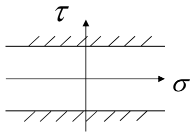
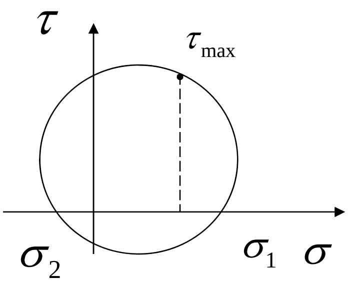
Chapter2. Yield Conditions

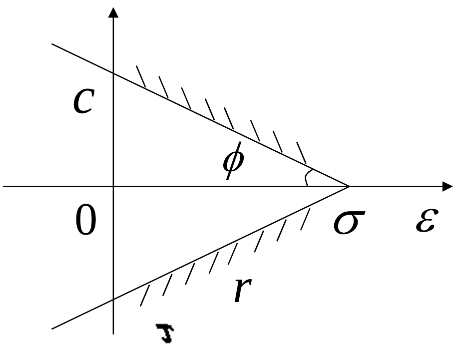
- Mohr-Column Criteria

  τ σ + tanφ– c=0

→ generalization of Tresca yield criteria → 



$$k = MAX\left(\frac{1}{2}|\sigma_1 - \sigma_2|,\ \frac{1}{2}|\sigma_2 - \sigma_3|,\ \frac{1}{2}|\sigma_3 - \sigma_1|\right)$$



$$|\tau| = c - \sigma \tan\phi = c - \mu\sigma$$

Where

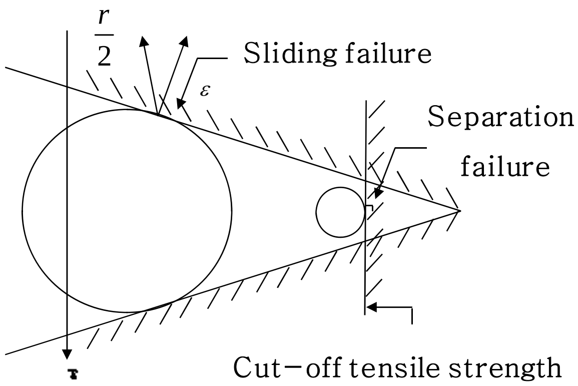
c = cohesion

μ = coefficient of friction

σ = tensile strength

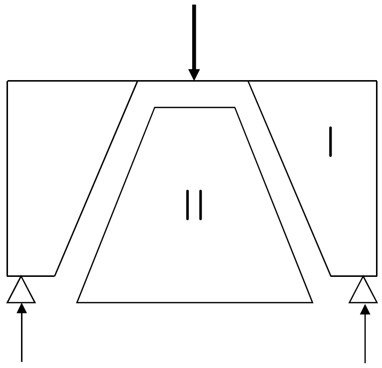
$\tan\phi$ = angle of friction

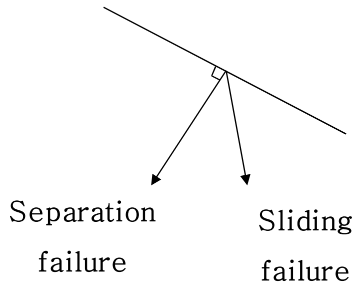
In concrete failure criteria is controlled by sliding failure



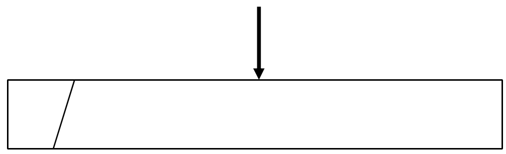
⟹ Modified coulomb criteria

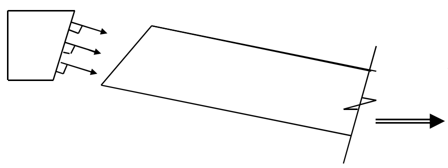
Deep beam





Shallow beam





균열면에 직각 (separation failure)
↓
Diagonal <u>tension</u> failure mode

• Failure mode of isotropic material

$$f\left(\sigma_1,\sigma_2,\sigma_3\right)=0$$

$$f\left(J_1,J_2,J_3\right)=0$$

where

σ_1 = the first invariant of the stress tensor σ_{ij}

J_1 = the second invariant of the deviatric stress tensor s_{ij}

Stress invariant $(\sigma_{ij}-\sigma\delta_{ij})n_j=0$

To find the principal stress

$$\left|\sigma_{ij} - \sigma\delta_{ij}\right| = 0$$

↑

determinant

$$\sigma^3 - I_1\sigma^2 + I_2\sigma \; - I_3 = 0$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z = \sigma_{ii}$$

$$I_2 = \left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x\right) - \tau_{xy}{}^2 - \tau_{yz}{}^2 - \tau_{zx}{}^2$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{yz} & \sigma_y & \tau_{yx} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix}$$

If the coordinate are chosen to coincide with the principal stress axis

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$

$$I_3 = \sigma_1\sigma_2\sigma_3$$

Deviation form

$$\sigma_{ij} = s_{ij} + \sigma_m\delta_{ij}$$

Where

$$\sigma_m = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) = \frac{1}{3}\sigma_{ij} = \frac{1}{3}I_1$$

$$s_{ij} = \sigma_{ij} - \sigma_m\delta_{ij}$$

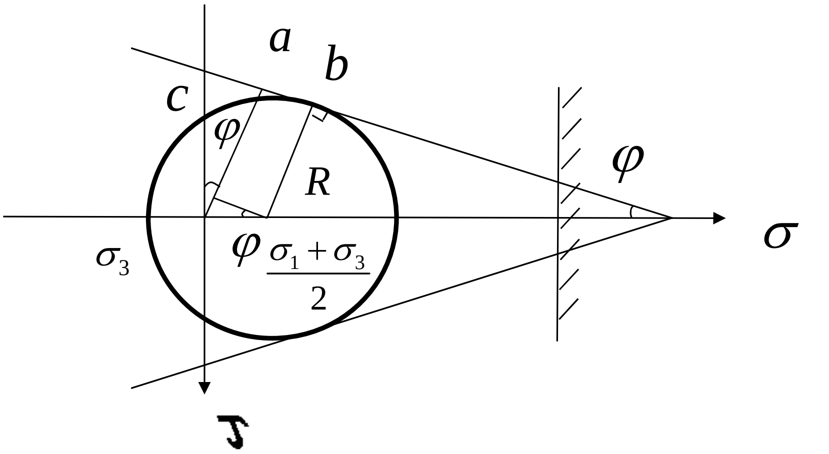
$$\left|s_{ij} - s\delta_{ij}\right| = 0$$

$$s^3 - J_1 s^3 - J_2 s - J_3 = 0$$

$$J_1 = 0$$

$$J_2 = \frac{1}{2}s_{ij}s_{ij} = \frac{1}{6}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2\right] + \tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2$$

$$J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki} = \begin{vmatrix} s_x & s_{xy} & s_{xz} \\ s_{yx} & s_y & s_{yz} \\ s_{zx} & s_{zy} & s_z \end{vmatrix}$$



$$R = c\cos\varphi - \frac{\sigma_1+\sigma_3}{2}\sin\varphi \quad \text{or} \quad R = \frac{\sigma_1-\sigma_3}{2}$$

$$\frac{\sigma_1-\sigma_3}{2} = c\cos\varphi - \frac{\sigma_1+\sigma_3}{2}\sin\varphi$$

$$\frac{\sigma_1}{2}\left(1+\sin\varphi\right) - \frac{\sigma_3}{2}\left(1-\sin\varphi\right) - c\cos\varphi = 0$$

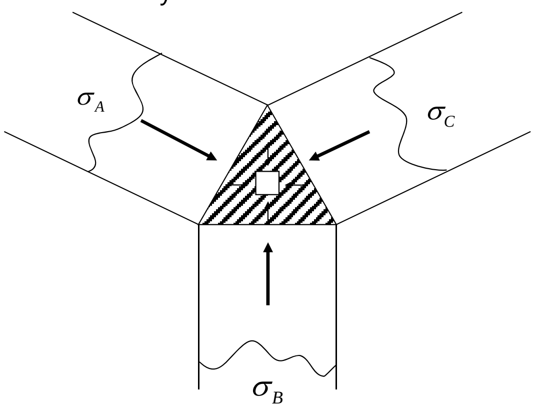
$$\frac{1+\sin\varphi}{1-\sin\varphi}\sigma_1 - \sigma_3 = \frac{2c}{1-\sin\varphi}\cos\varphi$$

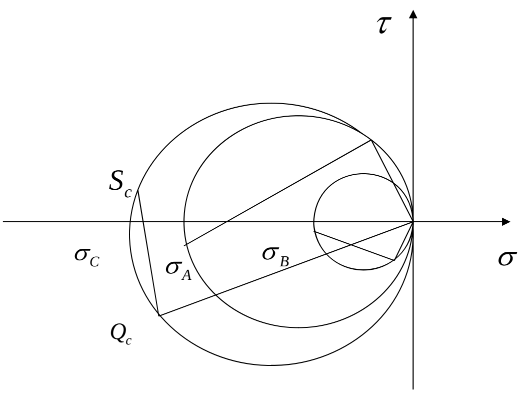
Let $k = \left(\frac{\cos\varphi}{1-\sin\varphi}\right)^2 = \tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) = \frac{1+\sin\varphi}{1-\sin\varphi}$

$\left(*\right) = k\sigma_1 - \sigma_3 = 2c\sqrt{k}$: sliding failure

At $\sigma_1 = f_A$:we expect separation failure

• discontinuity of stress fields



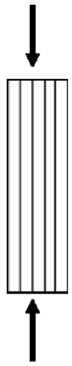


$k\sigma_1 - \sigma_3 = 2c\sqrt{k}$: sliding

$\sigma_1 = f_t$: separation

How to find k & c (or f_t)

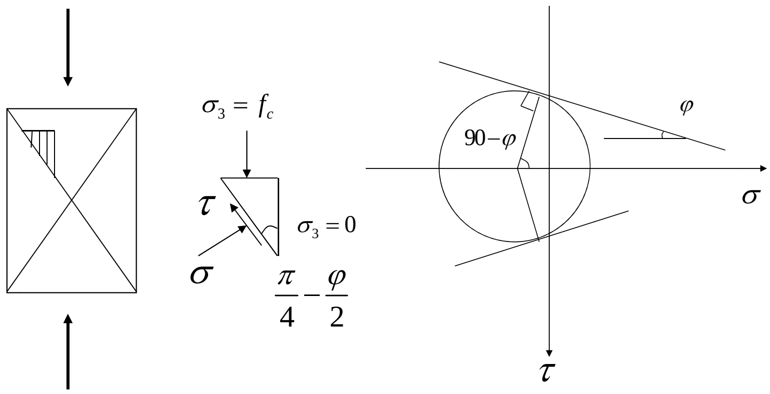
1) by compression test



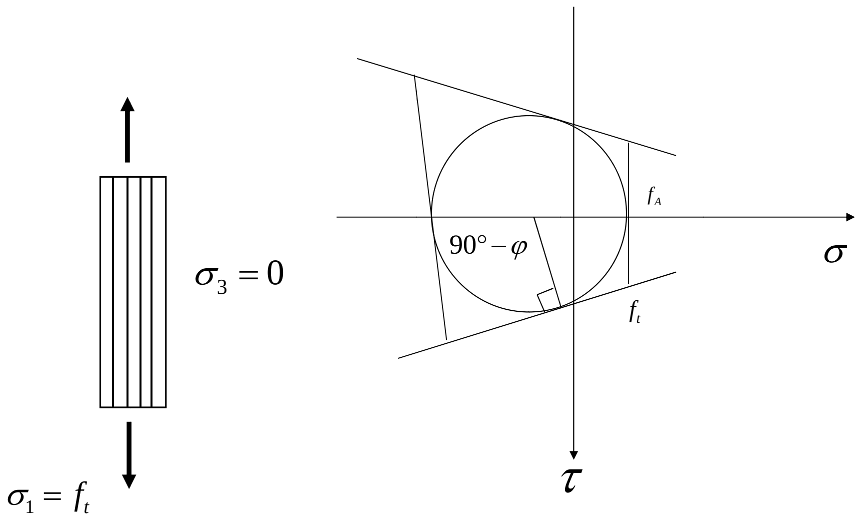
$k\sigma_1 - \sigma_3 = 2c\sqrt{k}$ - (1)

$\sigma_1 = 0$, $-\sigma_3 = f_c$ - (2)

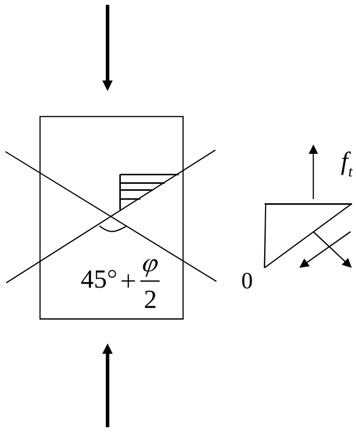
(2)→(1) $f_c = 2c\sqrt{k}$ (*)



2) by tension test



• by sliding failure



$\sigma_1 = f_t, \sigma_3 = 0$ -(3)

(3)→(1) and (*)

$$k\sigma_1 = kf_t = f_c \qquad \therefore k = \frac{f_c}{f_t}$$

• by separation failure

$$f_A = f_t$$

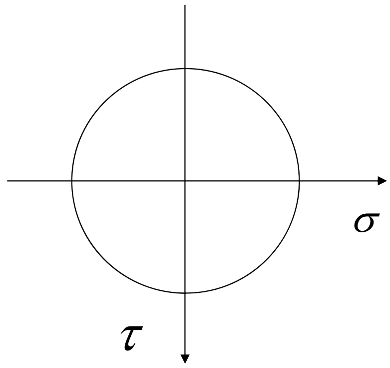
For sliding failure

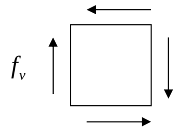
$$\frac{1}{k} f_c < f_A$$

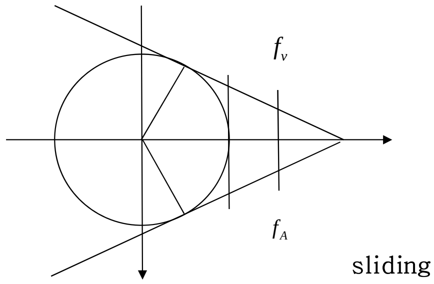
For separation failure

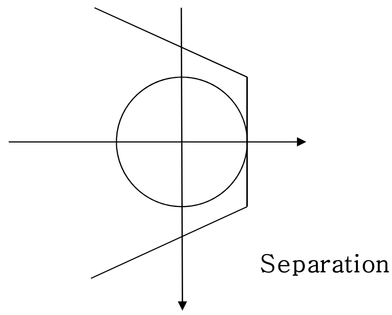
$$\frac{1}{k} f_c > f_A$$

3) by pure shear test









$$\sigma_1 = -\sigma_3 = f_v \quad \text{- (5)}$$

(5)→(1)

$$k\sigma_1 - \sigma_3 = (k+1)\sigma_1 = f_c$$

$$(k+1)\, f_v = f_c \rightarrow f_v = \frac{f_c}{k+1}$$

- For sliding failure if

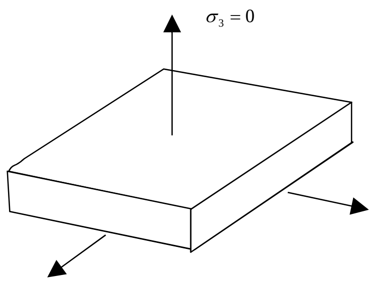
$$f_v < f_A$$

$$\frac{f_c}{k+1} < f_A$$

- For separation failure if

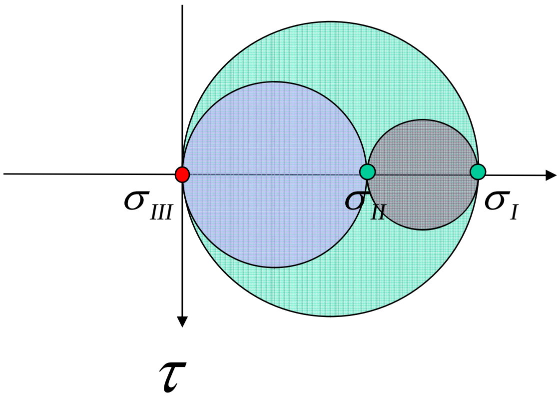
$$f_A < \frac{f_c}{k+1}$$

In plane stress (member stress)



$$\sigma_{III} = 0$$

1) $\sigma_{III} < \sigma_{II} < \sigma_{I}$



$\sigma_1 = \sigma_I, \ \sigma_3 = 0$

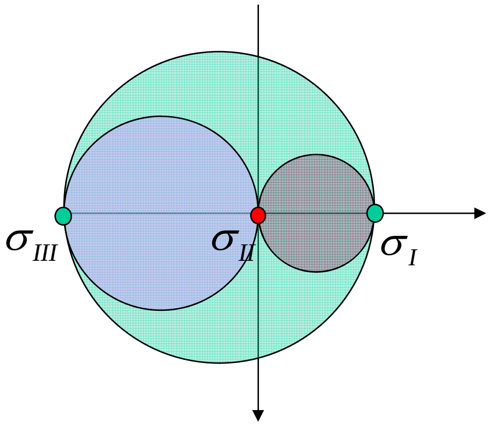
- For sliding failure

  $k\sigma_1 = f_c$ - ①

- For separation failure

  $\sigma_I = f_A$ - ②

2) $\sigma_{II} < 0 < \sigma_{I}$



$\sigma_1 = \sigma_I, \ \sigma_3 = \sigma_{II}$

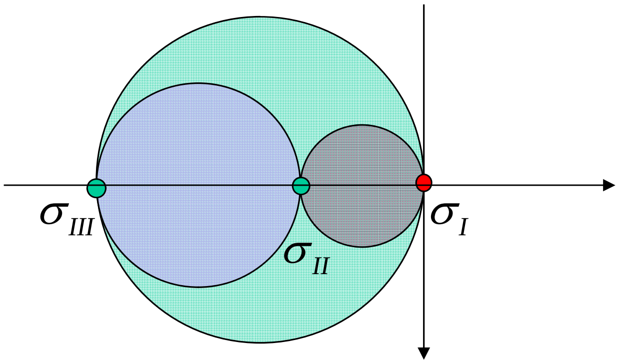
- For sliding failure

  $k\sigma_I - \sigma_{II} = f_c$ - ③

- For separation failure

  $\sigma_I < f_A$ - ④

2) $\sigma_{II} < 0 < \sigma_{I}$



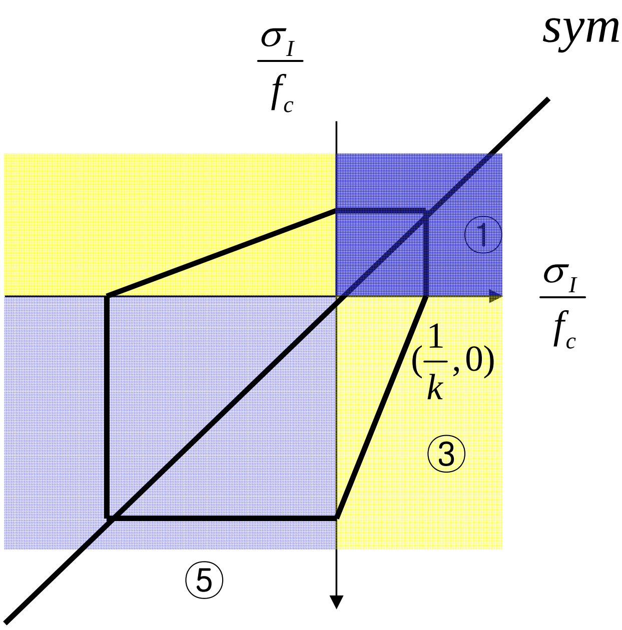
$\sigma_1 = 0$, $\sigma_3 = \sigma_{II}$

- For sliding failure

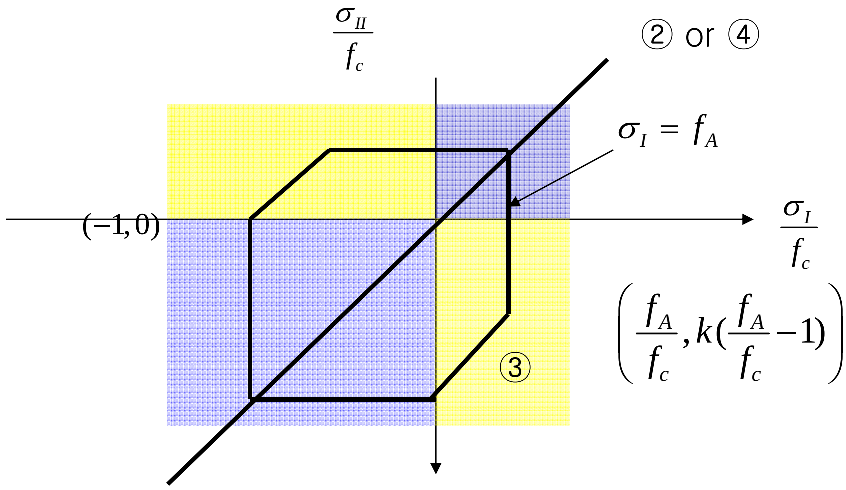
  $-\sigma_{III} = f_c$ - ⑤

- For separation failure

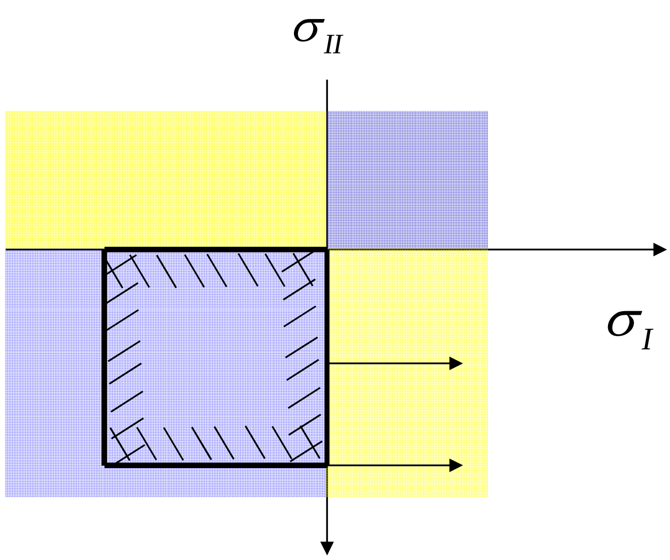
  Non exist



①,③,⑤ = sliding failure only



If $f_A = 0$ (즉, concrete의 tensile strength =0)

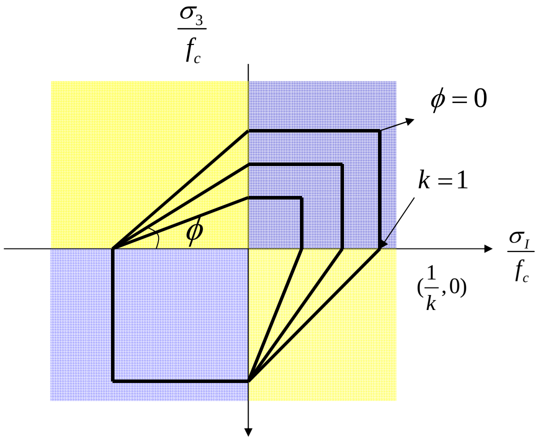


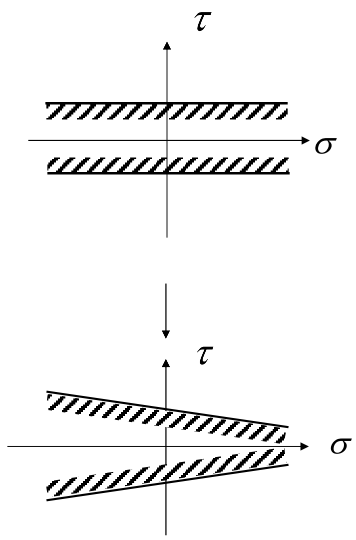
2.1.3 the rupture criteria for concrete

If concrete has to be identified with a modified coulum material, we can conclude the parameter k has a value of about 4

$$k = \frac{1+\sin\varphi}{1-\sin\varphi} = 4 \quad \rightarrow \quad \tan\varphi = 0.75 \quad \therefore \ \varphi = 37°$$

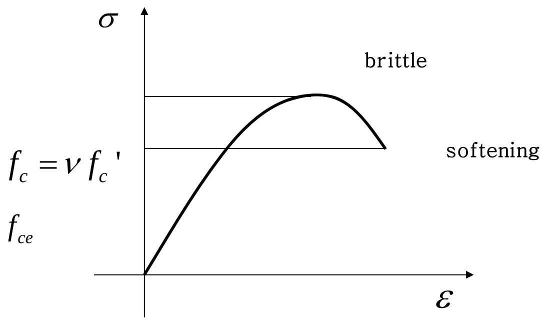
$$f_c = 2c\sqrt{k} \quad \rightarrow \quad c = \frac{f_c}{4}$$





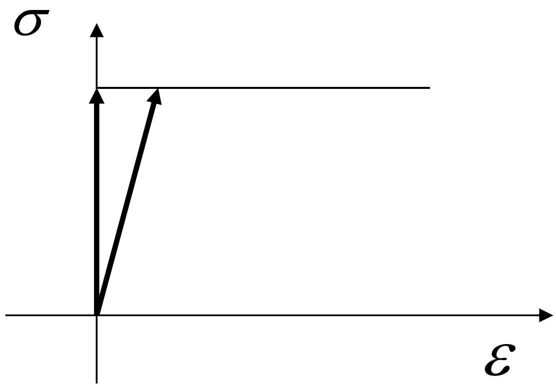
2.1.4 The plastic strength of concrete

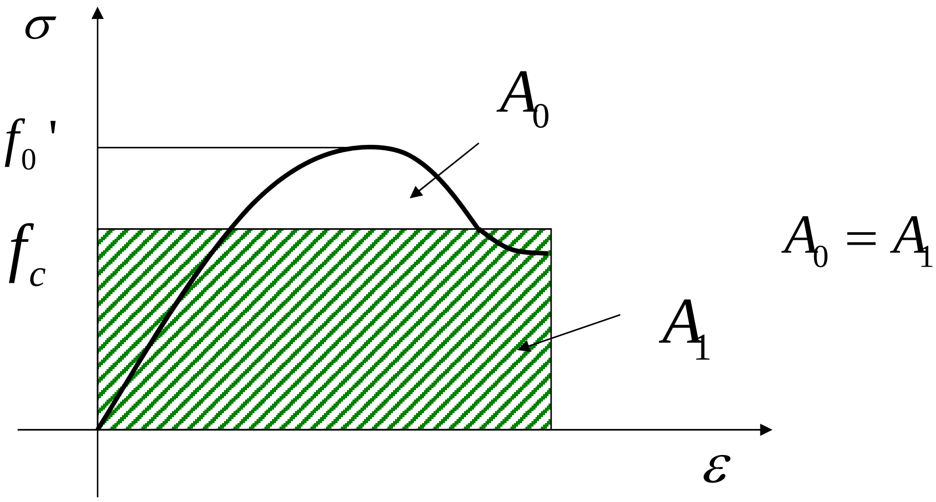
f_{ce} → effective strength of concrete ν : effectiveness factor



Effective strength of concrete

ν : Effective factor





The decreasing v-value for increasing strength seems to hold generally

- Geometric effects

  Stress concentration

  Absolute value of dimension

- Loading condition

  a/d ratio

1) 1978 CEB Model Code

   $v = 0.6$

2) M. P Nielsen

   $$f_{ce} = \left(0.7 - \frac{f_c^{'}}{29000}\right) f_c^{'} \quad (psi)$$

3) Ramirez

   $$f_{ce} = 30\sqrt{f_c^{'}} \quad (psi)$$

4) Schlaich & Weischede

   $v = 0.67$

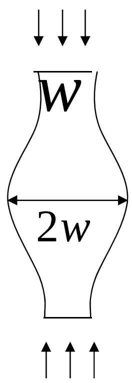
2.1.3 Failure criteria for concrete

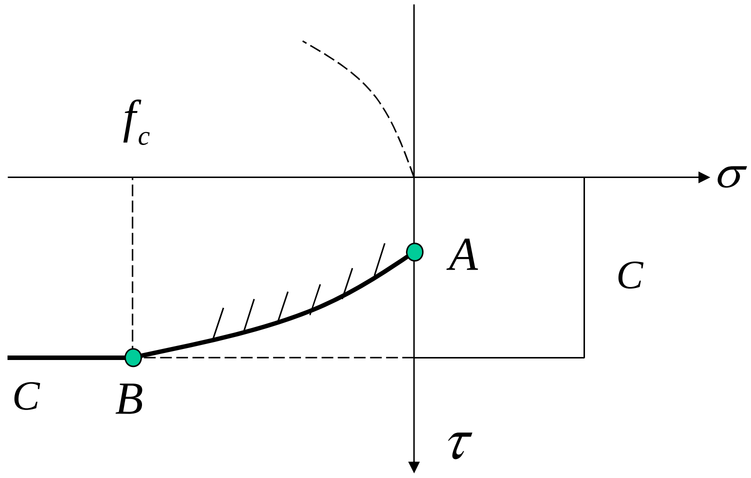
Concrete two phase : cement paste, aggregates

$K_{aqq} > K_{paste}$

$\sigma_{aqq} > \sigma_{paste}$

Coment paste : isotropic plastic material with $\varphi = 0$





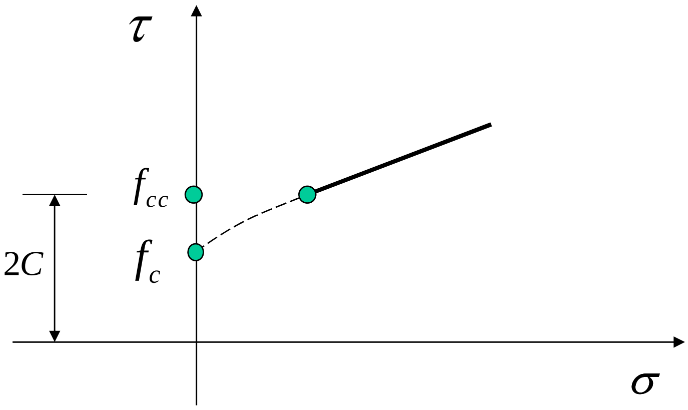
f_c

$\varphi = 0 \;\rightarrow\; k = 1$

Hydraulic test $\sigma_1 + \sigma_3 = P$

$k\sigma_1 - \sigma_3 = f_c$

$\sigma_1 = f_c + P$ (ideal)



$P = 0$

$\sigma = f_c$

$P \neq 0$

$\sigma = f_{cc} + P$

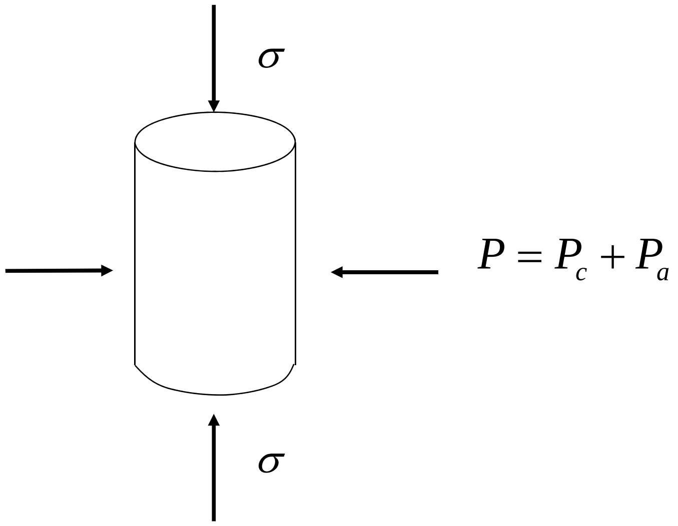
Eq(2.1.7)

$\sigma_1 - \sigma_3 = 2c$

$\sigma = 2c + P$

Failure condition for concrete

- Two phase material
- For granular material. c=0



$\sigma_c = P_c + 2c$ for cement paste

$\sigma_a = kP_a$

$$\sigma = \sigma_c + \sigma_a = P_c + 2c + kP_a$$
$$= P_c(1-k) + 2c + kP$$

When $P_c = 0$ $\sigma \rightarrow$ max

$\sigma = 2c + kP$ (2c is uniaxial compressive strength of the composite material)

$\sigma = f_c + kP$ (friction angle from aggregates, cohesion from cement paste)

For sufficiently high confining pressure the yield lines in cement paste tend to develop roughly under 45 degree.

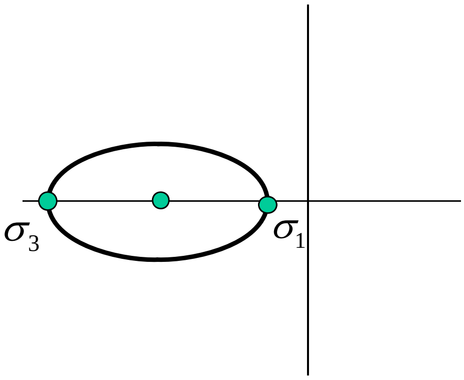
① uniaxial compressive strength is not affected by the aggregates

② increase in strength above for cement paste is due to the displacement caused by the aggregate particles of the yield lines in the cement paste.

f_c: real uniaxial strength

f_{cc}: apparent uniaxial compressive strength

Average stresses



$$\sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3)$$

$$\tau_m = \frac{1}{2}(\sigma_1 - \sigma_3)$$

Since $\sigma_1 = \sigma_m + \tau_m$, $\sigma_3 = \sigma_m - \tau_m$

Sub into (2.1.10) and (2.1.8)

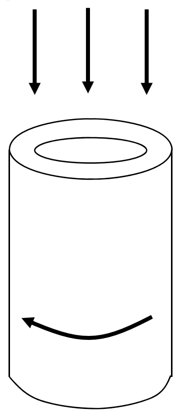
$$k\sigma_1 - \sigma_3 = f_c,\ \sigma_1 = f_A$$

$$(k-1)\sigma_m + (k+1)\tau_m = f_c,\ \sigma_m + \tau_m = f_A$$

Modified coulomb material in σ_m - τ_m coordinate system

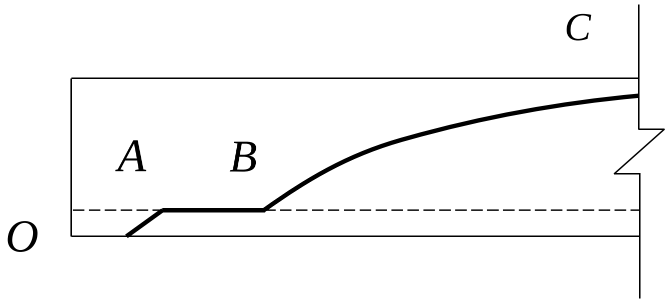
Test result

① thin – walled tube



② Biaxial test

Tensile stress $f_t = \sqrt{0.1 f_c}$



Law sliding resistance along a OA, BC separation along AB

- Shear failure of orthotropic panels
- Sliding resistance of a crack waloven
- Effect of softening , transverse pressure
- Geometry of structural element and loading

2.1.4 Structural concrete strength

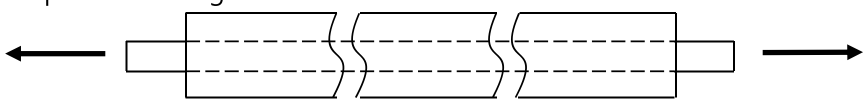
$f_{ccf} = vf_c$

$c_{ef} = vc$

$f_{tef} = v_t f_t$

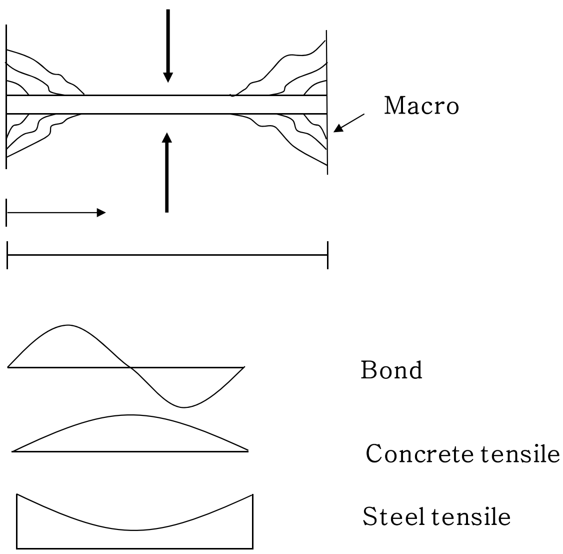
Softening phenomena is unable to take into account the softening in a rational manner

Compressive strength of cracked concrete



Micro crack

Macro crack



$A_e \cdot f_t = a_{\min} u \Sigma_0$ - ①

Where A_e effective area for tension

a= $= 2a_{\min}$ - ②

since $\Sigma_0 = \dfrac{4A_s}{d_b}$ - ③

③ & ② → ①

$$a = 2\frac{A_e f_t}{u\Sigma_0} = \frac{f_t d_b}{2u\dfrac{A_s}{A_e}}$$

Let $r = \dfrac{A_s}{A_e}$

$u = 4\lambda f_t$ ($\lambda = 2$ for deformed, $\lambda = 1$ for plastic)

$\beta = \dfrac{d_b}{4r}$

So $a = \dfrac{f_t d_b}{4\lambda f_t r} = \dfrac{\beta}{\lambda}$ (2.1.54)

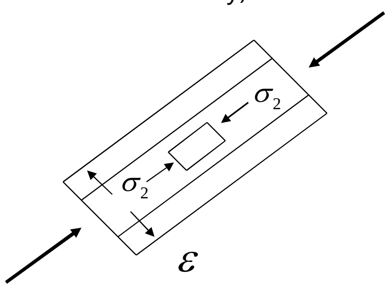
Assume $l_0 \propto \sigma_y \dfrac{d_b}{f_t} = c\sigma_y \dfrac{d_b}{f_t}$

$$\chi = \frac{l_0}{a} = \frac{c\sigma_s \dfrac{d_b}{f_t}}{\dfrac{1}{\lambda}\dfrac{d_b}{4r}} = \frac{4c\lambda r\sigma_s}{f_t}$$

$$\chi = \frac{r\sigma_s}{1.41\sqrt{f_c^{'}}} \quad (2.1.57)$$

5) Collins & Mitchell (compression field theory)

$$f_{ce} = \frac{f_c^{'}}{0.8 + 170\varepsilon_1}$$



Reading assignment p81~91 (Mutteni's book)

Refer to Dr. Yun's paper

By M.P. Nielson

$$v \simeq \frac{2}{\sqrt{f_c}} \text{ (in MPa)}$$