 0.0018 & 0.0873 & 0.0063 & 0.1106 & 0.0143 \\ 0.0018 & 0.0581 & 0.0063 & 0.0935 & 0.0143 & 0.1249 \\ 0.0873 & 0.0063 & 0.1106 & 0.0143 & 0.1252 & 0.0263 \\ 0.0063 & 0.0935 & 0.0143 & 0.1249 & 0.0263 & 0.1516 \end{bmatrix}$$

Eigensystem Realization Algorithm

Example

- SVD

$$\mathbf{H}(0) = \mathbf{R}\boldsymbol{\Sigma}U^{\mathrm{T}} = [\mathbf{R}_{2n} \;\vdots\; \mathbf{R}_0]\begin{bmatrix} \boldsymbol{\Sigma}_{2n} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_0 \end{bmatrix}[\mathbf{U}_{2n} \;\vdots\; \mathbf{U}_0]^{\mathrm{T}}$$

$$\mathbf{R}_{2n} = \begin{bmatrix} 0.0196 & 0.0002 & 0.0563 & 0.0018 & 0.0873 & 0.0063 \\ 0.0002 & 0.0198 & 0.0018 & 0.0581 & 0.0063 & 0.0935 \\ 0.0563 & 0.0018 & 0.0873 & 0.0063 & 0.1106 & 0.0143 \\ 0.0018 & 0.0581 & 0.0063 & 0.0935 & 0.0143 & 0.1249 \\ 0.0873 & 0.0063 & 0.1106 & 0.0143 & 0.1252 & 0.0263 \\ 0.0063 & 0.0935 & 0.0143 & 0.1249 & 0.0263 & 0.1516 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_{2n} = diag\begin{bmatrix} 0.3146 & 0.2429 & 0.0310 & 0.0294 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U}_{2n} = \begin{bmatrix} -0.1890 & 0.3457 & 0.6939 & 0.4410 & 0.0492 & -0.4079 \\ -0.3057 & -0.2136 & -0.4289 & 0.7136 & -0.4069 & -0.0465 \\ -0.2936 & 0.4997 & 0.0951 & 0.0828 & -0.0882 & 0.8003 \\ -0.4751 & -0.3088 & -0.0588 & 0.1340 & 0.8046 & 0.1003 \\ -0.3930 & 0.5953 & -0.4828 & -0.2739 & 0.0596 & -0.4237 \\ -0.6359 & -0.3679 & 0.2984 & -0.4432 & -0.4162 & -0.0390 \end{bmatrix}$$

Eigensystem Realization Algorithm

Example

- **H(1)**

$$\mathbf{H}(1) = \begin{bmatrix} \mathbf{Y}_2 & \mathbf{Y}_3 & \mathbf{Y}_4 \\ \mathbf{Y}_3 & \mathbf{Y}_4 & \mathbf{Y}_5 \\ \mathbf{Y}_4 & \mathbf{Y}_5 & \mathbf{Y}_6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0563 & 0.0018 & 0.0873 & 0.0063 & 0.1106 & 0.0143 \\ 0.0018 & 0.0581 & 0.0063 & 0.0935 & 0.0143 & 0.1249 \\ 0.0873 & 0.0063 & 0.1106 & 0.0143 & 0.1252 & 0.0263 \\ 0.0063 & 0.0935 & 0.0143 & 0.1249 & 0.0263 & 0.1516 \\ 0.1106 & 0.0143 & 0.1252 & 0.0263 & 0.1308 & 0.0422 \\ 0.0143 & 0.1249 & 0.0263 & 0.1516 & 0.0422 & 0.1730 \end{bmatrix}$$

Eigensystem Realization Algorithm

Example

- Calculate the A_d, B_d, C_c and D_c

$$\hat{\mathbf{A}}_{\mathbf{d}} = \mathbf{\Sigma}_{2n}^{-1/2}\mathbf{R}_{2n}^{T}\mathbf{H}(1)\mathbf{U}_{2n}\mathbf{\Sigma}_{2n}^{-1/2} = \begin{bmatrix} 1.3089 & -0.0000 & 0.0000 & -0.3431 \\ 0.0000 & 1.1647 & 0.3916 & 0.0000 \\ -0.0000 & -0.3916 & 0.6831 & 0.0000 \\ 0.3431 & -0.0000 & -0.0000 & 0.6683 \end{bmatrix}$$

$$\hat{\mathbf{B}}_{\mathbf{d}} = \text{the first } r\textit{th} \text{ columns of } \mathbf{Q}_{\gamma}(= \mathbf{\Sigma}_{n}^{1/2}\mathbf{U}_{n}^{T}) = \begin{bmatrix} -0.1060 & -0.1715 \\ 0.1704 & -0.1053 \\ 0.1222 & -0.0755 \\ 0.0757 & 0.1225 \end{bmatrix}$$

$$\hat{\mathbf{C}}_{\mathbf{c}} = \text{the first } m\textit{th} \text{ rows of } \mathbf{P}_{p}(= \mathbf{R}_{n}\mathbf{\Sigma}_{n}^{1/2}) = \begin{bmatrix} -0.1060 & 0.1704 & -0.1222 & -0.0757 \\ -0.1715 & -0.1053 & 0.0755 & -0.1225 \end{bmatrix}$$

$$\mathbf{D}_{\mathbf{c}} = \mathbf{Y}_{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Brief procedure of NExT-ERA

1. **Calculate the auto/cross correlation function of measured response.**

$$\mathbf{r}_{ij}(k\Delta t) = \frac{1}{N-k}\sum_{l=1}^{N-k} a_i(l)a_j(l+k) \quad (k = 0, 1, 2, 3, \dots, N)$$

2. **Build the Hankel matrix H(0) and H(1)**

$$\mathbf{H}(\mathrm{k}) = \begin{bmatrix} R_{ij}(k) & R_{ij}(k+1) & \cdots & R_{ij}(k+m-1) \\ R_{ij}(k+1) & R_{ij}(k+2) & \cdots & R_{ij}(k+m) \\ \vdots & \vdots & \ddots & \vdots \\ R_{ij}(k+n-1) & R_{ij}(k+n) & \cdots & R_{ij}(k+m+n-1) \end{bmatrix}$$

Brief procedure of NExT-ERA

3. **Singular value decomposition of Hankel matrix**

$$\mathbf{H}(0) = \mathbf{U\Sigma V}^{\mathbf{T}}$$

4. **Calculate the system matrices and modal components**

$$\mathbf{A} = \mathbf{\Sigma}^{-1/2}\,\mathbf{U}^{\mathbf{T}}\mathbf{H}(1)\mathbf{V}\mathbf{\Sigma}^{-1/2}$$

$$s_i = \gamma_i + j\omega_i = \ln z_i\,/\Delta t$$

$$\xi_i = -\frac{\gamma_i}{|s_i|}$$

Numerical simulation

▶ **Structure & Material condition**

- 2-DOF spring supported beam
- All properties are based on the paper "Damping estimation using free decays and ambient vibration tests" (2012).

$$K = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} (kN/m)\ ; \quad M = \begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1.25 \end{bmatrix} (ton)\ ; \quad C = \begin{bmatrix} 0.3275 & -0.0725 \\ -0.0725 & 0.3275 \end{bmatrix} (kN.s/m)$$

$f_1 = 1.2995$ Hz $\quad \xi_1 = 1.0410$ % $\quad \phi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$f_2 = 1.5915$ Hz $\quad \xi_2 = 2.0000$ %

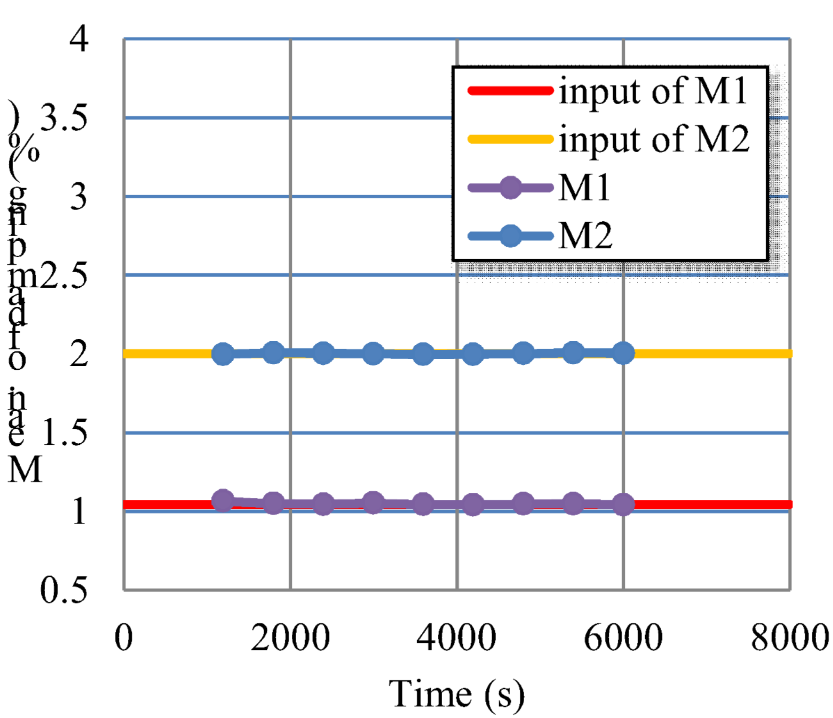
▶ **100 cases of simulation!**

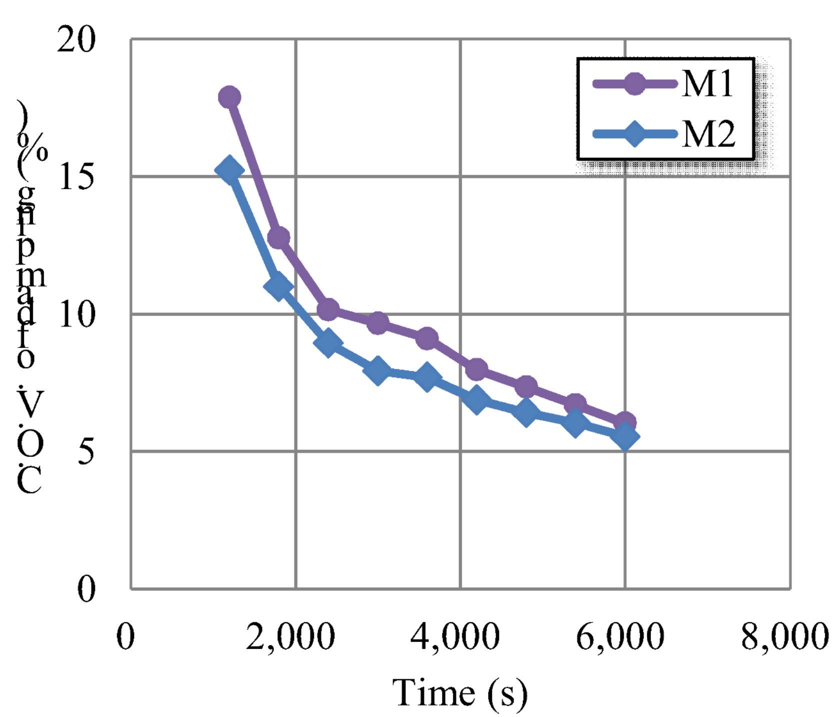
- Mean & C.O.V of damping ratio are evaluated.

Magalhães, F., Cunha, Á., Caetano, E., & Brincker, R. (2010). Damping estimation using free decays and ambient vibration tests. *Mechanical Systems and Signal Processing*, *24*(5), 1274-1290.

Parameter (1): Data length

- Effect of longer data length
  - More averaging is possible in long data under same Nfft.
  - Appropriate minimum data length: 20min??





Parameter (2): Nfft

- **Length of IRF (T_{IRF})**

$$T_{IRF} = \frac{\boldsymbol{Nfft} \cdot dt}{2}$$

- **Frequency resolution (df)**

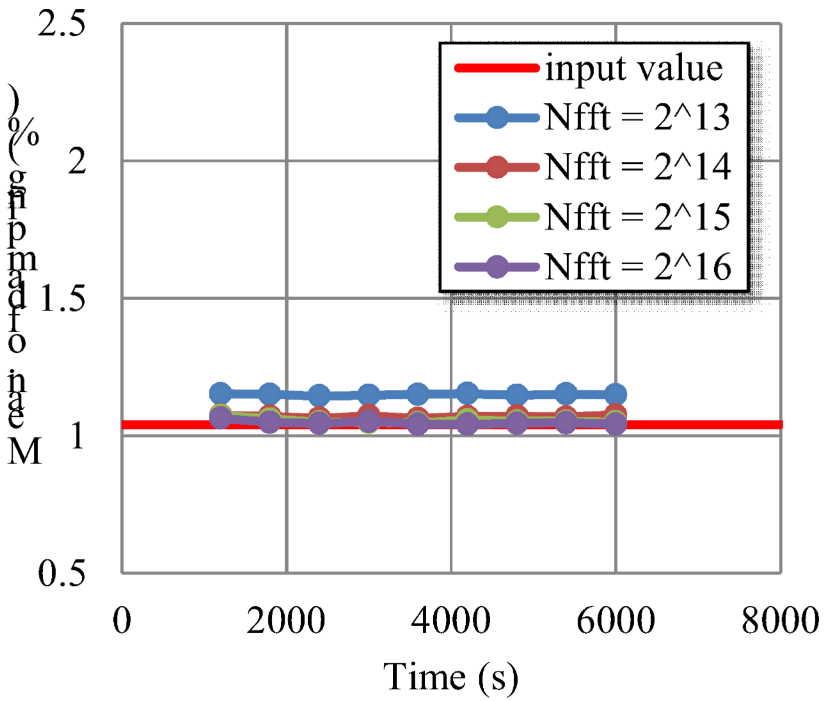
$$df = \frac{1}{dt \cdot \boldsymbol{Nfft}}$$

- **Number of averaging (n_{aver})**

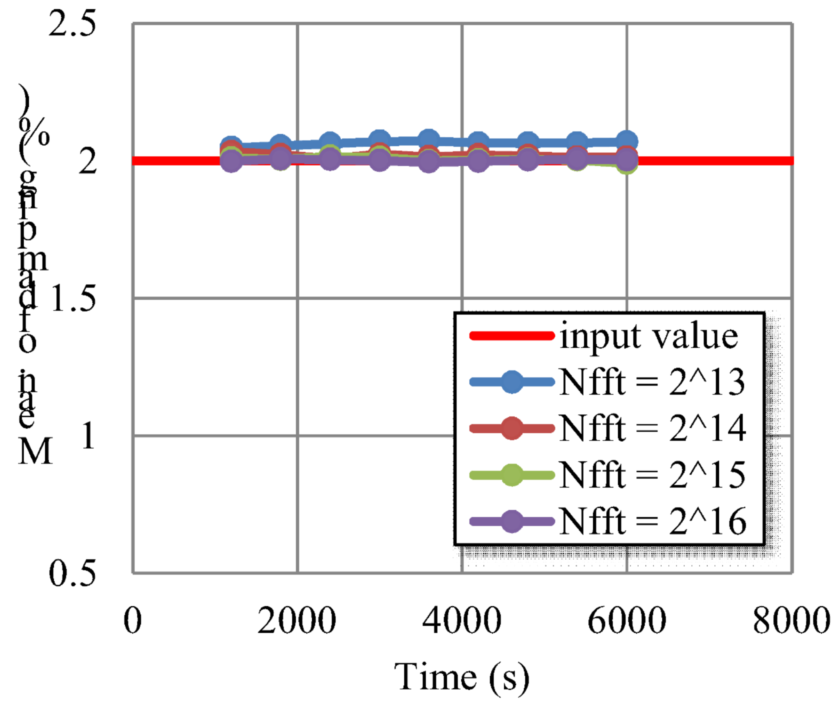
$$n_{aver} = \frac{2 \cdot ndata}{\boldsymbol{Nfft}} - 1$$

Parameter (2): Nfft

- **Effect of Nfft**
  - Nfft ↑ = IRF length ↑, frequency resolution ↑, averaging number ↓
  - Nfft ↓ = IRF length ↓, frequency resolution ↓, averaging number ↑



Mode 1 (M1)

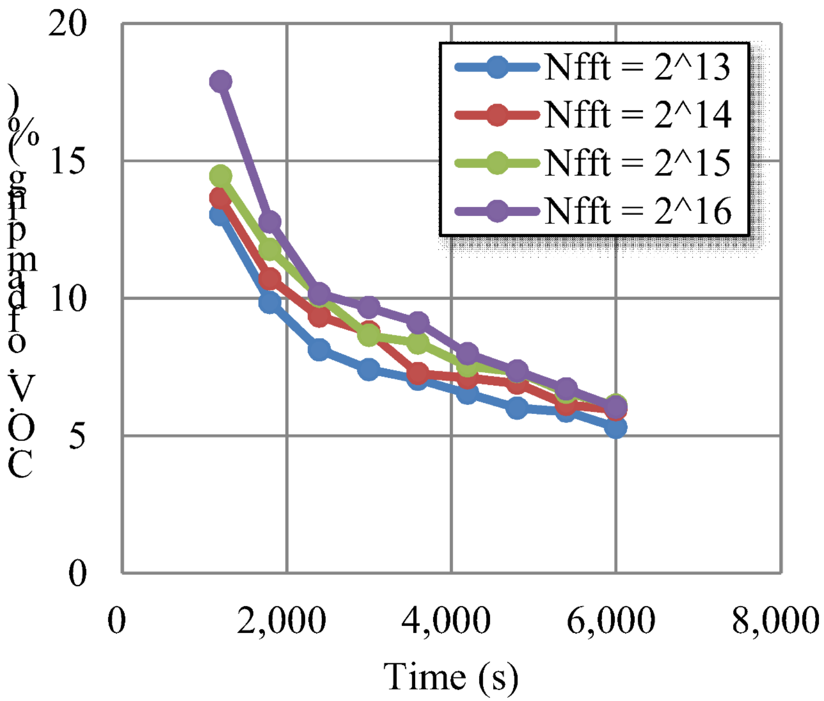


Mode 2 (M2)

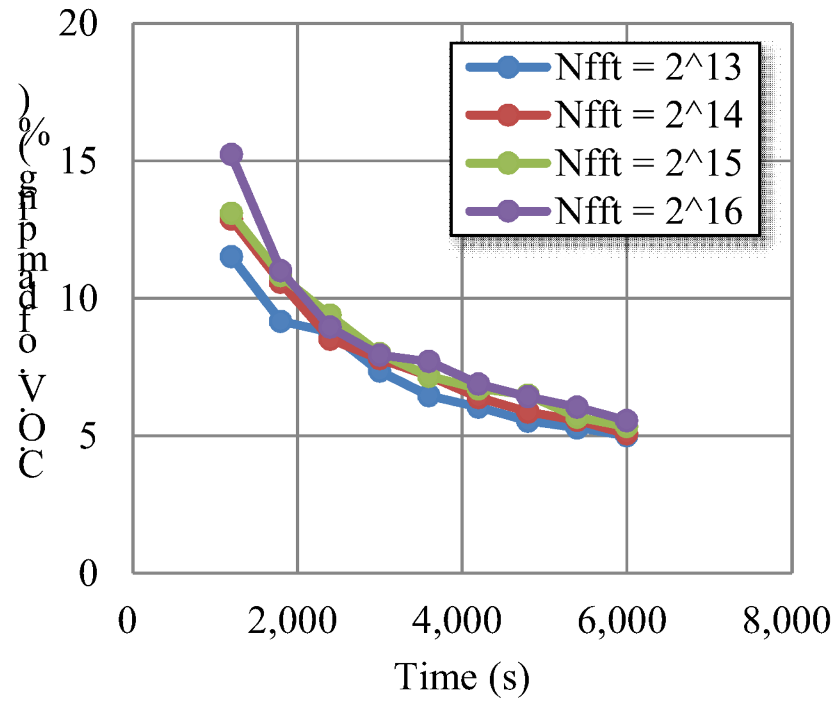
Parameter (2): Nfft

▶ **Effect of Nfft**

- Nfft ↑ = IRF length ↑, frequency resolution ↑, averaging number ↓
- Nfft ↓ = IRF length ↓, frequency resolution ↓, averaging number ↑



Mode 1 (M1)



Mode 2 (M2)

Parameter (2): Example of optimum Nfft

- Case (1): sampling frequency 100Hz, data length 1,200s

Power of NFFT (2^n)	Length of IRF (s)	Frequency resolution (Hz)	Averaging number
7	0.64	0.7813	937
8	1.28	0.3906	468
9	2.56	0.1953	234
10	5.12	0.0977	117
11	10.24	0.0488	58
12	20.48	0.0244	29
13	40.96	0.0122	14
14	81.92	0.0061	7
15	163.84	0.0031	3
16	327.68	0.0015	1
17	655.36	0.0008	0
18	1310.72	0.0004	0

(2) Parameter of ERA: Size of Hankel Matrix

▶ **Hankel matrix**

- Matrix composed with the IRF (Markov parameter)
- System information is extracted using SVD.

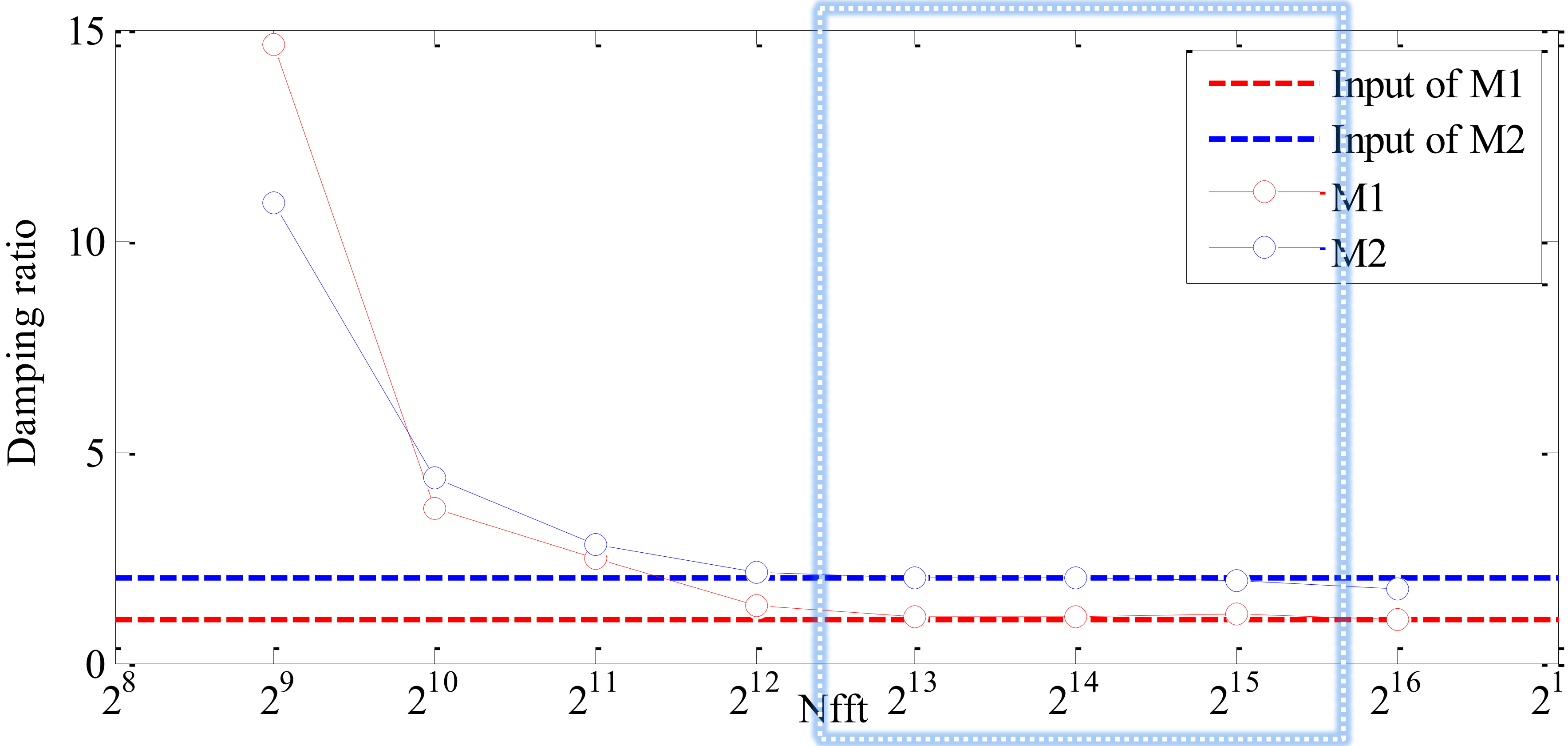
$$\mathbf{H(0)} = \begin{bmatrix} \mathbf{Y_1} & \mathbf{Y_2} & \cdots & \mathbf{Y_\gamma} \\ \mathbf{Y_2} & \mathbf{Y_3} & \cdots & \mathbf{Y_{\gamma+1}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y_p} & \mathbf{Y_{p+1}} & \cdots & \mathbf{Y_{p+\gamma-1}} \end{bmatrix} = \begin{bmatrix} \mathbf{C_cB_d} & \mathbf{C_cA_dB_d} & \cdots & \mathbf{C_cA_d^{\gamma-1}B_d} \\ \mathbf{C_cA_dB_d} & \mathbf{C_cA_d^2B_d} & \cdots & \mathbf{C_cA_d^{\gamma}B_d} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C_cA_d^{p-1}B_d} & \mathbf{C_cA_d^{p}B_d} & \cdots & \mathbf{C_cA_d^{p+\gamma-2}B_d} \end{bmatrix}$$

▶ **Size of Hankel matrix**

- **As much number of IRF** from the spectral density function as possible **without including noisy signals** is required for a successful damping identification.

Parameter (2): Example of optimum Nfft

- **Single case simulation**
  - Data length: 1,200s



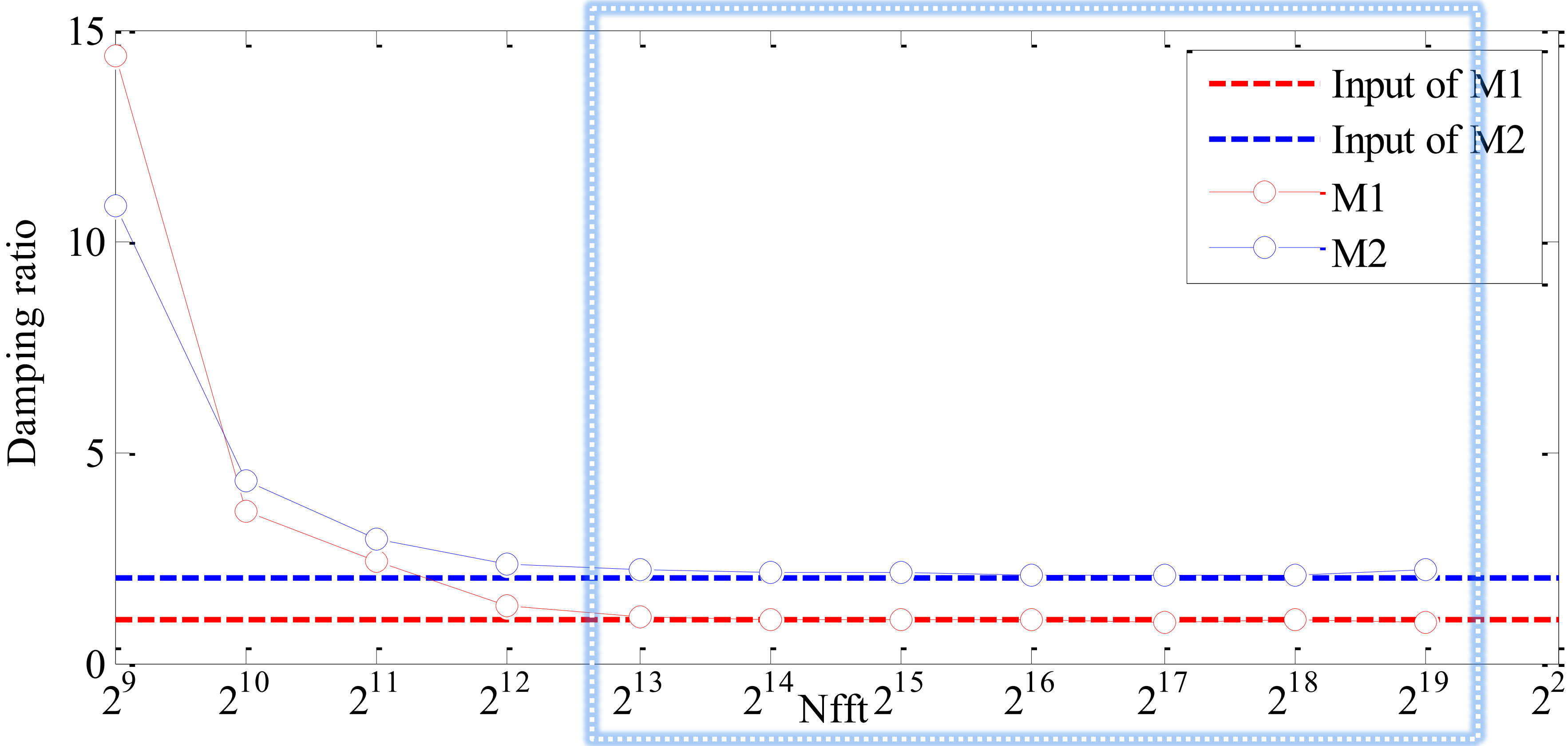
Parameter (2): Example of optimum Nfft

- Case (2): sampling frequency 100Hz, data length 6,000s

Power of NFFT (2^n)	Length of IRF (s)	Frequency resolution (Hz)	Averaging number
7	0.64	0.7813	4687
8	1.28	0.3906	2343
9	2.56	0.1953	1171
10	5.12	0.0977	585
11	10.24	0.0488	292
12	20.48	0.0244	146
13	40.96	0.0122	73
14	81.92	0.0061	36
15	163.84	0.0031	18
16	327.68	0.0015	9
17	655.36	0.0008	4
18	1310.72	0.0004	2

Parameter (2): Example of optimum Nfft

- **Single case simulation**
  - Data length: 6,000s



Parameter (3): Size of Hankel Matrix

▶ Hankel matrix

- Matrix composed with the IRF (Markov parameter)
- System information is extracted using SVD.

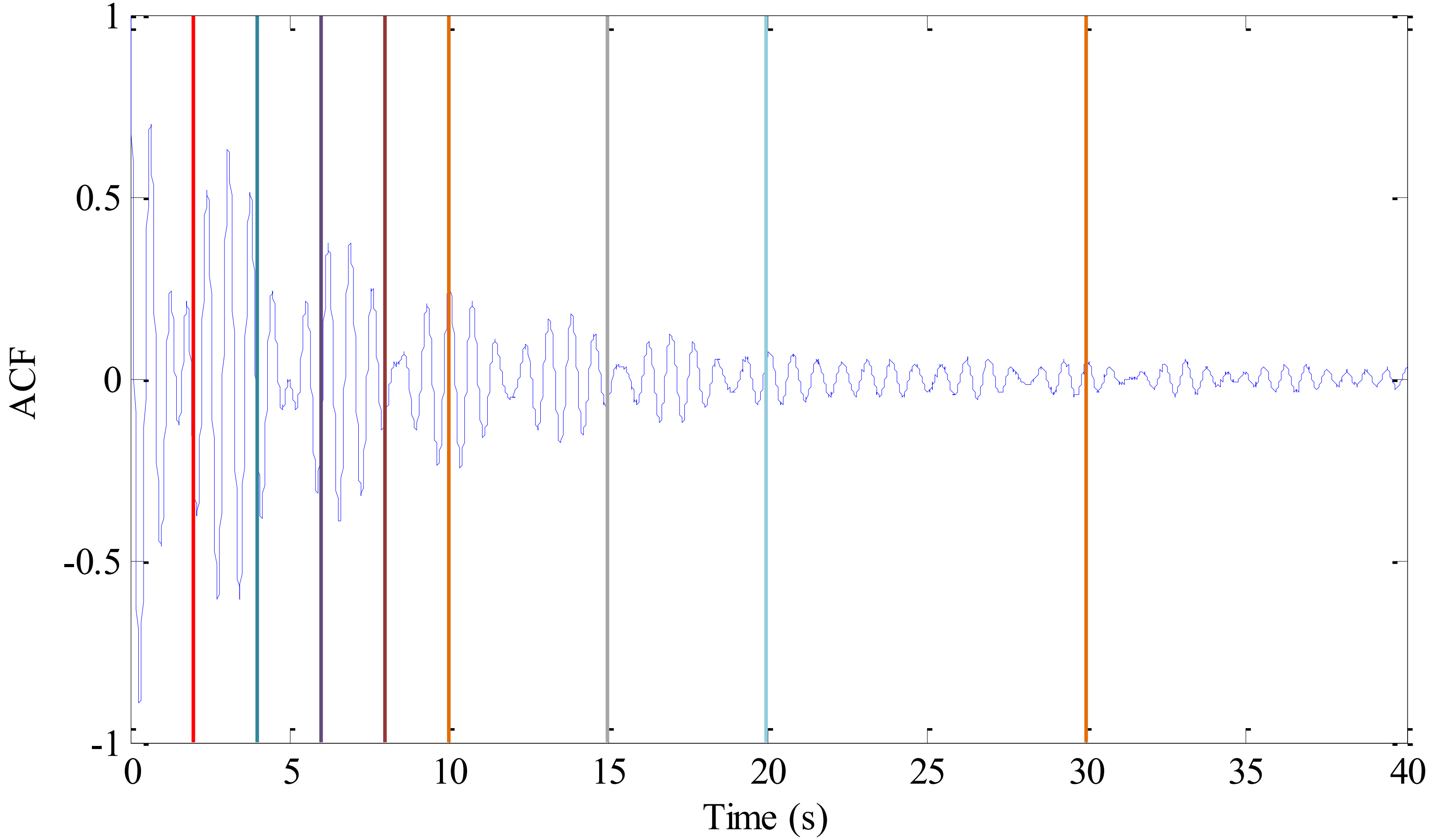
$$\mathbf{H(0)} = \begin{bmatrix} \mathbf{Y_1} & \mathbf{Y_2} & \cdots & \mathbf{Y_\gamma} \\ \mathbf{Y_2} & \mathbf{Y_3} & \cdots & \mathbf{Y_{\gamma+1}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y_p} & \mathbf{Y_{p+1}} & \cdots & \mathbf{Y_{p+\gamma-1}} \end{bmatrix} = \begin{bmatrix} \mathbf{C_c B_d} & \mathbf{C_c A_d B_d} & \cdots & \mathbf{C_c A_d^{\gamma-1} B_d} \\ \mathbf{C_c A_d B_d} & \mathbf{C_c A_d^2 B_d} & \cdots & \mathbf{C_c A_d^{\gamma} B_d} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C_c A_d^{p-1} B_d} & \mathbf{C_c A_d^{p} B_d} & \cdots & \mathbf{C_c A_d^{p+\gamma-2} B_d} \end{bmatrix}$$

▶ Effect of size of Hankel matrix

- Data number of utilized IRF.
- Too short? Little cycle would be included in ERA analysis
- Too long? meaningless noisy can be utilized.

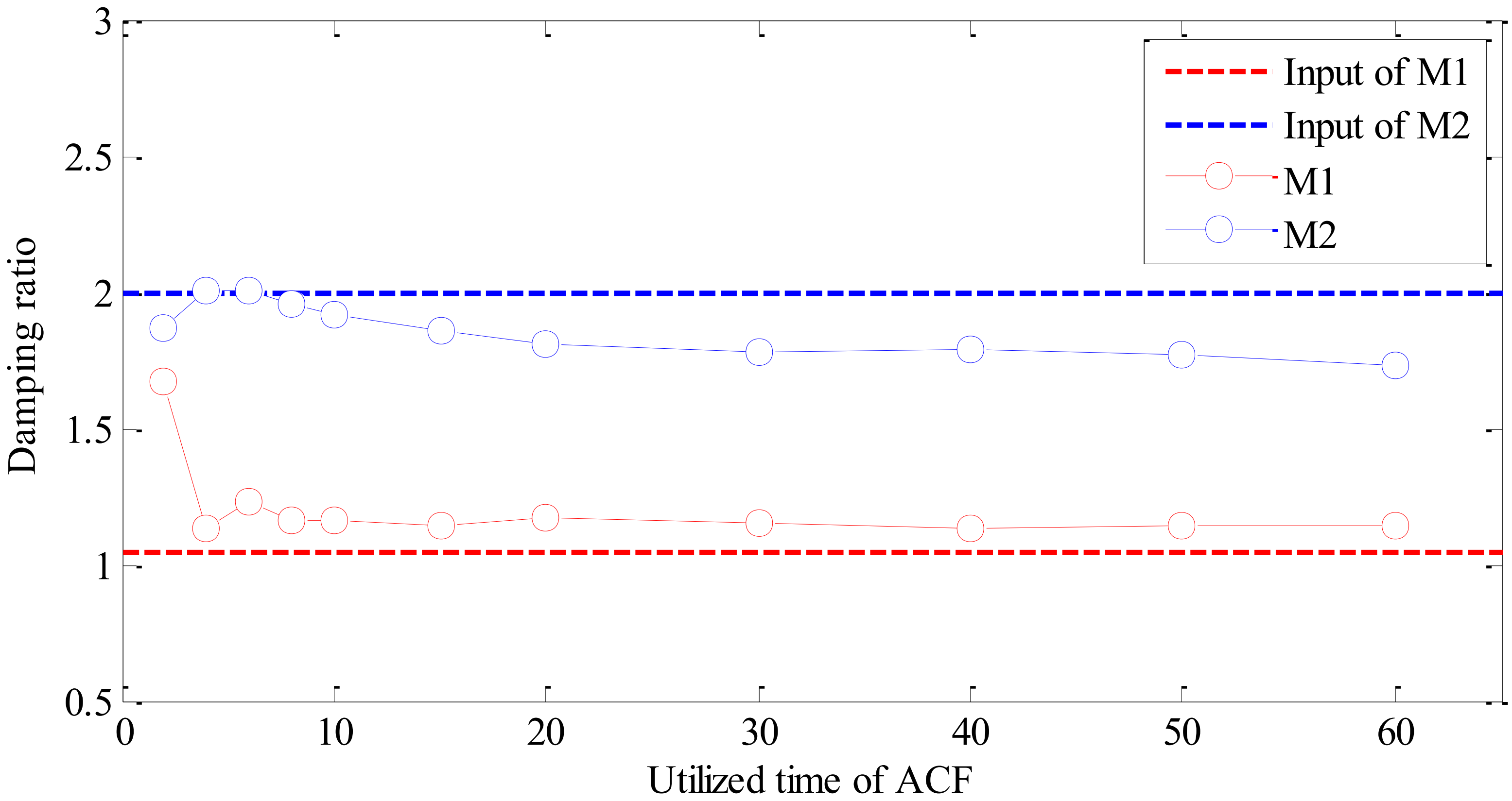
Parameter (3): Size of Hankel Matrix

▶ **Data length: 3,000s / Nfft = 2^{14}**



Parameter (3): Size of Hankel Matrix

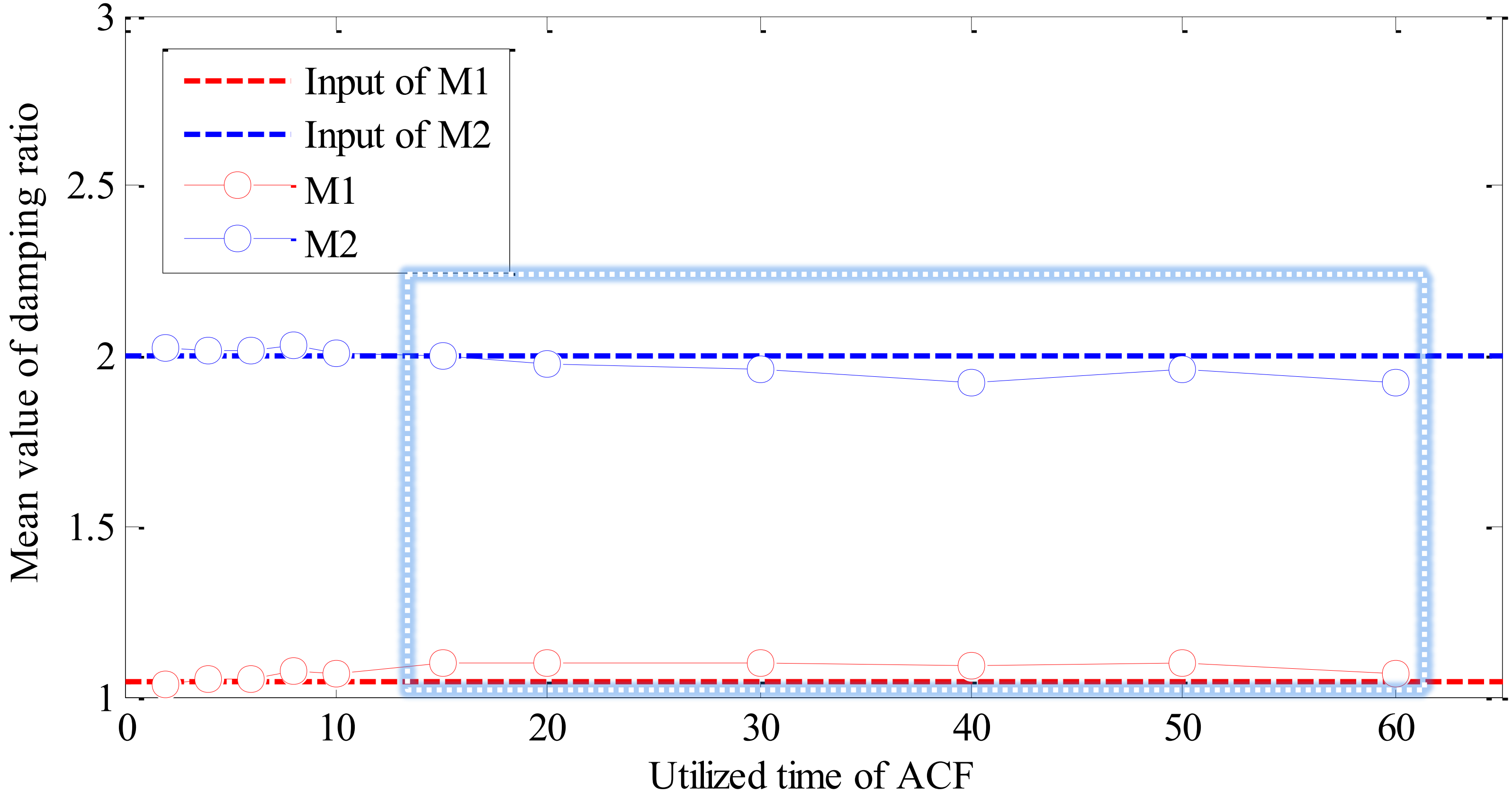
- **Single case**
  - Data length: 3,000s / Nfft = 2^{14}



Parameter (3): Size of Hankel Matrix

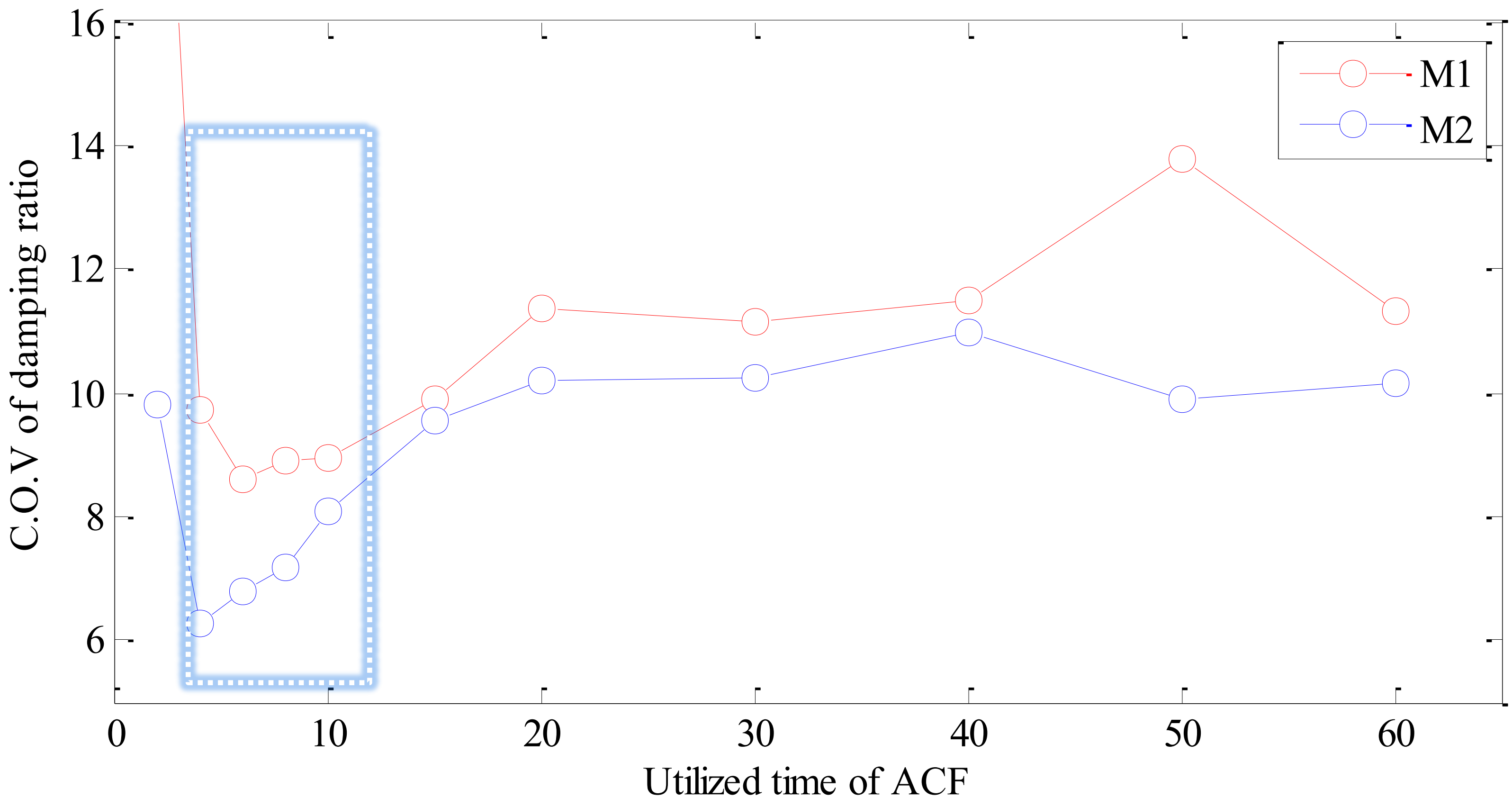
▶ **C.O.V of 100 cases simulation**

- Data length: 3,000s / Nfft = 2^{14}



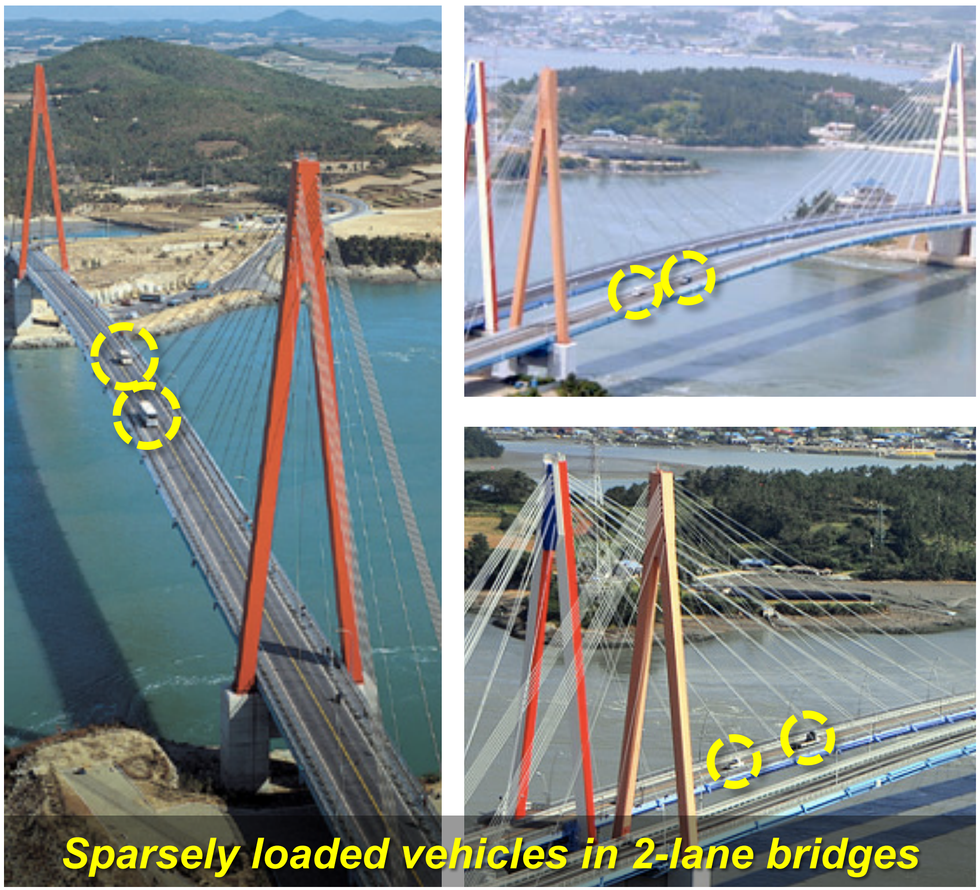
Parameter (3): Size of Hankel Matrix

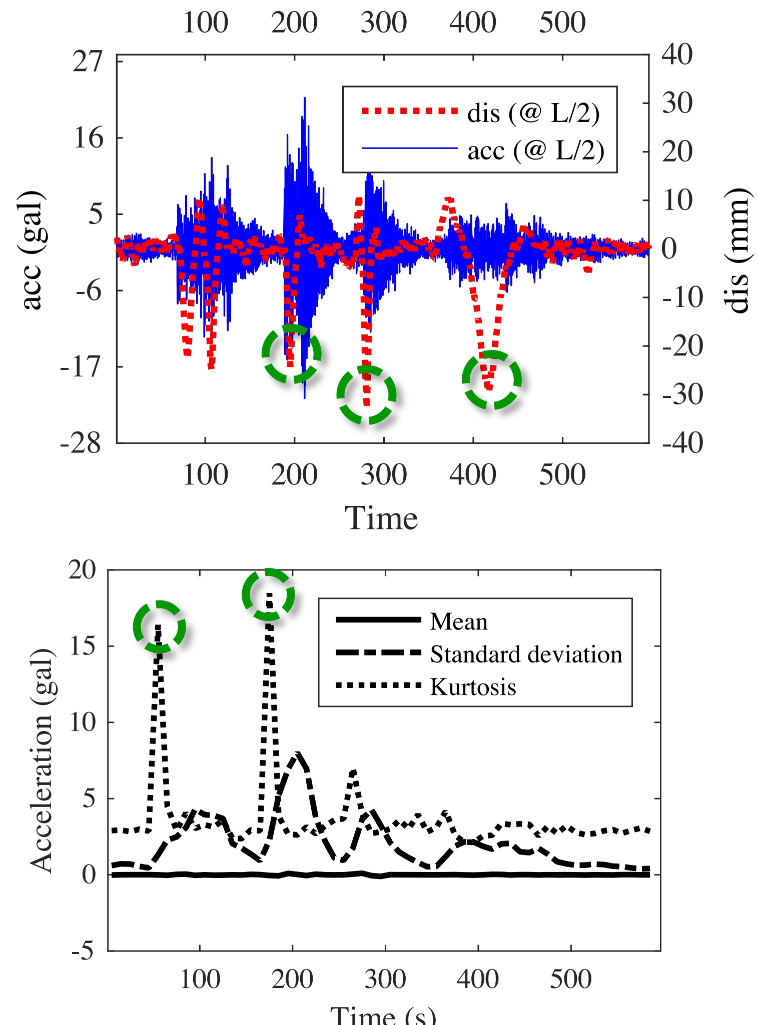
- **C.O.V of 100 cases simulation**
  - Data length: 3,000s / Nfft = 2^{14}



TIV characteristic (1): Nonstationarity

▶ **Nonstationarity in TIV**



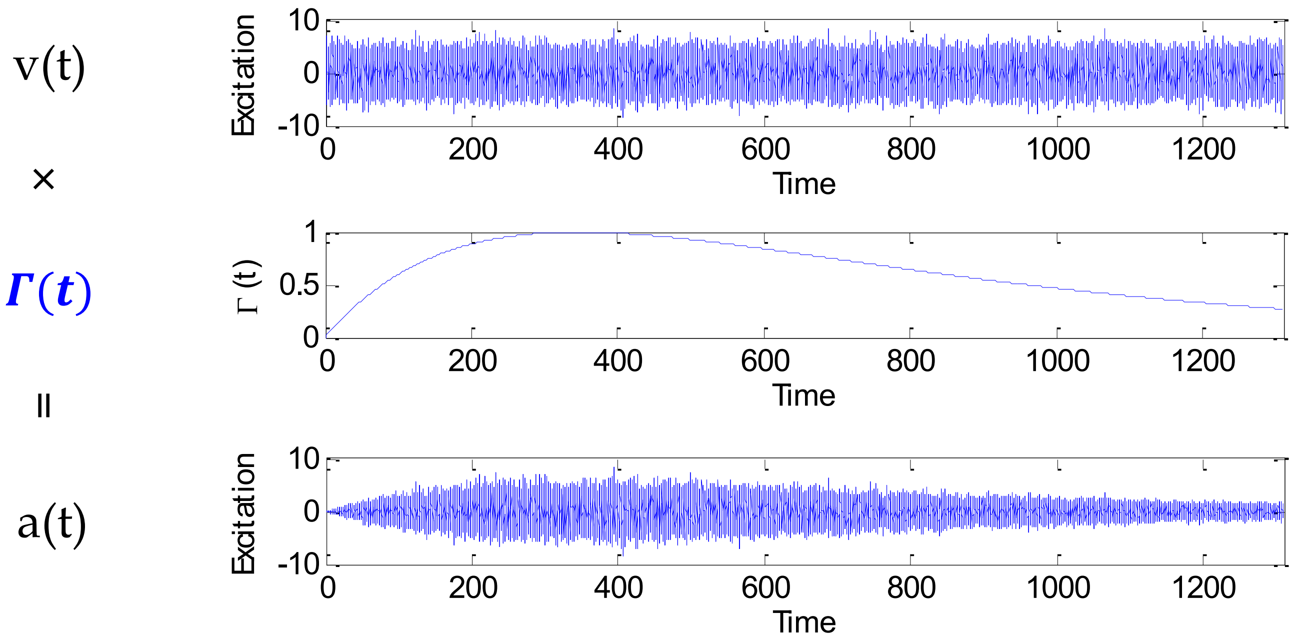


Signal stationarization

- **Amplitude modulating function (Chiang and Lin, 2008)**
  - Nonstationary process can be expressed as product form

$$a(t) = \Gamma(t)\ddot{v}(t)$$

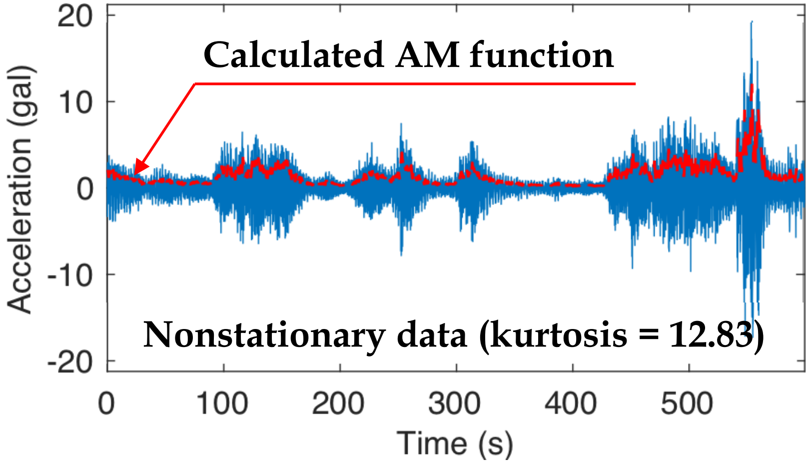
where $a(t)$ is the nonstationary measurement, $\Gamma(t)$ is amplitude-modulating (AM) function and $w(t)$ is stationary process.



Amplitude-modulating

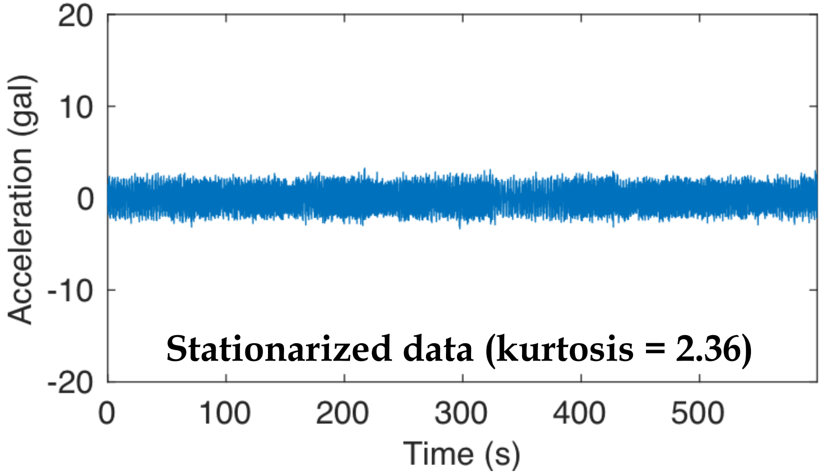
Calculate **amplitude-modulating (AM)** function by **temporal root mean square** of nonstationary acceleration

$$\Gamma(t) = C\sqrt{\frac{1}{T}\int_{t-T/2}^{t+T/2} a^2(\tau)d\tau}$$



Modulating of vehicle-induced acceleration for **approximated stationary acceleration**

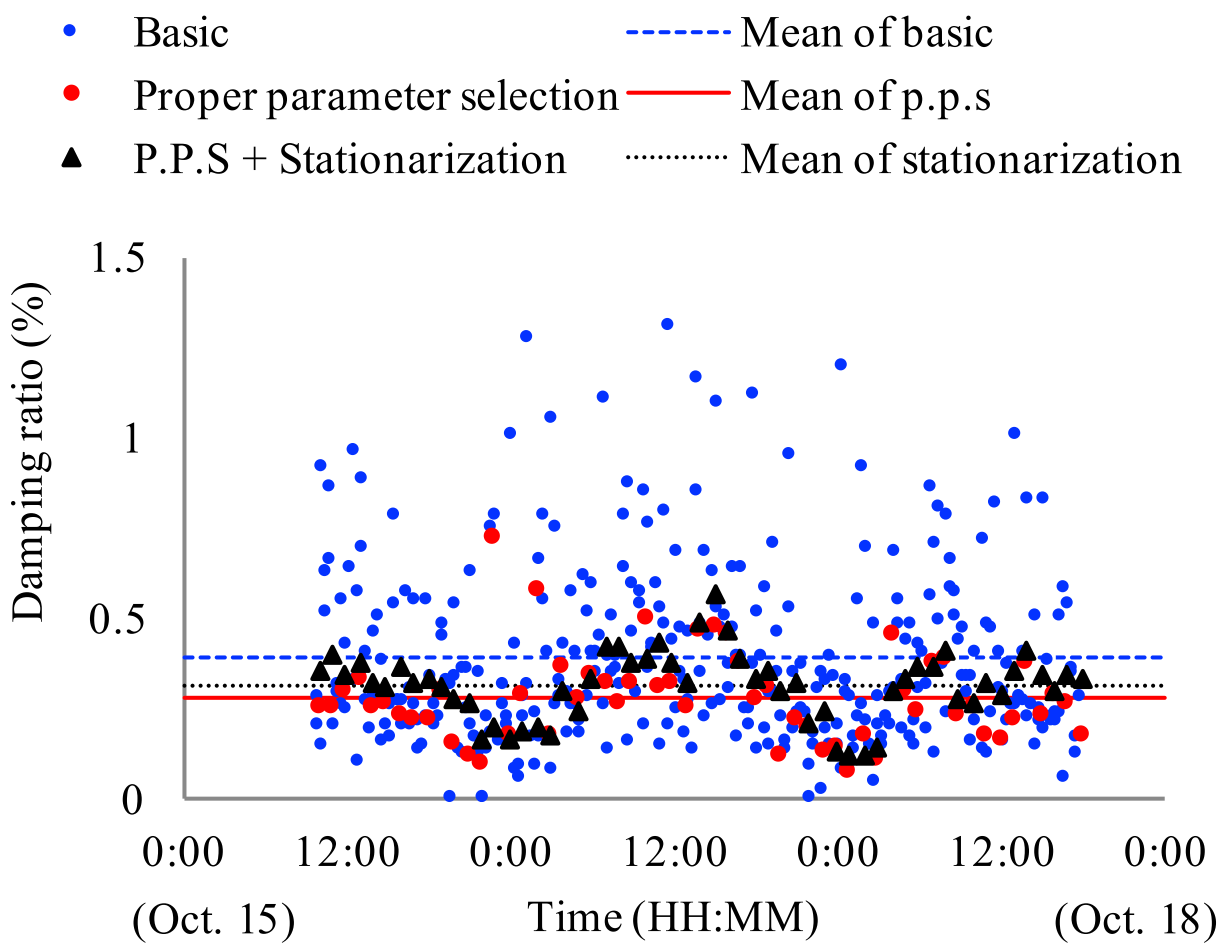
$$a(t) \simeq \Gamma(t)\hat{a}(t) \;\rightarrow\; \hat{\mathbf{a}}(\mathbf{t}) \simeq \mathbf{a}(\mathbf{t})/\boldsymbol{\Gamma}(\mathbf{t})$$



Chiang and Lin (2008). Identification of modal parameters from nonstationary ambient vibration data using correlation technique. AIAA journal, 46(11), 2752-2759.

Application to damping estimation

▶ **Estimated damping ratio according to signal stationarization**



Thanks for your attention!

I welcome your questions, ideas, suggestion and comments!

KSJ (fanta909@snu.ac.kr)