Engineering Economic Analysis

2019 SPRING

Prof. D. J. LEE, SNU

Chap. 22

COST CURVES

Average cost & Marginal cost

- Total cost function can be derived from the cost min. problem: $c(w_1,\ldots,w_n, y)$
- Various costs can be defined based on the total cost function
  - variable cost, fixed cost,
  - average cost, marginal cost,
  - long-run cost, short-run cost

Average cost & Marginal cost

- Let x_f : vector of fixed inputs, x_v : vector of variable factors

  $w = \left(w_v, w_f\right)$

- Short-run cost function

  $$c\left(w, y, x_f\right) = w_v \cdot x_v\left(w, y, x_f\right) + w_f \cdot x_f$$

  - Short-run average cost (SAC): $\dfrac{c\left(w, y, x_f\right)}{y}$
  - Short-run average variable cost (SAVC): $\dfrac{w_v \cdot x_v\left(w, y, x_f\right)}{y}$
  - Short-run average fixed cost (SAFC): $\dfrac{w_f \cdot x_f}{y}$
  - Short-run marginal cost (SMC): $\dfrac{\partial c\left(w, y, w_f\right)}{\partial y}$

Average cost & Marginal cost

- Long-run cost function

$$c(w,y) = w_v \cdot x_v(w,y) + w_f \cdot x_f(w,y)$$

There is no fixed input

  - Long-run average cost (LAC): $\dfrac{c(w,y)}{y}$
  - Long-run marginal cost (LMC): $\dfrac{\partial c(w,y)}{\partial y}$
  - Note that 'long-run average cost' equals 'long-run average variable cost' and 'long-run fixed costs' are zero'

Average cost & Marginal cost

- Example: Short-run Cobb-Douglas cost function
  - In a short-run, $x_2 = k$
  - Cost-min. problem: $\min \; w_1 x_1 + w_2 k$ $\; s.t. \;\; y = x_1^a k^{1-a}$ ⇒ $x_1 = k^{\frac{a-1}{a}} \cdot y^{\frac{1}{a}}$
  - SR Cost function: $c(w, y, k) = w_1 k^{\frac{a-1}{a}} \cdot y^{\frac{1}{a}} + w_2 k$

$$SAC = w_1 \left(\frac{y}{k}\right)^{\frac{1-a}{a}} + \frac{w_2 k}{y}$$

$$SAVC = w_1 \left(\frac{y}{k}\right)^{\frac{1-a}{a}}$$

$$SAFC = \frac{w_2 k}{y}$$

$$SMC = \frac{1}{a} w_1 \left(\frac{y}{k}\right)^{\frac{1-a}{a}}$$

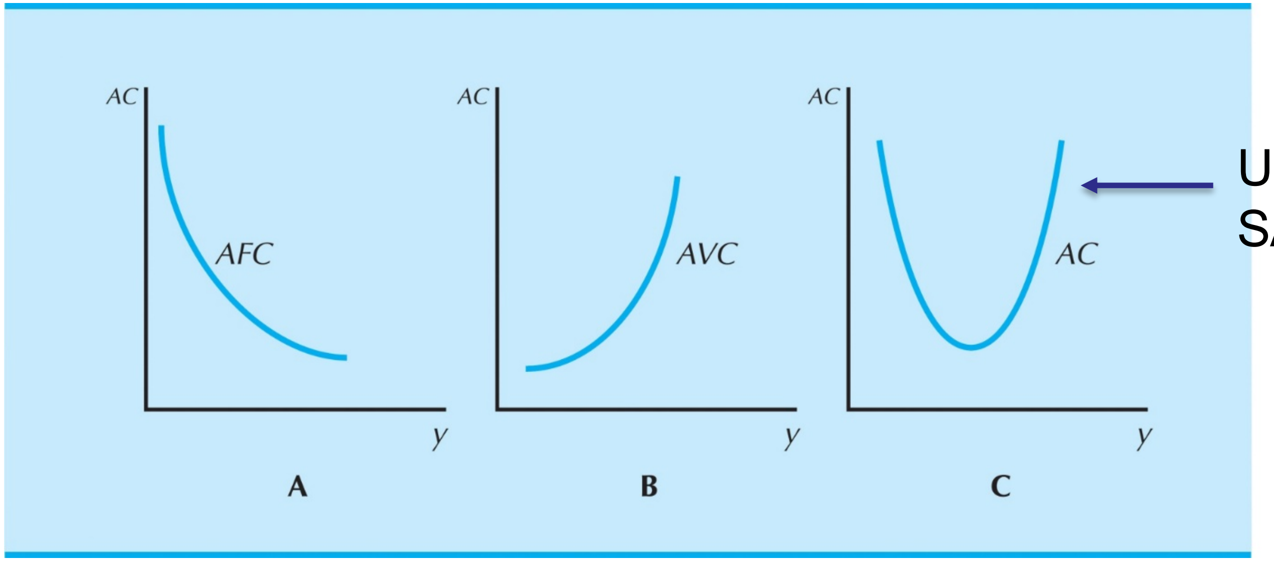
Cost curves (Geometry of costs)

- Total cost function can be derived from the cost min. problem: $c(w_1,\ldots,w_n, y)$
- Assume that the factor prices to be fixed, then $c(y)$.
- Total cost, $c(y)$, is assumed to be monotonic in y.
- Various cost curves: Average cost curve, Marginal cost curve, LR cost curve, SR cost curve etc.
- How are these cost curves related to each other?

Cost curves (Geometry of costs)

- SR Average cost
  - SR total cost = VC + FC: $c(y) = c_v(y) + F$
  - SAC = SAVC + SAFC

$$\frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} = \frac{w_v \cdot x_v(w, y, x_f)}{y} + \frac{w_f \cdot x_f}{y}$$



U-shaped SAC

Cost curves (Geometry of costs)

- Marginal cost vs. Average cost

$$\frac{d\mathrm{AC}(y)}{dy} = \frac{d}{dy}\left(\frac{c(y)}{y}\right) = \frac{c'(y)y - c(y)}{y^2}$$

$$= \frac{1}{y}\left(c'(y) - \frac{c(y)}{y}\right) = \frac{1}{y}\left(\mathrm{MC}(y) - \mathrm{AC}(y)\right)$$

- Thus, AC is increasing $(\mathrm{AC}'(y) > 0) \Leftrightarrow MC(y) > AC(y)$

  AC is decreasing $(\mathrm{AC}'(y) < 0) \Leftrightarrow MC(y) < AC(y)$

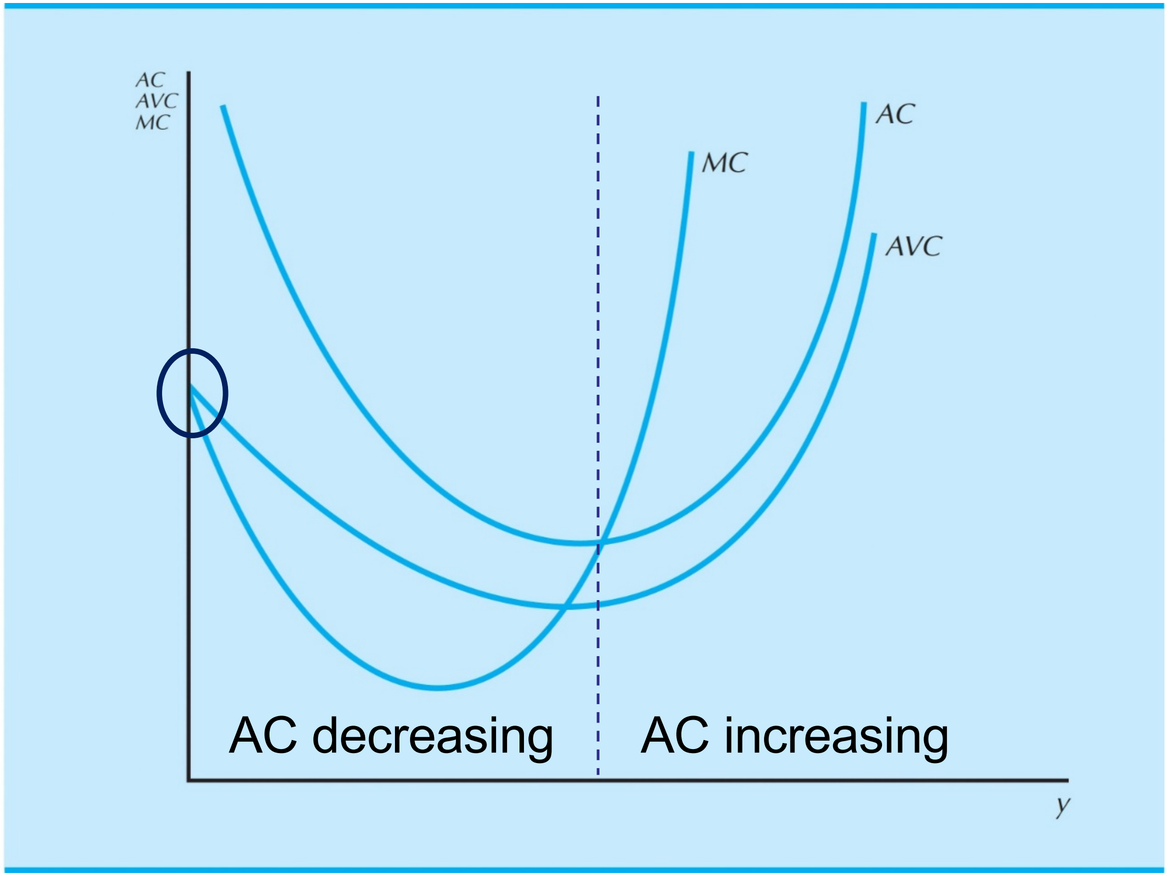
  AC is minimum $(\mathrm{AC}'(y) = 0) \Leftrightarrow MC(y) = AC(y)$

- MC for the first small unit of amount equals AVC for a single unit of output

$$MC(1) = \frac{TC(1) - TC(0)}{1} = \frac{c_v(1) + F - c_v(0) - F}{1} = \frac{c_v(1)}{1} = AVC(1)$$

Cost curves (Geometry of costs)

- Marginal cost vs. Average cost

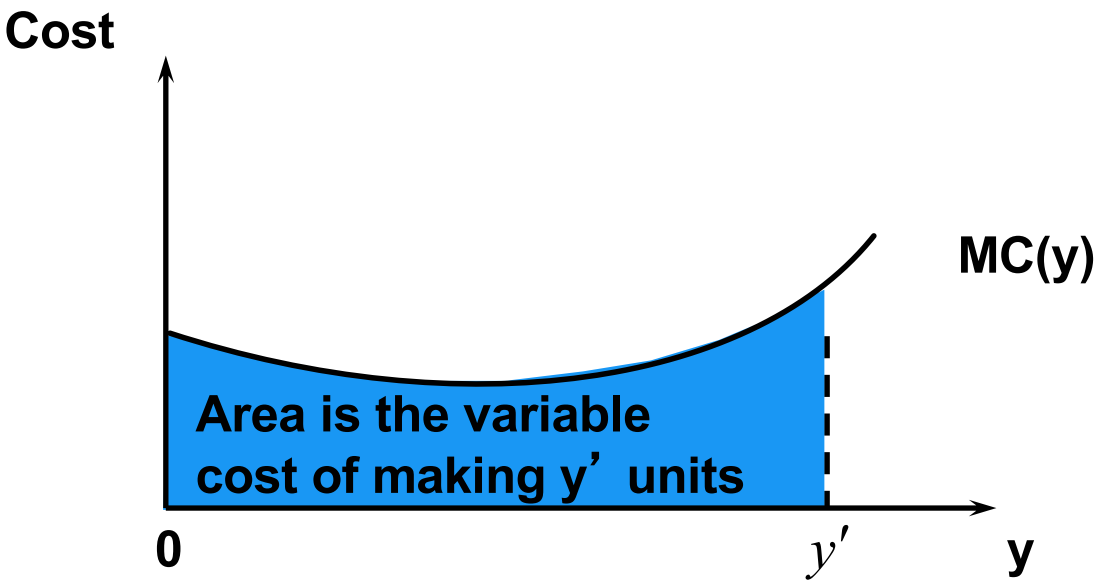


Cost curves (Geometry of costs)

- Marginal cost vs. Variable cost
  - Since

$$MC(y) = \frac{dc_v(y)}{dy} \Rightarrow c_v(y) = \int_0^y MC(z)dz$$

  - The area beneath the MC curve up to y gives us the variable cost of producing y units of output



Cost curves (Geometry of costs)

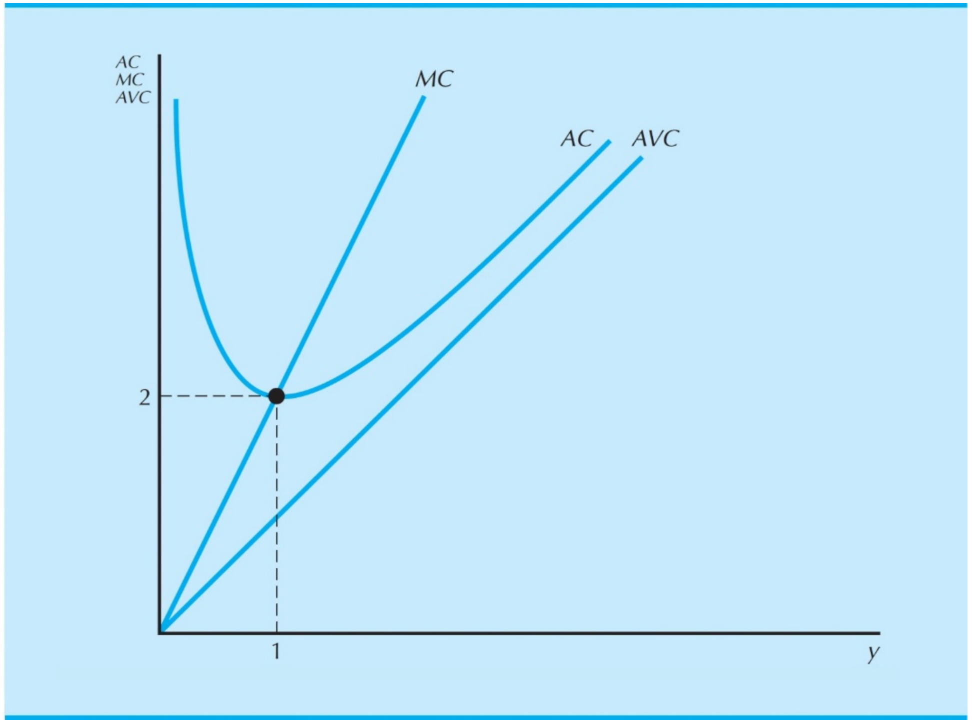
- Example
  - SR total cost function: $c(y) = y^2 + 1$
  - Variable cost: $c_v(y) = y^2$
  - Fixed cost: $c_f(y) = 1$

$$\mathrm{AVC}(y) = y^2 / y = y$$

$$\mathrm{AFC}(y) = 1 / y$$

$$\mathrm{AC}(y) = y + 1 / y$$

$$\mathrm{MC}(y) = 2y$$



Cost curves (Geometry of costs)

- Example: C-D Technology
  - Recall that $$c(w_1,w_2,y)=A^{\frac{-1}{a+b}}\left[\left(\frac{a}{b}\right)^{\frac{b}{a+b}}+\left(\frac{a}{b}\right)^{\frac{-a}{a+b}}\right]w_1^{\frac{a}{a+b}}w_2^{\frac{b}{a+b}}y^{\frac{1}{a+b}}$$
  - For a fixed w_1, w_2, $c(y)=Ky^{\frac{1}{a+b}},\ a+b\le 1$

$$AC(y)=Ky^{\frac{1-a-b}{a+b}}$$

$$MC(y)=\frac{K}{a+b}y^{\frac{1-a-b}{a+b}}$$

  - In the short-run, recall that $c(w,y,k)=w_1k^{\frac{a-1}{a}}\cdot y^{\frac{1}{a}}+w_2k$

$$c(y)=Ky^{\frac{1}{a}}+F$$

$$AC(y)=Ky^{\frac{1-a}{a}}+\frac{F}{y}$$

Cost curves (Geometry of costs)

- Example: MC curves for two plant

  - How much should you produce in each plant?

$$\min_{\{y_1, y_2\}} c_1(y_1) + c_2(y_2) \quad s.t. \quad y_1 + y_2 = \bar{y}$$

$$y_2 = \bar{y} - y_1 \;\Rightarrow\; \min_{y_1} c_1(y_1) + c_2(\bar{y} - y_1)$$

$$\frac{\partial c}{\partial y_1} = \frac{dc_1(y_1)}{dy_1} + \frac{dc_2(y_2)}{dy_2}\frac{dy_2}{dy_1} = 0 \quad \text{and} \quad \frac{dy_2}{dy_1} = -1$$

  - Therefore, the optimality condition is

$$MC_1(y_1^*) = MC_2(y_2^*)$$

Long-run vs. Short-run Cost Curves

- In the long run, all inputs are variable
  - LR problem: Planning the type and scale investment
  - SR problem: Optimal operation
- Given a fixed factor: Plant size k
  - SR cost function: $c_s(y,k)$
  - In LR, let the optimal plant size to produce y be $k(y)$
    - ➢ firm's conditional factor demand for plant size!
- Then LR cost function is

$$c(y) = c_s(y, k(y))$$

How this looks graphically?

Long-run vs. Short-run Cost Curves

- For some given level of output y^*

  $\Rightarrow k^* = k(y^*)$: optimal plant size for y^*

  $\Rightarrow c_s(y, k^*)$: SR cost function for a given k^*

  $\Rightarrow c_s(y, k(y))$: LR cost function

- Since SR cost min problem is just a constrained version of the LR cost min problem, SR cost curve must be at least as large as the LR cost curve for all y

$$c(y) \le c_s(y, k^*) \text{ for all level of } y$$

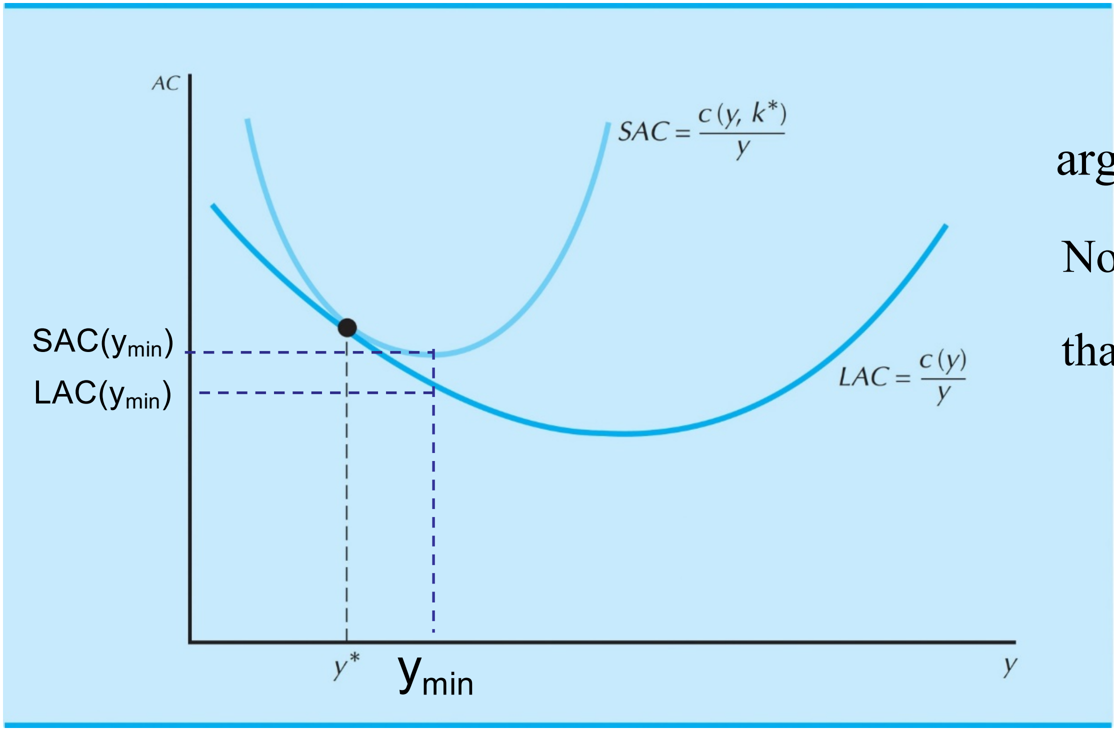
$$c(y^*) = c_s(y^*, k^*) \text{ when } k^* = k(y^*)$$

Long-run vs. Short-run Cost Curves

- Also $\text{LAC}(y) \leq \text{SAC}(y, k^*)$ for all level of y

  $\text{LAC}(y^*) = \text{SAC}(y^*, k^*)$ when $k^* = k(y^*)$

- Hence SR and LR cost curves must be tangent at y^*

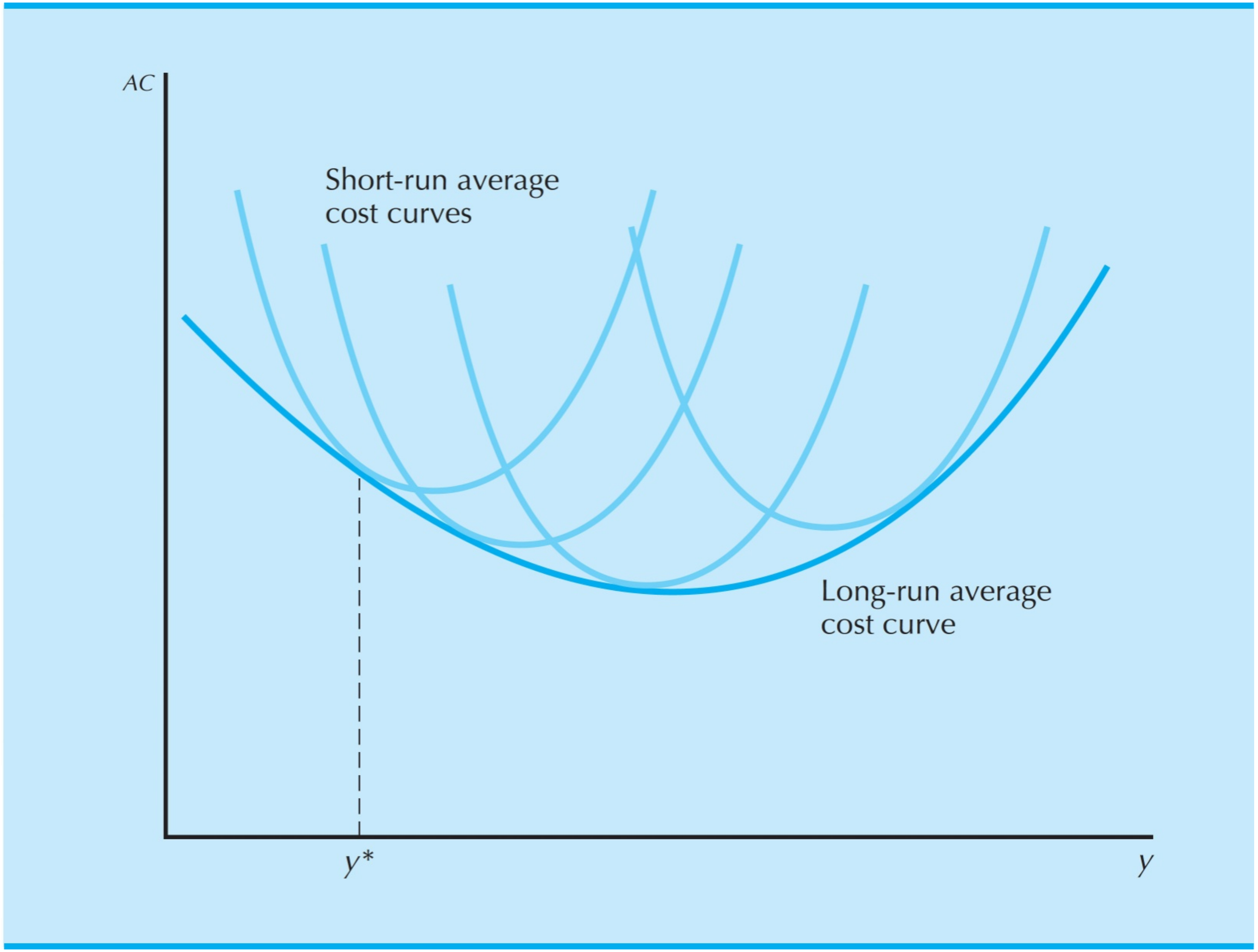


$\operatorname{argmin} \text{SAC}(y, k^*) = y_{\min}$

Note that $\text{LAC}(y_{\min}) \neq \text{SAC}(y_{\min}, k^*)$,

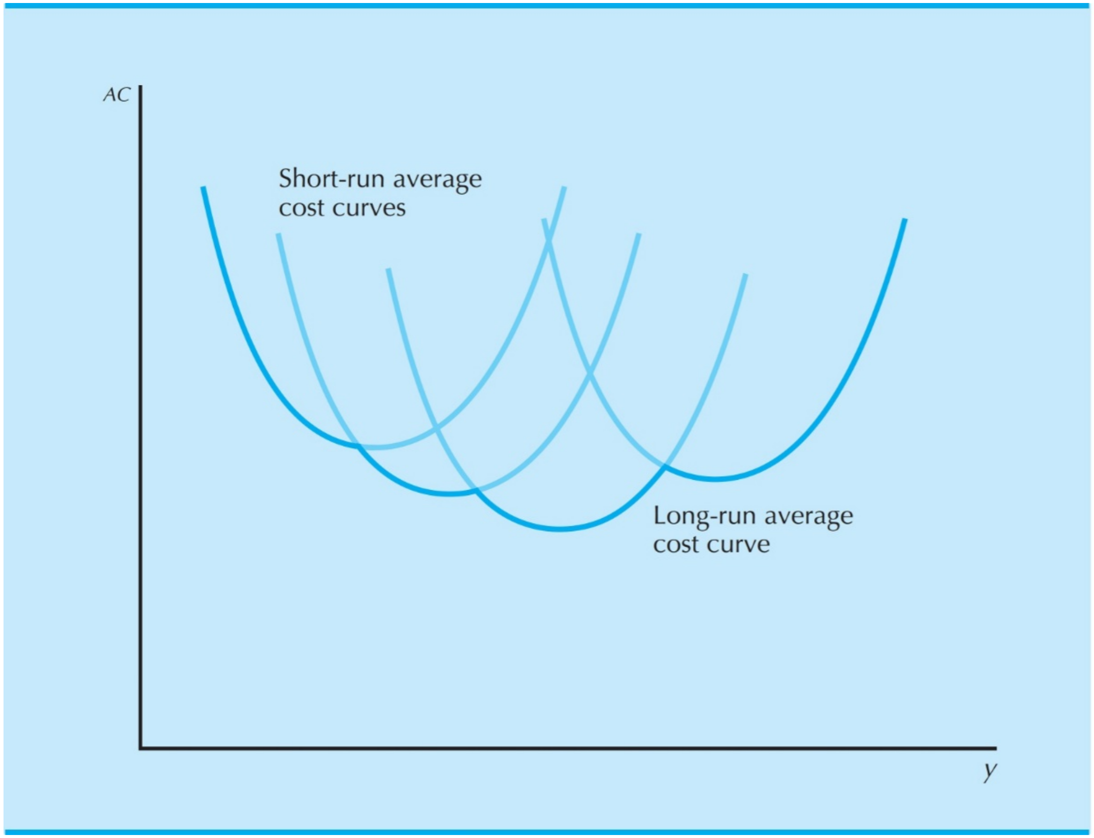
that is, $\text{LAC}(y_{\min}) \leq \text{SAC}(y_{\min}, k^*)$

Long-run vs. Short-run Cost Curves



Long-run vs. Short-run Cost Curves

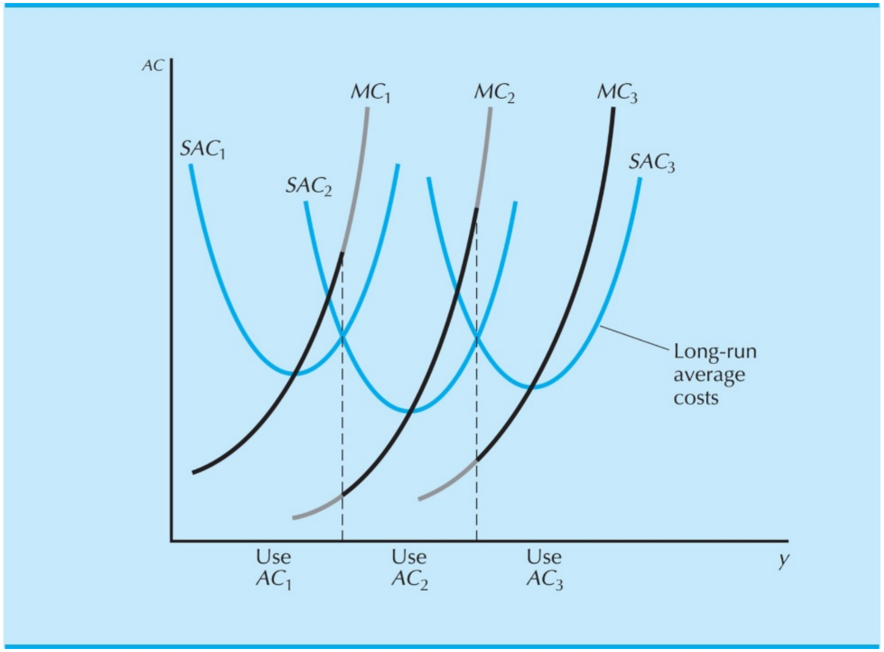
- Discrete levels of plant size: k_1, k_2, k_3, k_4



- LR average cost curve is the lower-envelop of SR average cost curves

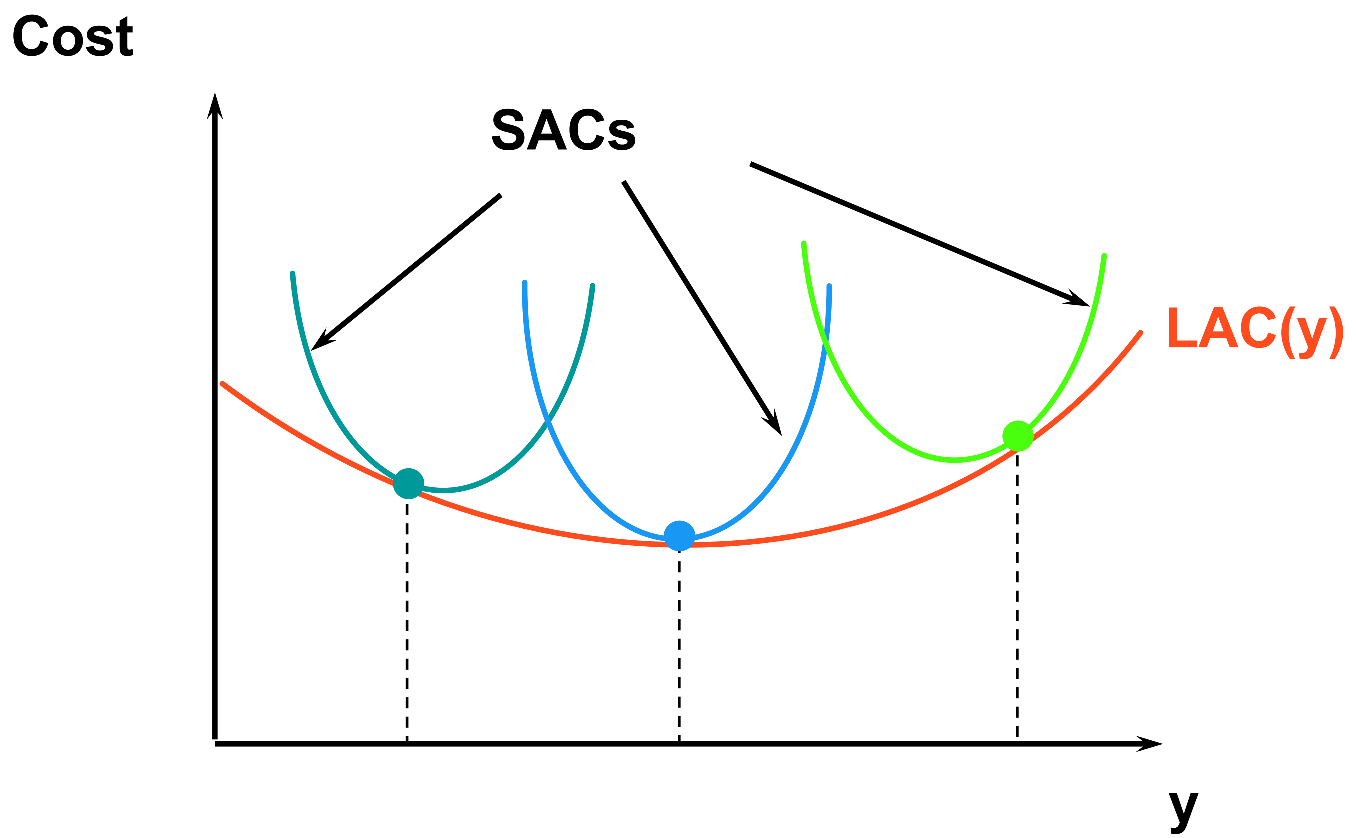
Long-run vs. Short-run Cost Curves

- LR marginal cost
  - When there are discrete levels of the fixed factor, the firm will choose the amount of the fixed factor to minimize costs.
  - Thus the LRMC curve will consist of the various segments of the SRMC curves associated with each different level of the fixed factor.

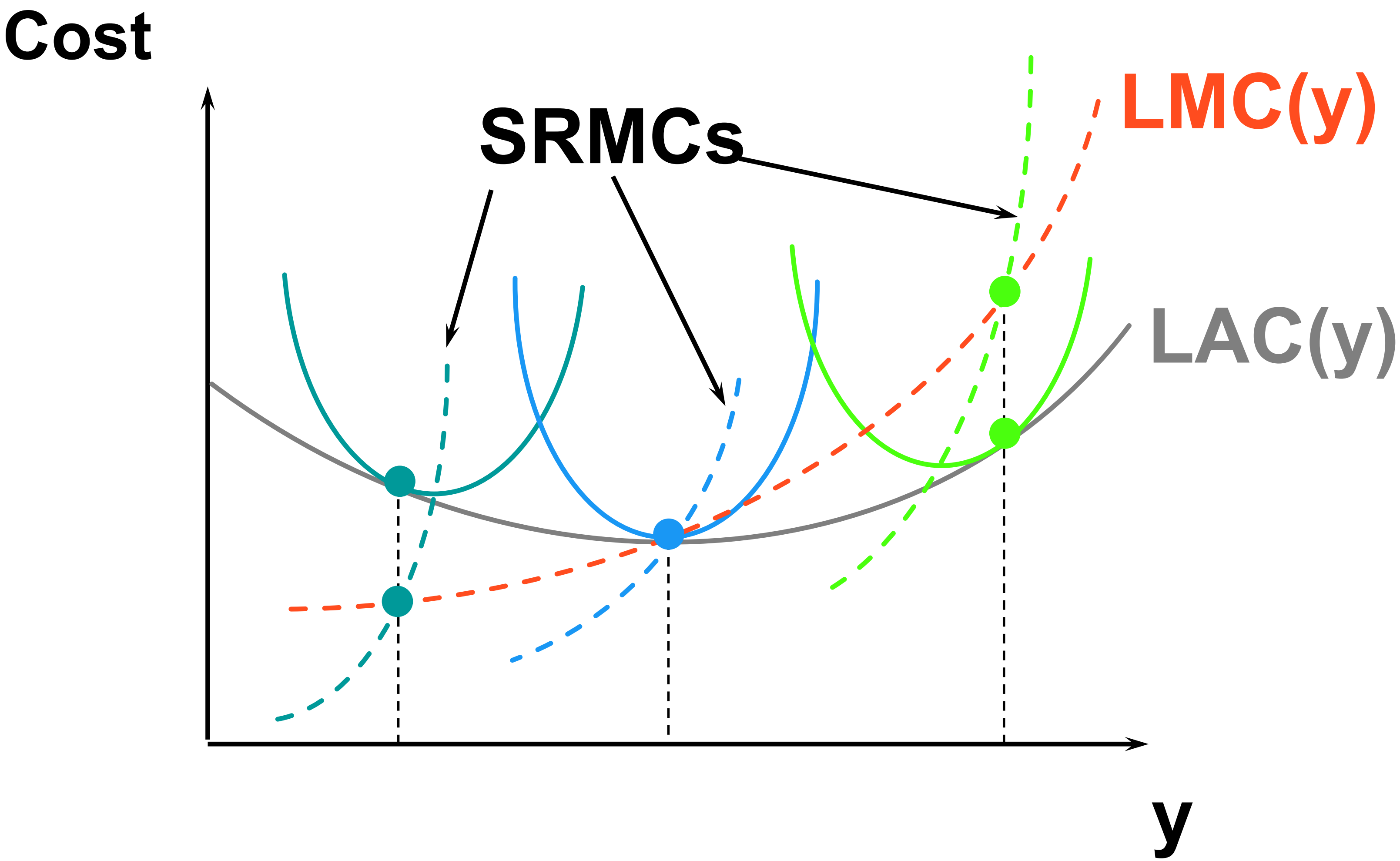


Long-run vs. Short-run Cost Curves

- LR marginal cost
  - This has to hold no matter how many different plant sizes there are !



Short-Run & Long-Run Marginal Cost Curves



Long-run vs. Short-run Cost Curves

- LR marginal cost
  - LR cost function

$$c(y) \equiv c_s(y, k(y))$$

  - Differentiating LR cost function w.r.t. y

$$\frac{dc(y)}{dy} = \frac{\partial c_s(y,k)}{\partial y} + \frac{\partial c_s(y,k)}{\partial k}\frac{\partial k(y)}{\partial y}$$

  - Since k^* is the optimal at $y=y^*$,

$$\frac{\partial c_s(y^*,k^*)}{\partial k} = 0 \quad \Rightarrow \quad \frac{dc(y^*)}{dy} = \frac{\partial c_s(y^*,k^*)}{\partial y}$$

  - Thus LRMC at y^* equals to SR marginal cost at (k^*, y^*)