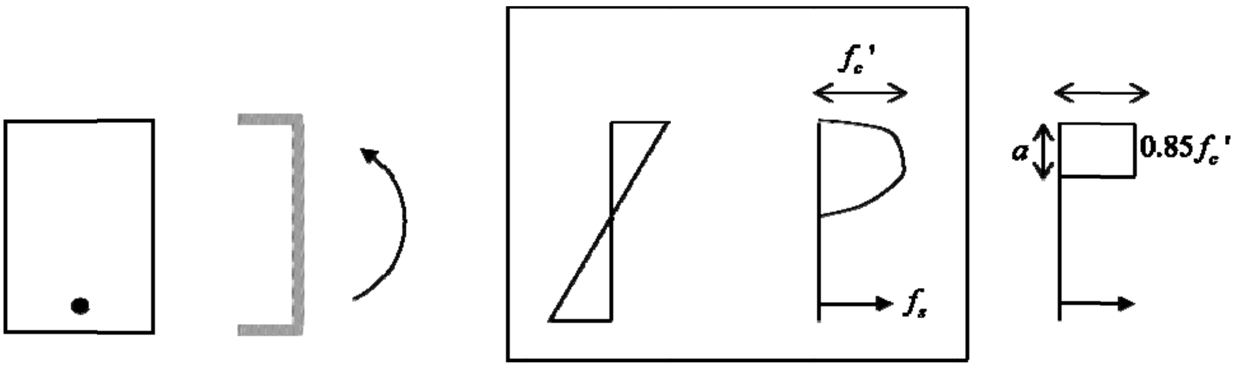
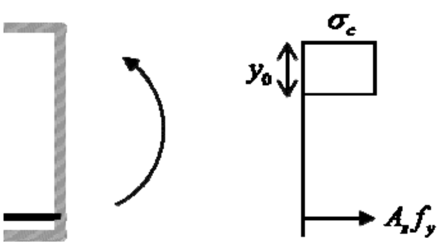
5. Beams

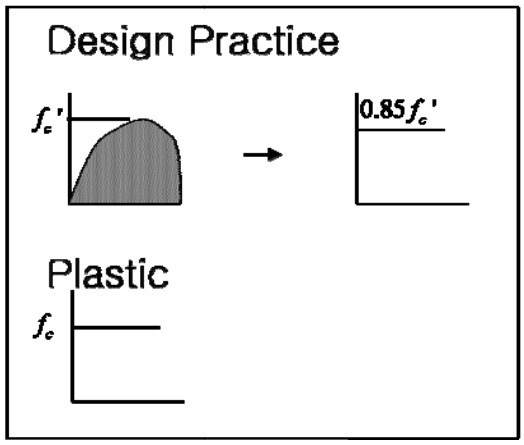
- bending
- shear
- torsion

5.1 Beams in Bending



Plastic approach





-equilibrium

$$\sigma_c b y_0 = A_s f_y$$

-yield condition

$$\sigma_c \le f_c$$

$$y_0 = \frac{A_s f_y}{b f_c}$$

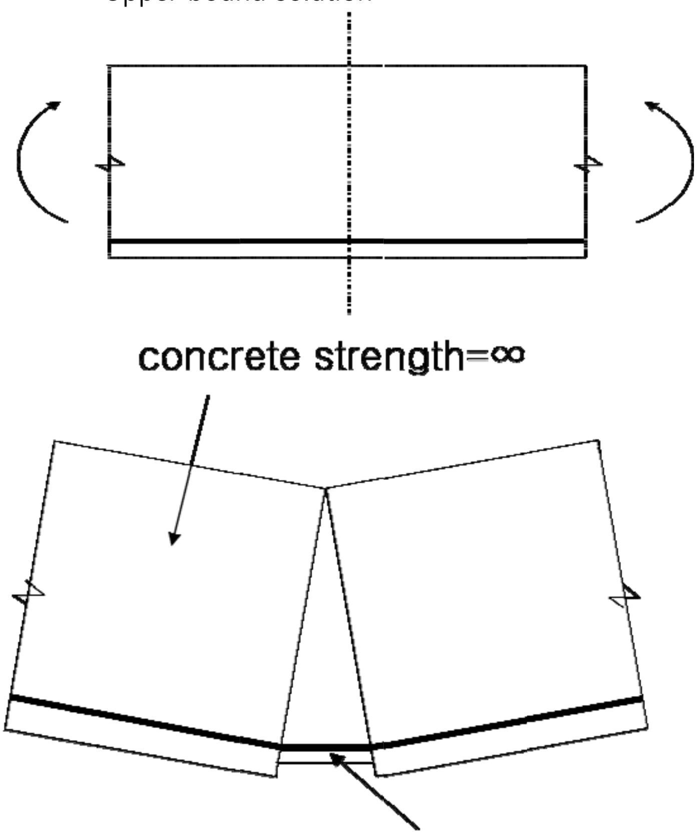
$$M = A_s f_y \left(d - \frac{y_0}{2} \right)$$

Let $\phi = \frac{A_s}{bd} \cdot \frac{f_y}{f_c}$ (w: mechanical reinf. ratio)

$$M_u = \left(1 - \frac{\phi}{2}\right)\phi b d^2 f_c$$

$$y_0 = \frac{A_s f_y}{b f_c} = \frac{A_s}{bd} \cdot \frac{f_y}{f_c} \cdot d = \phi d$$

- Upper bound solution



For finite compressive strength

Assume two lines of discontinuity
OA&OB

1) Constant strain expansion of region OAB

2) 2^{nd} mechanism : rigid body rotation

Dissipation work

We know that the first principal direction of strain is found to bisect the angle between the displacement vector and n-axis.

∴ direction of plastic displacement vector along OA is $\alpha = 90 - 2\beta$

i.e. $|v| = |v_2|$

$$W_l = \frac{1}{2} f_c v b (1 - \sin\alpha)$$

적분→ $W_l = \frac{1}{2} b f_c \frac{y}{\sin\beta} w_1 \left[1 - \sin(90° - 2\beta)\right] \frac{d_y}{\sin\beta}$

$$W_{right} = \int_0^{y_0} W_l dl = \frac{1}{2} b f_c \frac{y_0^2}{2} \frac{1 - \cos 2\beta}{\sin^2\beta} w_1 = \frac{1}{2} b f_c y_0^2 w_1$$

$$W_{total} = W_{right} + W_{rebar} = \frac{1}{2} b f_c y_0^2 w_1 + A_s f_y \left(d - \frac{y_0}{2}\right) w_1$$

$$W_{er} = M w_1$$

$$W_d = (w_1 + w_2)\left[\frac{1}{2} b f_c y_0^2 + A_s f_y \left(d - \frac{y_0}{2}\right)\right]$$

$$W_l = M(w_1 + w_2)$$

$$W_d = W_l$$

$$\rightarrow M_p = \frac{1}{2} b f_c y_0^2 + A_s f_y (d - y_0)$$

$$\frac{\partial M_p}{\partial y_0} = b f_c y_0 - A_s f_y = 0$$

¡Å $y_0 = \frac{A_s f_y}{b f_c}$

Lowest upper bound solution

$$M_p = A_s f_y \left(d - \frac{y_0}{2}\right)$$

5.2 Beams in shear

ACI code

$$V_n = V_c + V_s = 2\sqrt{f_c'}bd + \frac{A_s f_y}{s} d\cot\theta \rightarrow \theta = 45°$$

-axial forces: stringers

-shear force: compression field

¨ç concrete stresses

$$\begin{cases} \sigma_{cx} = -\sigma_c \cos^2\theta \rightarrow 1 \\ \sigma_{cy} = -\sigma_c \sin^2\theta \rightarrow 2 \\ \tau_{cxy} = \sigma_c \sin\theta\cos\theta \rightarrow 3 \end{cases}$$

② steel stress

$$\sigma_{sy} = \frac{A_s}{cb}\sigma_s = r\sigma_s \rightarrow 4$$

- Total stresses

$$\begin{cases} \sigma_x = \sigma_{cx} + \sigma_{sx} = -\sigma_c \cos^2\theta \rightarrow 5 \\ \sigma_y = \sigma_{cy} + \sigma_{sy} = -\sigma_c \sin^2\theta + r\sigma_s \rightarrow 6 \\ \tau = \sigma_c \sin\theta\cos\theta \rightarrow 7 \end{cases}$$

From 7,

$$\sigma_c = \frac{\tau}{\sin\theta\cos\theta} = \frac{\tau(\sin^2\theta + \cos^2\theta)}{\sin\theta\cos\theta} = \tau(\tan\theta + \cot\theta) \rightarrow 8$$

- Boundary conditions at the top face

$$\begin{cases} \sigma_y = 0 \\ r\sigma_s = \sigma_c \sin^2\theta(\rightarrow 10) = \dfrac{\tau}{\sin\theta\cos\theta}\sin^2\theta = \tau\tan\theta \rightarrow 10' \\ \sigma_x = -\dfrac{\tau}{\sin\theta\cos\theta}\cos^2\theta = -\tau\cot\theta \rightarrow 11 \end{cases}$$

8, $10'$ & 11 → expressions I . t . o. τ

- Yield conditions

$$\begin{cases} \sigma_c = \tau(\tan\theta + \cot\theta) \le f_c \\ \sigma_s \le f_Y \\ T \le A_s f_Y \quad \& \quad c \le c_f \end{cases}$$

Failure modes

-① stirrup yielding

-② web concrete crushing

Max. shear capacity

→ web crushing failure mode(stirrup yielding first)

$$\begin{cases} \sigma_c = \tau(\tan\theta + \cot\theta) \\ r\sigma_s = \tau\tan\theta \end{cases} \rightarrow \begin{cases} f_c = \tau(\tan\theta + \cot\theta) \rightarrow 16 \\ rf_Y = \tau\tan\theta \qquad \rightarrow 17 \end{cases}$$

16→17

$$\tan\theta = \frac{rf_Y}{\tau} = \frac{rf_Y}{f_c}\left(\tan\theta + \frac{1}{\tan\theta}\right) \rightarrow 18$$

Let, $\psi = \dfrac{rf_Y}{f_c} w$

Eq.18 is written as

$$\psi = \frac{\tan\theta}{\tan\theta + \dfrac{1}{\tan\theta}} = \frac{\tan^2\theta}{\tan^2\theta + 1}$$

$$\psi\tan^2\theta + \psi = \tan^2\theta$$

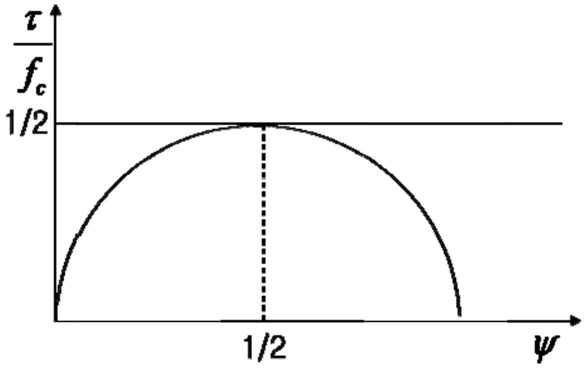
$$\tan^2\theta = \frac{\psi}{1-\psi} \rightarrow \tan\theta = \sqrt{\frac{\psi}{1-\psi}} \rightarrow 19 \qquad (5.2.14)$$

Hence eq. 16 is expressed as

$$\frac{\tau}{f_c} = \frac{1}{\tan\theta + \dfrac{1}{\tan\theta}} = \frac{\tan\theta}{\tan^2\theta + 1} \rightarrow 20$$

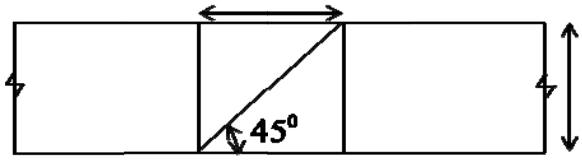
19→20

$$\frac{\tau}{f_c} = \sqrt{\frac{\psi}{1-\psi}} \cdot \frac{1}{\frac{\psi}{1-\psi}+1} = \sqrt{\psi(1-\psi)}$$



Design practice

-Min. stirrup: s<d



-Max. stirrup: $V_s \le 8\sqrt{f_c'}bd$

$$f_d = \tau(\tan\theta + \cot\theta) < f_c$$

$$= \frac{V}{\sin\theta\cos\theta} \cdot \frac{1}{bd}$$

$$= \frac{A_s f_y d \cot\theta}{s} \cdot \frac{1}{\sin\theta\cos\theta} \cdot \frac{1}{bd}$$

Assume $\theta = 45°$

$$f_d = \frac{A_s f_y d}{s}\frac{2}{bd} < f_c$$

$$V_s = \frac{A_s f_y d}{s} < \frac{1}{2} f_c bd = 8\sqrt{f_c'}bd$$

$$D = \sigma_c bd \cos\theta$$

$$V = D\sin\theta = \sigma_c bd \sin\theta\cos\theta$$

- Shift rule

$$C = \frac{M}{d} - \frac{V}{2}\cot\theta$$

$$T = \frac{M}{d} + \frac{V}{2}\cot\theta$$

Let, $\cot\theta$ =k

Free body diagram

-concrete stresses(1)

$$\begin{cases} \sigma_{cx} = -\sigma_c \cos^2\theta \\ \sigma_{cy} = -\sigma_c \sin^2\theta \\ \tau_{cxy} = \sigma_c \sin\theta\cos\theta \end{cases}$$

-steel stresses(2)

$$\begin{cases} \sigma_{sx} = 0 \\ \sigma_{sy} = r\sigma_s \\ \tau_{cxy} = 0 \end{cases}$$

-total stresses(3) ¡ç (1)+(2)

$$\begin{cases} \sigma_x = -\sigma_c \cos^2\theta \\ \sigma_y = -\sigma_c \sin^2\theta + r\sigma_s \\ \tau_{cxy} = \sigma_c \sin\theta\cos\theta \end{cases}$$

Eq's (3) are rewritten (@ top)

$$\begin{cases} \sigma_x = -\tau_{xy}\dfrac{\cos\theta}{\sin\theta} = -\tau_{xy}\cot\theta(\leftarrow \tau = p\cot\theta) = -p\cot^2\theta \\ \sigma_y = -\tau_{xy}\tan\theta + r\sigma_s = -p + r\sigma_s \\ \sigma_c = \tau\left(\tan\theta + \cot\theta\right) = p(1+\cot^2\theta) \end{cases}$$

Boundary along (1')-(2)-(2') & (2')-(2)-(3)

$$\begin{cases} \sigma_s = 0 \\ \sigma_x = -p\cot^2\theta \\ \sigma_y = -p \\ \sigma_c = p(1+\cot^2\theta) = p(1+k^2) < f_c \end{cases}$$

$$\sigma_y = 0, -p + r\sigma_s = 0$$

since $\begin{cases} r\sigma_s = p \\ \sigma_c = p(1+k^2) \end{cases}$

Region (2′)-(3)-(3′)

$$\begin{cases} \sigma_x = -2pk^2 \\ \sigma_y = -p + r\sigma_s = -2p \\ \sigma_c = 2p(1+k^2) \end{cases}$$

$$\tau = 2p\cot\theta$$

$$\tau = \frac{pa}{h} \quad (39)$$

$$\sigma_s = \frac{\tau\tan\theta - p}{r} \quad (40)$$

$$\sigma_x = -\tau\cot\theta \quad (41)$$

$$\sigma_y = -\tau\tan\theta \quad (42)$$

$$\sigma_c = \tau(\tan\theta + \cot\theta) \quad (43)$$

For web crushing failure

$$\begin{cases} \sigma_c = f_c \\ \sigma_s = f_y \end{cases} \quad \text{(a)}$$

Substituting (a) into (40) yields

$$f_y = \frac{\tau \tan\theta - p}{r} \quad \text{(b)}$$

Eq(43) becomes

$$\frac{f_c h}{pa} = \tan\theta + \cot\theta \quad \text{(c)}$$

Sub. (39) into (b) yields

$$f_y = \frac{\frac{pa}{h}\tan\theta - p}{r} \;\rightarrow\; \tan\theta = (f_y r + p)\frac{h}{pa}$$

Let $\psi = \frac{r f_y}{f_c}$

$$\tan\theta = \frac{f_c}{p}\frac{h}{a}\left(\psi + \frac{p}{f_c}\right) \quad \text{(d)}$$

Sub. (c) into (d) yields

$$\tan\theta = \left(\tan\theta + \cot\theta\right)\left(\psi + \frac{p}{f_c}\right)$$

$$\psi + \frac{p}{f_c} = \frac{\tan^2\theta}{\tan^2\theta + 1}$$

$$\tan^2\theta\left[\left(\psi + \frac{p}{f_c}\right) - 1\right] = -\left(\psi + \frac{p}{f_c}\right) \quad \text{(e)}$$

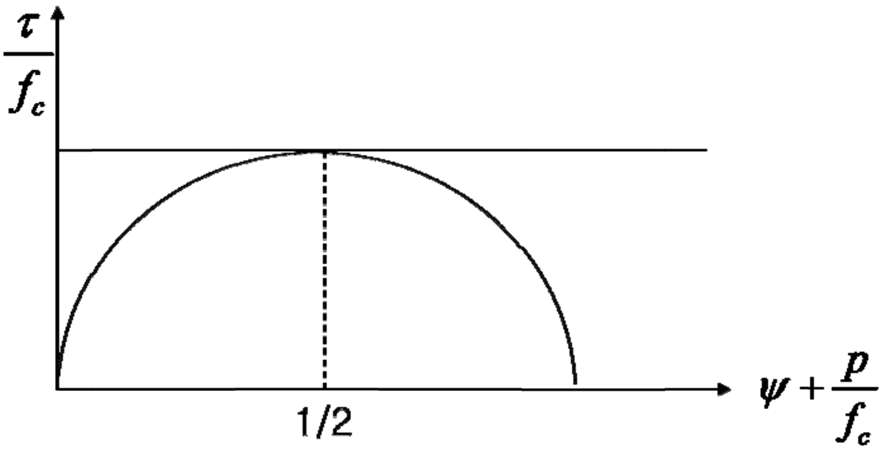
Eq(5.2.33) becomes

$$\frac{\tau}{f_c} = \frac{1}{\tan\theta + \cot\theta} = \frac{\tan\theta}{\tan^2\theta + 1} \quad \text{(f)}$$

Sub. (e) into (f) yields

$$\frac{\tau}{f_c} = \left(\psi + \frac{p}{f_c}\right)\frac{1}{\tan\theta} = \left(\psi + \frac{p}{f_c}\right)\sqrt{\frac{1 - \left(\psi + \frac{p}{f_c}\right)}{\psi + \frac{p}{f_c}}}$$

$$\therefore \frac{\tau}{f_c} = \sqrt{\left(\psi + \frac{p}{f_c}\right)\left[1 - \left(\psi + \frac{p}{f_c}\right)\right]} \quad (5.2.34)$$



Derivation of (5.2.37)

We know $\tau = \dfrac{pa}{h}$ (*)

Sub. (8) into (5.2.34) yields

$$\left(\frac{\tau}{f_c}\right)^2\left[1+\left(\frac{h}{a}\right)^2\right] + \frac{\tau}{f_c}\left[2\frac{h}{a}\psi - \frac{h}{a}\right] + \psi^2 - \psi = 0$$

$$\frac{\tau}{f_c} = \frac{1}{2}\frac{\frac{a}{h}}{1+\left(\frac{a}{h}\right)^2}\left[1 - 2\psi + \sqrt{1+4\left(\frac{h}{a}\right)^2\psi(1-\psi)}\right]$$

Eq (5.2.37) is valid for $\psi \le \frac{1}{2}\left(1 - \frac{h}{a}\right)$

$$\psi + \frac{p}{f_c} = \psi + \frac{\tau}{f_c}\frac{a}{h} \le 0.5$$

$$\psi \le \frac{1}{2}\left(1 - \frac{2h}{a}\frac{\tau}{f_c}\right) \quad (*)$$

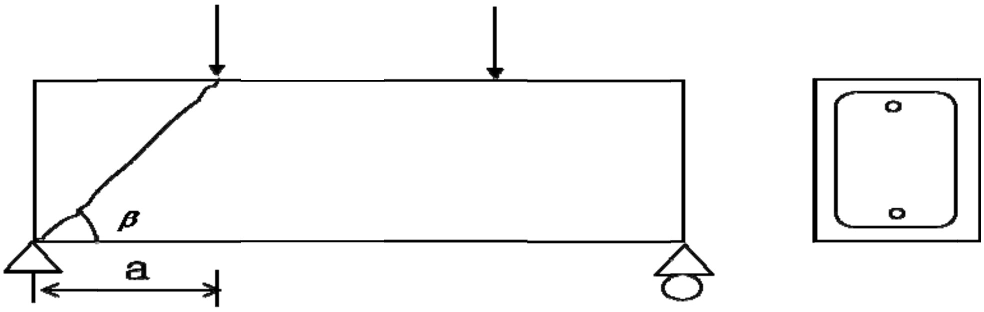
Sub.(47) into(*) yields

$$\psi \le \frac{1}{2}\left[1 - \frac{1 - 2\psi + \sqrt{1+4\left(\frac{h}{a}\right)^2\psi(1-\psi)}}{\left(\frac{a}{h}\right)^2 + 1}\right]$$

The result is

$$\psi \le \frac{1}{2}\left(1 - \frac{h}{a}\right)$$

Upper bound solutions



① $\frac{h}{a} \le \tan\beta \le \infty$

$$w_e = w_i$$

$$Pu = rf_y bh\cos\beta \cdot u + \frac{1}{2} f_c b(1-\cos\beta)\frac{h}{\sin\beta} \cdot u$$

$$\left(\frac{\tau}{f_c}\right)_{\min} = \sqrt{\psi(1-\psi)}$$

Where $\tan\psi = \frac{2\sqrt{\psi(1-\psi)}}{1-2\psi}$

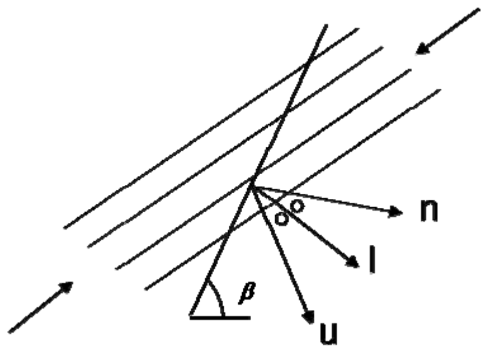
$$\beta = 2\theta$$

θ : angle of uniform diagonal stress field

β : angle of crack

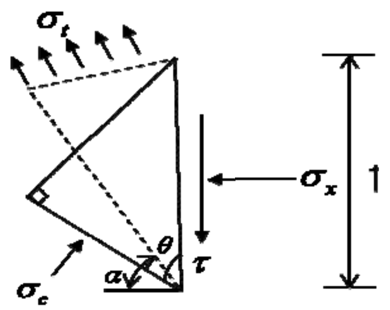
② $\tan\beta = \frac{h}{a}$

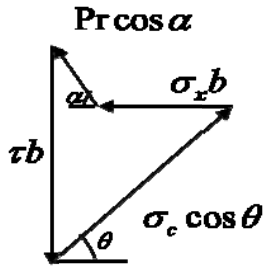
$$\frac{\tau}{f_c} = \frac{1}{2}\left[\sqrt{1+\left(\frac{h}{a}\right)^2} - \frac{a}{h}\right] + \psi\frac{a}{h}$$



5.2.2 Maximum shear capacity inclined shear reinforcement

Ⅰ-infinitesimal element





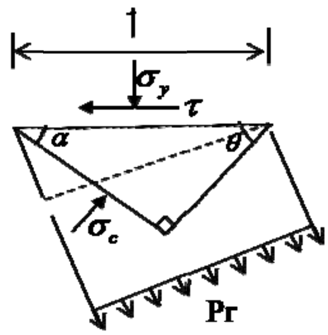
$P_r \cos\alpha$

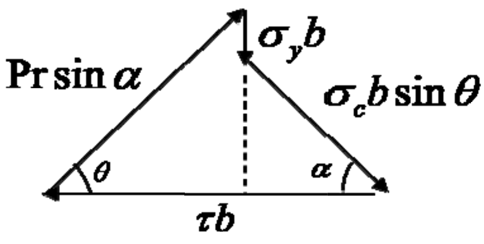
$P_r = \dfrac{A_s\sigma_s}{c}$

$\sigma_x b = -\sigma_c b\cos\theta + \Pr\cos^2\alpha$

$\sigma_x = -\sigma_c\cos^2\theta + \dfrac{\Pr}{b}\cos^2\alpha = -\sigma_c\cos^2\theta + \dfrac{A_s\sigma_s}{cb}\cos^2\alpha = -\sigma_c\cos^2\theta + r\sigma_s\cos^2\alpha$ (5.2.47)

Ⅱ- infinitesimal element





$\sigma_y b = -\sigma_c \sin\theta\cdot\sin\theta + pr\sin\alpha\cdot\sin\alpha$

$\sigma_y = -\sigma_c\sin^2\theta + r\sigma_s\sin^2\alpha$ (5.2.48)

$\tau = \sigma_c\cos\theta\cdot\sin\theta + r\sigma_s\cos\alpha\cdot\sin\alpha$ (5.2.49)

Setting $\sigma_y = 0$

$$r\sigma_s = \frac{\sigma_c \sin^2\theta}{\sin^2\alpha} \quad (5.2.50)$$

(50) → (49)

$$\tau = \sigma_c\left(\cos\theta\cdot\sin\theta + \frac{\sin^2\theta}{\sin\alpha}\cos\alpha\right)$$

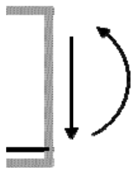
$$\sigma_c = \frac{\tau}{\sin^2\theta(\cot\theta+\cot\alpha)} \quad (5.2.51)$$

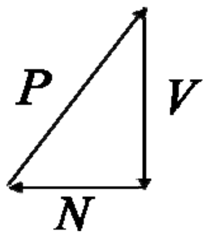
(51) → (50)

$$r\sigma_s = \frac{\tau}{\sin^2\theta}\cdot\frac{1}{\cot\theta+\cot\alpha}\cdot\frac{\sin^2\theta}{\sin^2\alpha} = \frac{\tau}{\sin^2\alpha(\cot\theta+\cot\alpha)} \quad (5.2.52)$$

(51) & (52) → (47)

$$\sigma_x = -\tau\left(\cot\theta - \cot\alpha\right)$$
$$N = \sigma_x bd$$
$$T = \frac{M}{h} + \frac{1}{2}V\left(\cot\theta-\cot\alpha\right)$$
$$C = \frac{M}{h} - \frac{1}{2}V\left(\cot\theta-\cot\alpha\right)$$





When $\sigma_c = f_c$

Maximum shear capacity

Eq(5.2.51) becomes

$$\frac{\tau}{f_c} = \sin^2\theta(\cot\theta+\cot\alpha)$$

Eq(5.2.52)

$$rf_y = f_c\sin^2\theta(\cot\theta+\cot\alpha)\cdot\frac{1}{\sin^2\alpha(\cot\theta+\cot\alpha)}$$

5.2.3 Maximum shear capacity, Beams without shear reinforcement

- Deep beam

Shear transfer = beam action+arch action

- Lower bound solution

Sigrist p.19

Marti (2.22) & (2.23)

$$\frac{\tau}{f_c} = \frac{1}{2}\left[\sqrt{1+\left(\frac{a}{h}\right)^2} - \frac{a}{h}\right]$$

The maximum load-carrying capacity corresponding to web crushing, $\sigma_c = f_c$

Since $\frac{\tau}{f_c} = \sin^2\theta(\cot\theta + \cot\alpha)$ &

$$f_y = f_c \sin^2\theta(\cot\theta + \cot\alpha)\cdot\frac{1}{\sin^2\alpha(\cot\theta + \cot\alpha)}$$

We have to find θ (free parameter)

Since $\psi = \frac{rf_y}{f_c}$

From (5.2.52)

$$\frac{rf_y}{f_c} = \psi = \frac{\sin^2\theta}{\sin^2\alpha}$$

$\therefore \sin\theta = \sqrt{\psi \sin^2\theta\alpha}$ - (A) or $\cos\theta = \sqrt{1-\psi\sin^2\alpha}$ - (B)

Then

$$\frac{\tau}{f_c} = \sin^2\theta(\cot\theta + \cot\alpha) = \sin\theta\cdot\cos\theta + \sin^2\theta\cdot\cot\alpha = \sqrt{\psi\sin^2\alpha}\sqrt{1-\psi\sin^2\alpha} + \psi\sin^2\alpha\cot\alpha$$

$$= \sqrt{\psi \sin^2 \alpha \left(1 - \psi \sin^2 \alpha\right)} + \psi \cos \alpha \sin \theta \quad (63)$$

From eqs (A) & (B)

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \sqrt{\frac{\psi \sin^2 \alpha}{1 - \psi \sin^2 \alpha}} \quad (64)$$

To find the max of eq (63)

$$\frac{\partial}{\partial \theta}\left(\frac{\tau}{f_c}\right) = \frac{\partial}{\partial \theta}\left[\sin \theta \cdot \cos \theta + \sin^2 \theta \cot \alpha\right] = \cos^2 \theta - \sin^2 \theta + 2 \sin \theta \cos \theta \cot \alpha = 0$$

$$\therefore -\cot \alpha = \frac{1}{2}\left(\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}\right)$$

$$\frac{2}{\tan \alpha} = \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} \quad (C)$$

(A) & (B) → (C)

$$\frac{2\psi \sin^2 \alpha - 1}{\sqrt{\psi \sin^2 \alpha \left(1 - \psi \sin^2 \alpha\right)}} = \frac{2}{\tan \alpha} \quad (C^*)$$

Let $\lambda = \psi \sin^2 \alpha$

Eq (C*) becomes

$$\lambda^2 - \lambda + \frac{\tan^2 \alpha}{4\left(1 + \tan^2 \alpha\right)} = 0$$

$$\lambda = \frac{1}{2}\left[1 \pm \sqrt{\frac{1}{1 + \tan^2 \alpha}}\right]$$

Since $\dfrac{1}{1 + \tan^2 \alpha} = \cos^2 \alpha$

$$\lambda = \frac{1}{2}\left[1 \pm \cos \alpha\right]$$

Therefore, if

$$\psi \sin^2 \alpha \le \frac{1}{2}\left(1 + \cos \alpha\right) \quad (5.2.55)$$

Eq (5.2.55) is valid

When $\lambda = \dfrac{1}{2}\left(1 + \cos \alpha\right)$ (D)

$$\frac{\tau}{f_c} = \sqrt{\frac{1}{2}\left(1 + \cos \alpha\right)\left(1 - \frac{1}{2} - \frac{\cos \alpha}{2}\right)} + \frac{1}{2}\left(1 + \cos \alpha\right)\cot \alpha = \frac{1}{2}\sqrt{1 - \cos^2 \alpha} + \frac{1}{2}\left(1 + \cos \alpha\right)\cot \alpha$$

$$=\frac{1}{2}\sqrt{1+\cos\alpha}\left[\sqrt{1-\cos\alpha}+\sqrt{1+\cos\alpha}\frac{\cos\alpha}{\sin\alpha}\right]=\frac{1}{2}\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}\left[1-\cos\alpha+\sin\alpha\frac{\cos\alpha}{\sin\alpha}\right]$$

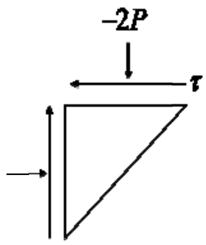
$$=\frac{1}{2}\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\frac{1}{2}\sqrt{\frac{1+\cos^2\frac{\alpha}{2}}{1-\cos^2\frac{\alpha}{2}}}=\frac{1}{2}\cot\frac{\alpha}{2} \quad (5.2.58)$$

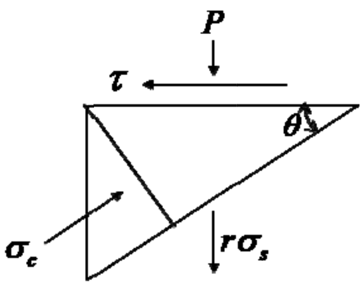
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

Sub $\psi^2\sin^2\alpha=\frac{1}{2}(1+\cos\alpha)$ into (5.2.59) yields

$$\tan\theta=\sqrt{\frac{\frac{1}{2}(1+\cos\alpha)}{1-\frac{1}{2}(1+\cos\alpha)}}=\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\sqrt{\frac{2\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}}=\cot\frac{\alpha}{2} \quad (5.2.59)$$

$$\sigma_s=\frac{p}{r}$$
$$\sigma_y=-2p$$
$$\sigma_x=-\tau\cot\theta$$





$$\sigma_c\sin^2\theta=p+r\sigma_s$$
$$\sigma_c\sin^2\theta=2p$$
$$\sigma_c\sin\theta\cos\theta=\tau$$

$$\cot\theta = \frac{\frac{a}{4}}{h} = \frac{a}{4h}$$

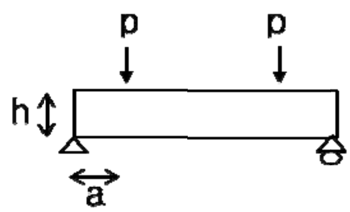
$$4p \cdot \frac{a}{4h} = \frac{pa}{h}$$

- Upper bound solution

- New flexural failure

5.2.4 Influence on shear capacity of longitudinal reinforcement

$$\begin{cases} T = \dfrac{M}{d} + \dfrac{V}{2}\cot\theta \\ C = -\dfrac{M}{d} + \dfrac{V}{2}\cot\theta \\ V = \dfrac{A_v f_y}{s} \cdot d\cot\theta \end{cases}$$



Yielding of longitudinal bars & stirrups

$$T_y = \frac{pa}{h} + \frac{p}{2}\cot\theta \quad \text{(a)}$$

$$\tau = \frac{V}{bh} = \frac{p}{bh} = \frac{A_v f_y}{bsh} \cdot h\cot\theta = rf_y\cot\theta \quad \text{(b)}$$

Let $\phi = \dfrac{A_{sl}}{bh}\dfrac{f_{rl}}{f_c}$ & $\psi = \dfrac{rf_y}{f_c}$

Eq(a) is rewritten as

$$A_{sl} f_{rl} = \frac{\tau bha}{h} + \frac{\tau bh}{2} \cdot \frac{\tau}{rf_y}$$

$$\phi = \frac{\tau}{f_c}\left(\frac{a}{h}\right) + \frac{\tau}{2f_c} \cdot \frac{\tau}{f_c} \cdot \frac{f_c}{rf_y} = \frac{\tau}{f_c}\left(\frac{a}{h}\right) + \frac{1}{2}\left(\frac{\tau}{f_c}\right)\frac{1}{\psi}$$

$$\frac{\tau}{f_c} = \psi \cdot \frac{a}{h} \cdot \left[\sqrt{1 + \frac{2\phi}{\psi\left(\frac{a}{h}\right)^2}} - 1\right] = \psi\left[\sqrt{\frac{2\phi}{\psi} + \left(\frac{a}{h}\right)^2} - \left(\frac{a}{h}\right)\right] = -\psi \cdot \frac{a}{h} + \sqrt{\left(\psi\frac{a}{h}\right)^2 + 2\psi\phi}$$

5.2.6 Design of shear reinforcement in beams

$$T = \frac{M}{d} + \frac{V}{2}\cot\theta$$

$$\sigma_c = \tau\left(\tan\theta + \cot\theta\right) \le v f_c'$$

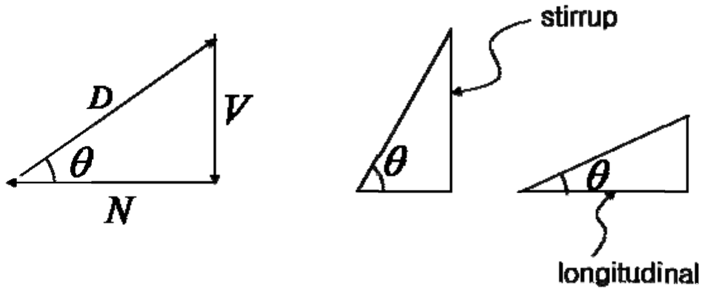
In ACI Code,

To avoid diagonal crushing failure

$$V_n = V_c + V_s$$

$$V_s \le 8\sqrt{f_c'} b_w d$$

$$\frac{50 b_w s}{f_y} \le A_v \le \left(V_s \le 8\sqrt{f_c'} b_w d\right)$$



$$D = f_d \cos\theta bd$$

$$V = D\sin\theta = f_d \cos\theta \sin\theta bd$$

$$\boxed{\frac{3}{5} \le \cot\theta \le \frac{5}{3}}$$