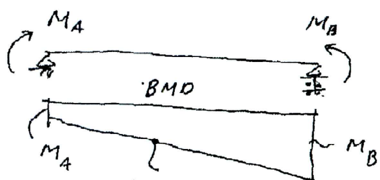
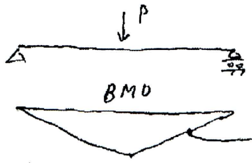
5.5 Beams with other loading conditions

ex) ┌ unequal end moments →
└ Central concentrated load



$(M_x)_{ext}$ = function of z



(1) Beams in practical situations will be subjected to a wide variety of loadings, thus producing non-uniform moment along the length of the beam

↳ the resulting governing D.E.

= Linear differential equations with Variable coefficients (not constant coefficient)

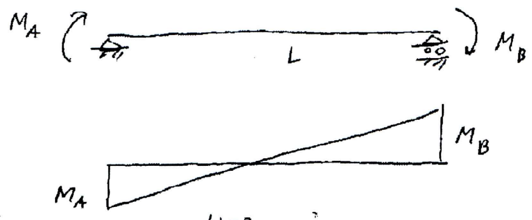
↳ 일반적으로 해석적 정해가 존재치 않으므로 수치해석적 근사법에 의해 M_{cr}을 산정해야

(Timoshenko and Gere 1961, Massonet 1947, Horne 1954, Salvadori 1955)

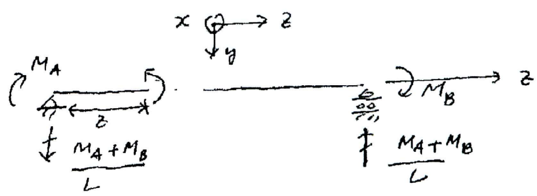
(2) A simple but effective method

↑ equivalent moment concept (보-기둥 부재의 등가균등모멘트 개념과 흡사)

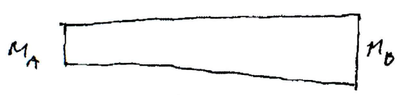
5.5.1 Unequal End Moments



: 복곡률



$$(M_x)_{ext} = M_A - \left(\frac{M_A+M_B}{L}\right)z \longleftrightarrow M_o$$



: 단곡률

따라서 (5.4.21)식의 미분방정식은 다음과 같은 꼴로 수정된다:

$$EC_w\frac{d^4\gamma}{dz^4} - GJ\frac{d^2\gamma}{dz^2} - \underbrace{\frac{1}{EI_y}}_{\text{a variable coeff.}}\left\{M_A - \left(\frac{M_A+M_B}{L}\right)z\right\}^2\cdot\gamma = 0$$

Solution 형태를 무한급수 형태로 표시한 후 근사해를 얻을 수 밖에 없음

ex) (Bessel function, Legendre ") →

(또는 Galerkin method ....)

↳ "quite cumbersome".

For design purposes, the effect of moment gradient on the critical moment can easily be accounted for by the use of an equivalent moment factor C_b (Salvadori 1955):

↑ Bending Coefficient

$$M_{cr} = C_b \times M_{ocr} \quad ---- \quad (5.5.1)$$

where $M_{ocr} = \frac{\pi}{L}\sqrt{EI_yGJ}\sqrt{1+\overline{W}^2}$ ← (5.4.34)

$$C_b = 1.75 + 1.05\left(\frac{M_A}{M_B}\right) + 0.3\left(\frac{M_A}{M_B}\right)^2 \le 2.3$$

「지금도 쓰이고 있음」

How replaced.

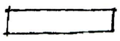
$|M_B| > |M_A|$

$\frac{M_A}{M_B}$ = positive for double curvature bending

$\frac{M_A}{M_B}$ = negative for single " "

「마치 Austa 접근법과 비슷」

★ Fig. 5.14 (P. 327) : 이론값과 설계식 (5.5.1) 비교 ↗ $C_b = 1.0$



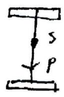
(1) Uniform moment의 경우 (즉 $M_A = M_B$, 단곡률)가 가장 불리

(2) $M_A = M_B$ 이고 복곡률의 경우는 2.3배의 강도내력 증가를 인정

upper bound

5.5.2 Central Concentrated Load

Fig. 5.16 (P. 329) 참조 :



(1) 집중하중이 shear center에 작용하는 상황

(2) 보 좌굴시 수직반력 P/2 외에도 $\frac{1}{2}(Pu_m)$ 크기의 비틀림 반력이 발생함에 유의

↳ 보 중앙부에서의 횡변위

「External moment components w.r.t. the x-y-z coord.」 →

$$(M_x)_{ext} = \frac{P}{2}\left(\frac{L}{2} - z\right)$$

$$(M_y)_{ext} = 0$$

$$(M_z)_{ext} = -\frac{P}{2}\left(u_m - u(z)\right)$$

비틀림모멘트

↳ 확인요

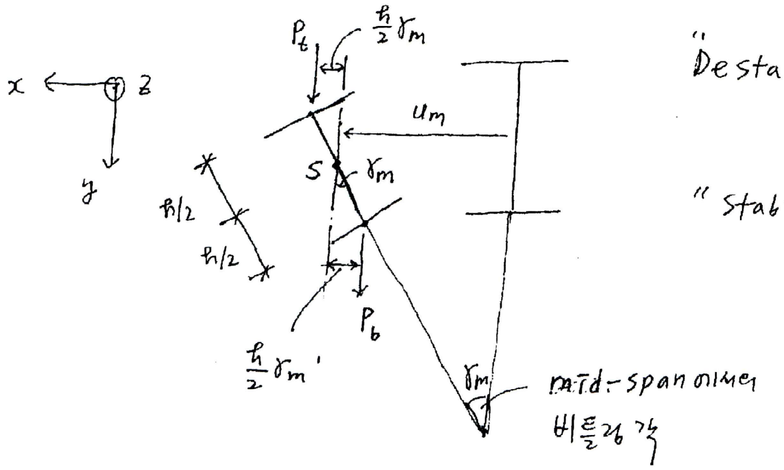
부호는 오른손 나사 규약에 의함

Eqs. (5.5.3) ~ (5.5.5)

(1) (5.5.3) ~ (5.5.5) 식에 표시된 'undeformed 좌표계'에 대한 external moment components를 'deformed 좌표계'로 projection하여 (5.5.6) ~ (5.5.8) 식이 얻어짐을 확인할 것 ← Fig. 5.17 참조

(2) (1)에서 얻어진 'deformed 좌표계'에 대한 'external moment components'를 'internal moment components'와 등치시킨 후 횡변위 성분 u를 소거하여, 최종적으로 governing D.E.가 (5.5.15) 식과 같이 얻어짐을 확인할 것. ↳ 제 3항 계속; Variable ↳ 무한급수해법 불가피 (Not that difficult)

★ 가력점 위치 · 횡하중에 대한 영향 : 「Fig. 5.18 참조 (p. 332)」 (Timoshenko-Gere)



"Destabilizing" moment $= P_t \times \left(\frac{h\gamma_m}{2}\right)$
↳ (5.5.16)

"Stabilizing" moment $= P_b \times \left(\frac{h\gamma_m}{2}\right)$
↳ (5.5.~~7~~ 17)

「이항을 $(M_z)_{ext}$에 반영하여 (5.5.14) 식을 수정하여 처리」

★ Nethercot and Rockey's Approximate Equation for the purpose of design (1971)

현존 Europe 최고 Steel researcher

$$M_{cr} = \frac{P_{cr} L}{4} = C_b \times \underbrace{M_{ocr}}_{\text{base strength}} \quad \cdots (5.5.18)$$

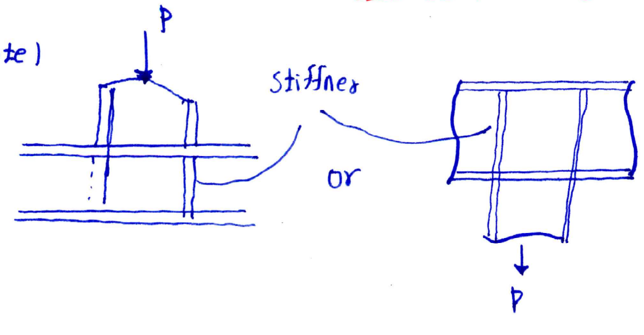
↑ "unified approach" (교과서 Fig. 5.18)

with $C_b = A \times B$ for the bottom flange loading
$= A$ for the shear center loading
$= A/B$ for the top flange loading

$\left(A = 1.35;\ B = 1 + 0.649\,\overline{W} - 0.180\,\overline{W}^2;\ \overline{W} = \frac{\pi}{L}\sqrt{EC_w/GJ}\right)$

Note : 등분포모멘트의 1.35배!

Note)



「그리고 반드시 가력점 부근 횡지지 해야 현명!」

5.5.3 Other loading conditions

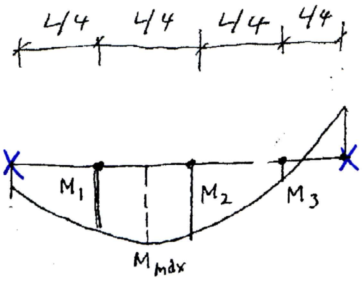
(1) Table 5.2 (a): based on numerical results by a number of researchers
(참고문헌 2, 8, 9, 10~15)

↳ shear center loading을 전제로

★ (2) Table 5.2 (b): Unified (approximate) formula by Kirby and Nethercot (1979)

★ 「"AISC-LRFD 최근 개정판에서 이 공식 채택"」 KBC, also

Euro 기준
AISC 기준



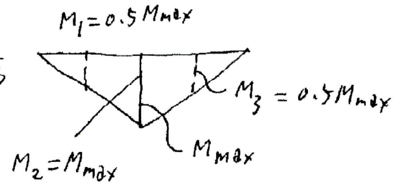
X: brace Point

= BMD

$$C_b = \frac{12}{3\left(\frac{M_1}{M_{max}}\right) + 4\left(\frac{M_2}{M_{max}}\right) + 3\left(\frac{M_3}{M_{max}}\right) + 2} \quad \cdots (5.5.22)$$

Ex) ① Uniform moment case: $M_1 = M_2 = M_3 = M_{max} \rightarrow C_b = \frac{12}{3+4+3+2} = 1.0$ (OK)

② 중앙부 집중하중



$$C_b = \frac{12}{3(0.5) + 4 + 3(0.5) + 2} = \frac{12}{9} = \frac{4}{3} = 1.33 \doteq 1.35$$

Table 5.2 (a)

HW #

↳ (5.5.22)식을 사용하여 C_b를 계산하고
이 결과가 Table 5.2 (a)의 값과 잘 일치함을 확인할 것.

(3) Table 5.3 (p.335) 및 Fig. 6.19 (p.336)

↳ 2등분점하중 및 등분포하중의 2가지 경우에 대한,
가력점에 따른 좌굴하중 산정 실용식 (Nethercot and Rockey 1971)

↳ 「사실은 가력점 횡지지하는 것이 타당」

권일중

① 캔틸레버
② 양단고정
③ 기타
④ 연속보

Ex) cantilever, both ends fixed, 연속보

(19)

5.6 Beams with other "support" conditions

(1) The discussion so far pertains only to beams that are torsionally simply supported.

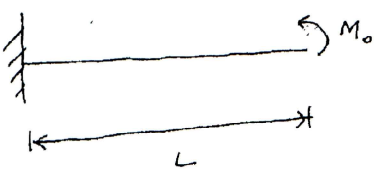
↳ (The ends of the beams are free to rotate and warp about the weak axis, but are restrained against rotation about the centroidal axis.

(2) The effective length concept to account for the support conditions.
유효횡좌굴길이 적용가능 (지지)

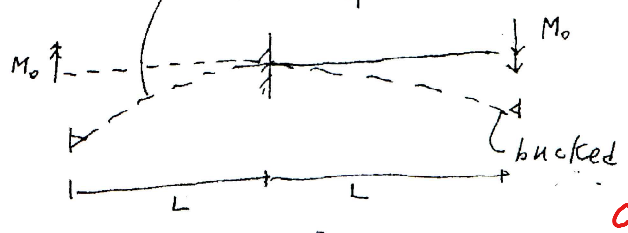
5.6.1 Cantilever Beams

(a) Uniform moment (가장불리)

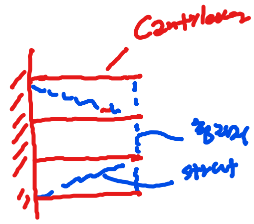
「길이가 "2L"인 양단 비틀림 단순지지 보가 균등모멘트 M_o를 받을때의 좌굴변형 모드와 동일」



: side view



: top view



: top view

Thus, from Eq (5.4.34) ← M_{ocr}

$$M_{cr} = \frac{\pi}{\text{"}2L\text{"}}\left(\sqrt{EI_y GJ}\right)\sqrt{1+\frac{\pi^2 EC_w}{(\text{"}2L\text{"})^2 GJ}} \quad \text{--- (6-1)}$$

(b) For other loadings, recourse must be made to numerical procedures to obtain solutions (e.g., Anderson and Trahair 1972, Nethercot 1973)
수치해석 불가피
(특수한 경우의 design aids
(실제로 tip을 횡지지할 것임) → diagonal strut)

① < Fig. 20 ← Cantilever beam with a concentrated load at the free end
Fig. 21 ← " " " " uniform load ← 위험!

② For other support conditions (Nethercot 1983),
(5.4.34)식의 일반화에 불과 (p. 324)

$$M_{cr} = \left(\frac{\pi}{KL}\right)\sqrt{EI_y GJ}\sqrt{1+\frac{\pi^2 E C_w}{(KL)^2 GJ}} \quad \text{--- (5.6.2)}$$

In which K is the effective length factor of the beam

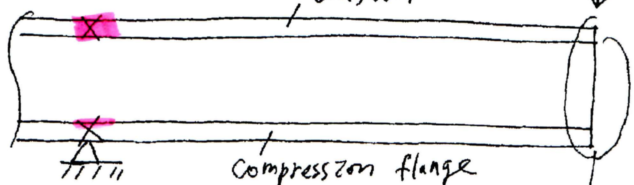
↳ Table 5.4 (p.339) 참조 ← conservative estimate
(단부 집중하중 또는 등분포하중에 적용가능)
↳ 물리적 feel과 부합하는가? (다음 쪽)

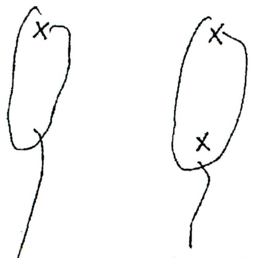
Several observations from Table 5.4

(1)

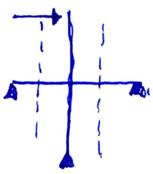
P (top flange loading 로 가정)

tension





가령
실험체
설계시
유의



가장 practical 한 지점조건

: x = lateral support

compression flange

No lateral support
(k = 2.5)

top flange
lateral bracing only
(k = 2.5)

「인장 flange 만 횡지지하는 의미가 없음을 알 수 있음」

both top and bottom flange bracing
(k = 1.5)
≠
2.5 vs. 1.5

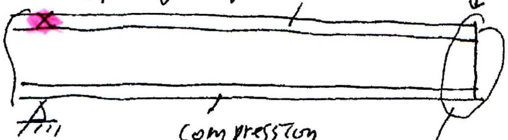
→ Top flange loading 이 아니면 k ≤ 1.0

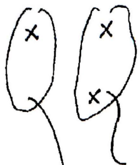
(2)

top flange bracing only

tension

P





compression

하단부 압축 flange 횡지지 없음

No lateral restraint
(k = 7.5)
≠
2.5 vs. 7.5
(좌굴강도가 급격히 저하함)

k = 4.5

k = 7.5

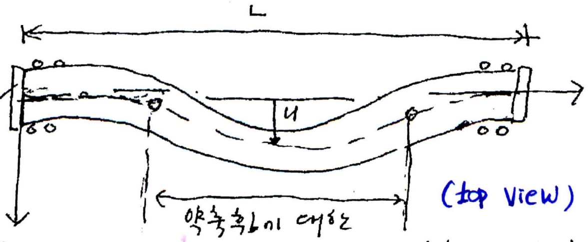
> 7.5 vs. 4.5

"Fundamentals of lateral bracing" ← 압축 flange 의 횡지지가 중요

5.6.2 Fixed End Beams

← Torsionally

L



Fixed against warping and twisting

약축휨에 대한 변곡점 사이거리 = L/2 = 0.50 L

(top view)

"경계조건"

For lateral bending

$$u|_{z=0} = u|_{z=L} = u'|_{z=0} = u'|_{z=L} = 0$$

For twisting

$$\gamma|_{z=0} = \gamma|_{z=L} = 0$$

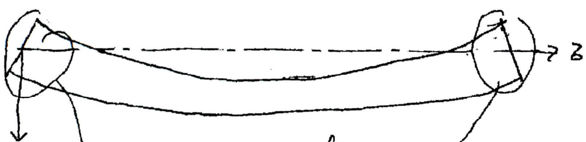
For warping

$$\left.\frac{d\gamma}{dz}\right|_{z=0} = \left.\frac{d\gamma}{dz}\right|_{z=L} = 0$$

왜 이러한가?

(∵ 상부상 모멘트 $T_{sv} = GJ\frac{d\gamma}{dz} = 0$ 이므로)

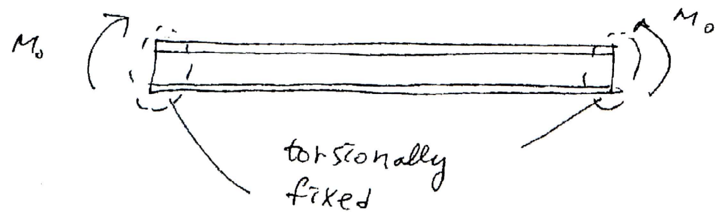
고정단부에서



Free to rotate about the strong axis.

(side View)

"Under a uniform moment"



5.4 절과 동일한 유도과정을 거쳐서 (or $(M_{z'})_{int} = (M_z)_{ext}$)

$$\frac{d^4\gamma}{dz^4} - 2q \cdot \frac{d^2\gamma}{dz^2} - b\gamma = 0 \quad \text{--- (5.4.23)}$$

In which $q = \dfrac{GJ}{2ECw}$; $b = \dfrac{M_o^2}{EI_y\, ECw}$

일반해는 $\gamma = A \sin mz + B \cos mz + Ce^{nz} + De^{-nz}$ --- (5.4.24)

In which $m = \sqrt{-q + \sqrt{q^2+b}}$; $n = \sqrt{q + \sqrt{q^2+b}}$

(5.4.24)에 경계조건 4가지, 즉 $\gamma|_{z=0} = \gamma|_{z=L} = \gamma'|_{z=0} = \gamma'|_{z=L} = 0$을 대입하여 정리하면, 다음의 특성방정식이 얻어짐.

$$\left(\frac{n^2-m^2}{2mn}\right) \sin mL \sinh nL + \cos mL \cosh nL - 1 = 0 \quad \text{--- (5.6.5)}$$

이 특성방정식은 간단히 풀리지 않음
(시행착오적 계산 필요)

★ 횡좌굴에 대한 등가비지지 거리 개념을 도입하여 구함

$KL = 0.50L$ (앞의 20쪽 2행 참조)

$$\therefore M_{cr} = \frac{\pi}{(L/2)} \sqrt{EI_y GJ} \sqrt{1 + \frac{\pi^2 ECw}{(L/2)^2 GJ}} \quad \text{---- (5.6.6)}$$

↳ The critical moment for a fixed end beam is considerably higher than that of a simply supported beam (Fig. 5.23 참조)

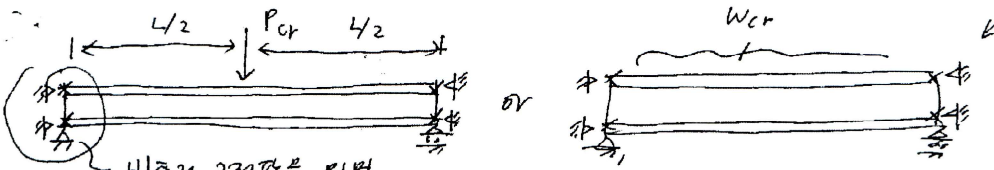
(양단 고정을 전제하고)

" For other types of loading " (producing non-uniform distribution of moments along the length of the beam)

↳ The D.E will have variable coefficients and recourse to numerical procedure is inevitable.

(5.5절 및 15 및 16쪽 참고)

Nethercot and Rockey (1971)



비틀림 고정단을 의미

$$M_{cr} = C_{bs}\, M_{ocr} \quad \text{--- (5.6.7)}$$

(5.4.34)식

where $$M_{cr} = \frac{P_{cr}}{4} \times L \quad \text{or} \quad \frac{w_{cr} L^2}{8} \quad \text{--- (5.6.8)}$$

and
$$\begin{aligned} C_{bs} &= A \times B \quad (\text{for bottom flange loading}) \\ &= A \quad (\text{for shear center} \quad ") \\ &= A/B \quad (\text{for upper flange} \quad ") \end{aligned}$$

↳ the expressions of A and B (Table 5.5 참고, p.342)

The effective length factor K for the beams shown in Table 5.5 can be obtained by equating (5.6.7) with (5.6.2).

$$\underbrace{\frac{\pi}{KL}\sqrt{EI_y GJ}\sqrt{1+\frac{\pi^2 E C_w}{(KL)^2 GJ}}}_{(5.6.2)} = \underbrace{C_{bs} M_{ocr}}_{(5.6.7)}$$

Solving for K,

$$K = \frac{\pi\sqrt{EI_y GJ}}{\sqrt{2}L\,(C_{bs}M_{ocr})}\left\{1+\sqrt{\frac{4(C_{bs}M_{ocr})^2}{EI_y GJ}\,\frac{EC_w}{GJ}+1}\right\} \quad \cdots (5.6.10)$$

↳ K 값에 영향을 미치는 인자들 : L, E, G, Cw, J

- L : 비지지 거리
- E, G : 재료성질
- Cw, J : 단면형상

the types of loading, 전단중심에 대한 하중 작용점의 상대위치 등등.

「실용 설계에 (5.6.10)식을 사용하기에는 너무 번거롭기 때문에 통상 K = 1.0 (보수적 값)을 사용함」

↳ 결론 (simplistic!)

★ 만일 횡방향 휨과 (u) 비틀림 (γ)에 대한 구속조건이 서로 다르다면, lateral bending에 대한 K_b 값과 twisting에 대응되는 K_t 값도 서로 달라질 것이다.

Uniform moment를 받는 경우 (Vlasov 1959), 러시아 연구자

$$M_{ocr} = \frac{\pi\sqrt{EI_y GJ}}{K_b L}\sqrt{1+\frac{\pi^2 E C_w}{(K_t L)^2 GJ}} \quad \cdots (5.6.11)$$

K_b 및 K_t 값 → Table 5.6 참고 (p.343)

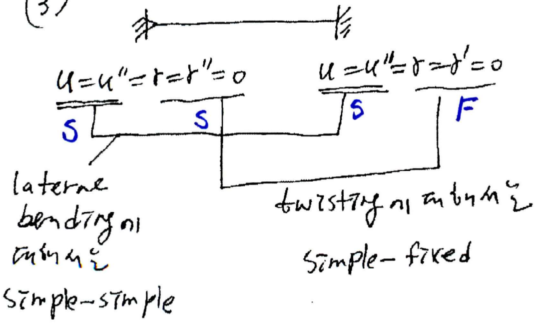
Note : ① $u = u'' = \gamma = \gamma'' = 0$ (양단 모두) ← torsionally simple support 조건에 상당

∴ 따라서 $K_b = 1.0$; $K_t = 1.0$ (첫번째 줄)

② $u = u' = \gamma = \gamma' = 0$ (양단 모두) ← torsionally fixed support 조건에 상당

따라서 $K_b = K_t = 0.492 \doteq 0.50$ (7번째 줄)

③



← $K_b = 0.904 \doteq 1.0$, $K_t = 0.693 \doteq 0.70$ (2번째 줄)

∴ For simplicity

- S − S → 1.0
- S − F → 0.70
- F − F → 0.50

} Bending 이든 twisting 이든. (In short)

→ 실지로 않은 경우에

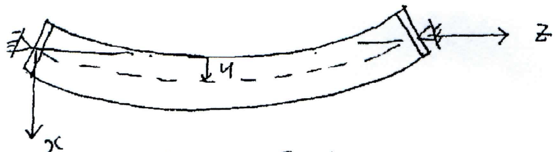
"However, if the degree of end fixity is questionable, a conservative measure is to use one for the effective length factor."

$K = 1.0$ 사용

5.6.3 Other End Conditions

(to make engineer's life easier)

(1) The ends of the beams prevented from warping, but unrestrained in bending about weak axis (Fig. 5.24)



: top view

The out-of-plane conditions for lateral bending ; $u|_{z=0} = u|_{z=L} = 0$

$$\frac{d^2u}{dz^2}\Big|_{z=0} = \frac{d^2u}{dz^2}\Big|_{z=L} = 0 \quad \leftarrow M_f = 0$$

For twisting ; $\gamma|_{z=0} = \gamma|_{z=L} = 0$

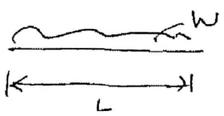
For warping ; $\frac{d\gamma}{dz}\Big|_{z=0} = \frac{d\gamma}{dz}\Big|_{z=L} = 0 \quad \leftarrow T_{sv} = 0$

$$M_{cr} = C_{bs} M_{ocr} \quad \text{--- (5.6.7)}$$

where $M_{cr} = \frac{P_{cr} \cdot L}{4}$ or $M_{cr} = \frac{W_{cr} L^2}{8}$ --- (5.6.8)

and $C_{bs} = \begin{cases} A \cdot B & \text{for bot. flange loading} \\ A & \text{for shear center "} \\ A/B & \text{for top flange "} \end{cases}$

"Expressions of A and B" → See Table 5.7 (P.344)



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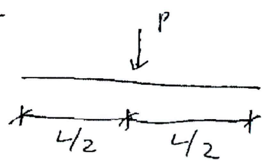


(2) The ends of the beams prevented from bending about weak axis, but free to warp (Fig. 5.25) → See Table 5.8 (P.346)

$$u|_{z=0} = u|_{z=L} = \frac{du}{dz}\Big|_{z=0} = \frac{du}{dz}\Big|_{z=L} = 0 ; \ \gamma|_{z=0} = \gamma|_{z=L} = \frac{d^2\gamma}{dz^2}\Big|_{z=0} = \frac{d^2\gamma}{dz^2}\Big|_{z=L} = 0$$

(3) An alternative approach to calculate the M_{cr} for beams of doubly symm. sections with out-of-plane bending and torsional simply supported as well as fixed end conditions by Clark and Hill (1962):

(비틀림 단순지지
〃 고정지지)

여전히 일반성은 부족, 실무 사용에 너무 복잡해 쓰기에 어려움

more simple form

$$M_{cr} = \frac{C_4}{L}\sqrt{EI_y GJ} \quad \text{------ (5.6.16)}$$

where

$$C_4 = C_1 \frac{\pi}{K}\left\{\sqrt{1+\left(\frac{\pi}{KL}\right)^2\left(\frac{EC_w}{GJ}\right)(C_2^2+1)} \pm C_2\left(\frac{\pi}{KL}\right)\sqrt{\frac{EC_w}{GJ}}\right\} \quad \text{--- (5.6.17)}$$

(여전히 일반성 부족)

「지점조건 및 하중패턴에 따른 K, C_1, C_2 값은 Table 5.9 (P. 347 참조)

See Table 5.9

C_1 = the coefficient accounting for the type of load and support conditions

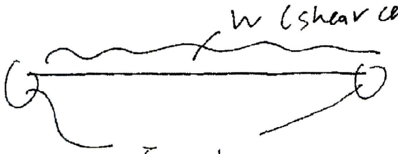
C_2 = the coefficient accounting for the location of the load vertically w.r.t. the shear center of the cross section;

- the plus sign → the bottom flange loading
- the minus sign → the top flange loading
- $C_2 = 0$ → the shear center loading or an end moment loading

Ex)

Double check

w (shear center loading 가정)



Fixed

Fig. 5.22 참조 $u = u' = 0$, $\gamma = \gamma' = 0$

$$\overline{W} = \frac{\pi}{L}\sqrt{\frac{EC_w}{GJ}} = 1.0 \text{ 인 때}$$

목차를 챙기기 위해

(i) Solution by Nethercot and Rockey's approach

$C_{bs} = A = 1.643 + 1.771 - 0.405 = 3.009$ (from Table 5.5)

$$\frac{w_{cr}L^2}{8} = C_{bs} M_{ocr} = 3.009 \, M_{ocr} \rightarrow w_{cr} = \frac{8}{L^2} \times 3.009 \, M_{ocr} = \frac{24}{L^2} M_{ocr}$$

$$= \frac{24}{L^2}\left\{\frac{\pi}{L}\sqrt{EI_y GJ} \times \sqrt{1+1^2}\right\}$$

$$\therefore w_{cr} = \frac{24\sqrt{2}\,\pi}{L^3}\sqrt{EI_y GJ}$$

$$= \frac{33.9\pi}{L^3}\sqrt{EI_y GJ}$$

(ii) Solution by Clark and Hill's approach

↳ Table 5.9 의 "Case No. 6"에 상당 (Fixed) ← "not case 7"

why? (강축휨에 대해서는 simple!)

$K = 0.50$; $C_1 = 1.0$; $C_2 = 0$ (shear center loading 가정)

$$M_{cr} = \frac{C_4}{L}\sqrt{EI_y GJ} \quad \leftarrow \text{Eq. (5.6.16)}$$

$$C_4 = (1.0)\left(\frac{\pi}{0.50}\right)\left\{\sqrt{1+(2)^2 \times \overline{W}^2 \times (0^2+1)}\right\} \quad \text{Eq. (5.6.17)}$$

$\overline{W} \approx 1.0$

$$= 2\pi\sqrt{5} = 4.47 \cdot \pi$$

$$\frac{w_{cr}L^2}{8} = \left(\frac{4.47\pi}{L}\right)\sqrt{EI_y GJ}$$

$$\therefore W_{cr} = \frac{35.8\pi}{L^3}\sqrt{EI_y GJ} \doteq \frac{33.9\pi}{L^3}\sqrt{EI_y GJ}$$

Note : 만일 강축휨에 대하여 양단고정의 조건이 추가된다면

vertical bending in y-z plane

「Table 5.9 에서 Case no. 7에 해당 될 것임.」

이경우 $K = 0.5$; $C_1 = 0.9$ 이므로

$$M_{cr} = \frac{w_{cr}L^2}{12} = \frac{(0.90)\times 4.47\pi}{L}\sqrt{EI_y EC_w}$$

$$\therefore W_{cr} = \frac{48.3\pi}{L^3}\sqrt{EI_y EC_w} \quad \leftarrow \frac{48.3}{35.8} = 1.35$$

(35% Increase)

큰 값인가?

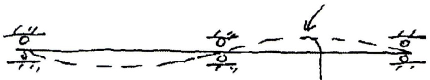
HW #

↳ 본 문제에 집중하중이 작용하는 경우에 대하여 두 방법에 의한 좌굴하중 P_{cr}을 산정하고 비교하여 볼 것. (Shear center 하중 / 단순지지 가정)

비틀림

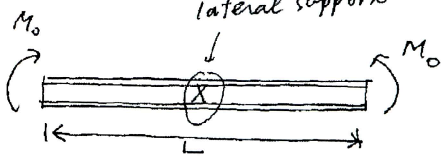
5.6.4 Continuous Beam

(1) Under uniform moment.



: top view

[complete sine wave]
(the 2nd buckling mode가 강제된 것 임)



: side view

$$m = \sqrt{-a + \sqrt{a^2 + b}} = \frac{2\pi}{L}$$

where

$a = GJ/2EC_w$

$b = \frac{M_o^2}{EI_y C_w}$

$\sin mL = 0$ ---- (5.4.31) ↙ Eigenproblem of simply supported I-section under pure bending (~~p.322~~) (p.324)

$mL = 2\pi$ ← [the second lowest value of m]

$$\therefore M_{ocr} = \frac{\pi}{(L/2)} \sqrt{EI_y GJ} \sqrt{1 + \frac{\pi^2 EC_w}{(L/2)^2 GJ}} \quad \text{---- (5.6.19)}$$

└ 결국 K= 0.50 사용한 결과 동일.

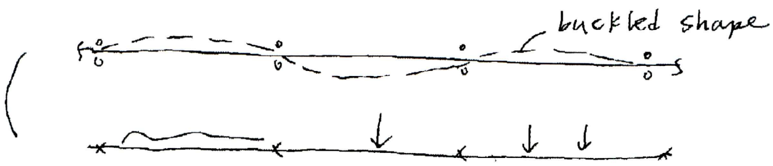
(2) For other types of loadings, numerical solutions for the critical loads are available (for example, Timoshenko and Gere (1961).

↳ Table 5.10 참고 (two-span continuous beam)

↳ Eqs. (5.6.7) ~ (5.6.9) 표현에 A, B 값 다 있음

(3) For a continuous beam with more than two spans,

↳



Influencing factors ; 구간별 상대강비, 하중 패턴 및 상대 크기, 중간 횡지지점의 구속상태 --- (참고문헌 21 ~ 24)

↳ 너무 복잡

For design purposes, it is common to evaluate the critical load for each span separately by assuming the ends of the span are simply supported.

↳ 각 스팬에서 얻어진 임계하중 가운데 최소치를 택하여 (보수적으로) 당해 연속보의 좌굴하중으로 취함

Conservative!

이러한 limitation을 잘 인지하고 있어야

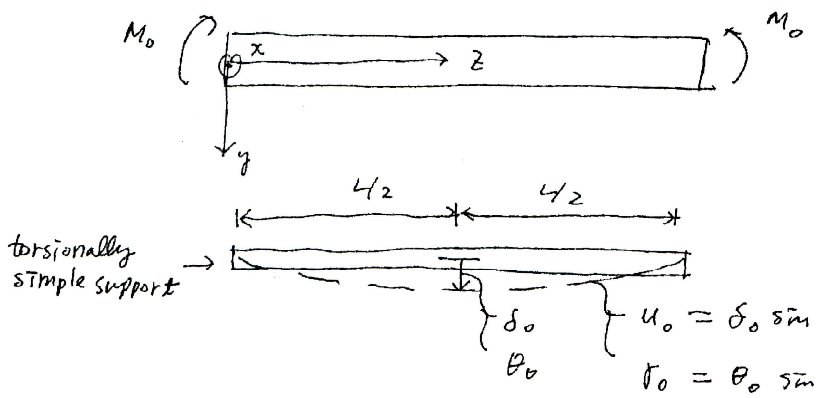
5.7 Initially Crooked Beams

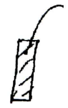
- Real beams are seldom perfect.

↑ The presence of initial curvature and twists will cause them to bend and twist at the beginning of loading

↳ 「Load-Deflection problem」

" Case study "





rectangular cross section (5.4절의 Ex. 5.1 문제)

torsionally simple support →

$$u_0 = \delta_0 \sin \frac{\pi}{L} z \quad \cdots (7.1)$$
$$\gamma_0 = \theta_0 \sin \frac{\pi}{L} z \quad - (7.2)$$

Initial imperfection 도입

Equating the corresponding external and internal moments,

Eq. (5.4.8) → $$EI_y \frac{d^2 u}{dz^2} + (\gamma + \gamma_0) M_0 = 0 \quad \cdots (7.3)$$

Eq. (5.4.9) → $$GJ \frac{d\gamma}{dz} - \left(\frac{du}{dz} + \frac{du_0}{dz}\right) M_0 = 0 \quad \cdots (7.4)$$

(P.320)

「external moments 가 수정됨」

(7.4)식을 z로 미분하고 (7.3)식에 대입하여 u를 소거하면

(7.3) 및 (7.4)식 대입

$$EI_y GJ \cdot \frac{d^2\gamma}{dz^2} + \gamma M_0^2 = EI_y M_0 \frac{d^2 u_0}{dz^2} - \gamma_0 M_0^2$$
$$= -\cancel{E}\cancel{I_y} M_0 \delta_0 \left(\frac{\pi}{\cancel{L}}\right)^2 \sin \frac{\pi}{L} z - \theta_0 M_0^2 \sin \frac{\pi}{L} z \quad \cdots (5.7.6)$$

$$\left[\frac{M_{ocr}}{\cancel{\pi^2 EI_y / L^2}}\right] \times \theta_0$$

" 초기불안정 요소량을 단일 변수로 만들기 위해 다음을 가정 "

$$\delta_0 : \theta_0 = M_{ocr} : \frac{\pi^2 EI_y}{L^2} = \left(\frac{1}{\pi^2 EI_y / L^2}\right) : \frac{1}{M_{ocr}} \quad \cdots (5.7.7)$$

(M_{ocr}: 비틀림 좌굴 강도) ($\frac{\pi^2 EI_y}{L^2}$: 약축 휨좌굴강도)

↑ reasonable 한 것인가?

즉 $\delta_0 = \left(\frac{M_{ocr}}{\pi^2 EI_y / L^2}\right) \times \theta_0$ 를 (6.7.6)식에 대입하여 정리하면,

「Constant, anyway」 그래도

↳ 전체적 경향 파악의 일반성 잃지 않음.

$$EI_yGJ\frac{d^2\gamma}{dz^2}+\gamma M_o^2=-M_o\cdot M_{ocr}\theta_o\sin\frac{\pi}{L}z-M_o^2\theta_o\sin\frac{\pi}{L}z$$

$$=-M_o^2\left(\frac{M_{ocr}}{M_o}+1\right)\cdot\theta_o\sin\frac{\pi}{L}z$$

$$\frac{d^2\gamma}{dz^2}+k^2\gamma=-k^2\theta_o\left(\frac{M_{ocr}}{M_o}+1\right)\sin\frac{\pi}{L}z \quad \text{---- (5.7.9)}$$

where $k^2=M_o^2/EI_yGJ$

$\gamma_p=C\cos(\frac{\pi}{L}z)+D\sin(\frac{\pi}{L}z)$

미정계수법에 의해 particular solution을 구하면

$$\gamma_p=\left[\frac{-k^2\theta_o(\frac{M_{ocr}}{M_o}+1)}{k^2-(\frac{\pi}{L})^2}\right]\sin\frac{\pi}{L}z \quad \text{---- (5.7.15)}$$

$k^2=\dfrac{M_o^2}{EI_yGJ}$; $M_{ocr}=\dfrac{\pi}{L}\sqrt{EI_yGJ}$ 대입

$$=\left[\frac{\theta_o\times\frac{M_o}{M_{ocr}}}{1-\frac{M_o}{M_{ocr}}}\right]\sin\frac{\pi}{L}z \quad \text{--- (5.7.16)}$$

Note: $\dfrac{P}{P_e}\leftrightarrow\dfrac{M_o}{M_{ocr}}$

따라서 일반해는

$$\gamma=A\sin kz+B\cos kz+\gamma_p \quad \text{------ (5.7.17)}$$

γ_p ← (5.7.16)식

경계조건: $\gamma(0)=0$; $\gamma(L)=0$

$B=0$; $A\sin kL=0$

$A=0$ or $\sin kL=0$

$\sin kL=0$ → 이 경우는 M_{ocr}이 결여지는 경우로서 관심 대상이 아님

따라서 $A=0$ and $B=0$

$$\therefore\ \gamma=\left[\frac{\theta_o\times(\frac{M_o}{M_{ocr}})}{1-(\frac{M_o}{M_{ocr}})}\right]\sin(\frac{\pi}{L}z) \quad \text{--- (5.7.18)}$$

(5.7.18)을 (5.7.3) 및 (5.7.4)식에 대입하여 u를 구하면

"HW# → 확인"

$$u=\frac{\delta_o\times(\frac{M_o}{M_{ocr}})}{1-(M_o/M_{ocr})}\times\sin(\frac{\pi}{L}z) \quad \text{--- (5.7.19)}$$

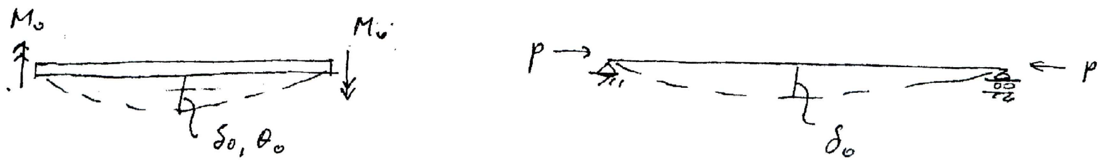
$$\gamma_{total}=\gamma_o+\gamma=\left[\frac{1}{1-M_o/M_{ocr}}\right]\times(\theta_o\sin\frac{\pi z}{L})=A_F\times\gamma_o \quad \text{--- (20)}$$

$$u_{total}=u_o+u=\left[\frac{1}{1-M_o/M_{ocr}}\right]\times(\delta_o\sin\frac{\pi z}{L})=A_F\times u_o \quad \text{--- (21)}$$

where $A_F=\left[\dfrac{1}{1-M_o/M_{ocr}}\right]$ ---- (22)

Notes: 1)

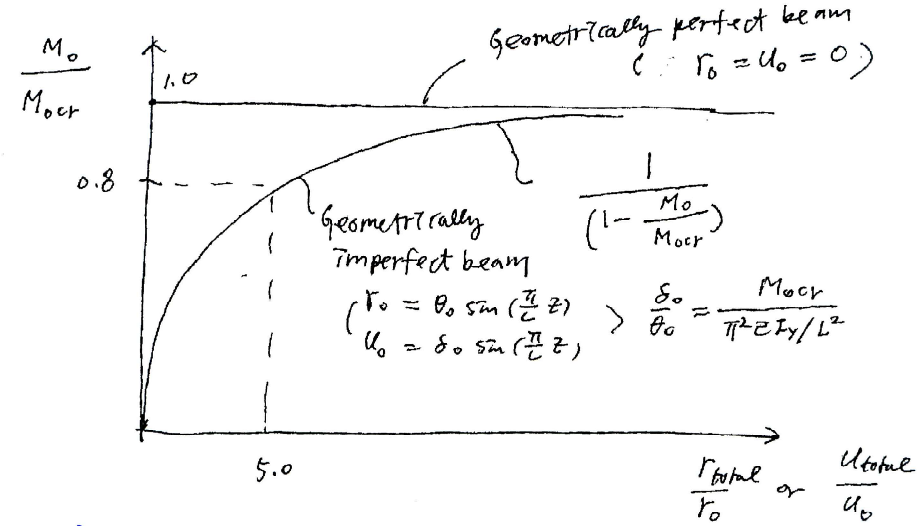
$$\underbrace{A_F = \frac{1}{1 - M_o/M_{ocr}}}_{\text{Eq. (5.7.22)}} \longleftrightarrow \underbrace{A_F = \frac{1}{1 - P/P_e}}_{\text{Eq. (2.6.20)}}$$

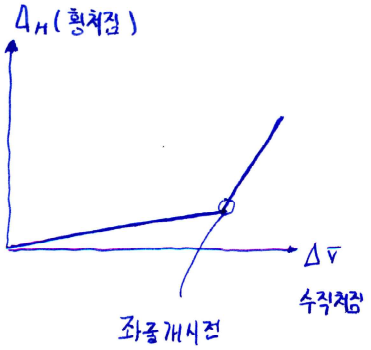


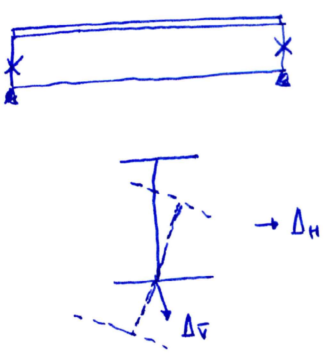
- The similarity of Eq (5.7.22) and Eq. (2.6.20) should be noted.
- An expression similar to that of Eq. (5.7.22) can be derived for a geometrically imperfect I-beam under a uniform moment.

2) Load-deflection relationship of an initially crooked beam (Fig. 5.26)

실측시 이런정도 인지해야







5.8 Inelastic Beams

(1) The solutions for the critical loads presented in the preceding sections are valid only for a fully elastic beam.

yielding of material does not take place anywhere in the beam

↑ reasonable assumption for beams of high slenderness ratio (L/r_y)

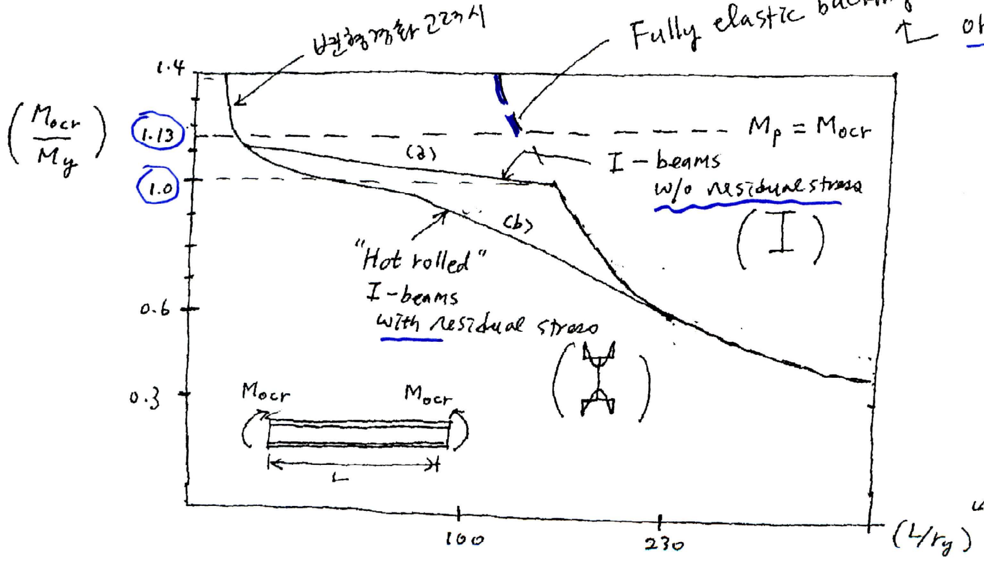
(2) For beams of intermediate slenderness ratios, yielding will occur in some fibers of the beam before the attainment of the critical load.

↳ Only elastic portion of the cross section will remain effective in providing resistance to lateral buckling. As a result, the critical load will be reduced.

↳ Fig. 5.27 Beam strength curve 참고

Fully elastic buckling curve (typical)

↑ 아래의 (*)식 참고



횡좌굴에 대한 세장비

Note: $M_{ocr} = \frac{\pi}{L}\sqrt{EI_yGJ\left(1+\frac{\pi^2}{L^2}\cdot\frac{EC_w}{GJ}\right)}$ ← (5.4.34)식

$$= \sqrt{\frac{\pi^2 EI_yGJ}{L^2} + \frac{\pi^4 EI_y\, E(I_y h^2/4)}{L^4}} \quad \leftarrow \begin{cases} I_y = Ar_y^2 \\ C_w = \frac{I_y h^2}{4} \end{cases}$$

$$\therefore \frac{M_{ocr}}{M_y} = \frac{1}{M_y}\times\sqrt{\frac{\pi^2 EAGJ}{(L/r_y)^2} + \frac{\pi^4 E A^2 h^2/4}{(L/r_y)^4}} \quad \text{---- } (*)$$

For a fully elastic beam

(3) For simply supported I-beams subjected to equal and opposite end moments, If there are no residual stresses the distribution of yielding across the section is symmetric about the horizontal principle axis and is constant along the entire length of the beam.

symm.

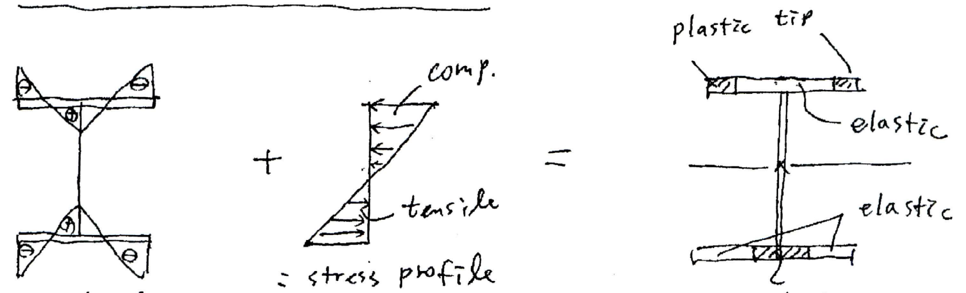
$$M_{ocr} = \frac{\pi}{L}\sqrt{(EI_y)_e (GJ)_e}\sqrt{1 + \frac{\pi^2}{L^2}\frac{(EC_w)_e}{(GJ)_e}} \quad \cdots (5.8.1)$$

EH Section approach →

where $(EI_y)_e$, $(GJ)_e$, and $(EC_w)_e$ are the effective bending rigidity, torsional rigidity, and warping rigidity, respectively.
(Flint 1953, 2.10절의 elastic core 개념 생각)

(4) However, the presence of residual stress greatly reduce the lateral torsional buckling strength of the beam in the inelastic range.
(Fig. 5.27 (b) 참조)

(5) Eq. (5.8.1) is not applicable for beams with residual stresses, since this equation is only valid doubly symmetric sections. If residual stresses are considered in the analysis, the distribution of yielding across the cross section will not be symmetric about the horizontal principal axis.



: residual stress distribution

: stress profile due to the applied moment

: possible yield pattern

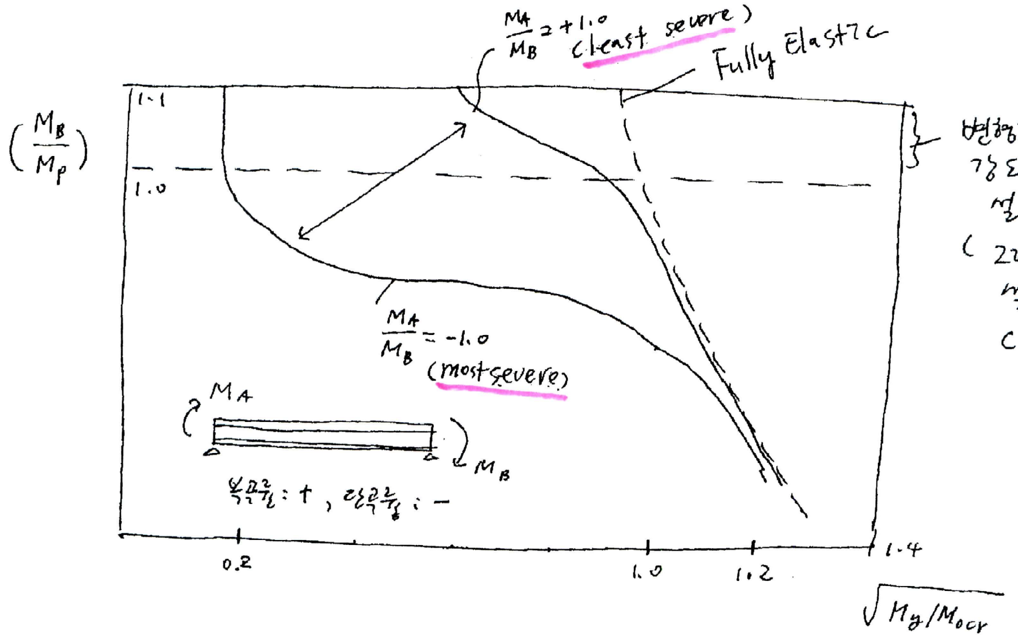
The discussions of this type of section → Johnston (1976)

(6) To obtain the inelastic buckling loads for beams with a more general loading case other than that of equal and opposite end moments, recourse must be had to numerical methods (Chen-Austa 1977; Galambos 1963; Lay-Galambos 1967).

↳ The in-plane bending moment will vary along the beam, so the distribution of yielding will also vary from cross section to cross section.

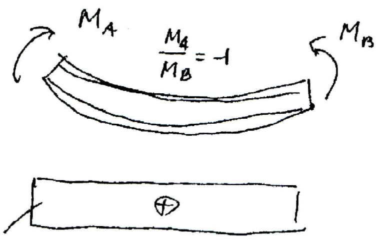
(7) Fig. 5.28 (P. 354)

↑ Numerical solutions for some hot-rolled I-beams with a number of different loading arrangement (by Nethercot and Trahair 1976)

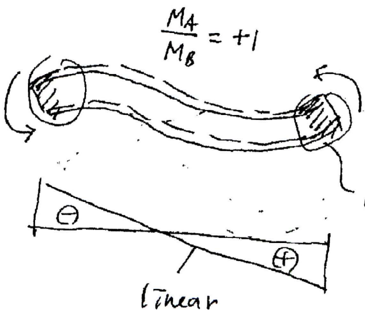


변형경화에 따른 강도증진은 보통 설계상은 무시 가능 (그러나 접합부 설계에는 고려해야, capacity design)

modified slenderness ((31)쪽의 (4)식 참조)



Constant BMD
(보의 전구간에 걸쳐서 항복이 일어남)
: most severe moment gradient



Confined yielding
↳ 재축방향으로 $(EI_y)_e$, $(GJ)_e$ … 값이 변동!!

linear BMD
: least severe moment gradient.

★ It is quite evident that the lateral instability behavior of real beams is complex.

↳ Certain simplifying assumptions are inevitable to make use of the analytical results for design purposes.

Note : 현재 실무설계에서 탄성해석에 의한 C_b factor를 비탄성 LTB의 강도에도 그대로 사용 → 확인해 볼 문제???

"Go to 보충자료"

5.9 Design curves for steel Beams.

Some of the important parameters of the LTB behavior

↳ ① L_b, ② the cross-sectional geometry, ③ the material behavior, (elastic vs. inelastic)
④ the magnitude, type, and location of the applied loads, and
⑤ the type of lateral supports, as well as the end conditions of the beam, etc.

↑ Simplifying inevitable for design purposes

5.9.1 AISC – ASD
5.9.2 AISC – PD
} 생략 (교과서 참조)

5.9.3 AISC – LRFD ← limit state – based

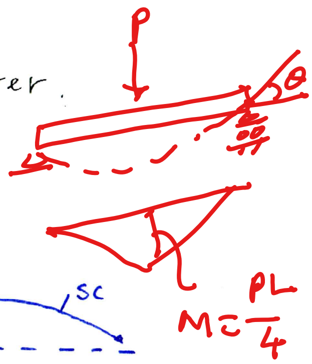
Three possible types of failure limits
- (1) plastic yielding
- (2) lateral instability ← "Global" buckling
- (3) local buckling

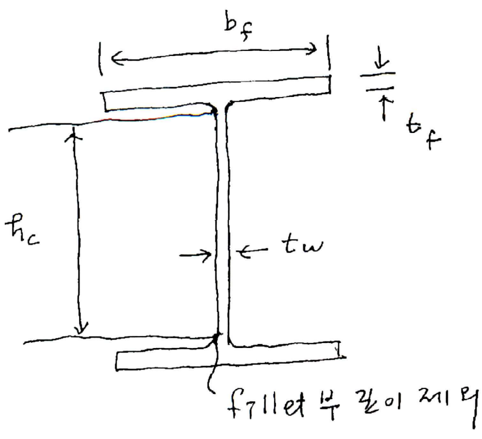
(1) To avoid local buckling,

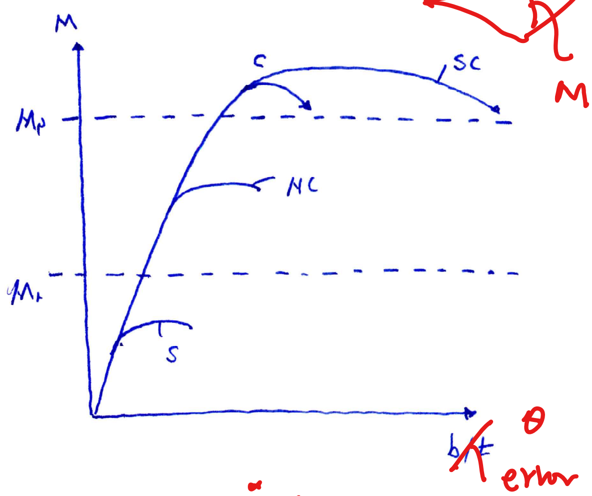
FLB ← $\dfrac{b_f}{2t_f} \le \dfrac{65}{\sqrt{F_y}}$ --- (5.9.31)

WLB ← $\dfrac{h_c}{t_w} \le \dfrac{640}{\sqrt{F_y}}$ --- (5.9.32)

} compact section criter.

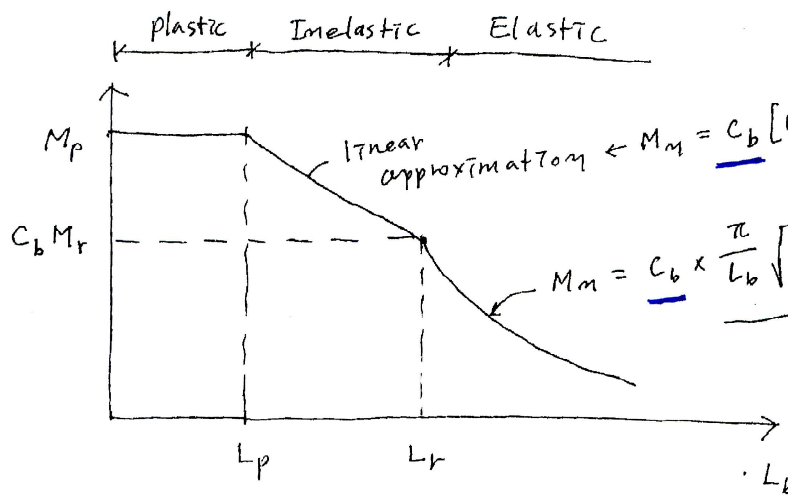






"go to 공학도강의 보충자료" ← 이미 "기둥 및 골조 안정론 핸들" ppt에서 언급

(2) LTB limit state strength (H형강의 강축휨의 경우)



$\leftarrow M_n = \underline{C_b}\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$

$M_n = \underline{C_b} \times \underbrace{\frac{\pi}{L_b}\sqrt{(EI_yGJ + \frac{\pi^2}{L_b^2}EI_yEC_w}}_{M_{ocr}\ (5.4.34\text{식})} \le C_bM_r$

L_b (= Unbraced length)

$$L_p = \frac{300 r_y}{\sqrt{F_y}} \quad ; \quad L_r = \frac{r_y X_1}{(F_y - F_r)}\sqrt{1 + \sqrt{1 + X_2(F_y - F_r)^2}} \quad \text{---} (5.9.34)$$

↑ (5.9.33)

$$C_b = 1.75 + 1.05\left(\frac{M_A}{M_B}\right) + 0.3\left(\frac{M_A}{M_B}\right)^2 \le 2.3 \quad \text{---} (5.5.2)$$

↑ Equivalent (Uniform) moment factor by Salvadori (1955) → Now Replaced by Kirby-Nethercot (1979) P.333

$M_r = S_x(F_y - F_r)$ ← the transition moment from elastic to inelastic behavior to account for the presence of residual stress (F_r).

↑ "탄성한계 횡좌굴 모멘트"

Notes : i) L_r 값은 $C_b = 1.0$으로 하여 "보수적으로" 산정됨.

$$M_r = S_x(F_y - F_r) = \underbrace{\frac{\pi}{L_r}\sqrt{EI_yGJ + \frac{\pi^2}{L_r^2}EI_yEC_w}}_{M_{ocr}}$$

M_{ocr} (균등모멘트를 받는 경우의 탄성 횡좌굴 모멘트)

↑ 고려식 (5.4.34)식

위 식에서 L_r에 대하여 풀면 $L_r = [\qquad]$ --- (5.9.34)

ii) C_b factor는 양단부 모멘트의 크기 및 부호가 다를 경우 이를 등가의 균등모멘트로 변환해주는 계수임.

↳ "최근의 AISC-LRFD 1999 에서는 좀더 일반성 있는 5.5.22식 (p.333)으로 개정
↑ Kirby-Nethercot (1979)"

HW # (5.9.34)의 L_r, X_1, X_2 (생략) expression 구할 것.

↳ 이미 있음

AISC-LRFD 최신판 (3rd ed.) : AISC-LRFD 1999

2010도 본질적으로 동일

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CHAPTER F

BEAMS AND OTHER FLEXURAL MEMBERS

This chapter applies to compact and noncompact prismatic members subject to flexure and shear. For members subject to combined flexure and axial force, see Section H1. For members subject to fatigue, see Section K3. For members with slender compression elements, see Appendix B5. For web-tapered members, see Appendix F3. For members with slender web elements (plate girders), see Appendix G.

F1. DESIGN FOR FLEXURE

The nominal flexural strength M_n is the lowest value obtained according to the limit states of: (a) yielding; (b) lateral-torsional buckling; (c) flange local buckling; and (d) web local buckling. For laterally braced compact beams with $L_b \le L_p$, only the limit state of yielding is applicable. For unbraced compact beams and noncompact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable. The lateral-torsional buckling limit state is not applicable to members subject to bending about the minor axis, or to square or circular shapes.

This section applies to homogeneous and hybrid shapes with at least one axis of symmetry and which are subject to simple bending about one principal axis. For simple bending, the beam is loaded in a plane parallel to a principal axis that passes through the shear center or the beam is restrained against twisting at load points and supports. Only the limit states of yielding and lateral-torsional buckling are considered in this section. The lateral-torsional buckling provisions are limited to doubly symmetric shapes, channels, double angles, and tees. For lateral-torsional buckling of other singly symmetric shapes and for the limit states of flange local buckling and web local buckling of noncompact or slender-element sections, see Appendix F1. For unsymmetric shapes and beams subject to torsion combined with flexure, see Section H2. For biaxial bending, see Section H1.

1. Yielding

$L_b \le L_p$ 및 for compact sections.

The flexural design strength of beams, determined by the limit state of yielding, is $\phi_b M_n$:

$$\phi_b = 0.90$$

$$M_n = M_p \qquad \text{(F1-1)}$$

where

M_p = plastic moment ($= F_y Z \le 1.5 M_y$ for homogeneous sections), kip-in. (N-mm)
M_y = moment corresponding to onset of yielding at the extreme fiber from an elastic stress distribution ($= F_y S$ for homogeneous section and $F_{yf} S$ for hybrid sections), kip-in. (N-mm)

(37)

See Section B10 for further limitations on M_n where there are holes in the tension flange.

2. Lateral-Torsional Buckling LTB

This limit state is only applicable to members subject to major axis bending. The flexural design strength, determined by the limit state of lateral-torsional buckling, is $\phi_b M_n$:

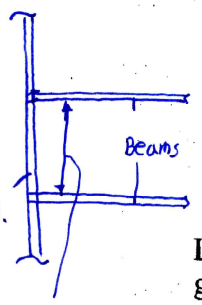
ϕ_b = 0.90
M_n = nominal flexural strength determined as follows

2a. Doubly Symmetric Shapes and Channels with $L_b \le L_r$

The nominal flexural strength is:

$$M_n = C_b \left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p \tag{F1-2}$$

where



L_b = distance between points braced against lateral displacement of the compression flange, or between points braced to prevent twist of the cross section, in. (mm) Simplistic 하게 규정하고 있음!
L_p = limiting laterally unbraced length as defined below, in. (mm)
L_r = limiting laterally unbraced length as defined below, in. (mm)
M_r = limiting buckling moment as defined below, kip-in. (N-mm)

L_b = Story height

In the above equation, C_b is a modification factor for non-uniform moment diagrams where, when both ends of the beam segment are braced:

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C} \tag{F1-3}$$

Kirby-Nethercot (1979)

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)
M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)
M_B = absolute value of moment at centerline of the unbraced beam segment, kip-in. (N-mm)
M_C = absolute value of moment at three-quarter point of the unbraced beam segment, kip-in. (N-mm)

Very Conservative

C_b is permitted to be conservatively taken as 1.0 for all cases. Equations F1-4 and F1-6 are conservatively based on C_b = 1.0. For cantilevers or overhangs where the free end is unbraced, C_b = 1.0.

See Table 5.4 in the text

The limiting unbraced length, L_p, shall be determined as follows.

(a) For I-shaped members including hybrid sections and channels:

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$L_p = \frac{300 r_y}{\sqrt{F_y}}$ (in ksi)

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}} \tag{F1-4}$$

AISC-LRFD 1999
개정판의 특징
(단위계와
무관하게
- 포맷을 수정)

(b) For solid rectangular bars and box sections:

$$L_p = \frac{0.13 r_y E}{M_p} \sqrt{JA} \tag{F1-5}$$

where

A = cross-sectional area, in.^{2} (mm^2)
J = torsional constant, in.^{4} (mm^4)

The limiting laterally unbraced length L_r and the corresponding buckling moment M_r shall be determined as follows.

(a) For doubly symmetric I-shaped members and channels:

$$L_r = \frac{r_y X_1}{F_L} \sqrt{1 + \sqrt{1 + X_2 F_L^2}} \tag{F1-6}$$

$$M_r = F_L S_x \tag{F1-7}$$

where

$$X_1 = \frac{\pi}{S_x} \sqrt{\frac{EGJA}{2}} \tag{F1-8}$$

$$X_2 = 4 \frac{C_w}{I_y} \left(\frac{S_x}{GJ} \right)^2 \tag{F1-9}$$

S_x = section modulus about major axis, in.^{3} (mm^3)
E = modulus of elasticity of steel, 29,000 ksi (200 000 MPa)
G = shear modulus of elasticity of steel, 11,200 ksi (77 200 MPa)
F_L = smaller of $(F_{yf} - F_r)$ or F_{yw}, ksi (MPa)
F_r = compressive residual stress in flange; 10 ksi (69 MPa) for rolled shapes, 16.5 ksi (114 MPa) for welded built-up shapes
F_{yf} = yield stress of flange, ksi (MPa)
F_{yw} = yield stress of web, ksi (MPa)
I_y = moment of inertia about y-axis, in.^{4} (mm^4)
C_w = warping constant, in.^{6} (mm^6)

(b) For solid rectangular bars and box sections:

NO warping

$$L_r = \frac{2 r_y E \sqrt{JA}}{M_r} \tag{F1-10}$$

$$M_r = F_{yf} S_x \tag{F1-11}$$

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2b. Doubly Symmetric Shapes and Channels with $L_b > L_r$

The nominal flexural strength is:

$$M_n = M_{cr} \le M_p \tag{F1-12}$$

where M_{cr} is the critical elastic moment, determined as follows.

(a) For doubly symmetric I-shaped members and channels:

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{E I_y G J + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \tag{F1-13}$$

$$= \frac{C_b S_x X_1 \sqrt{2}}{L_b / r_y} \sqrt{1 + \frac{X_1^2 X_2}{2 (L_b / r_y)^2}}$$

(b) For solid rectangular bars and symmetric box sections:

$$M_{cr} = \frac{57\,000 C_b \sqrt{JA}}{L_b / r_y} \tag{F1-14}$$

2c. Tees and Double Angles

For tees and double-angle beams loaded in the plane of symmetry:

$$M_n = M_{cr} = \frac{\pi \sqrt{E I_y G J}}{L_b} \left[B + \sqrt{1 + B^2} \right] \tag{F1-15}$$

where

$M_n \le 1.5 M_y$ for stems in tension
$M_n \le 1.0 M_y$ for stems in compression
$B = \pm 2.3 (d / L_b) \sqrt{I_y / J}$ (F1-16)

The plus sign for B applies when the stem is in tension and the minus sign applies when the stem is in compression. If the tip of the stem is in compression anywhere along the unbraced length, use the negative value of B.

3. Design by Plastic Analysis → 첨부한 보충자료 참고 (철골강의 노트, 대학원)

Design by plastic analysis, as limited in Section A5.1, is permitted for a compact section member bent about the major axis when the laterally unbraced length L_b of the compression flange adjacent to plastic hinge locations associated with the failure mechanism does not exceed L_{pd}, determined as follows.

(a) For doubly symmetric and singly symmetric I-shaped members with the compression flange equal to or larger than the tension flange (including hybrid members) loaded in the plane of the web:

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$$L_{pd} = \left[0.12 + 0.076\left(\frac{M_1}{M_2}\right)\right]\left(\frac{E}{F_y}\right)r_y \tag{F1-17}$$

where

F_y = specified minimum yield stress of the compression flange, ksi (MPa)
M_1 = smaller moment at end of unbraced length of beam, kip-in. (N-mm)
M_2 = larger moment at end of unbraced length of beam, kip-in. (N-mm)
r_y = radius of gyration about minor axis, in. (mm)

(M_1 / M_2) is positive when moments cause reverse curvature and negative for single curvature

(b) For solid rectangular bars and symmetric box beams:

$$L_{pd} = \left[0.17 + 0.10\left(\frac{M_1}{M_2}\right)\right]\left(\frac{E}{F_y}\right)r_y \geq 0.10\left(\frac{E}{F_y}\right)r_y \tag{F1-18}$$

There is no limit on L_b for members with circular or square cross sections nor for any beam bent about its minor axis.

In the region of the last hinge to form, and in regions not adjacent to a plastic hinge, the flexural design strength shall be determined in accordance with Section F1.2.

F2. DESIGN FOR SHEAR

This section applies to unstiffened webs of singly or doubly symmetric beams, including hybrid beams, and channels subject to shear in the plane of the web. For the design shear strength of webs with stiffeners, see Appendix F2 or Appendix G3. For shear in the weak direction of the shapes above, pipes, and unsymmetric sections, see Section H2. For web panels subject to high shear, see Section K1.7. For shear strength at connections, see Sections J4 and J5.

1. Web Area Determination

The web area A_w shall be taken as the overall depth d times the web thickness t_w.

2. Design Shear Strength

The design shear strength of unstiffened webs, with $h / t_w \leq 260$, is $\phi_v V_n$,

where

$\phi_v = 0.90$
V_n = nominal shear strength defined as follows.

(a) For $h / t_w \leq 2.45\sqrt{E / F_{yw}}$

$$V_n = 0.6 F_{yw} A_w \tag{F2-1}$$

(b) For $2.45\sqrt{E / F_{yw}} < h / t_w \leq 3.07\sqrt{E / F_{yw}}$

(맨 뒤로 보내기)

5.10 Other design approaches

→ 한일독 (SSRC, European ...)

∵ Rondal-Maquoi form에 근거 ← Eq. (2.11.17)

(보의 횡좌굴 거동 역시 기둥의 stability 거동과 흡사하다는 가정하에 $\frac{P}{P_y}$ 를 $\frac{M_n}{M_p}$ 으로 대체한 형식을 제안)

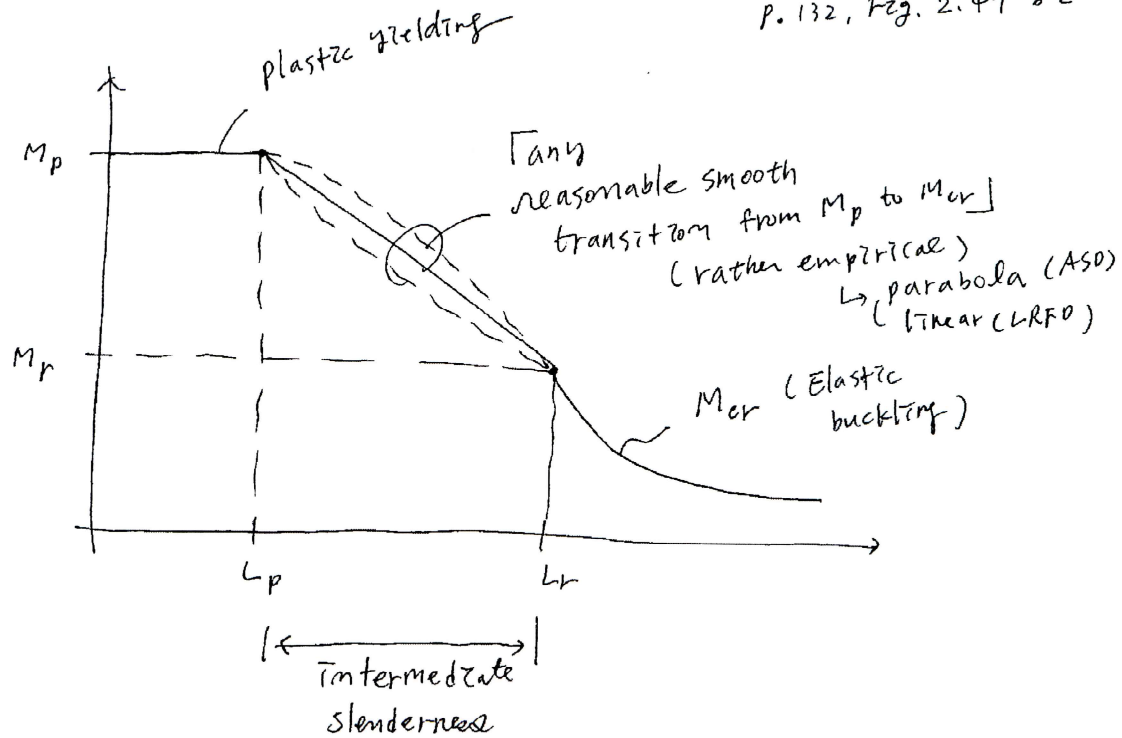
→ Eq. (5.10.1)

and $\eta = \alpha(\lambda - 0.15)$

$\alpha = 0.293$ (SSRC curve 2)

p. 132, Fig. 2.47 참조

Note:



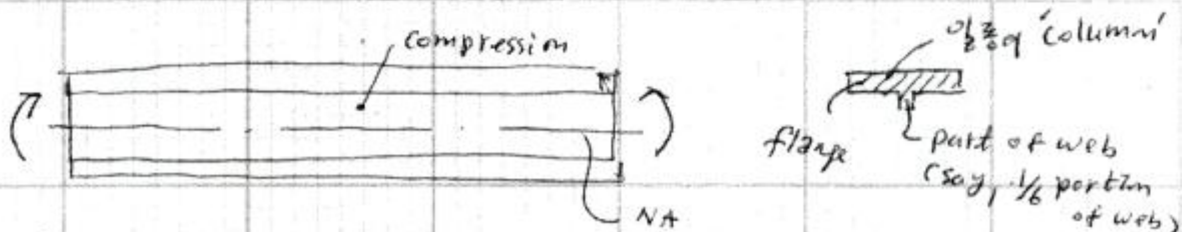
☆ The LRFD beam curve is generally more "liberal" and much simpler than that of the ASD beam curve.

<비탄성 횡좌굴 Inelastic LTB 보충자료>

- The same concepts of extending the "elastic buckling theory" for axially loaded members to the cases where portions of the column cross section are yielded also apply to the LTB of beams
  ↳ Effective section approach

- The theoretical relationships between the tangent modulus, the reduced modulus, and the ultimate loads are also valid for perfectly straight beams, as are the effects of initial imperfections of geometry and of residual stresses

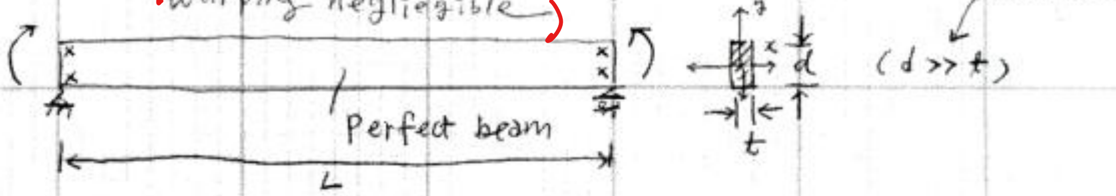
ex)



- Far more difficult because of the more complex geometry and the presence of both lateral and torsional deformations
  ↳ Inelastic LTB studies (for example, Galambos 1998, Trahair 1993

- A simple analytical solution of thin rectangular beam using "effective section concept"

Warping negligible



(i)

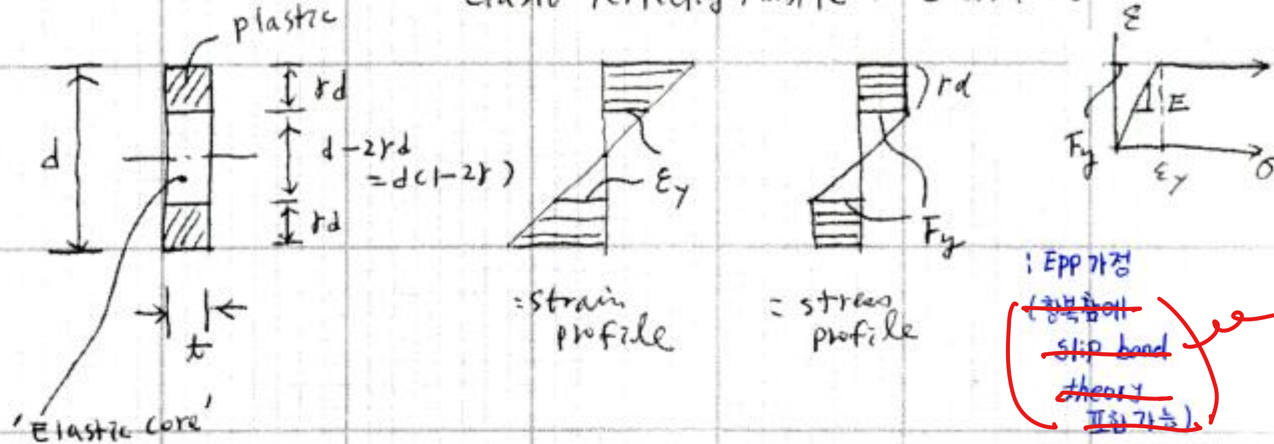
M_{cr} (= elastic LTB strength)

('이의 유도')

$$= \frac{\pi}{L}\sqrt{EI_y\,GJ} \quad ----(0)$$

(ii) Concept of elastic core (or effective section)

"Elasto-Perfectly Plastic σ-ε assumed"



: EPP 가정
~~(항복후에 slip band theory 포함 가능)~~

M_r (= partially plastic strength)

$$= 2 \times t \left[\underbrace{rd\left(\frac{d}{2} - rd\right)}_{\text{plastic}} + \underbrace{\frac{1}{2}\left(\frac{d}{2} - rd\right) \times \left\{\frac{2}{3} \times \frac{1}{2}\right\}\left(\frac{d}{2} - rd\right)}_{\text{elastic}} \right] F_y$$

$$= (1 + 2r - 2r^2)\left(\frac{td^2}{6} F_y\right) \quad M_y \quad = (1 + 2r - 2r^2) M_y$$

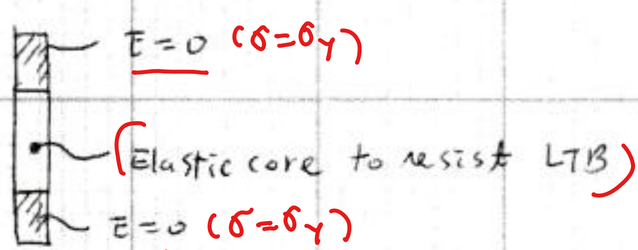
기호 혼동치 말것

when $r = 0$, $M_r = M_y = \frac{td^2}{6} F_y \quad (= S \cdot F_y)$

$r = 0.5$, $M_r = M_p = \frac{td^2}{4} F_y \quad (= Z \cdot F_y) = \frac{3}{2} M_y$ shape factor

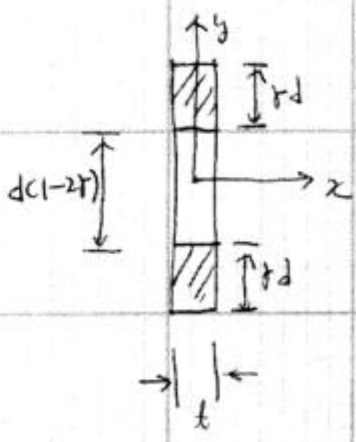
$$\therefore \frac{M_r}{M_p} = \frac{2}{3}\left[1 + 2r(1-r)\right] \quad \text{-----} \quad (1)$$

The cross section available ~~for~~ to resist LTB will be the elastic core.



(iii) Effective section properties and LTB strength

Recall Eq (0), $M_{cr} = \frac{\pi}{L}\sqrt{E I_y \times G J}$ ---(0)



$G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+0.3)} = 0.385E$ → steel (typ.)

$(I_y)_e = \frac{d(1-2\gamma)t^3}{12}$ ← 「약축에 대한 유효 단면 2차 모멘트」

$(J)_e = \frac{1}{3}d(1-2\gamma)t^3$ ← "「유효 비틀림 상수」"

「손상화가 'γ' 비율만큼 진행시의 유효 단면 특성」

$(I_y)_e, (J)_e$ 대입

$M_{cr} = \frac{\pi}{L}\sqrt{E(I_y)_e \times (0.385)E\,(J)_e}$

(HW# 확인요)

$\left(\frac{M_{cr}}{M_p}\right) = 1.30\left(\frac{E}{F_y}\right)\left(\frac{t}{d}\right)\frac{(1-2\gamma)}{(L/t)}$ ---(2)

「"탄성 한계 횡지지거리"」 ← $\gamma = 0$ & $\frac{M_{cr,I}}{M_p} = \frac{2}{3}$

$M_{cr} = M_y$

$(L/t)_{limit} = \frac{3}{2}(1.30)\left(\frac{E}{F_y}\right)\left(\frac{t}{d}\right)$

$= 1.95\left(\frac{E}{F_y}\right)\left(\frac{t}{d}\right)$ ---(3)

(i) 즉 $(L/t) > (L/t)_{limit}$ 이면 "Elastic" LTB

곧, $(I_y)_e$ 및 $(J)_e$는 단면전체의 값을 사용할 수 있으므로

(즉 $r=0$인 경우에 상당하므로)

(2)식에서 $r=0$을 대입하면 됨;

$\left(\frac{M_{cr}}{M_p} \le \frac{2}{3}\right)$

$$\frac{M_{cr}}{M_p} = \frac{1.30(E/F_y)(t/d)}{(L/t)} \quad \text{if} \quad \frac{L}{t} \ge 1.95\left(\frac{E}{F_y}\right)\left(\frac{t}{d}\right) \quad \text{---} (4)$$

(ii) $(L/t) \le (L/t)_{limit}$ 이면 'Inelastic' LTB가 발생하므로

'곧 $0 < r \le 0.5$ 인 경우'에 상당한다.

(2)식을 적용하되

무차원세장비 (L/t) 대 무차원 좌굴강도 (M_{cr}/M_p)의 관계를 (형지지) 유도하기 위해서는 (2)식의 r을 소거해야 한다.

(1)식의 분자 M_r는 소성화가 'r'만큼 진행되었을 때의 LTB 강도에 대응하는 값이므로 $(M_r = M_{cr})$ (1)식을 r에 대해서 풀면,

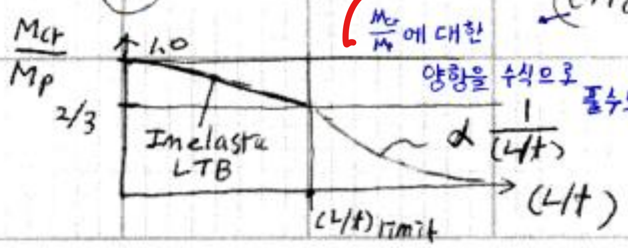
$$r = \frac{1}{2}\left[1 - \sqrt{3 - 3\left(\frac{M_{cr}}{M_p}\right)}\right] \quad \text{----} (5)$$

HW# 확인요

(5) → (2)에 대입하고 (L/t)에 대해 풀면

$\left(\frac{2}{3} \le \frac{M_{cr}}{M_p} \le 1.0\right)$

$$\frac{L}{t} = 1.30\left(\frac{F_y}{E}\right)\left(\frac{t}{d}\right)\frac{\sqrt{3-3(M_{cr}/M_p)}}{(M_{cr}/M_p)} \quad \text{if} \quad \frac{L}{t} < 1.95\left(\frac{F_y}{E}\right)\left(\frac{t}{d}\right) \quad \text{---} (6)$$



($\frac{M_{cr}}{M_p}$에 대한 양함을 수식으로 풀수도)

ex) $E/F_y = 29000/50$, $d/t = 10$

$(L/t)_{limit} \doteq 115$

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