Chap 10. Techniques of Irregular Wave Analysis

10.1 Statistical Quantities of Wave Data

10.1.1 Analysis of Analogue Data

Read text for Tucker's method to calculate η_{rms}.

10.1.2 Analysis of Digital Data

(A) Data length and time interval of data sampling

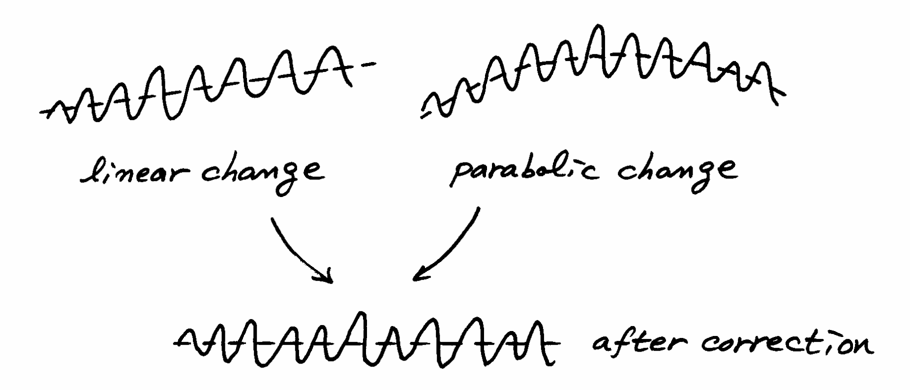
<u>Data length</u>

Field measurement: 20~30 min

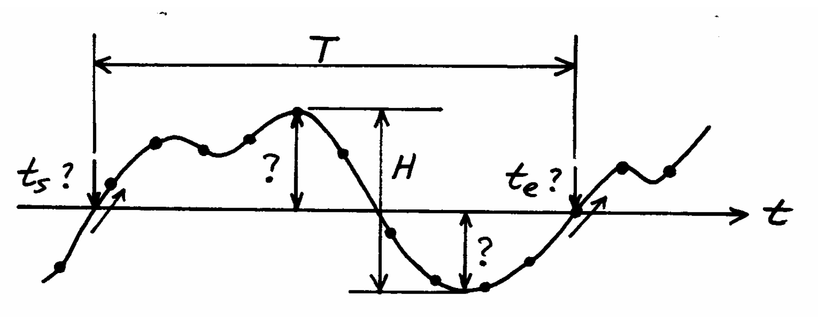
Laboratory: 200 waves or more

<u>Sampling interval</u>: 1/10 ~ 1/20 of T_s

(B) Correction of mean water level



(C) Analysis of zero-upcrossing points, maxima, and minima



(D) Calculation of correlation coefficient between wave height and period
Important for distribution of wave periods and wave grouping analysis

(E) Calculation of spectral parameters

(F) Frequency distribution of surface elevations, wave heights and periods

10.2 Frequency Spectrum of Irregular Waves

10.2.1 Theory of Spectral Analysis

Time series $\eta(t):\ \eta(\Delta t), \eta(2\Delta t), \cdots, \eta(N\Delta t)$

↓

$\boxed{\text{harmonic analysis}}$

↓

$$\eta(t_*) = \frac{A_0}{2} + \sum_{k=1}^{N/2-1}\left(A_k \cos\frac{2\pi k}{N} t_* + B_k \sin\frac{2\pi k}{N} t_*\right) + \frac{A_{N/2}}{2}\cos \pi t_*$$

where

$$t_* = t/\Delta t, \quad t_* = 1, 2, \cdots, N$$

$$A_k = \frac{2}{N}\sum_{t_*=1}^{N} \eta(t_*\Delta t)\cos\frac{2\pi k}{N} t_*, \quad 0 \le k \le N/2$$

$$B_k = \frac{2}{N}\sum_{t_*=1}^{N} \eta(t_*\Delta t)\sin\frac{2\pi k}{N} t_*, \quad 1 \le k \le N/2-1$$

↓

$\boxed{\text{Calculate periodogram}}$

↓

$$I_k = \begin{cases} N(A_k^2 + B_k^2), & 1 \le k \le N/2-1 \\ NA_0^2, & k = 0 \\ NA_{N/2}^2, & k = N/2 \end{cases}$$

↓

$\boxed{\text{Calculate spectral density}}$

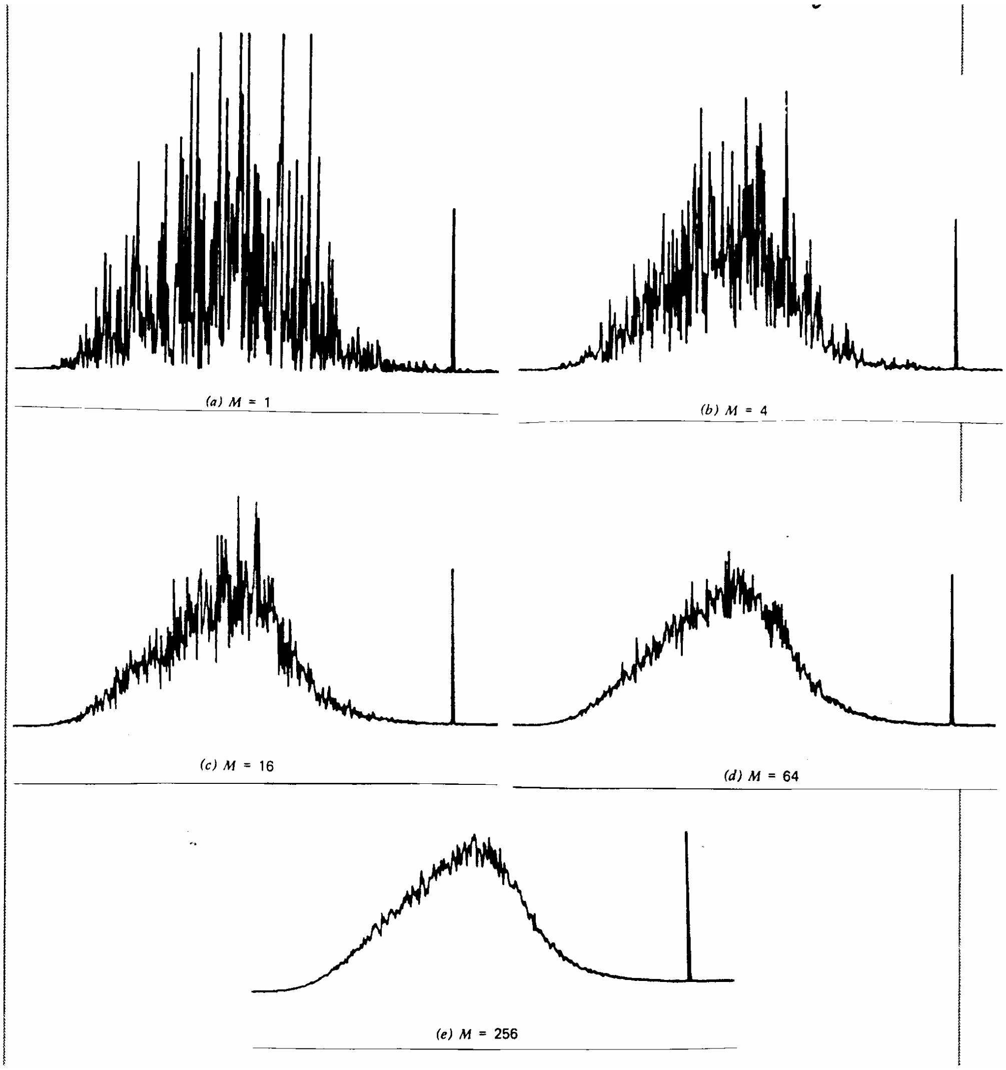
↓

$$E(I_k) = \frac{2}{\Delta t} S(f_k) \quad \rightarrow \quad S(f_k) = \frac{\Delta t}{2} E(I_k)$$

$$Var(I_k) = \frac{4}{(\Delta t)^2} S^2(f_k)$$

$$S.D.(I_k) = \sqrt{Var(I_k)} = E(I_k)$$

But I_k varies greatly, or $S(f_k)$ fluctuates greatly. Use autocorrelation method (Blackman and Turkey, 1958) or smoothed periodogram method (FFT method) to suppress the fluctuation, which uses smoothing over a certain frequency band, as in Eq. (10.32).



Ensemble averaging is also used (see Fig. 10.3):

2hr 30 min record $\rightarrow$ 5 × 30 min $\rightarrow$ $\hat{S}(f_k) = \frac{1}{5}\sum_{i=1}^{5} S_i(f_k)$

10.2.2 Spectral Estimate with Smoothed Periodograms

(A) Record length and data sampling interval

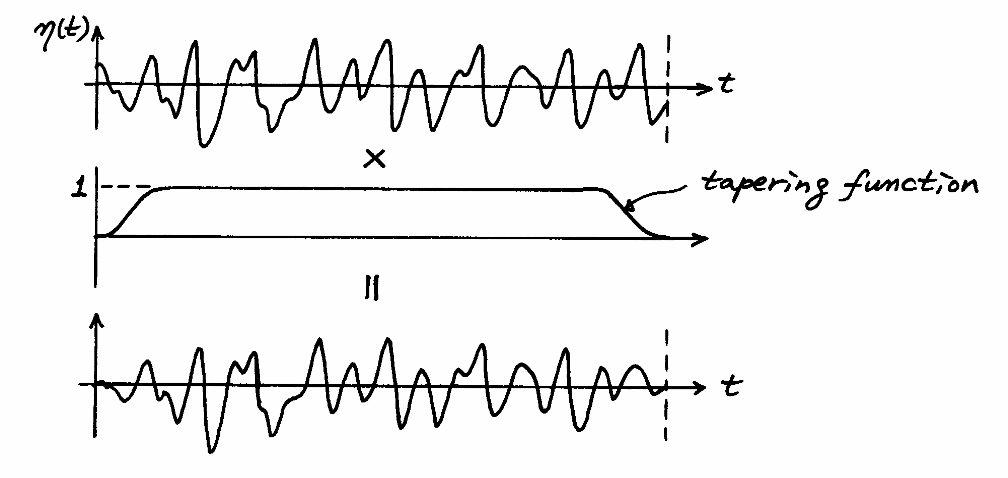
$\uparrow$ $\uparrow$

as long as possible $(1/10 \sim 1/20)T_s$

(B) Correction for mean water level

(C) Data window

Fourier analysis assumes periodicity of wave record, but in general $\eta(1) \neq \eta(N+1)$ $\rightarrow$ discontinuity $\rightarrow$ tapering is used to round off the discontinuity.



Correction is made to recover the energy loss at both ends.

(D) Computation of Fourier coefficients

FFT (fast Fourier transform) is used with $N = 2^m$.

(E) Calculation of periodogram

(F) Smoothing of periodogram

Different smoothing functions are available

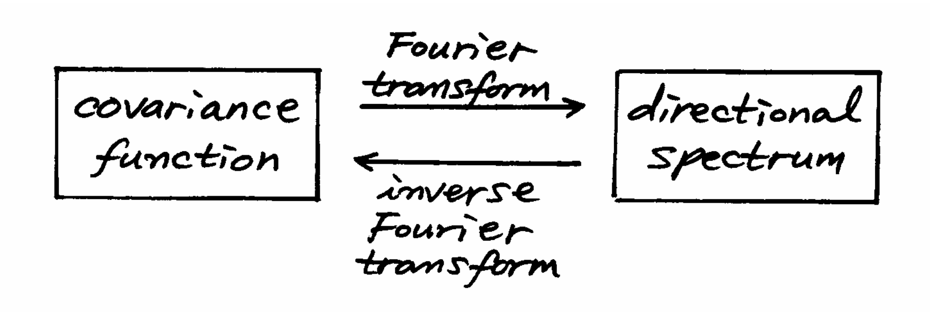
(G) Final adjustment of energy level

$$S(f) = S(f) \times \frac{\eta_{rms}^2}{m_0}$$

10.3 Directional Spectra of Random Sea Waves

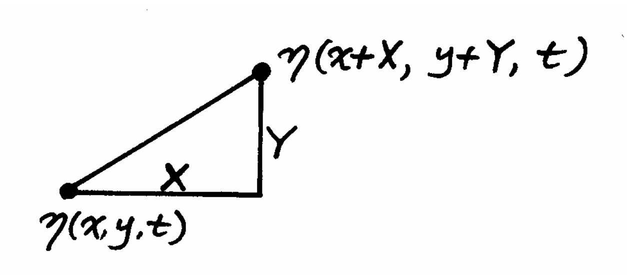
Directional spectra need simultaneous recording of several wave components at several locations, while frequency spectrum needs a single point measurement. There are direct methods and remote sensing methods for directional spectrum.

10.3.1 Relation between Directional Spectrum and Covariance Function



10.3.2 Estimate of Directional Spectra with a Wave Gauge Array

(A) Direct Fourier transform method



$$\Psi'(\tau \mid X,Y) = \frac{1}{T}\int_0^T \eta(t \mid x,y)\eta(t+\tau \mid x+X, y+Y)dt$$

$$\Psi'(\tau \mid X,Y) \quad \Leftrightarrow \quad \Phi_0(f \mid X,Y) = C_0 - iQ_0, \quad -\infty < f < \infty$$

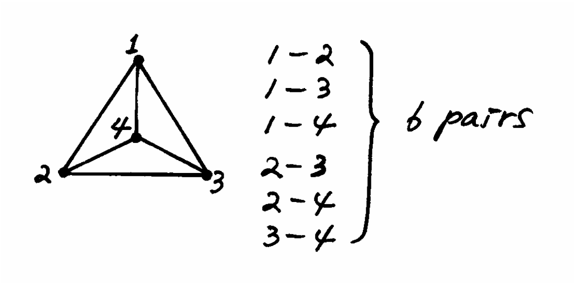
$$S_{k_0}(u,v \mid f_0) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi_0^*(X,Y \mid f_0)e^{-i(uX+vY)}dXdY$$

where $u = k\cos\theta$, $v = k\sin\theta$, and f_0 is fixed.

For a finite number of wave gauges, N,

$$\hat{S}_{k_0}(u,v\,|\,f_0) = \frac{1}{(2\pi)^2}\sum_{n=-M}^{M}\Phi_0^*(X_n,Y_n\,|\,f_0)e^{-i(uX_n+vY_n)}$$

where $M = N(N-1)/2$ = number of pairs of wave gauges, e.g., if $N = 4$, then $M = 6$.



In terms of real variables,

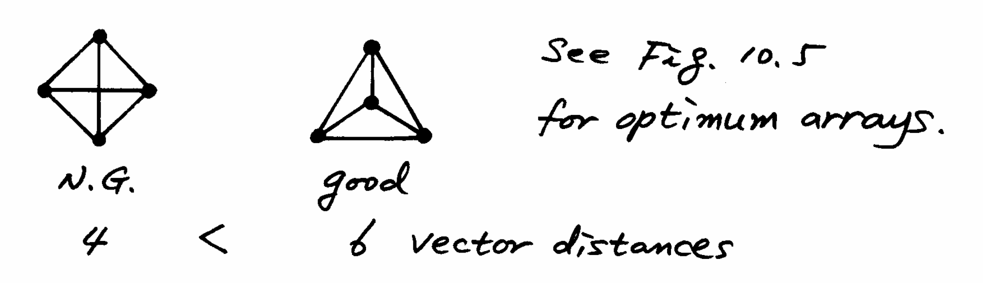
$$\hat{S}_{k_0}(u,v\,|\,f_0) = \frac{1}{(2\pi)^2}\Big\{C_0(0,0\,|\,f_0) + 2\sum_{n=1}^{M}[C_0(X_n,Y_n\,|\,f_0)\cos(uX_n+vY_n)$$
$$+ Q_0(X_n,Y_n\,|\,f_0)\sin(uX_n+vY_n)]\Big\}$$

$$S(f_0,\theta) = \underset{\substack{\uparrow\\(10.75)}}{S(f_0)}\,\underset{\substack{\uparrow\\(10.76)}}{G(\theta\,|\,f_0)}$$

(B) Maximum likelihood method

$G(\theta\,|\,f_0)$ is given by Eq. (10.79). Better directional resolution, see Fig. 10.4, but splitting of the peak in directional spreading function for relatively broad peaks.

(C) Layout of wave gauge arrays



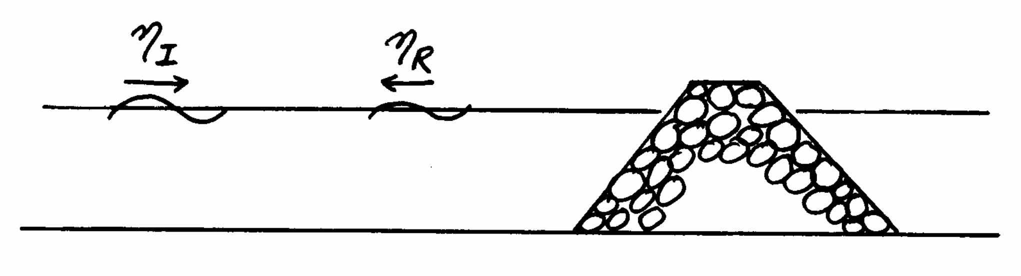
10.3.3 Estimate of Directional Wave Spectra with a Directional Buoy and with a Two-Axis Current Meter

Pitch-roll buoy

Measure η (heave), $\partial\eta/\partial x$ (pitch), $\partial\eta/\partial y$ (roll) at a fixed point.

10.4 Resolution of Incident and Reflected Waves of Irregular Profiles

10.4.1 Measurement of the Reflection Coefficient in a Wave Flume



Incident wave profile: $\eta_I(x,t) = \sum_{n=1}^{N} a_I^n \cos(k_n x - \omega_n t + \varepsilon_I^n)$

Reflected wave profile: $\eta_R(x,t) = \sum_{n=1}^{N} a_R^n \cos(k_n x + \omega_n t + \varepsilon_R^n)$

For each frequency, $\omega_n = 2\pi f_n$, and $\omega_n^2 = g k_n \tanh(k_n h)$

Measure $\eta(x,t) = \eta_I(x,t) + \eta_R(x,t)$ at $x = x_1$ and $x_2 = x_1 + \Delta l$

Measured time series: $\eta_1(t) = \eta(x_1,t) = \eta_I(x_1,t) + \eta_R(x_1,t)$

$$\eta_2(t) = \eta(x_2,t) = \eta_I(x_2,t) + \eta_R(x_2,t)$$

Using FFT, $\eta_1(t) = \sum_{n=1}^{N} \left(A_1^n \cos\omega_n t + B_1^n \sin\omega_n t \right)$

$$\eta_2(t) = \sum_{n=1}^{N} \left(A_2^n \cos\omega_n t + B_2^n \sin\omega_n t \right)$$

Need to satisfy $\eta_1(t) = \eta_I(x_1,t) + \eta_R(x_1,t)$ and $\eta_2(t) = \eta_I(x_2,t) + \eta_R(x_2,t)$ for each frequency ω_n for any t.

$$\begin{aligned}
\eta_1(t) &= \sum_{n=1}^{N} a_I^n \cos(k_n x_1 - \omega_n t + \varepsilon_I^n) + \sum_{n=1}^{N} a_R^n \cos(k_n x_1 + \omega_n t + \varepsilon_R^n) \\
&= \sum_{n=1}^{N} a_I^n \left\{ \cos(k_n x_1 + \varepsilon_I^n)\cos\omega_n t + \sin(k_n x_1 + \varepsilon_I^n)\sin\omega_n t \right\} \\
&\quad + \sum_{n=1}^{N} a_R^n \left\{ \cos(k_n x_1 + \varepsilon_R^n)\cos\omega_n t - \sin(k_n x_1 + \varepsilon_R^n)\sin\omega_n t \right\} \\
&= \sum_{n=1}^{N} [a_I^n \cos\phi_I^n + a_R^n \cos\phi_R^n]\cos\omega_n t + \sum_{n=1}^{N} [a_I^n \sin\phi_I^n - a_R^n \sin\phi_R^n]\sin\omega_n t
\end{aligned}$$

where $\phi_I^n = k_n x_1 + \varepsilon_I^n$, $\phi_R^n = k_n x_1 + \varepsilon_R^n$.

Therefore,

$$A_1^n = a_I^n \cos\phi_I^n + a_R^n \cos\phi_R^n\,, \qquad B_1^n = a_I^n \sin\phi_I^n - a_R^n \sin\phi_R^n$$

Similarly,

$$A_2^n = a_I^n \cos(\phi_I^n + k_n \Delta l) + a_R^n \cos(\phi_R^n + k_n \Delta l)$$

$$B_2^n = a_I^n \sin(\phi_I^n + k_n \Delta l) - a_R^n \sin(\phi_R^n + k_n \Delta l)$$

We have 4 equations for 4 unknowns, a_I^n, a_R^n, ϕ_I^n, ϕ_R^n, which are given by Eqs. (10.111) and (10.112).

Note that

$$a_I^n, a_R^n \propto \frac{1}{|\sin(k_n \Delta l)|} \qquad (10.111)$$

If $|\sin(k_n \Delta l)| \cong 0$, small errors will be amplified. Goda recommends

$$0.05 \cong \frac{\Delta l}{L_{\max}} \le \frac{\Delta l}{L_n} \le \frac{\Delta l}{L_{\min}} \cong 0.45\,, \qquad L_{\max} \ge L_n \ge L_{\min}$$

Since $k_n = 2\pi / L_n$,

$$k_{\min}\Delta l = 0.1\pi \le k_n \Delta l \le 0.9\pi = k_{\max}\Delta l$$

for which $|\sin(k_n \Delta l)| \ge 0.309$.

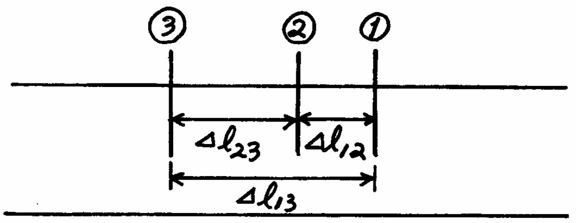
Find the effective range of resolution $f_{\min} \le f_n \le f_{\max}$ for given depth h, Δl = distance between two gauges using

$$(2\pi f_{\min})^2 = gk_{\min}\tanh(k_{\min}h) \quad \text{with} \quad k_{\min}\Delta l = 0.1\pi$$

$$(2\pi f_{\max})^2 = gk_{\max}\tanh(k_{\max}h) \quad \text{with} \quad k_{\max}\Delta l = 0.9\pi$$

Make sure almost all incident wave energy is in the effective range $f_{\min} \le f_n \le f_{\max}$ (see Fig. 10.6).

Reliability of measurements will increase if we use 3 wave gauges.



Gages 1 and 2: $(a_I^n)_{12}$, $(a_R^n)_{12}$ for $(f_{\min})_{12} \le f_n \le (f_{\max})_{12}$

Gages 2 and 3: $(a_I^n)_{23}$, $(a_R^n)_{23}$ for $(f_{\min})_{23} \le f_n \le (f_{\max})_{23}$

Gages 1 and 3: $(a_I^n)_{13}$, $(a_R^n)_{13}$ for $(f_{\min})_{13} \le f_n \le (f_{\max})_{13}$

may use average a_I^n, a_R^n
may increase the effective range for $f_{\min} \le f_n \le f_{\max}$

Overall reflection coefficient $K_R = \sqrt{\dfrac{E_R}{E_I}} = \sqrt{\dfrac{(m_0)_R}{(m_0)_I}} = \dfrac{(H_{m0})_R}{(H_{m0})_I}$

Goda suggests to use for any wave height H (e.g., $H_{1/3}$, $\overline{H}$, H_{rms})

$$H_I = \frac{1}{(1+K_R^2)^{1/2}} H_S, \qquad H_R = \frac{K_R}{(1+K_R^2)^{1/2}} H_S$$

where H_S = mean value of wave heights at two gages, that is

$$K_R = \frac{H_R}{H_I}, \qquad H_I^2 + H_R^2 = H_S^2$$