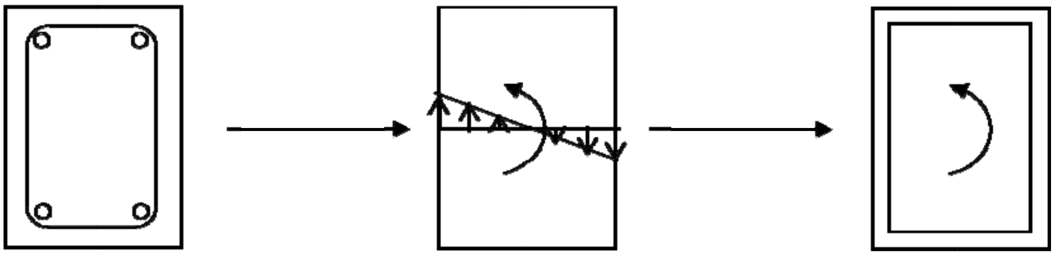
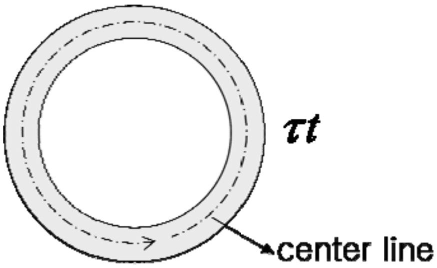
5.3 Beams on Torsion

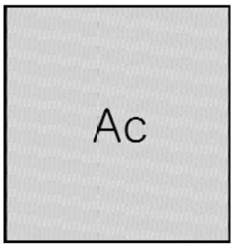


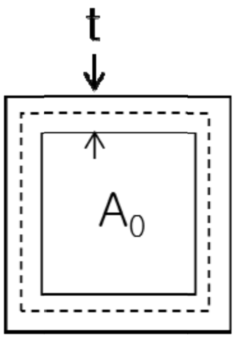
Bredt's Formula



$$\tau t = \frac{T}{2A_o}$$

- Equivalent thin tube thickness



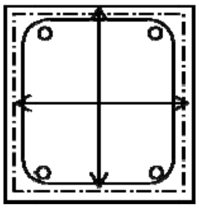


Collins & Mitchell

$$\begin{cases} t = \frac{3}{4}\frac{A_c}{P_c} \\ A_o = \frac{2}{3}A_c \end{cases}$$

ACI Code

$$\begin{cases} t = \frac{A_{oh}}{P_h} \\ A_o = 0.85A_{oh} \end{cases}$$



$g = \tau t = \dfrac{T}{2A_o}$ ①

g : uniform shear flow

A_o : area enclosed by g

P_o : perimeter of A_o

$g = f_d \cos\theta \sin\theta \cdot t$ ②

$N_T = f_d \cos^2\theta \cdot t \cdot P_o$ ③

① → ③

$N_T = \dfrac{TP_o}{2A_o}\cot\theta$ ④

Global Equilibrium condition

$\Sigma M_{3-4} = 0$: $T_b = \dfrac{M}{d} + \dfrac{1}{2}N_T$ ⑤

$\Sigma T = 0$: $T = 2A_o g = 2A_o f_d \sin\theta\cos\theta \cdot t$ ⑥

Since

$\dfrac{A_t f_s}{s} = f_d \sin^2\theta \cdot t$ ⑧

Eq. ⑥ becomes

$T = 2A_o \dfrac{A_t f_s}{s}\cot\theta$ ⑨

-Yield condition

$$\begin{cases} T_b \le T_y \\ T_t \le T_y \\ f_d \le \nu f_c' = f_{ce} \\ \dfrac{A_t f_s}{s} \le \dfrac{A_t f_y}{s} \end{cases}$$ ⑩

-Flexure-torsion interaction

a) Hoops and bot. long. Steel yield

$$\begin{cases} T_b = T_y \\ \dfrac{A_t f_s}{s} = \dfrac{A_t f_y}{s} \end{cases} \quad ⑪$$

Eq. ⑤ becomes

$$T_{by} = \frac{M_n}{d} + \frac{1}{2}\frac{T_n P_o}{2A_o}\cot\theta \quad ⑫$$

Eq. ⑨ becomes

$$T_n = 2A_o \frac{A_t f_y}{s}\cot\theta \quad ⑬$$

$$\cot\theta = \frac{T_n}{2A_o}\frac{s}{A_t f_y} \quad ⑬'$$

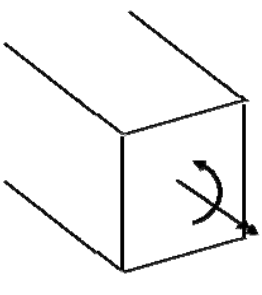
⑬' → ⑫

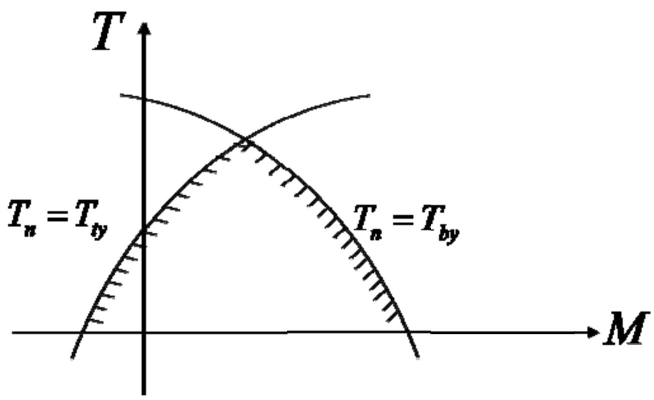
$$T_{by} = \frac{M_n}{d} + \frac{1}{2}\cdot\frac{T_n P_o}{2A_o}\cdot\frac{T_n}{2A_o}\cdot\frac{s}{A_s f_y}$$

$$\frac{M_n}{d} + \left(\frac{T_n}{2A_o}\right)^2 \cdot \frac{P_o}{2T_{by}} \cdot \frac{s}{A_t f_y} = 1 \quad \text{(parabolic interaction)}$$

b) Hoop & top long steel yields

$$-\frac{M_n}{T_{ty} d} + \left(\frac{T_n}{2A_0}\right)^2 \frac{P_0}{2T_{ty}} \cdot \frac{s}{A_t f_y} = 1$$



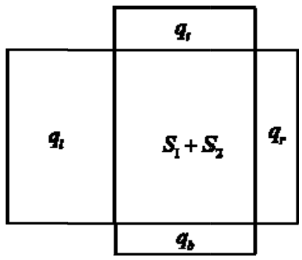


- Shear + torsion

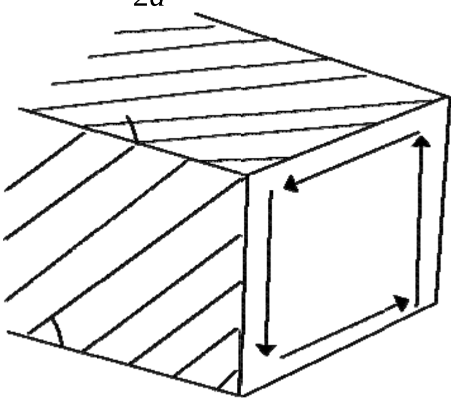
Combined result

$$q_l = \frac{V}{2d} + \frac{T}{2A_0} \quad ①$$

$$q_r = -\frac{V}{2d} + \frac{T}{2A_0} \quad ②$$



$$q_t = g_b = \frac{V}{2d} \quad ③$$



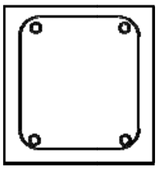
$$\frac{A_t f_y}{s} = q \tan\theta \quad ④$$

$$\cot\theta_l = \frac{s}{A_t f_s}\left[\frac{V}{2d} + \frac{T}{2A_0}\right] ⑤$$

$$\cot\theta_r = \frac{s}{A_t f_s}\left[-\frac{V}{2d} + \frac{T}{2A_0}\right] ⑥$$

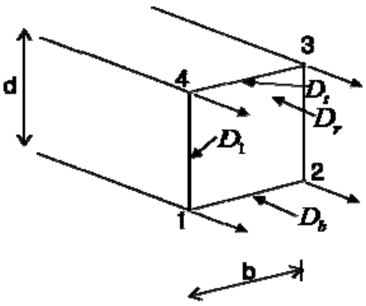
$$\cot\theta_t = \cot\theta_b = \frac{s}{A_t f_s} \cdot \frac{T}{2A_0} ⑦$$

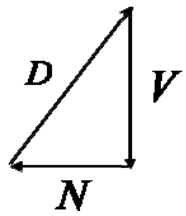
- First failure mode



$$T_b = T_{by}$$

$$f_s = f_y$$





$$\Sigma M_{3-4} = 0$$

$$T_{by} d = M + N_l \frac{d}{2} + N_r \frac{d}{2} + N_b d$$

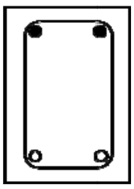
$$T_{by} = \frac{M}{d} + q_l d \cot\theta_l \frac{1}{2} + q_r d \cot\theta_r \frac{1}{2} + q_b b \cot\theta_b \quad ⑧$$

Sub ⑤ ⑥ & ⑦ into ⑧ yields $P \geq R$

$$T_{by} = \frac{M}{d} + \left[\frac{V}{2d} + \frac{T}{2A_0}\right]\frac{d}{2}\frac{s}{A_t f_y}\left[\frac{V}{2d} + \frac{T}{2A_0}\right] + \left[\frac{V}{2d} + \frac{T}{2A_0}\right]\frac{d}{2}\frac{s}{A_t f_y}\left[\frac{V}{2d} + \frac{T}{2A_0}\right] + \frac{T}{2A_0} \cdot b \cdot \frac{s}{A_t f_y}\frac{T}{2A_0}$$

$$\frac{M}{T_{by} d} + \left(\frac{V}{2d}\right)^2 \frac{d}{T_{by}}\frac{s}{A_t f_y} + \left(\frac{T}{2A_0}\right)^2 \frac{b t_d}{T_{by}}\frac{s}{A_t f_y} = 1$$

- second failure mode



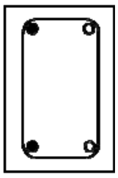
$$T_t = T_{ty}$$

$$A_s f_s = A_s f_y$$

$$\Sigma M_{1-2} = 0$$

$$-\frac{M}{T_{by} d} + \left(\frac{V}{2d}\right)^2 \frac{d}{T_{ty}}\frac{s}{A_t f_y} + \left(\frac{T}{2A_0}\right)^2 \frac{d_t b}{T_{ty}}\frac{s}{A_t f_y} = 0$$

- 3 rd



- 4 rd