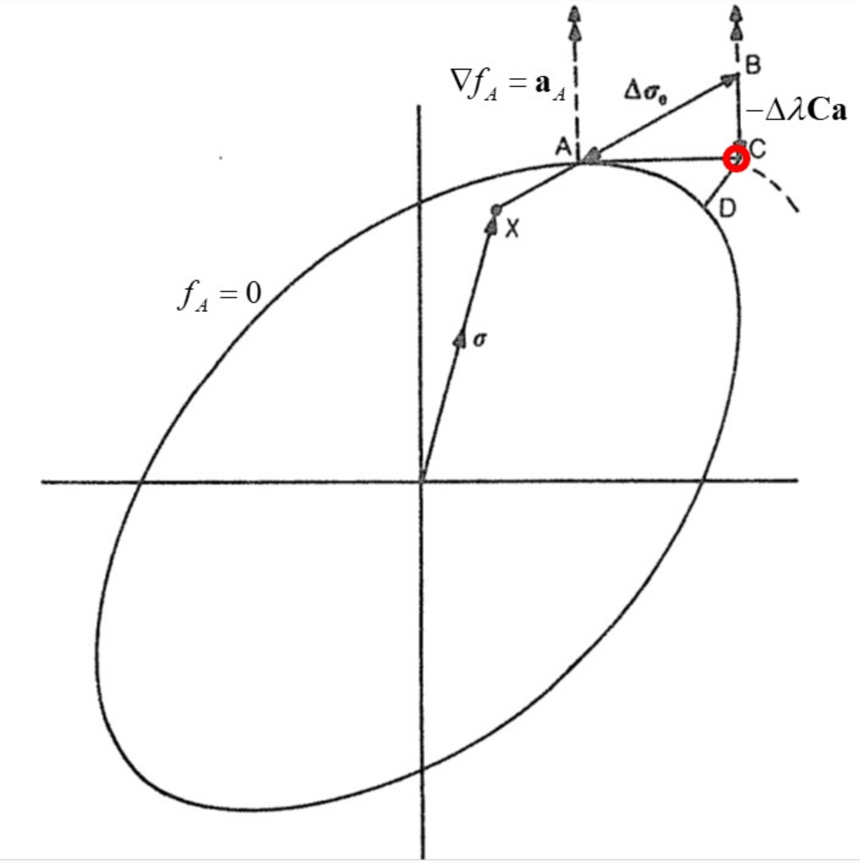
Chapter 6: Basic Plasticity
- Ch. 6.6

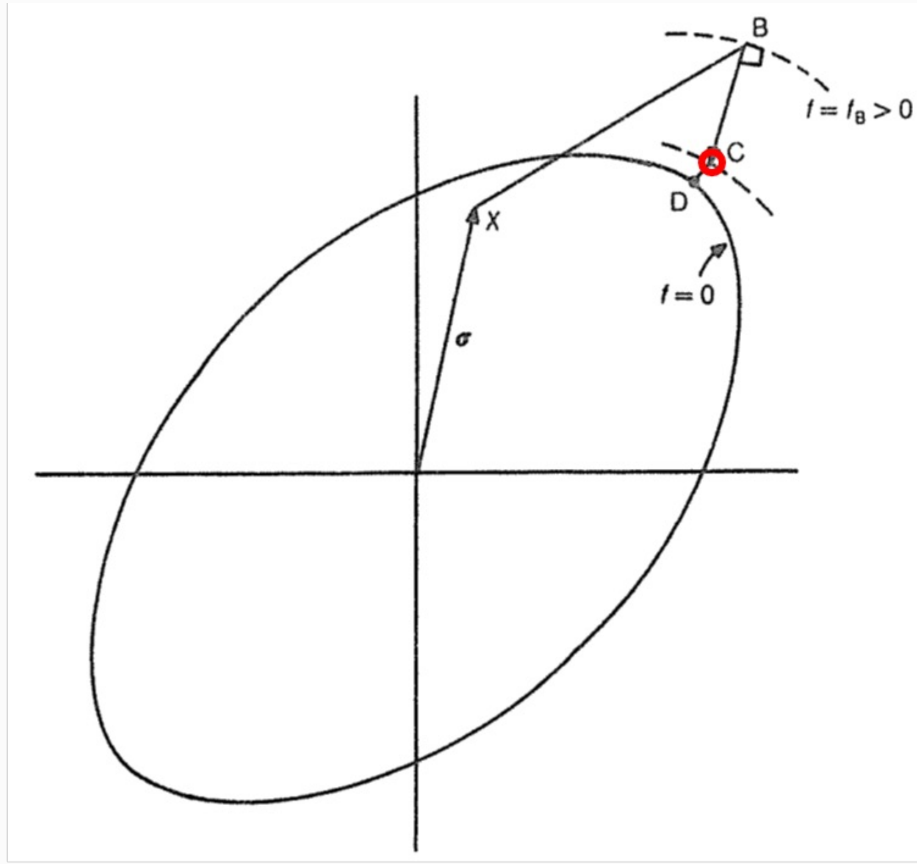
Myoung-Gyu Lee

TA: Gyu Jang Sim (gyujang95@snu.ac.kr)

6.6.3 Returning to the yield surface



[Fig 6.9(b) predictor by Owen]



[Fig 6.10 alternative predictor]

Each predictor (point C) should return to yield surface where $f = 0$.

※ Size of the yield surface is not fixed if hardening is considered.

- **N-R** from **eq.6.60** can be repeated to return to the yield surface, for each predictor (point C in fig.6.9(b) or fig.6.10.

Fig 6.9(b) $\boldsymbol{\sigma}_C = \boldsymbol{\sigma}_A + \Delta\boldsymbol{\sigma}_e - \Delta\lambda \mathbf{C}\mathbf{a}_A$ [eq. 6.57] where $\Delta\lambda = \dfrac{\mathbf{a}_A^T \mathbf{C} \Delta\boldsymbol{\varepsilon}}{\mathbf{a}_A^T \mathbf{C}\mathbf{a}_A + A'_A}$

Fig 6.10 $\boldsymbol{\sigma}_C = \boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_B$ [eq. 6.60] where $\Delta\lambda = \dfrac{f_B}{\mathbf{a}_B^T \mathbf{C}\mathbf{a}_B + A'_B}$ [eq. 6.59]

⬇

$\boldsymbol{\sigma}_D = \boldsymbol{\sigma}_C - \Delta\lambda \mathbf{C}\mathbf{a}_C$ where $\Delta\lambda = \dfrac{f_C}{\mathbf{a}_C^T \mathbf{C}\mathbf{a}_C + A'_C}$

$\boldsymbol{\sigma}_E = \boldsymbol{\sigma}_D - \Delta\lambda \mathbf{C}\mathbf{a}_D$ where $\Delta\lambda = \dfrac{f_D}{\mathbf{a}_D^T \mathbf{C}\mathbf{a}_D + A'_D}$

$\boldsymbol{\sigma}_F = \boldsymbol{\sigma}_E - \Delta\lambda \mathbf{C}\mathbf{a}_E$ where $\Delta\lambda = \dfrac{f_E}{\mathbf{a}_E^T \mathbf{C}\mathbf{a}_E + A'_E}$

⬇

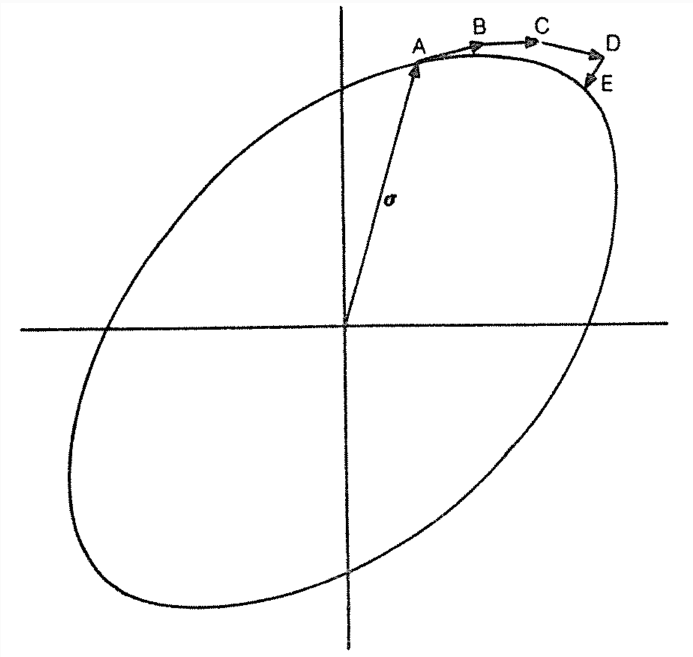
Repeat until $|f| > tolerence$

6.6.4 Sub-incrementation

- Instead of introducing return to the yield surface, the incremental strain $\Delta\boldsymbol{\varepsilon}$ is divided into m sub-steps each of $q\Delta\boldsymbol{\varepsilon}$, where $q = 1/m$, then simple forward Euler method can be used.

- To estimate **proper** m , proper **error analysis** should be done.

- Some workers used **'two-step Euler procedure'** for the **error analysis**.



[Fig 6.11 Sub-incrementation]

$$\boldsymbol{\sigma}_{B1} = \boldsymbol{\sigma}_A + \mathbf{C}_{tA}\Delta\boldsymbol{\varepsilon} = \boldsymbol{\sigma}_A + \Delta\boldsymbol{\sigma}_1$$ [eq. 6.64]

$$\boldsymbol{\sigma}_{B2} = \boldsymbol{\sigma}_A + \frac{1}{2}\left(\mathbf{C}_{tA} + \mathbf{C}_{tB1}\right)\Delta\boldsymbol{\varepsilon} = \boldsymbol{\sigma}_A + \frac{1}{2}\left(\Delta\boldsymbol{\sigma}_1 + \Delta\boldsymbol{\sigma}_2\right)$$ [eq. 6.65]

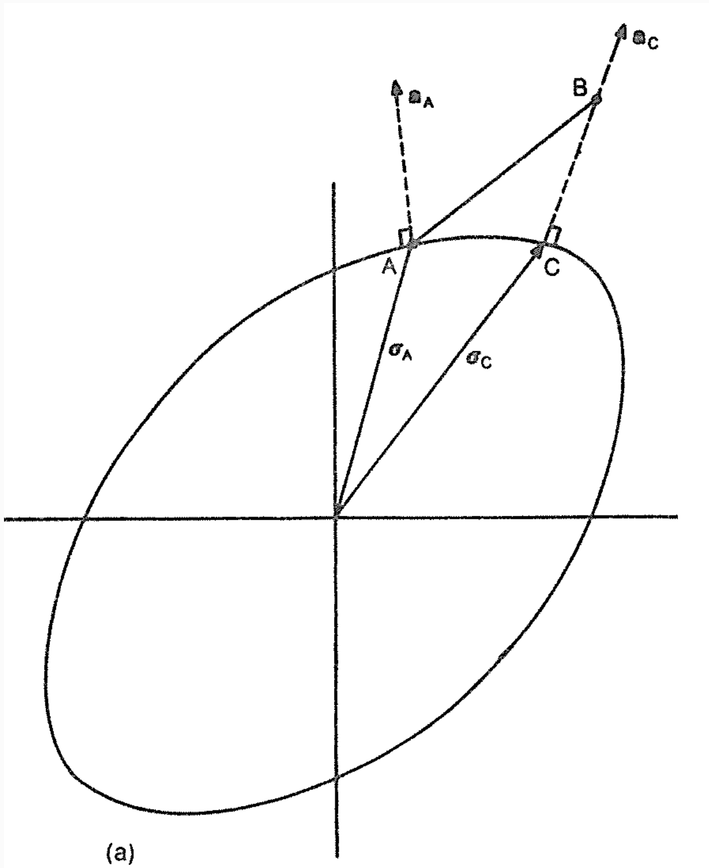
where $\Delta\boldsymbol{\sigma}_2 = \mathbf{C}_{tB1}\Delta\boldsymbol{\varepsilon}$ [eq. 6.66]

The error is: $\delta\boldsymbol{\sigma} = \boldsymbol{\sigma}_{B2} - \boldsymbol{\sigma}_{B1} = \frac{1}{2}\left(\Delta\boldsymbol{\sigma}_2 - \Delta\boldsymbol{\sigma}_1\right)$ [eq. 6.67]

Nyssen's estimation: $m = \dfrac{2\sigma_e\left(\delta\boldsymbol{\sigma}\right)}{\beta\sigma_o}$ [eq. 6.67] $\beta : tolerance$

6.6.5 Generalized trapezoidal or mid-point algorithms

- A number of different integration algorithms can be included in the generalized algorithm:



[Fig 6.12 (a) General return, flow vectors]

$$\boldsymbol{\sigma}_C = \boldsymbol{\sigma}_A + \mathbf{C}\left(\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}_p\right) = \boldsymbol{\sigma}_B - \mathbf{C}\Delta\boldsymbol{\varepsilon}_p$$ [eq. 6.73]

$$\Delta\boldsymbol{\varepsilon}_p = \Delta\lambda\left[\left(1-\eta\right)\mathbf{a}_A + \eta\mathbf{a}_c\right]$$ [eq. 6.74a]

or

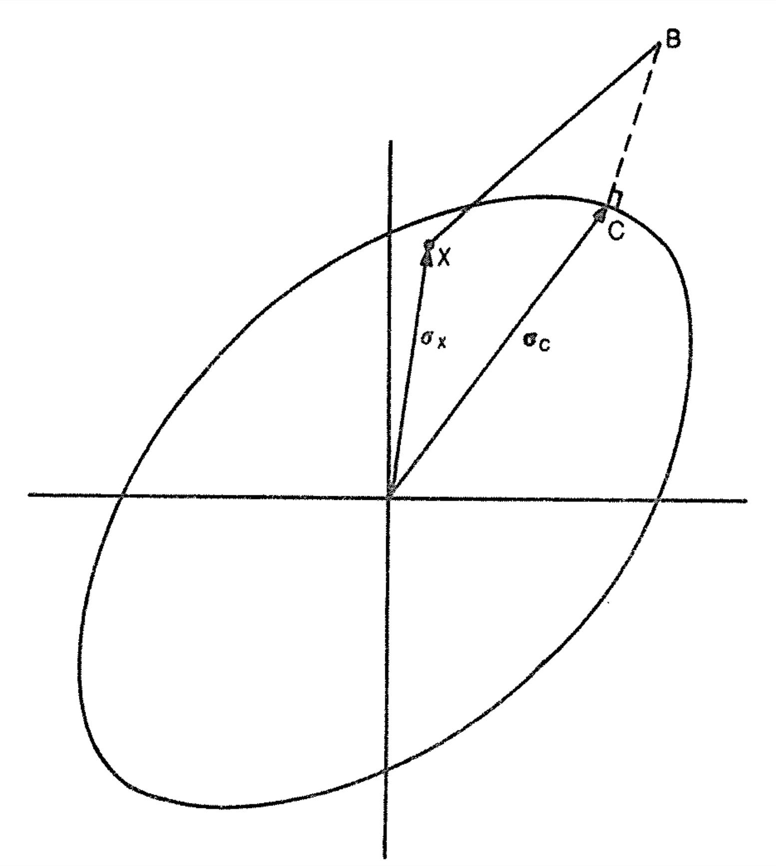
$$\Delta\boldsymbol{\varepsilon}_p = \Delta\lambda\,\underbrace{\mathbf{a}\left(\left(1-\eta\right)\boldsymbol{\sigma}_A + \eta\boldsymbol{\sigma}_C\right)}_{function\ \mathbf{a}\ w.r.t\ stress}$$ [eq. 6.74b]

$$f_C = \sigma_{eC}\left(\boldsymbol{\sigma}_C\right) - \sigma_{0C}\left(\varepsilon_{psC}\right) = \sigma_{eC}\left(\boldsymbol{\sigma}_C\right) - \sigma_{0C}\left(\varepsilon_{psB} + \Delta\varepsilon_{ps}(\Delta\varepsilon_p)\right)$$ [eq. 6.75]

- $\eta = 0$ ➡ Forward-Euler (tangential) algorithm
- $\eta = 1$ ➡ **Backward-Euler algorithm**

6.6.6 A backward-Euler return

- Backward-Euler return algorithm is the most popular and commonly used algorithm for stress integration for metal, due to its **stability, accuracy and effectiveness**.



[Fig 6.12 (b) Backward-Euler return]

Set of unknowns (independent variables)

$\{\Delta\lambda, \mathbf{a}_C\}$

or $\{\Delta\lambda, \boldsymbol{\sigma}_C\}$ since $\mathbf{a}_C = [\partial f / \partial \boldsymbol{\sigma}]_C$

or $\{\Delta\lambda, \Delta\boldsymbol{\varepsilon}_p\}$ since $\boldsymbol{\sigma}_C = \mathbf{C}(\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}_p)$

In this textbook, $\{\Delta\lambda, \boldsymbol{\sigma}_C\}$ is chosen.

➡ **7** unknowns: requires **7** equations

$$\begin{cases} \boldsymbol{\sigma}_C = \boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C & \text{[eq. 6.78]} \\ f(\boldsymbol{\sigma}_C) = 0 \end{cases}$$

- To use N-R, eq.6.78 is reformulated as residual value, that needs convergence to zero.

$$\boldsymbol{\sigma}_C = \boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C$$ [eq. 6.78] ➡ $$\mathbf{r} = \boldsymbol{\sigma}_C - \left(\boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C\right) = 0$$ [eq. 6.79]

- Then, the two set of equations can be expressed as:

$$\begin{cases} \mathbf{r} = \boldsymbol{\sigma}_C - \left(\boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C\right) = 0 \\ f = 0 \end{cases} \Rightarrow \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ f \end{pmatrix} = 0$$

- For the set of unknowns $\{\Delta\lambda, \boldsymbol{\sigma}_C\}$, the N-R can be expressed as:

$$0 = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ f \end{pmatrix} + \begin{pmatrix} \Delta r_1 \\ \Delta r_2 \\ \Delta r_3 \\ \Delta r_4 \\ \Delta r_5 \\ \Delta r_6 \\ \Delta f \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ f \end{pmatrix} + \begin{bmatrix} \frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \frac{\partial r_1}{\partial \sigma_3} & \frac{\partial r_1}{\partial \sigma_4} & \frac{\partial r_1}{\partial \sigma_5} & \frac{\partial r_1}{\partial \sigma_6} & \frac{\partial r_1}{\partial \Delta\lambda} \\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \frac{\partial r_2}{\partial \sigma_3} & \frac{\partial r_2}{\partial \sigma_4} & \frac{\partial r_2}{\partial \sigma_5} & \frac{\partial r_2}{\partial \sigma_6} & \frac{\partial r_2}{\partial \Delta\lambda} \\ \frac{\partial r_3}{\partial \sigma_1} & \frac{\partial r_3}{\partial \sigma_2} & \frac{\partial r_3}{\partial \sigma_3} & \frac{\partial r_3}{\partial \sigma_4} & \frac{\partial r_3}{\partial \sigma_5} & \frac{\partial r_3}{\partial \sigma_6} & \frac{\partial r_3}{\partial \Delta\lambda} \\ \frac{\partial r_4}{\partial \sigma_1} & \frac{\partial r_4}{\partial \sigma_2} & \frac{\partial r_4}{\partial \sigma_3} & \frac{\partial r_4}{\partial \sigma_4} & \frac{\partial r_4}{\partial \sigma_5} & \frac{\partial r_4}{\partial \sigma_6} & \frac{\partial r_4}{\partial \Delta\lambda} \\ \frac{\partial r_5}{\partial \sigma_1} & \frac{\partial r_5}{\partial \sigma_2} & \frac{\partial r_5}{\partial \sigma_3} & \frac{\partial r_5}{\partial \sigma_4} & \frac{\partial r_5}{\partial \sigma_5} & \frac{\partial r_5}{\partial \sigma_6} & \frac{\partial r_5}{\partial \Delta\lambda} \\ \frac{\partial r_6}{\partial \sigma_1} & \frac{\partial r_6}{\partial \sigma_2} & \frac{\partial r_6}{\partial \sigma_3} & \frac{\partial r_6}{\partial \sigma_4} & \frac{\partial r_6}{\partial \sigma_5} & \frac{\partial r_6}{\partial \sigma_6} & \frac{\partial r_6}{\partial \Delta\lambda} \\ \frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2} & \frac{\partial f}{\partial \sigma_3} & \frac{\partial f}{\partial \sigma_4} & \frac{\partial f}{\partial \sigma_5} & \frac{\partial f}{\partial \sigma_6} & \frac{\partial f}{\partial \Delta\lambda} \end{bmatrix} \begin{pmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\sigma_3 \\ \Delta\sigma_4 \\ \Delta\sigma_5 \\ \Delta\sigma_6 \\ \Delta^2\lambda \end{pmatrix}$$

$$\Rightarrow 0 = \begin{pmatrix} \mathbf{r} \\ f \end{pmatrix} + \begin{bmatrix} \frac{\partial \mathbf{r}}{\partial \boldsymbol{\sigma}} & \frac{\partial \mathbf{r}}{\partial \Delta\lambda} \\ \left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T & \frac{\partial f}{\partial \Delta\lambda} \end{bmatrix} \begin{pmatrix} \Delta\boldsymbol{\sigma} \\ \Delta^2\lambda \end{pmatrix} \Rightarrow$$

repeat $\begin{pmatrix} \boldsymbol{\sigma} \\ \Delta\lambda \end{pmatrix} \leftarrow \begin{pmatrix} \boldsymbol{\sigma} \\ \Delta\lambda \end{pmatrix} + \begin{pmatrix} \Delta\boldsymbol{\sigma} \\ \Delta^2\lambda \end{pmatrix}$

until $\begin{cases} \|\mathbf{r}\|^2 > tolerance\,1 \\ \text{and} \\ |f| > tolerance\,2 \end{cases}$

- Calculations should be done for

$$\begin{bmatrix} \dfrac{\partial \mathbf{r}}{\partial \boldsymbol{\sigma}} & \dfrac{\partial \mathbf{r}}{\partial \Delta\lambda} \\ \left(\dfrac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T & \dfrac{\partial f}{\partial \Delta\lambda} \end{bmatrix}$$

where $\mathbf{r} = \boldsymbol{\sigma}_C - \left(\boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C\right) = 0$

and $f = \sigma_e(\boldsymbol{\sigma}_C) - \sigma_0(\varepsilon_{ps})$

- Note that:

$$\varepsilon_{ps} = \int \dot{\varepsilon}_p dt = \int d\varepsilon_p = \int d\lambda$$

$$\Delta\lambda = \Delta\varepsilon_{ps}$$

$$\Rightarrow \quad \frac{\partial \sigma_0}{\partial \varepsilon_{ps}} = \frac{\partial \sigma_0}{\partial \lambda} = \frac{\partial \sigma_0}{\partial \Delta\lambda}$$

※ This is valid only for certain conditions.
Refer to remark #13.1 in Kwansoo Chung, Myoung-Gyu Lee, “Basics of Continuum Plasticity”.

1. $\dfrac{\partial \mathbf{r}}{\partial \boldsymbol{\sigma}}$

$$\mathbf{r} = \boldsymbol{\sigma}_C - \left(\boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C\right) = 0$$ [eq. 6.79]

$\boldsymbol{\sigma}_B$ is fixed, so

$$\Rightarrow \frac{\partial \mathbf{r}}{\partial \boldsymbol{\sigma}_C} = \mathbf{I} + \Delta\lambda \mathbf{C}\left(\frac{\partial \mathbf{a}}{\partial \boldsymbol{\sigma}}\right) = \mathbf{I} + \Delta\lambda \mathbf{C}\left(\frac{\partial^2 f}{\partial \boldsymbol{\sigma}^2}\right)$$

$\mathbf{I}$: *symmetric* 4^{th} *order unit tensor*

In Voigt notation,

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2. $\dfrac{\partial \mathbf{r}}{\partial \Delta\lambda}$

$$\mathbf{r} = \boldsymbol{\sigma}_C - \left(\boldsymbol{\sigma}_B - \Delta\lambda \mathbf{C}\mathbf{a}_C\right) = 0$$

$$\Rightarrow \frac{\partial \mathbf{r}}{\partial \Delta\lambda} = \mathbf{C}\mathbf{a}$$

3. $\dfrac{\partial f}{\partial \boldsymbol{\sigma}}$

$$\Rightarrow \frac{\partial f}{\partial \boldsymbol{\sigma}} = \mathbf{a}$$

4. $\dfrac{\partial f}{\partial \Delta\lambda}$

$$f = \sigma_e(\boldsymbol{\sigma}_C) - \sigma_0(\varepsilon_{ps})$$

$$\Rightarrow \frac{\partial f}{\partial \Delta\lambda} = \frac{\partial \sigma_0}{\partial \varepsilon_{ps}} = A'$$

$$0=\begin{pmatrix}\mathbf{r}\\ f\end{pmatrix}+\begin{bmatrix}\dfrac{\partial\mathbf{r}}{\partial\boldsymbol{\sigma}} & \dfrac{\partial\mathbf{r}}{\partial\Delta\lambda}\\ \left(\dfrac{\partial f}{\partial\boldsymbol{\sigma}}\right)^T & \dfrac{\partial f}{\partial\Delta\lambda}\end{bmatrix}\begin{pmatrix}\Delta\boldsymbol{\sigma}\\ \Delta^2\lambda\end{pmatrix}$$

For simplicity, let $\mathbf{Q}\doteq\mathbf{I}+\Delta\lambda\mathbf{C}\left(\dfrac{\partial^2 f}{\partial\boldsymbol{\sigma}^2}\right)$

$$\begin{bmatrix}\mathbf{Q} & \mathbf{Ca}\\ \mathbf{a} & A'\end{bmatrix}\begin{pmatrix}\Delta\boldsymbol{\sigma}\\ \Delta^2\lambda\end{pmatrix}=-\begin{pmatrix}\mathbf{r}\\ f\end{pmatrix}\ \Rightarrow\ \begin{cases}\mathbf{0}=\mathbf{r}+\mathbf{Q}\Delta\boldsymbol{\sigma}+\Delta^2\lambda\mathbf{Ca} & \text{[variation of eq.6.80]}\\ 0=f+\mathbf{a}^T\Delta\boldsymbol{\sigma}+A'\Delta^2\lambda & \text{[variation of eq.6.82]}\end{cases}$$

$$\mathbf{0}=\mathbf{r}+\mathbf{Q}\Delta\boldsymbol{\sigma}+\Delta^2\lambda\mathbf{Ca}\ \Rightarrow\ \Delta\boldsymbol{\sigma}=-\mathbf{Q}^{-1}\left(\mathbf{r}+\Delta^2\lambda\mathbf{Ca}\right)$$

Substitution to 6.82 $\Rightarrow$ $0=f-\mathbf{a}^T\mathbf{Q}^{-1}\left(\mathbf{r}+\Delta^2\lambda\mathbf{Ca}\right)+A'\Delta^2\lambda=f-\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{r}-\left(\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{Ca}-A'\right)\Delta^2\lambda$

$$\Rightarrow\ \Delta^2\lambda=\frac{f-\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{r}}{\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{Ca}-A'}\quad\text{and}\quad\Delta\boldsymbol{\sigma}=-\mathbf{Q}^{-1}\left(\mathbf{r}+\frac{f-\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{r}}{\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{Ca}-A'}\mathbf{Ca}\right)$$

[variation of eq.6.83]

Final term project:

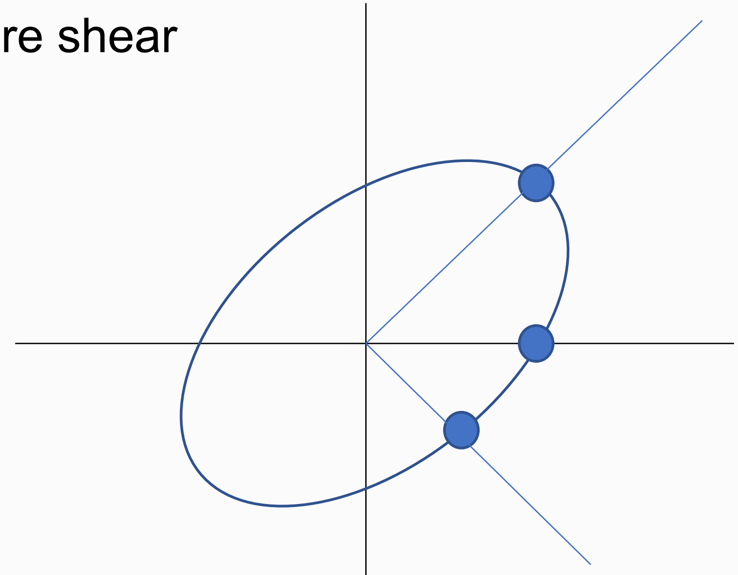
- Implement stress update algorithms to predict stress.

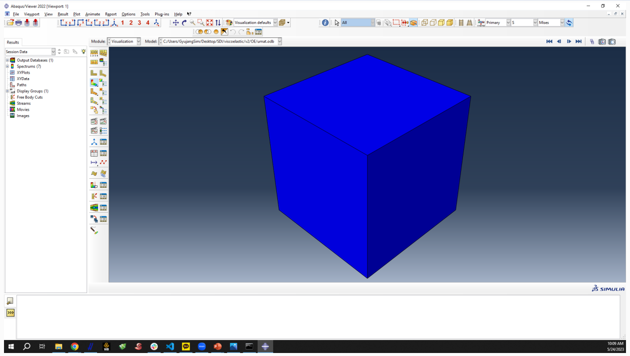
  (1) Euler forward (Explicit), (2) Backward Euler (Closest point projection)
  With/without sub-increments (10-1000 increment)

- Apply the algorithm to the given strain history, and check if your stress history matches with Abaqus result.

- Von Moses, linear isotropic hardening

- Three strain paths: uniaxial, balanced biaxial, and pure shear





	A	B	C	D	E	F
1	LE11	LE22	LE33	LE12	LE13	LE23
2	0	0	0	0	0	0
3	0	1.00E-04	0	0	0	0
4	0	0.0002	0	0	0	0
5	0	0.0003	0	0	0	0
6	0	0.0004	0	0	0	0
7	0	0.0005	0	0	0	0
8	0	0.0006	0	0	0	0
9	0	0.0007	0	0	0	0
10	0	0.0008	0	0	0	0
11	0	0.0009	0	0	0	0
12	0	0.001	0	0	0	0
13	0	0.001099	0	0	0	0
14	0	0.001199	0	0	0	0
15	0	0.001299	0	0	0	0
16	0	0.001399	0	0	0	0
17	0	0.001499	0	0	0	0
18	0	0.001599	0	0	0	0
19	0	0.001699	0	0	0	0
20	0	0.001798	0	0	0	0
21	0	0.001898	0	0	0	0
22	0	0.001998	0	0	0	0
23	0	0.002098	0	0	0	0
24	0	0.002198	0	0	0	0
25	0	0.002297	0	0	0	0
26	0	0.002397	0	0	0	0
27	0	0.002497	0	0	0	0
28	0	0.002597	0	0	0	0
29	0	0.002696	0	0	0	0
30	0	0.002796	0	0	0	0
31	0	0.002896	0	0	0	0
32	0	0.002996	0	0	0	0
33	0	0.003095	0	0	0	0
34	0	0.003195	0	0	0	0

strain history

	A	B	C	D	E	F
1	Sxx	Syy	Szz	Sxy	Sxz	Syz
2	0	0	0	0	0	
3	4.94E-12	99942.6	-8.47E-13	2.12E-13	-8.47E-13	3.25E-
4	-2.32E-13	199771	3.84E-14	-8.88E-15	3.84E-14	-1.60E-
5	-5.25E-14	299485	1.00E-14	-4.10E-15	1.00E-14	-2.48E-
6	9.30E-14	399085	-1.60E-14	6.27E-15	-1.60E-14	4.98E-
7	-1.00E-13	498573	1.94E-14	-7.69E-15	1.94E-14	-4.96E-
8	9.10E-14	597948	-2.10E-14	8.41E-15	-2.10E-14	3.53E-
9	-7.08E-14	697212	2.26E-14	-9.09E-15	2.26E-14	-1.21E-
10	5.39E-14	796364	-2.29E-14	9.20E-15	-2.29E-14	-6.11E-
11	-5.27E-14	895404	2.34E-14	-9.37E-15	2.34E-14	7.26E-
12	6.36E-14	994335	-2.28E-14	9.04E-15	-2.28E-14	4.75E-
13	-9.38E-14	1.09E+06	2.26E-14	-8.84E-15	2.26E-14	-3.70E-
14	1.17E-13	1.19E+06	-2.14E-14	8.22E-15	-2.14E-14	6.27E-
15	-1.30E-13	1.29E+06	2.08E-14	-7.80E-15	2.08E-14	-7.87E-
16	1.37E-13	1.39E+06	-1.92E-14	7.03E-15	-1.92E-14	9.01E-
17	-1.41E-13	1.49E+06	1.83E-14	-6.50E-15	1.83E-14	-9.77E-
18	1.42E-13	1.59E+06	-1.66E-14	5.66E-15	-1.66E-14	1.02E-
19	-1.39E-13	1.68E+06	1.56E-14	-5.11E-15	1.56E-14	-1.04E-
20	1.35E-13	1.78E+06	-1.39E-14	4.28E-15	-1.39E-14	1.03E-
21	-1.29E-13	1.88E+06	1.29E-14	-3.76E-15	1.29E-14	-1.01E-
22	1.22E-13	1.98E+06	-1.13E-14	3.00E-15	-1.13E-14	9.70E-
23	-9.00E-14	2.08E+06	1.04E-14	-2.56E-15	1.04E-14	-6.88E-
24	4.88E-14	2.17E+06	-8.84E-15	1.89E-15	-8.84E-15	3.08E-
25	-5.36E-15	2.27E+06	8.07E-15	-1.54E-15	8.07E-15	9.81E-
26	-2.92E-14	2.37E+06	-6.69E-15	9.77E-16	-6.69E-15	-4.17E-
27	5.49E-14	2.47E+06	6.08E-15	-7.39E-16	6.08E-15	6.50E-
28	-8.63E-14	2.56E+06	-4.86E-15	2.78E-16	-4.86E-15	-9.43E-
29	1.11E-13	2.66E+06	4.40E-15	-1.40E-16	4.40E-15	1.17E-
30	-1.16E-13	2.76E+06	-3.34E-15	-2.23E-16	-3.34E-15	-1.21E-
31	1.17E-13	2.85E+06	3.04E-15	2.70E-16	3.04E-15	1.20E-
32	-4.63E-12	2.95E+06	-2.13E-15	-5.47E-16	-2.13E-15	-4.63E-
33	3.17E-13	3.05E+06	1.96E-15	5.19E-16	1.96E-15	3.18E-
34	-4.43E-14	3.14E+06	-1.18E-15	-7.26E-16	-1.18E-15	-4.42E-

stress history


Thank you!