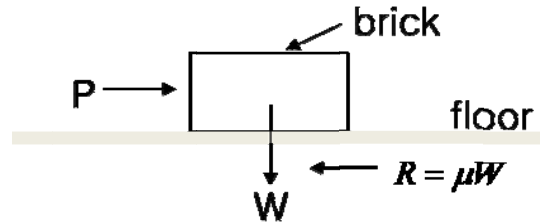
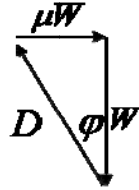


## 8. Shear in joints

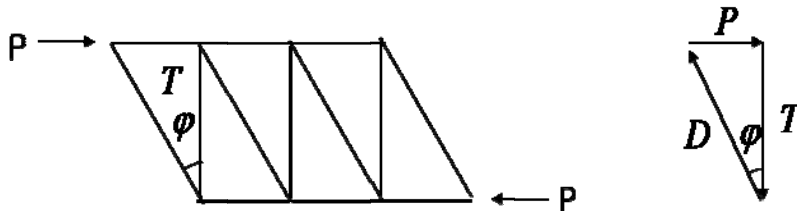
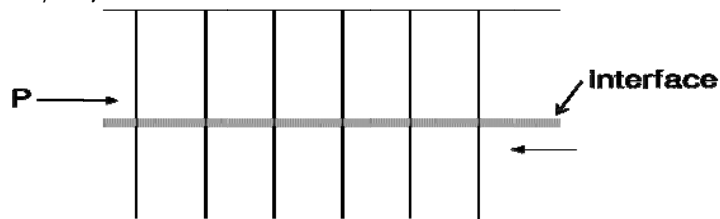
→ shear friction theory



$P \geq R \rightarrow$  Sliding



$\tan \phi = \mu$



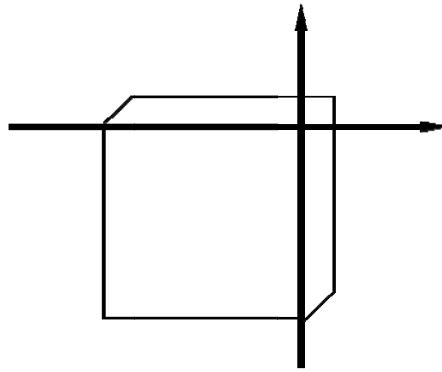
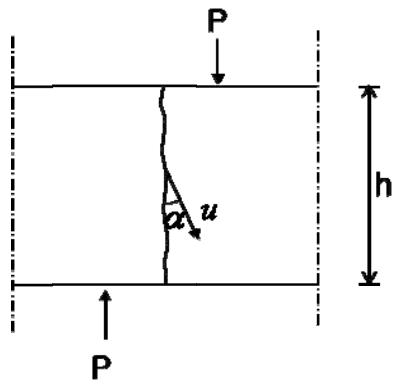
$$T \tan \phi \geq P \leftrightarrow \mu T \geq P \leftrightarrow \mu A_s f_y \geq P \leftrightarrow A_s \geq \frac{P}{\mu f_y}$$

$$D = \frac{T}{\cos \phi} = \frac{P \cot \phi}{\cos \phi}$$

$$\therefore \sigma_c = \frac{P}{\sin \phi \cdot bt} \leq f$$

### 8.2 Upper bound solutions

#### 8.2.1 Monolithic concrete



- i)  $\alpha = 0$
- ii)  $0 < \alpha < \varphi$
- iii)  $\alpha = \varphi$
- iv)  $\varphi < \alpha$

$$\begin{cases} W_E = Pu \cos \alpha \\ D_R = A_s f_y u \sin \alpha \\ D_C = W_t h \end{cases}$$

- i)  $\alpha = 0$

$$Pu = \frac{1}{2} f_c (1 - \sin \alpha) u h t$$

$$\tau h t u = \frac{1}{2} f_c u h t$$

$$\frac{\tau}{f_c} = \frac{1}{2} \quad \textcircled{1}$$

- ii)  $0 < \alpha < \varphi$

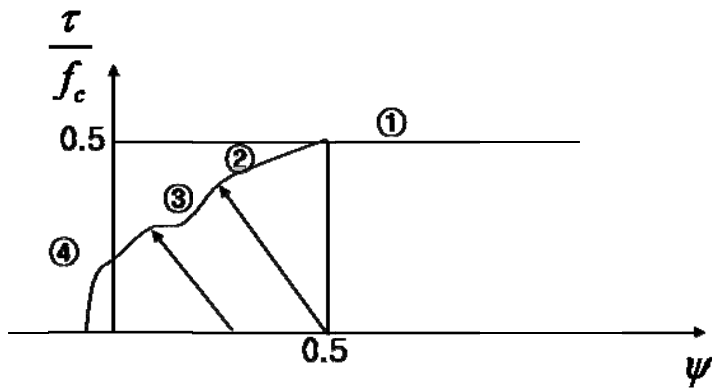
$$\frac{\tau}{f_c} = \sqrt{\psi(1-\psi)} \quad \text{where } \psi = \frac{A_s f_y}{h t f_c} \quad \textcircled{2}$$

- iii)  $\alpha = \varphi$

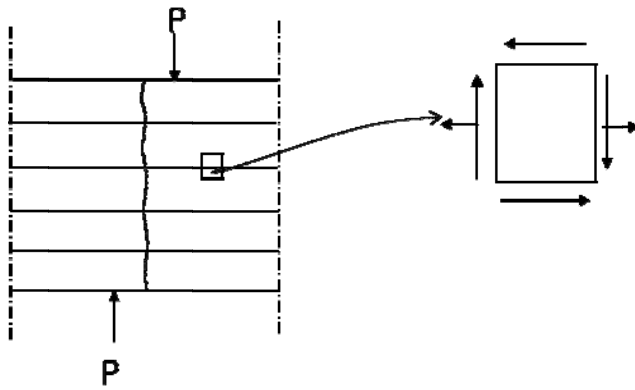
$$\frac{\tau}{f_c} = \frac{1 - \sin \varphi}{2 \cos \varphi} + \psi \tan \alpha$$

- iv)  $\varphi < \alpha$

$$\frac{\tau}{f_c} = \sqrt{\left(\psi + \frac{f_t}{f_c}\right) \left[1 - 2 \frac{f_t}{f_c} \frac{\sin \varphi}{1 - \sin \varphi} - \left(\psi + \frac{f_t}{f_c}\right)\right]}$$

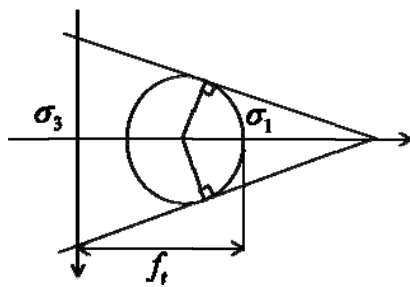


### 8.3 Lower bound solution



$$\tau = \frac{P}{ht}$$

$$\sigma_x = \frac{T}{ht} = \frac{A_s f_y}{ht}$$



$$k\sigma_1 - \sigma_3 = 2c\sqrt{k}$$

$$\sigma_3 = k\sigma_1 + 2c\sqrt{k}$$

$$\sigma_3 = \frac{1 + \sin \varphi}{1 - \sin \varphi} f_t - f_c$$

In Mohr's circle

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2}$$

$$\text{Radius of } M, C = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sigma_m = \frac{f_t}{2} + \frac{f_t}{2} \frac{1 + \sin \varphi}{1 - \sin \varphi} - \frac{f_c}{2} = \frac{f_t}{1 - \sin \varphi} - \frac{f_c}{2}$$

$$R = \frac{f_c}{2} - f_t \frac{\sin \varphi}{1 - \sin \varphi}$$

$$(\sigma - \sigma_m)^2 + \tau^2 = R^2$$

Let  $\sigma = \psi f_c$

$$\left( \psi f_c + \frac{f_c}{2} - \frac{f_t}{1 - \sin \varphi} \right)^2 + \tau^2 = \left( \frac{f_c}{2} - f_t \frac{\sin \varphi}{1 - \sin \varphi} \right)^2$$

$$\left( \frac{\tau}{f_c} \right)^2 = \left( \frac{1}{2} - \frac{f_t}{f_c} \frac{\sin \varphi}{1 - \sin \varphi} \right)^2 - \left( \psi + \frac{1}{2} - \frac{f_t}{f_c} \frac{1}{1 - \sin \varphi} \right)^2$$