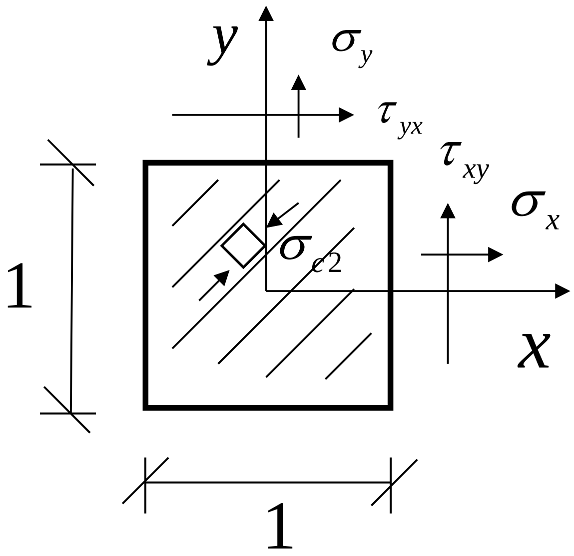
2.2.2 Disk of orthogonal reinforcement

Consider a square disk of a unit length under shear and normal stresses



Concrete : $\sigma_{cx}, \sigma_{cy}, \tau_{cxy}$

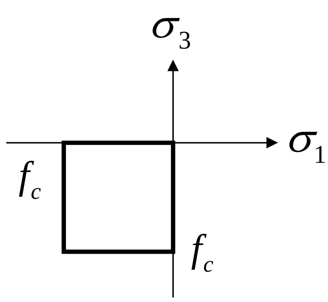
Steel : σ_{sx}, σ_{sy}

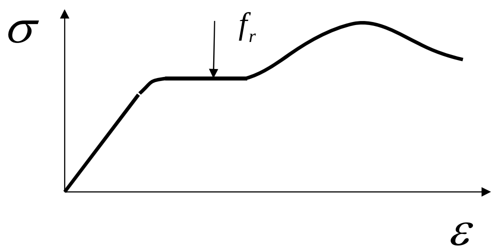
• Total stresses are decomposed into concrete and steel stresses

$$\sigma_x = \sigma_{cx} + \sigma_{sx}$$
$$\sigma_y = \sigma_{cy} + \sigma_{sy}$$
$$\tau_{xy} = \tau_{cxy}$$

• principal concrete stresses are denoted as σ_{c1}, σ_{c2}

• reinforcement ratio



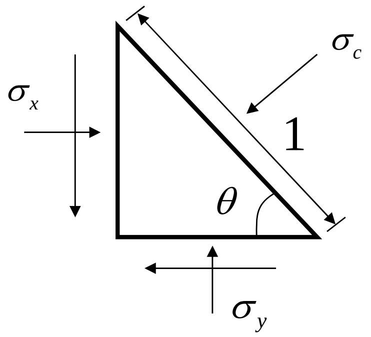


$$\Phi = \frac{A_{sx}}{t \cdot 1} \frac{f_r}{f_c}$$

Cf : $\omega = \frac{A_s}{bd} \frac{f_y}{0.85 f_c^{'}}$

① Tensile strength

$$f_{tx} = \frac{A_{sx} f_Y}{t \cdot 1} = \Phi_x f_c$$



$$f_{ty} = \frac{A_{sy} f_Y}{t \cdot 1} = \Phi_y f_c$$

② compressive strengths

$$f_{cx} = f_Y + f_c = (1+\Phi_x)\, f_c$$

$$f_{cy} = f_Y + f_c = (1+\Phi_y)\, f_c$$

③ pure shear

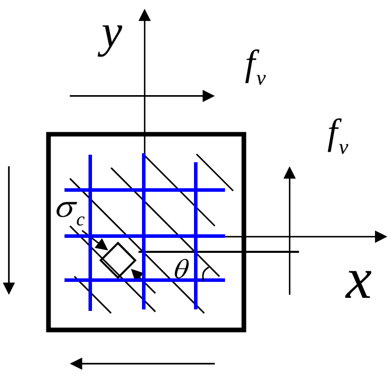
- concrete : uniaxial stress state

$$\sigma_{c1} = 0, \quad \sigma_{c2} = -\sigma_c$$

- steel : $f_{sx} = f_Y$, $f_{sy} = f$ → ductile

$$\sigma_{cx} = -\frac{\sigma_c}{2} - \frac{\sigma_c}{2}\cos 2\theta$$
$$= -\frac{\sigma_c}{2}(1+\cos 2\theta)$$
$$= -\sigma_c \sin^2\theta$$

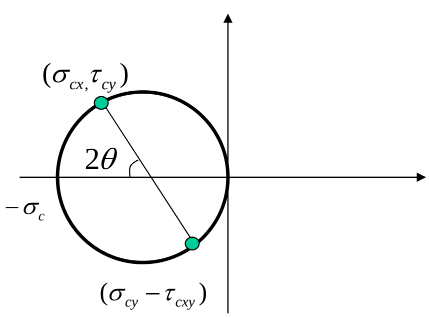
$$\sigma_{cy} = -\frac{\sigma_c}{2} - \frac{\sigma_c}{2}\cos 2\theta$$
$$= -\frac{\sigma_c}{2}(1+\cos 2\theta)$$
$$= -\sigma_c \sin^2\theta$$



$$\tau_{cxy} = \frac{\sigma_c}{2}\sin 2\theta = \sigma_c \sin\theta\cos\theta$$

$$\Sigma F_x = 0 \ : \ \begin{array}{l} \sigma_{cx} \cdot t = A_{sx} f_Y \\ \sigma_c \cos^2\theta \cdot t = A_{sx} f_Y \end{array} \quad - ④$$

$$\Sigma F_y = 0 \ : \ \begin{array}{l} \sigma_{cy} \cdot t = A_{sy} f_Y \\ \sigma_c \sin^2\theta \cdot t = A_{sy} f_Y \end{array} \quad - ⑤$$



From Eqs. ④ & ⑤

$$\sigma_c = \frac{A_{sx} f_Y}{t\cos^2\theta} = \Phi_x \frac{1}{\cos^2\theta} f_c \quad - ⑥$$

$$\sigma_c = \frac{A_{sy} f_Y}{t\sin^2\theta} = \Phi_y \frac{1}{\sin^2\theta} f_c \quad - ⑦$$

Since ⑥ = ⑦

$$\Phi_x \frac{1}{\cos^2\theta} f_c = \Phi_y \frac{1}{\sin^2\theta} f_c$$

$$\frac{\Phi_y}{\Phi_x} = \tan^2\theta = \mu \quad \text{- ⑧}$$

$$\sigma_c = \Phi_x f_c \frac{1}{\cos^2\theta} = \Phi_x (1+\mu) f_c$$

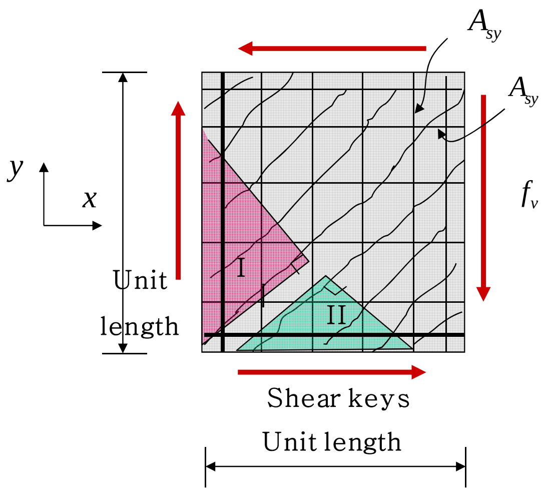
$$f_v = \sigma_c \sin\theta\cos\theta \quad \text{- ⑨}$$

Substituting ⑥ into ⑨ yields

$$f_v = \frac{A_{sx} f_Y}{t\cos^2\theta}\sin\theta\cos\theta = \Phi_x f_c \tan\theta \quad \text{- ⑩}$$

$$= \Phi_x f_c \sqrt{\mu}$$

$$f_v = \Phi_x f_c \sqrt{\frac{\Phi_y}{\Phi_x}} = f_c\sqrt{\Phi_x \Phi_y} \quad \text{- ⑪}$$



First yield condition (No crushing failure of concrete)

$$\sigma_c < f_c$$

$$\frac{2 f_v}{\sin\theta\cos\theta} = \Phi_x (1+\mu) f_c < f$$

$$\Phi_x + \frac{\Phi_y}{\Phi_x}\Phi_x < 1$$

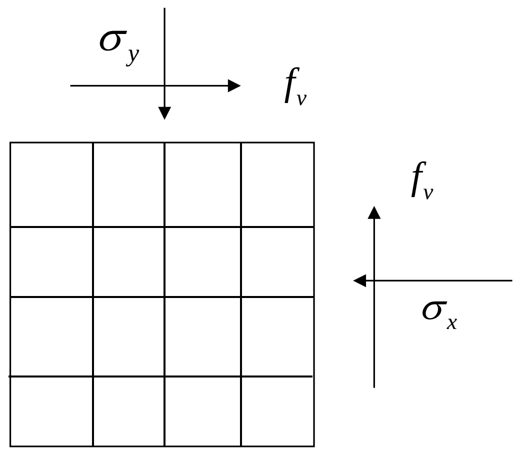
$\Phi_x + \Phi_y < 1$ Max reinforcement

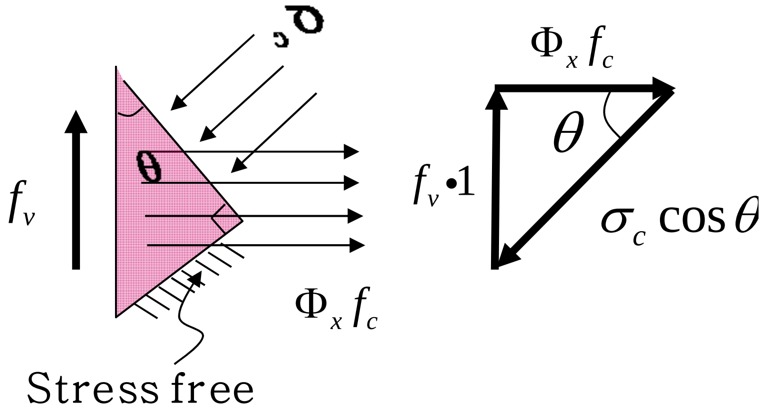
Under pure shear → different approach!

Aim is to find the shear strength f_v

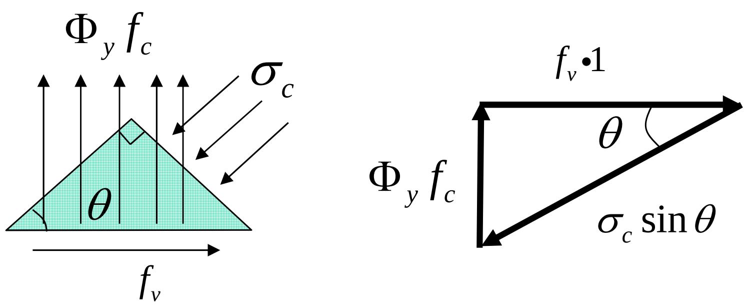
Subject to $\sigma_c < f_c$

$$f_s = f_{sx} = f_{sy} = f_y$$





$$f_v = \Phi_x f_c \tan\theta$$
$$f_v = \sigma_c \cos\theta \sin\theta$$

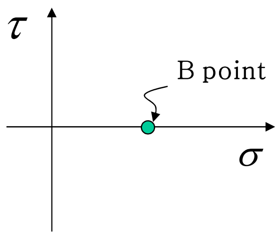


$$f_v = \sigma_c \cos\theta \sin\theta$$
$$f_v = \Phi_y f_c \tan\theta$$

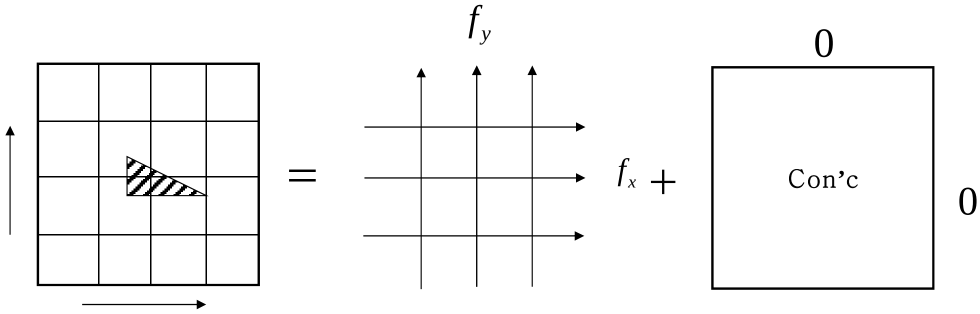
$$f_v \cdot f_v = \Phi_x \Phi_y f_c^2$$
$$\therefore \quad f_v = \sqrt{\Phi_x \Phi_y} f_c$$

to find f_v under combined stresses

(i.e. σ_x or/and σ_y)



Mohr's circle

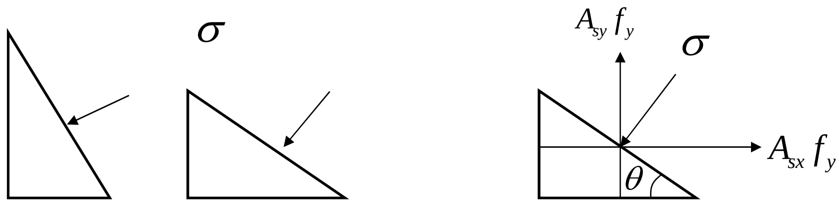


$\Phi_x = \Phi_y = \Phi$

At B Point

$\sigma_{sx} = \sigma_{sy} = f_y$

$\sigma_{cx} = \sigma_{cy} = 0$



$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta = \frac{A_s f_y}{t}$$

(First term : $\dfrac{A_{sx} f_y}{t}$, second term : $\dfrac{A_{sy} f_y}{t \cdot 1}$)

Equivalent stress

$\sigma = \Phi f_c (\Phi_x = \Phi_y)$

$\sigma_1 = \sigma_2 =$ principal stress $\rightarrow$ $\tau = 0$

$\sigma_{c1} = \sigma_{c2} = 0$

$\tau = 0$

Keeping the steel stress $= f_Y$

And varying $-f_c \le \sigma_{c2} \le 0$

$\sigma_1 = \Phi f_c$ - ①

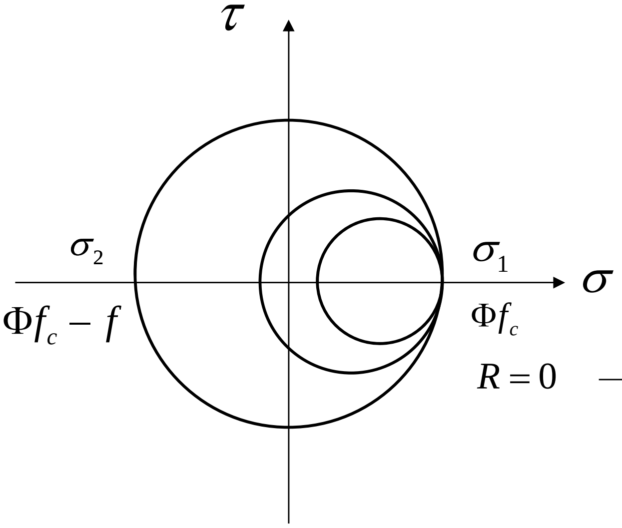
$\Phi f_c - f_c \le \sigma_{c2} \le \Phi f_c$ - ②

$\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\dfrac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_{xy}{}^2}$ - ③

$\sigma 2 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\dfrac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_{xy}{}^2}$ - ④

Since ① = ③

$$\Phi f_c = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_{xy}{}^2}$$



$R = 0 \longrightarrow \tau = 0$

$$\left(\Phi f_c - \frac{\sigma_x + \sigma_y}{2}\right)^2 = \frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_{xy}{}^2$$

- ⑤

$$-\left(\Phi f_c - \sigma_x\right)\left(\Phi f_c - \sigma_y\right) + \tau_{xy}{}^2 = 0$$

- ⑥

Substituting Eq ⑤ into ② yields

Eq ③

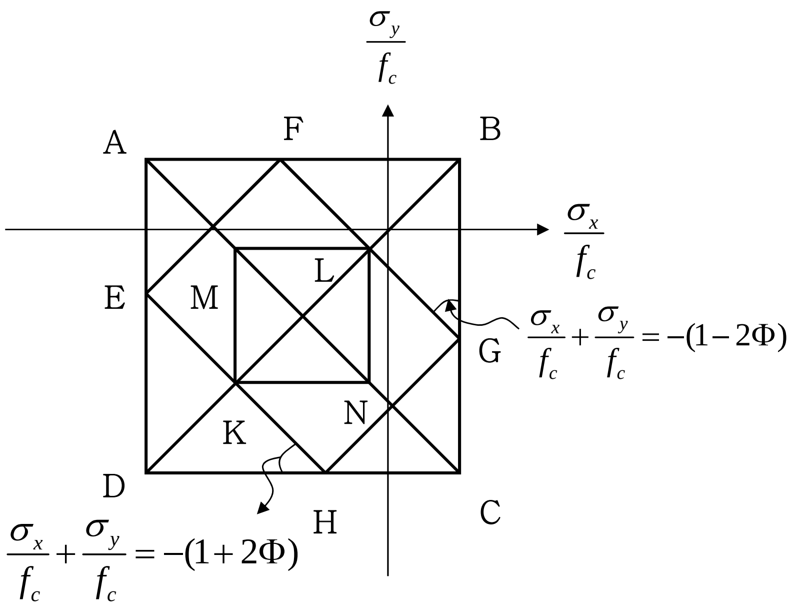
$$\sqrt{\frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_{xy}{}^2} = \Phi f_c - \frac{1}{2}\left(\sigma_x + \sigma_y\right)$$

Eq ②

$$\left(\Phi - 1\right) f_c \le \frac{1}{2}\left(\sigma_x + \sigma_y\right) - \Phi f_c + \frac{1}{2}\left(\sigma_x + \sigma_y\right) \le \Phi f_c$$

$$\left(\Phi - 1\right) f_c \le \sigma_x + \sigma_y - \Phi f_c \le \Phi f_c$$

$$-\left(1 - 2\Phi\right) f_c \le \sigma_x + \sigma_y \le 2\Phi f_c \quad \text{(FG Line)}$$

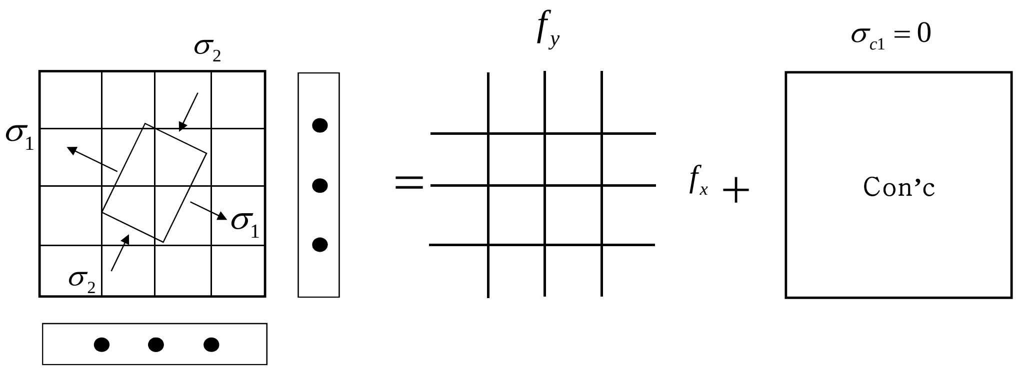


- BFG Region

at Point B

$f_{sx} = f_{sy} = f_Y$

$\sigma_{c1} = \sigma_{c2} = 0$



$\sigma_1 = \Phi f_c$

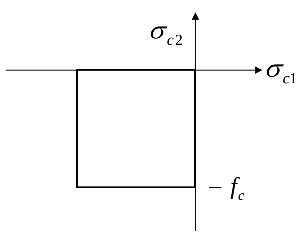
$(\phi - 1)\, f_c \le \sigma_2 \le \phi f_c$

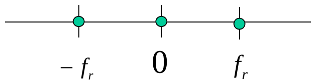
in BFG region : $f_{sx} = f_{sy} = f_Y$

$-f_c < \sigma_{c2} < 0$

Conical

$$-\left(\Phi f_c - \sigma_x\right)\left(\Phi f_c - \sigma_y\right) + \tau_{xy}{}^2 = 0$$



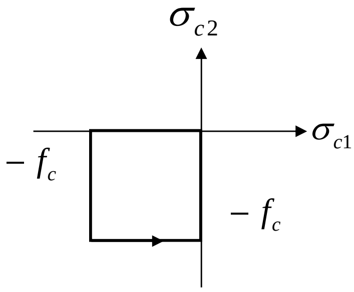


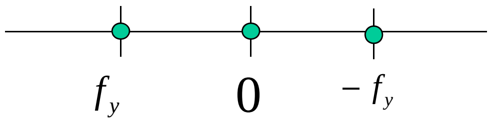
FG Line

$\sigma_x + \sigma_y = -\left(1 - 2\Phi\right) f_c$

- In DEH Region

  At D Point





$f_{sx} = f_{sy} = -f_Y$

Keep : $\sigma_{c2} = -f_c$

$-f_c \le \sigma_{c1} \le 0$

Equivalent

$\sigma_2 = -(\Phi+1)\, f_c$

$\sigma_1 = \sigma_{s1} + \sigma_{c1}$

$= -\Phi f_c - f_c$

$-f_c \le \sigma_{c1} \le 0 \;\;\&\;\; \sigma_{s1} = -f_Y$

$\therefore\; -f_c - f_Y < \sigma_{s1} + \sigma_{c1} < -f_Y$

$-(\Phi+1)\, f_c < \sigma_{c1} < -\Phi f_c$ - ①

$\sigma_2 = -(\Phi+1)\, f_c$ - ②

Since

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^{\;2}} \quad \text{- ③}$$

③ = ②

$$-(\Phi+1)\, f_c = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^{\;2}}$$

$$-\left[(\Phi+1)\, f_c + \sigma_x\right]\left[(\Phi+1)\, f_c + \sigma_y\right] + \tau_{xy}^{\;2} = 0 \quad \text{- ④}$$

We know that

$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$

$\sigma_1 = (1+\Phi)\, f_c + \sigma_x + \sigma_y$ - ⑤

Substituting Eq ⑤ into ① yields

$-(1+\Phi)\, f_c \le (1+\Phi)\, f_c + \sigma_x + \sigma_y \le -\Phi f_c$

$-2(1+\Phi)\, f_c \le \sigma_x + \sigma_y \le -(1+2\Phi)\, f_c$

At Line LG → Line NC ← Line KH

$f_{sx} = f_{sy} = f_Y$ $\quad$ $f_{sx} = f_Y$, $f_{sy} = -f_Y$ $\quad$ $f_{sx} = f_{sy} = -f_Y$

$\sigma_{c1} = 0,\ \sigma_{c2} = -f_c$

: only change in steel stresses result in no variation of τ_{xy}

τ_{xy} along KH is constant in KHNC

In Region LMKN

At point L $\sigma_x = \sigma_y$

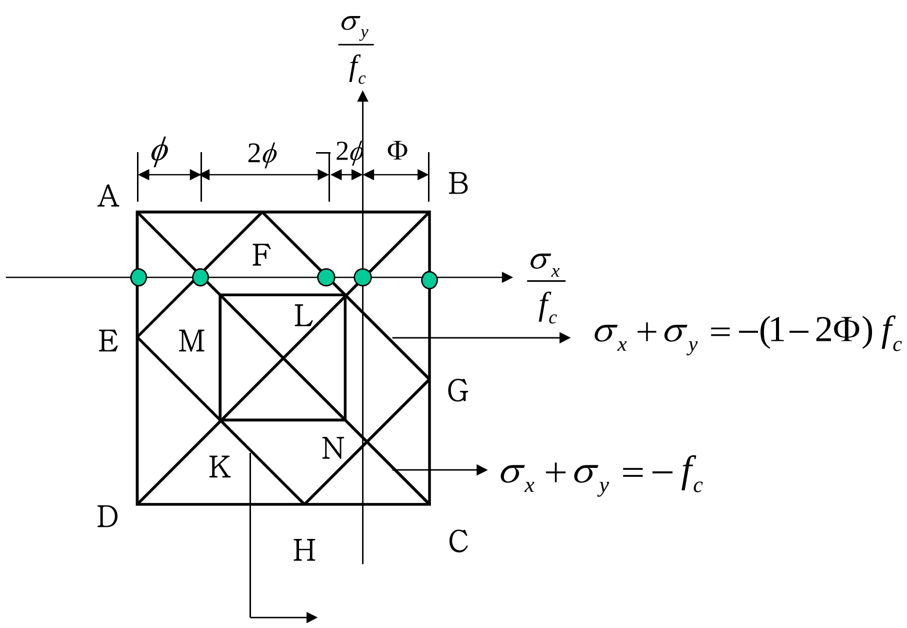
$$2\frac{\sigma_x}{f_c} = -(1-2\Phi)$$

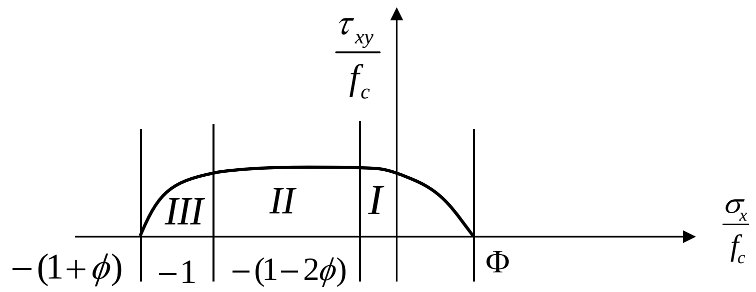
$$\frac{\sigma_x}{f_c} = -\frac{1}{2}(1-2\Phi)$$

$$-(\Phi f_c - \sigma_x)^2 + \tau_{xy}{}^2 = 0$$

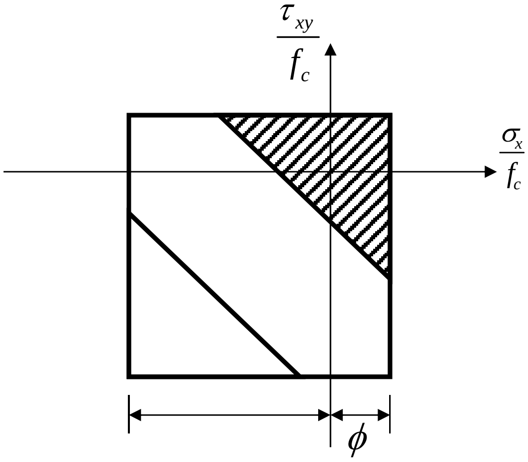
$$\rightarrow\ \tau_{xy}{}^2 = \Phi f_c + \frac{1-2\Phi}{2} f_c = \frac{f_c}{2}$$

$$(\tau_{xy})_{\max} = \frac{f_c}{2}$$





Substituting $\frac{\sigma_x}{f_c}$ =0, into the equations representing yield surface yields



1) $-(1-2\Phi) \le \frac{\sigma_x}{f_c} \le \Phi$

$$-(\Phi f_c - \sigma_x)(\Phi f_c - \sigma_y)^2 + \tau_{xy}{}^2 = 0$$

$$\tau_{xy}{}^2 = \Phi f_c(\Phi f_c - \sigma_y)$$

$$\tau_{xy} = \sqrt{\Phi f_c(\Phi f_c - \sigma_y)}$$

$$\frac{\tau_{xy}}{f_c} = \sqrt{\left(\Phi - \frac{\sigma_x}{f_c}\right)\Phi}$$

2) For the second region (constant τ_{xy}) $\frac{\sigma_x}{f_c} = -1 + 2\Phi$

$$\frac{\tau_{xy}}{f_c} = \sqrt{(1-\Phi)\Phi}$$

3) For the third region

$$-\left[(1+\Phi) f_c + \sigma_x\right]\left[(1+\Phi) f_c + \sigma_y\right] + \tau_{xy}{}^2 = 0 \text{ w} - (**)$$

$\sigma_x + \sigma_y = -(1+2\Phi) f_c$: EH Line

$\sigma_y = -\sigma_x - (1+2\Phi) f_c$ - (*)

Substituting (*) into (**) yields

$$\left[(1+\Phi) f_c + \sigma_x\right]\left[\Phi f_c + \sigma_x\right] + \tau_{xy}{}^2 = 0$$

$$\left[\sigma_x + \left(\frac{1}{2} + \Phi\right) f_c\right]^2 + \left[\left(\frac{1}{2} + \Phi\right)^2 - \Phi(1+\Phi)\right] f_c{}^2 = \tau_{xy}{}^2$$

$$\rightarrow \tau_{xy} = \sqrt{\frac{1}{4} f_c{}^2 - \left[\sigma_x + \left(\frac{1}{2} + \Phi\right) f_c\right]^2}$$

If we disregard the special condition for $\Phi \approx 0$

$$\sigma_x + \sigma_y \geq -(1-2\Phi)\, f_c$$
$$-(\Phi f_c - \sigma_x)(\Phi f_c - \sigma_y) + \tau_{xy}{}^2 = 0$$

In remaining region

$$\sigma_x + \sigma_y \leq -(1-2\Phi)\, f_c$$
$$-\left[(1+\Phi)\, f_c + \sigma_x\right]\left[(1+\Phi)\, f_c + \sigma_y\right] + \tau_{xy}{}^2 = 0$$

For more approximation

$$1+\Phi \approx 1$$
$$-(1-2\Phi) = -1+2\Phi = -1+(1+\Phi)-1+\Phi \cong -1+\Phi$$

① $\sigma_x + \sigma_y \geq -(1-\Phi)\, f_c \;:\; -(\Phi f_c - \sigma_x)(\Phi f_c - \sigma_y) + \tau_{xy}{}^2 = 0$

Setting $\sigma_y = 0 \;\rightarrow\; \tau_{xy} = \sqrt{(\Phi f_c - \sigma_x)}$

② $\sigma_x + \sigma_y \leq -(1-\Phi)\, f_c$

$\tau_{xy} = \sqrt{(f_c + \sigma_x)\, f_c}$

Shear strength of

Think Process

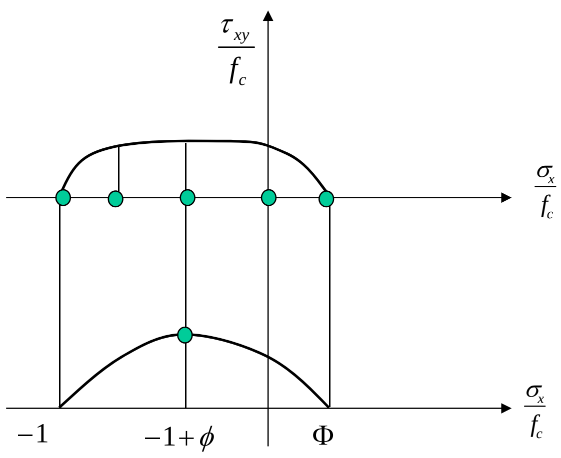
① Consider isotropic

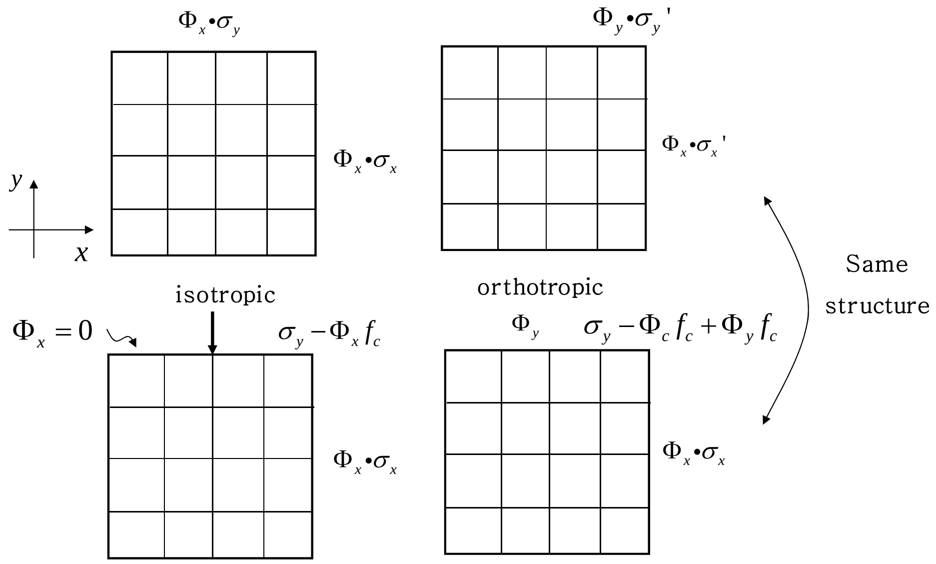
② Delete steel in y-dir, add tensile force $\Phi_x f_c$

③ Add steel Φ_y in y-dir, substract

The yield condition in the orthotropic disk

$\Phi_x \neq \Phi_y$





$$\sigma_x^{'} = \sigma_x \quad \text{- ①}$$

$$\sigma_y^{'} = \sigma_y - \Phi_x f_c + \Phi_y f_c$$

$$= \sigma_y - (1-\mu)\Phi_x f_c \quad \text{- ②}$$

Where $\mu = \dfrac{\Phi_y}{\Phi_x}$

Remind Eq's (75) and (76)

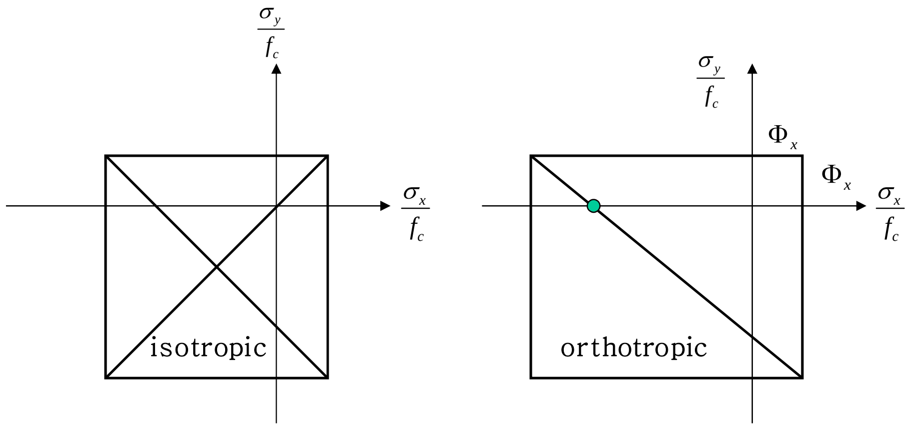
$$-(\Phi f_c - \sigma_x)(\Phi f_c - \sigma_y) + {\tau_{xy}}^2 = 0 \quad \text{- ③}$$

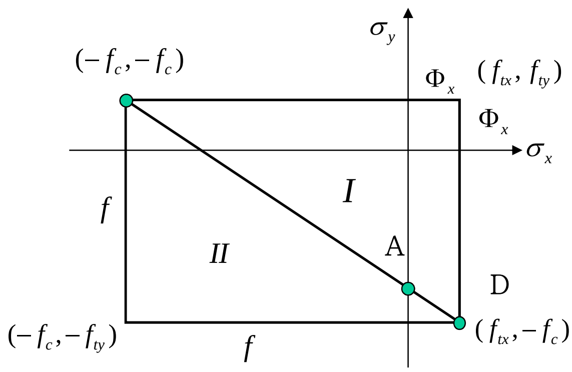
Substituting ①&② into ③ yields

$$-(\Phi_x f_c - \sigma_x^{'})(\Phi_x f_c - \sigma_y^{'} - (1-\mu)\Phi_x f_c) + {\tau_{xy}}^2 = 0$$

$$-(\Phi_x f_c - \sigma_x^{'})(\mu\Phi_x f_c - \sigma_y^{'}) + {\tau_{xy}}^2 = 0$$

$$-(\Phi_x f_c - \sigma_x)(\mu\Phi_x f_c - \sigma_y) + {\tau_{xy}}^2 = 0$$





$$\sigma_y = -\frac{f_c + f_{ty}}{f_c + f_{tx}}\sigma_x + A$$

$$A = \eta f_{tx} - f_c$$

Where $\eta = \dfrac{f_c + f_{ty}}{f_c + f_{tx}}$

$$\sigma_y = -\eta\sigma_x + \eta f_{tx} - f_c$$

For Ⅰ region : $\sigma_y \geq -\eta\sigma_x + \eta f_{tx} - f_c$

For Ⅱ region : $\sigma_y \leq -\eta\sigma_x + \eta f_{tx} - f_c$