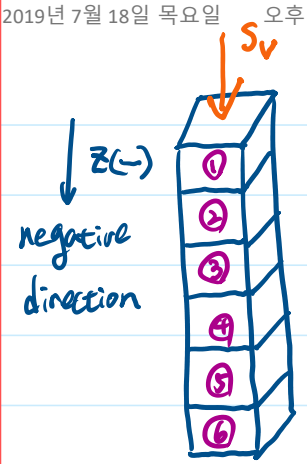
Formulation for a 1D Example

2019년 7월 18일 목요일 오후 8:02



Only z-direction is considered.

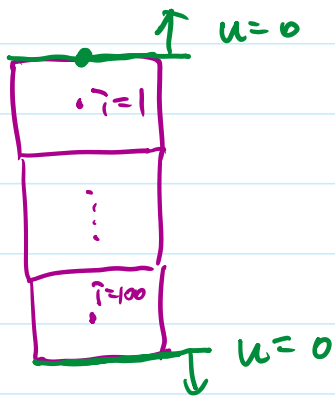
x-, y-directions are ignored.

$\Delta T = 0$ (isothermal)

Only $\underset{=\sigma}{\sigma_{zz}}$ and $\underset{\varepsilon}{\varepsilon_{zz}}$ are unknown.

bi-lateral constraints : $\varepsilon_{11} = \varepsilon_{22} = 0$

move only vertically



① Force Equilibrium Equation

$$\frac{dS}{dz} - \rho_b g = 0$$

② Strain and Displacement

$$\varepsilon = \frac{du}{dz}$$

③ Total and Effective Stresses

$$S = \sigma + \alpha P$$

④ Constitutive relation

$$\sigma = E\varepsilon$$

1 2 3

1 : $\sigma = E\dfrac{du}{dz}$

2 : $S = E\dfrac{du}{dz} + \alpha P$

3 : $E\dfrac{d^2u}{dz^2} + \alpha\dfrac{dP}{dz} + \rho_b g = 0$ Displacement equation

Describe the above equation verbally

When a gravitational force is given to a rock formation in a 1D domain (z-direction), the 2nd spatial derivative of the displacement can be expressed in terms of

the displacement can be expressed in terms of E, α, ρ_b, g, 1st spatial derivate of pore pressure.

Approximation of the derivates

$$\frac{d^2u}{dz^2} = \frac{\frac{u_{i+1}-u_i}{\Delta z} - \frac{u_i - u_{i-1}}{\Delta z}}{\Delta z}$$

$$= \frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta z^2}$$

$$\frac{dp}{dz} = \frac{p_{i+1} - p_i}{\Delta z}$$

$$E\,\frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta z^2} + \alpha\,\frac{p_{i+1} - p_i}{\Delta z} - \rho_b g = 0$$

$$u_{i-1} - 2u_i + u_{i+1} = -\frac{\alpha \Delta z}{E}(p_{i+1} - p_i) + \frac{\Delta z^2 \rho_b g}{E}$$

① Calculate pore pressures in flow simulation
② Calculate displacements
③ " bulk volumes
④ " ΔS_m
⑤ " S_v
⑥ " σ
⑦ " k
⑧ " ϕ_n^*
⑨ " ε_v
⑩ " C_p, C_ϕ
⑪ " ϕ_{n+1}^*
⑫ Update k and ϕ in flow simulation