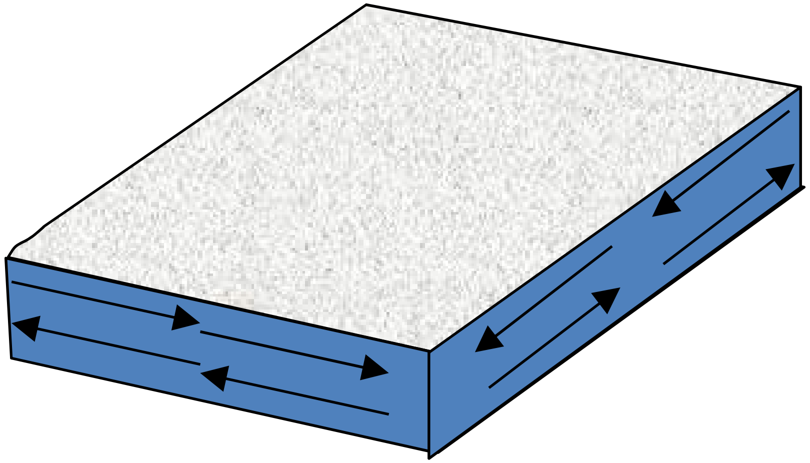
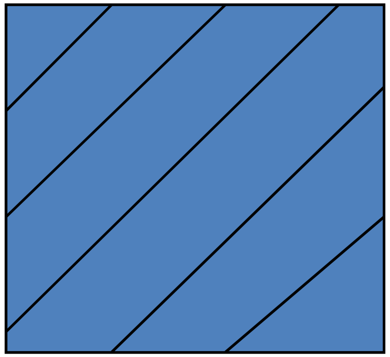
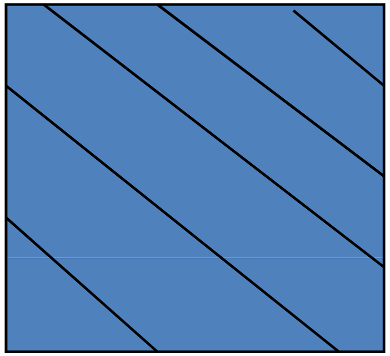
Torsion in slabs





Top panel



bottom panel

$$t_p = \left(1 - 2\Phi_o\right) A_s f_Y h$$

The Theory of Plain Concrete

- Constitutive Equations
- Dissipation Works
- Lines of Discontinuity
- Stress Fields
- Strain Fields
- Applications
- Concentrated Loadings

Constitutive Equations

- Coulomb Material
- Yield Condition **6 Surfaces**
- Strain Vectors

$$k\sigma_1 - \sigma_3 = f_c, \quad \sigma_1 \geq \sigma_2 \geq \sigma_3$$

$$k\sigma_3 - \sigma_1 = f_c, \quad \sigma_3 \geq \sigma_2 \geq \sigma_1$$

$$k\sigma_1 - \sigma_2 = f_c, \quad \sigma_1 \geq \sigma_3 \geq \sigma_2$$

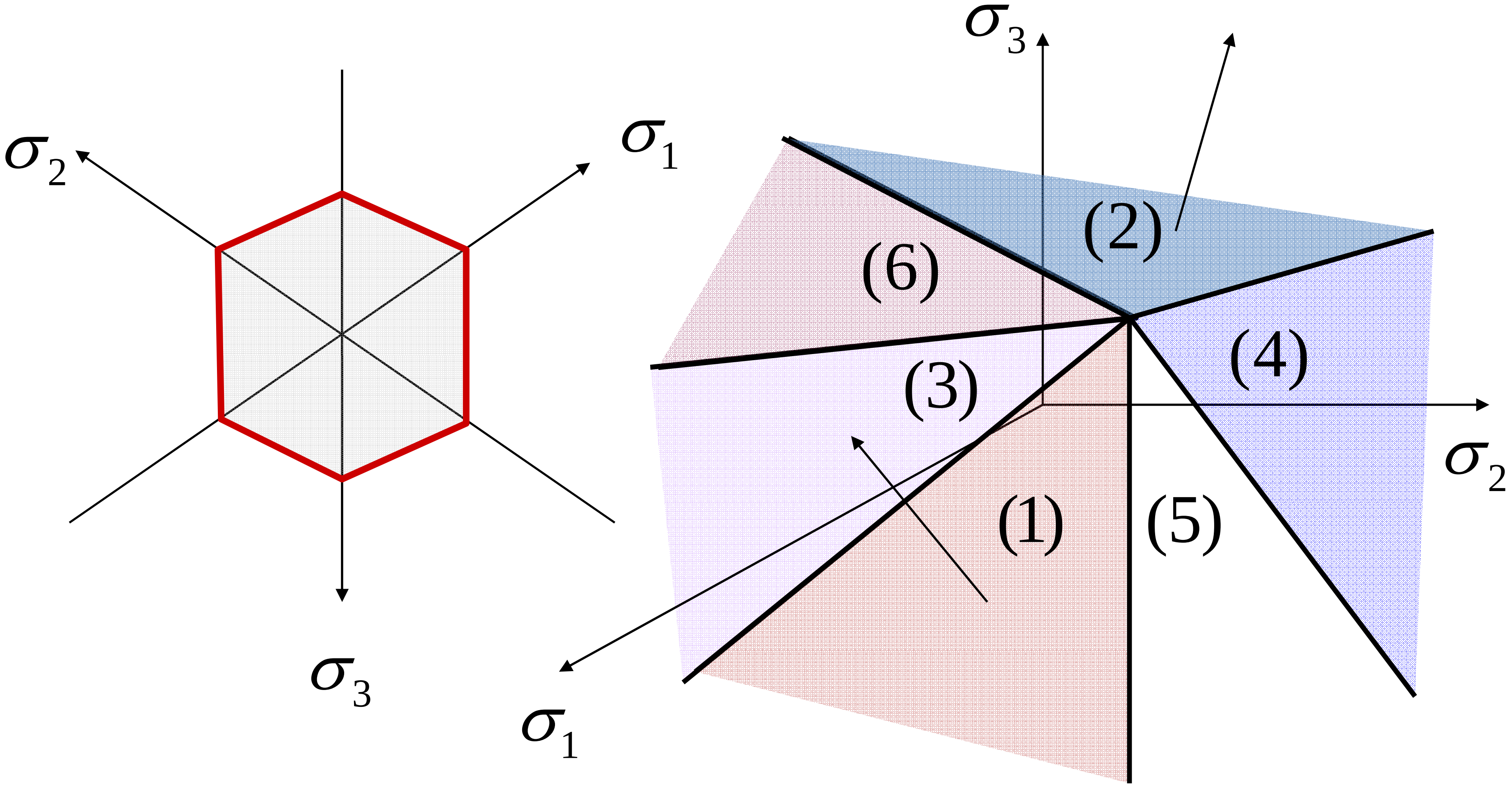
$$k\sigma_2 - \sigma_1 = f_c, \quad \sigma_2 \geq \sigma_3 \geq \sigma_1$$

$$k\sigma_2 - \sigma_3 = f_c, \quad \sigma_2 \geq \sigma_1 \geq \sigma_3$$

$$k\sigma_3 - \sigma_2 = f_c, \quad \sigma_3 \geq \sigma_1 \geq \sigma_2$$

$$\Rightarrow \quad q_i = \lambda \frac{\partial f}{\partial Q_i}$$

Yield Surface in 3-D



Plastic Strain Vector

- Along planes

$$k\sigma_1 - \sigma_3 = f_c, \quad \sigma_1 \ge \sigma_2 \ge \sigma_3$$

$$q_i = \lambda \frac{\partial f}{\partial Q_i}$$

	ε_1	ε_2	ε_3
1	λk	0	$-\lambda$
2	$-\lambda$	0	λk
3	λk	$-\lambda$	0
4	$-\lambda$	λk	0
5	0	λk	$-\lambda$
6	0	$-\lambda$	λk

	ε_1	ε_2	ε_3
1	λk	0	$-\lambda$
2	$-\lambda$	0	λk
3	λk	$-\lambda$	0
4	$-\lambda$	λk	0
5	0	λk	$-\lambda$
6	0	$-\lambda$	λk

$$\frac{\sum \varepsilon^{+}}{\sum \left|\varepsilon^{-}\right|} = k$$

- Along edges

	ε_1	ε_2	ε_3
1/5	$\lambda_1 k$	$\lambda_2 k$	$-(\lambda_1+\lambda_2)$
4/5	$-\lambda_1$	$(\lambda_1+\lambda_2)k$	$-\lambda_2$
4/2	$-(\lambda_1+\lambda_2)$	$\lambda_2 k$	$\lambda_1 k$
2/6	$-\lambda_1$	$-\lambda_2$	$(\lambda_1+\lambda_2)k$
3/6	$\lambda_1 k$	$-(\lambda_1+\lambda_2)$	$\lambda_2 k$
1/3	$(\lambda_1+\lambda_2)k$	$-\lambda_2$	$-\lambda_1$

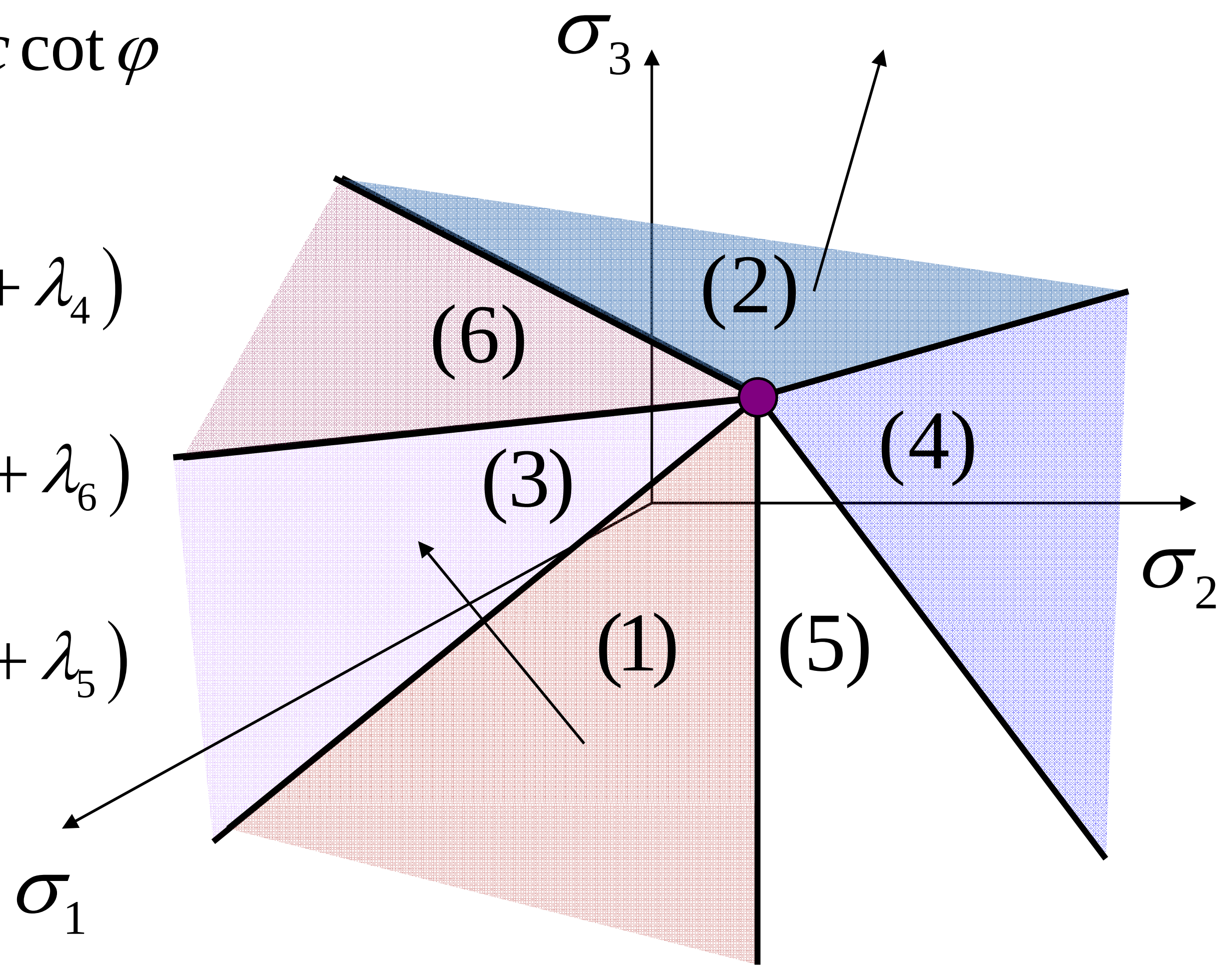
Plastic Strain Vector at Apex

$$\sigma_1 = \sigma_2 = \sigma_3 = \frac{f_c}{k-1} = c\cot\varphi$$

$$\varepsilon_1 = (\lambda_1 + \lambda_3)k - (\lambda_2 + \lambda_4)$$

$$\varepsilon_2 = (\lambda_4 + \lambda_5)k - (\lambda_3 + \lambda_6)$$

$$\varepsilon_3 = (\lambda_2 + \lambda_6)k - (\lambda_1 + \lambda_5)$$



Dissipation Work

Along plane 1

$$W = \sigma_1 \varepsilon_1 + \sigma_1 \varepsilon_1 + \sigma_1 \varepsilon_1 = \sigma_1 \lambda k - \sigma_3 \lambda = \lambda f_c$$

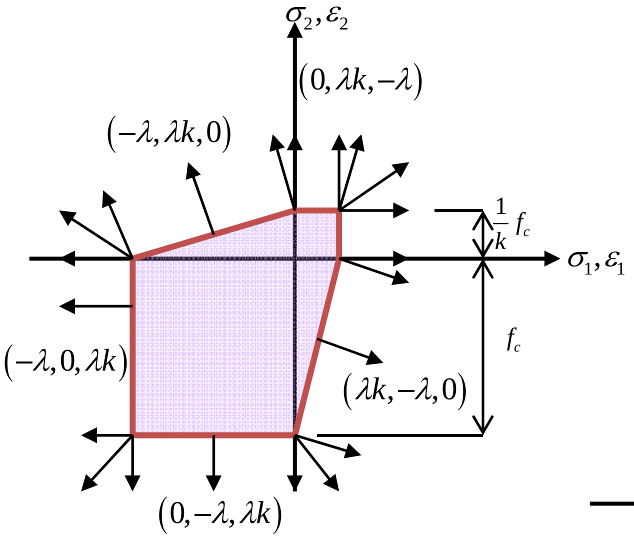
Along the edge 1/5

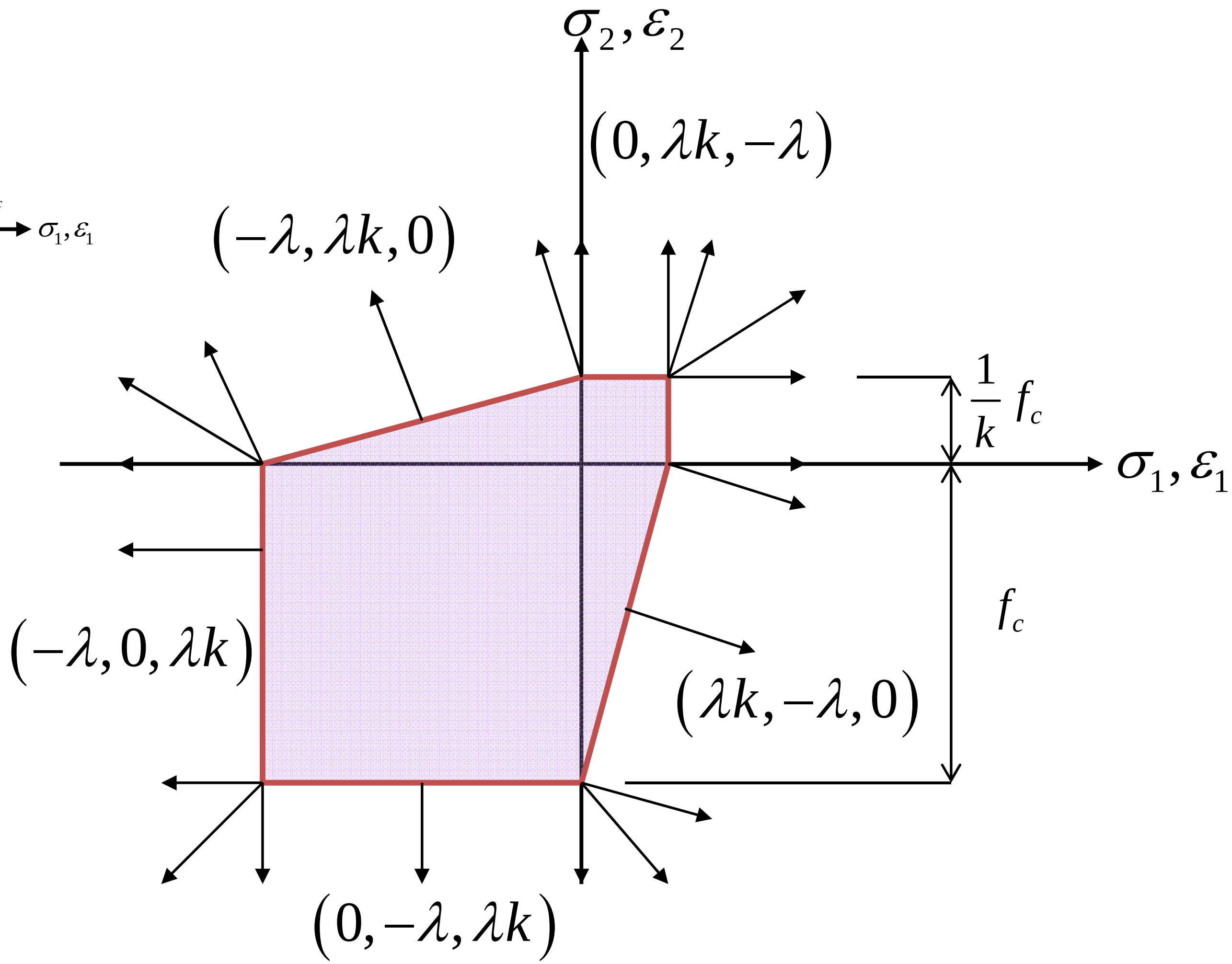
$$W = \left(\lambda_1 + \lambda_2 \right) f_c$$

At the apex

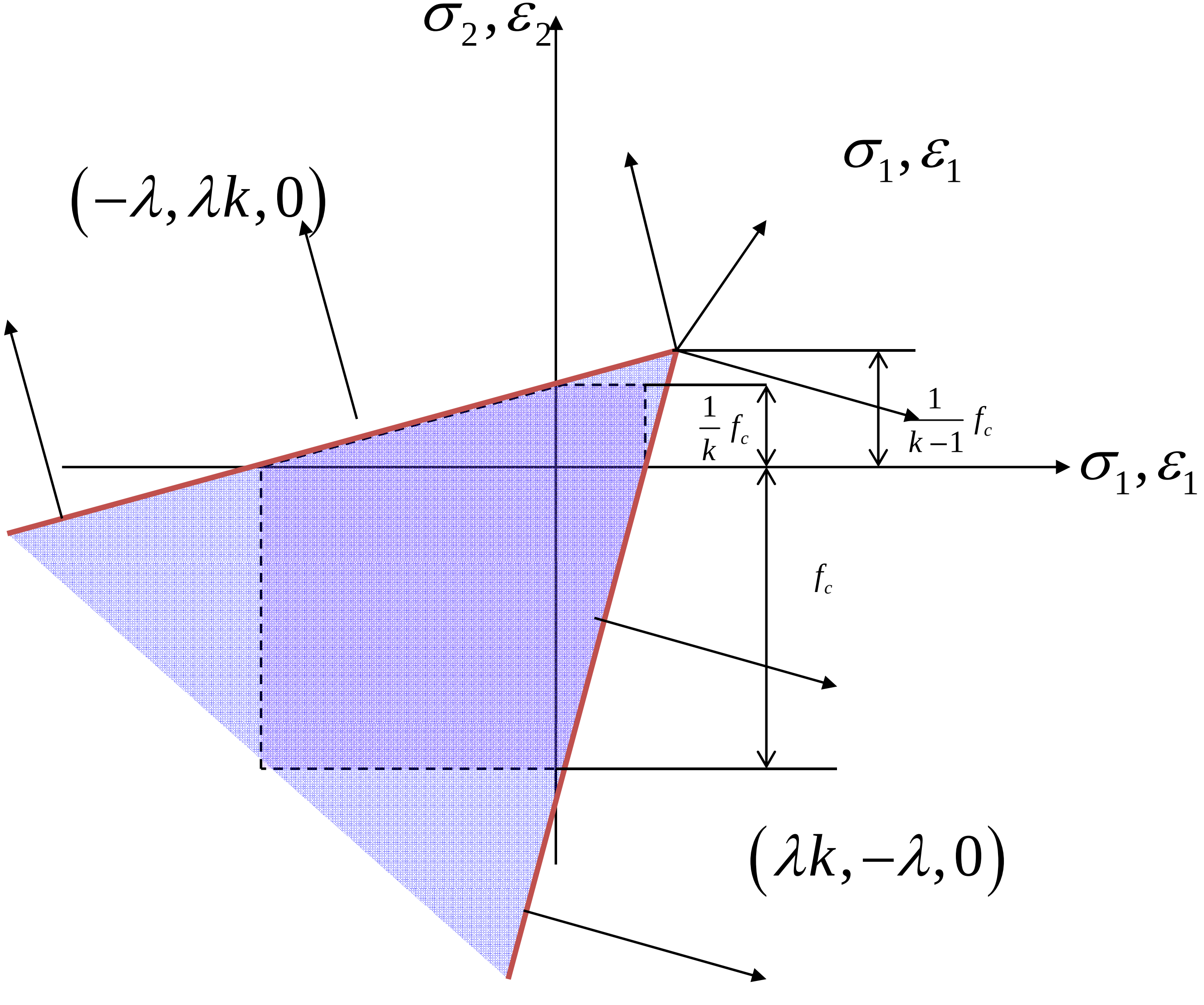
$$W = \left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 \right) f_c$$

Yield Condition and Flow Rule for Coulomb Material in Plane Stress

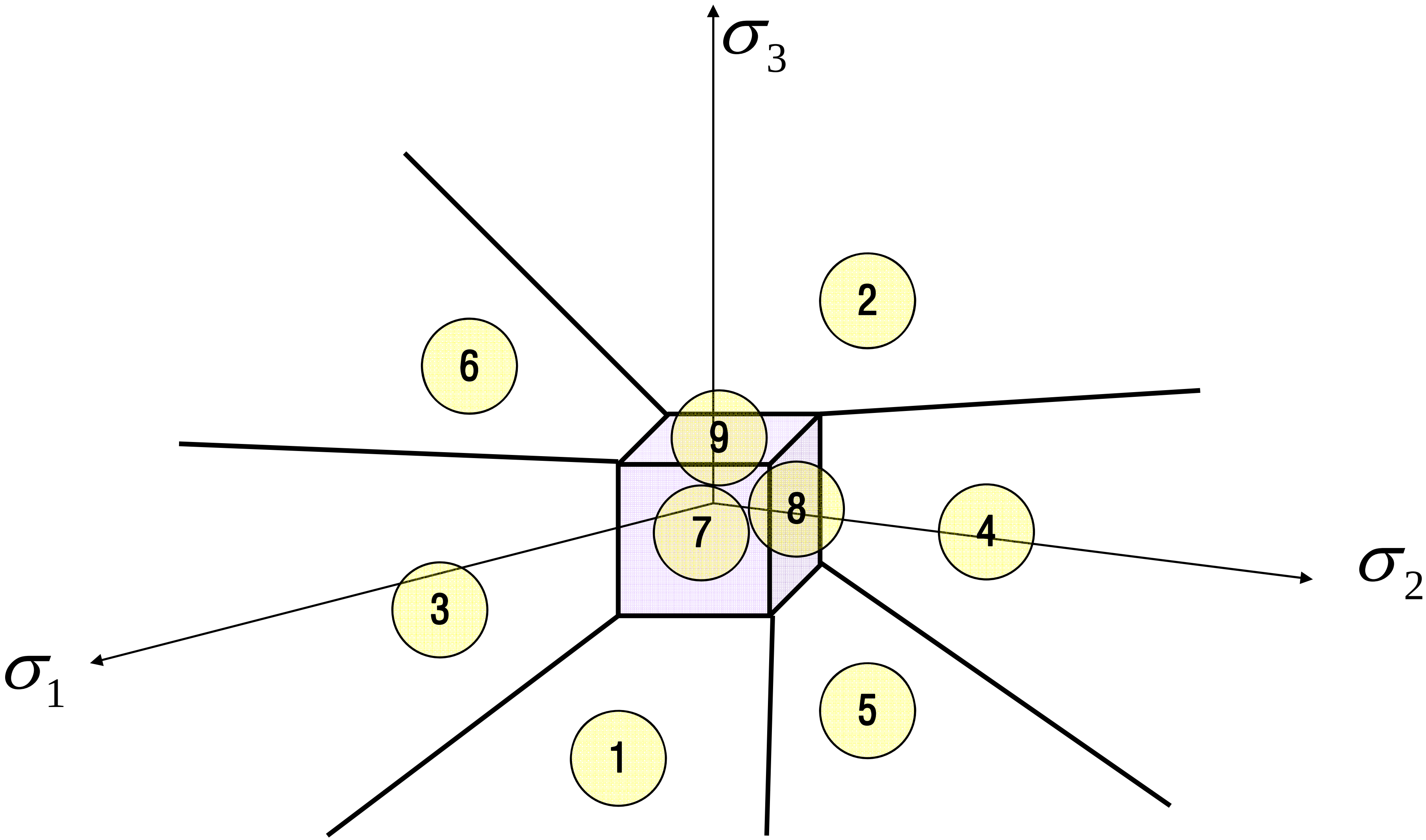




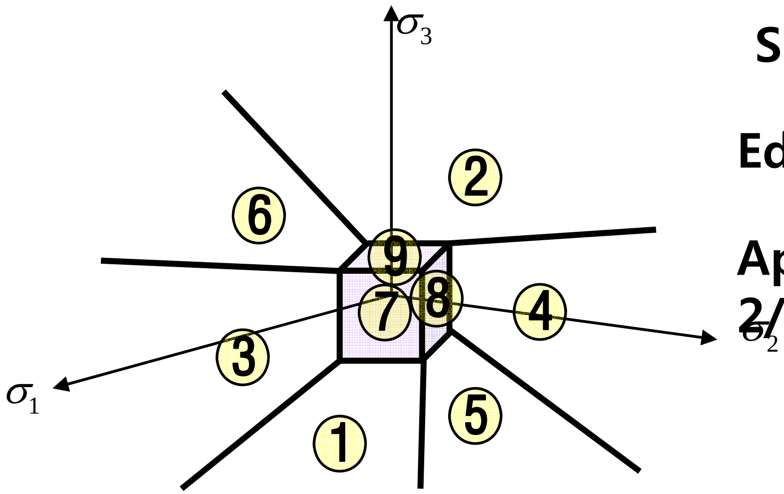
Plane Strain



Modified Coulomb Material



Modified Coulomb Material



Surfaces: 7, 8, 9

Edges: 1/7, 3/7, 6/9, 2/9, 4/8, 5/8, 8/7, 7/9, 8

Apex: 4/5/8, 7/8/9/1, 1/3/7, 3/7/6/9, 2/6/9,

8/9/2/4, 7/8/9

$$\frac{\sum \varepsilon^+}{\sum \left| \varepsilon^- \right|} > k$$

Dissipation Works

$$When \quad \frac{\sum \varepsilon^{+}}{\sum \left| \varepsilon^{-} \right|} = k$$

$$W = f_c \sum \left| \varepsilon^{-} \right|$$

$$When \quad \frac{\sum \varepsilon^{+}}{\sum \left| \varepsilon^{-} \right|} > k$$

$$W = f_c \sum \left| \varepsilon^{-} \right| + f_t \left(\sum \varepsilon^{+} - k \sum \left| \varepsilon^{-} \right| \right)$$