High-power Pulsed Electron and Ion Diodes

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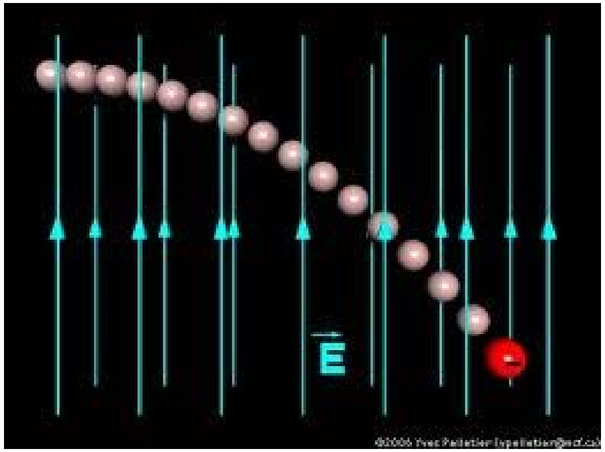
Motions in uniform electric field

- Equation of motion of a charged particle in fields

$$m\frac{d\boldsymbol{v}}{dt} = q[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r},t)], \qquad \frac{d\boldsymbol{r}}{dt} = \boldsymbol{v}(t)$$

- Motion in constant electric field



✓ For a constant electric field **E** = $\mathbf{E_0}$ with **B** = 0,

$$\boldsymbol{r}(t) = \boldsymbol{r}_{\mathbf{0}} + \boldsymbol{v}_{\mathbf{0}}t + \frac{q\boldsymbol{E}_{\mathbf{0}}}{\mathbf{2}m}t^2$$

✓ Electrons are easily accelerated by electric field due to their smaller mass than ions.

✓ Electrons (Ions) move against (along) the electric field direction.

✓ The charged particles get kinetic energies.

Motions in uniform magnetic field

- Motion in constant magnetic field

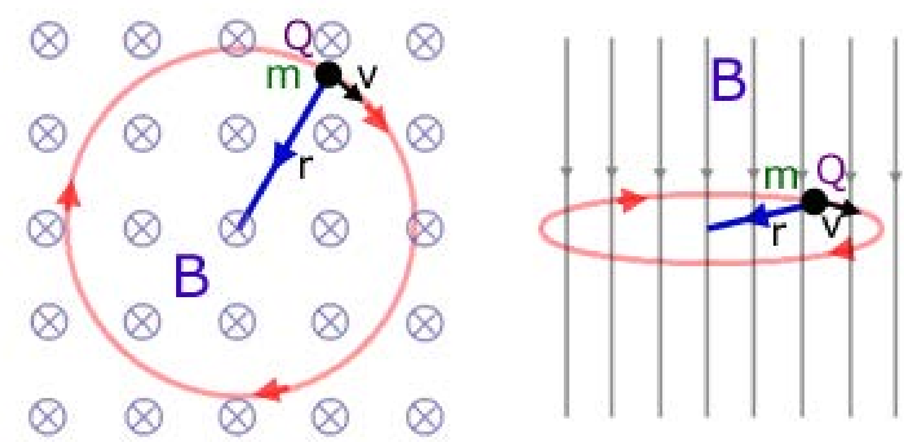
$$m\frac{d\boldsymbol{v}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}$$

- For a constant magnetic field **B** = B_0**z** with **E** = 0,

$$m\frac{dv_x}{dt} = qB_0v_y$$

$$m\frac{dv_y}{dt} = -qB_0v_x$$

$$m\frac{dv_z}{dt} = 0$$



- Cyclotron (gyration) frequency

$$\frac{d^2v_x}{dt^2} = -\omega_c^2 v_x$$

$$\boxed{\omega_c = \frac{|q|B_0}{m}}$$

Motions in uniform magnetic field

- Particle velocity

$$v_x = v_\perp \cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp \sin(\omega_c t + \phi_0)$$
$$v_z = 0$$

- Particle position

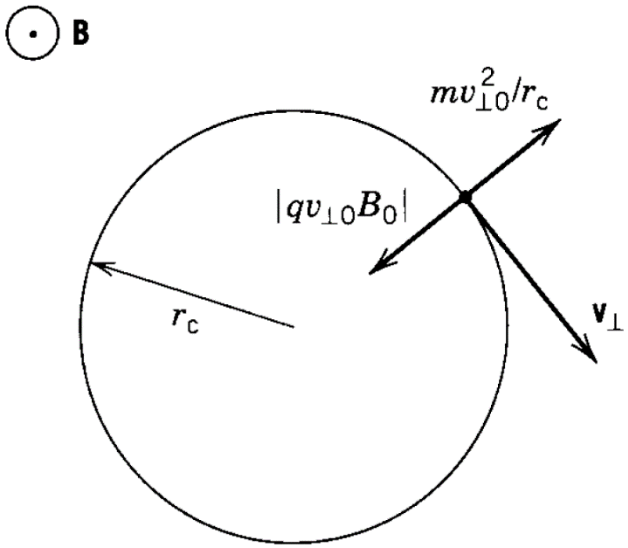
$$x = r_c \sin(\omega_c t + \phi_0) + (x_0 - r_c \sin\phi_0)$$
$$y = r_c \cos(\omega_c t + \phi_0) + (y_0 - r_c \cos\phi_0)$$
$$z = z_0 + v_{z0} t$$

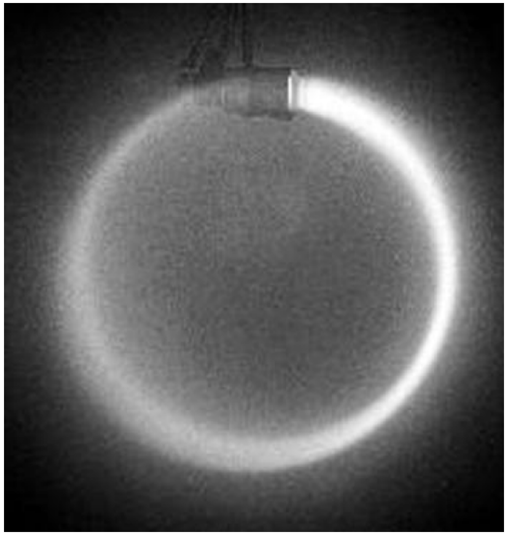
- Guiding center

$$(x_0, y_0, z_0 + v_{z0} t\ )$$

- Larmor (gyration) radius

$$\boxed{r_c = \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| B_0}}$$





Gyro-frequency and radius

- The direction of gyration is always such that the magnetic field generated by the charged particle is opposite to the externally imposed field. → diamagnetic

- For electrons

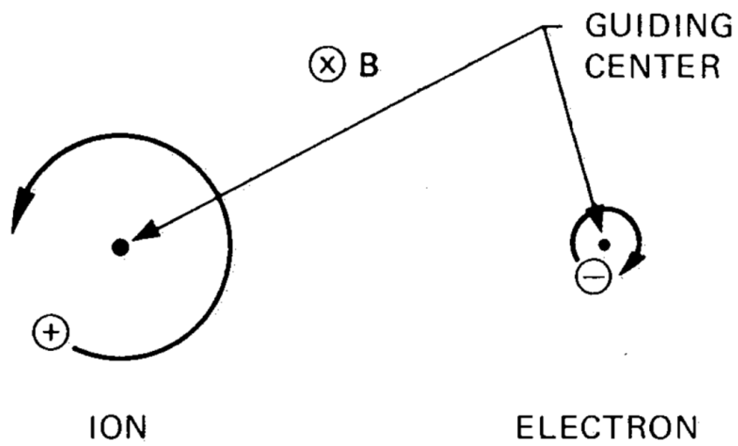
$$f_{ce} = 2.80 \times 10^6\, B_0 \text{ [Hz]} \quad (B_0 \text{ in gauss})$$

$$r_{ce} = \frac{3.37\sqrt{E}}{B_0} \text{ [cm]} \quad (E \text{ in volts})$$

- For singly charged ions

$$f_{ci} = 1.52 \times 10^3\, B_0/M_A \text{ [Hz]} \quad (B_0 \text{ in gauss})$$

$$r_{ci} = \frac{144\sqrt{EM_A}}{B_0} \text{ [cm]} \quad (E \text{ in volts}, M_A \text{ in amu})$$

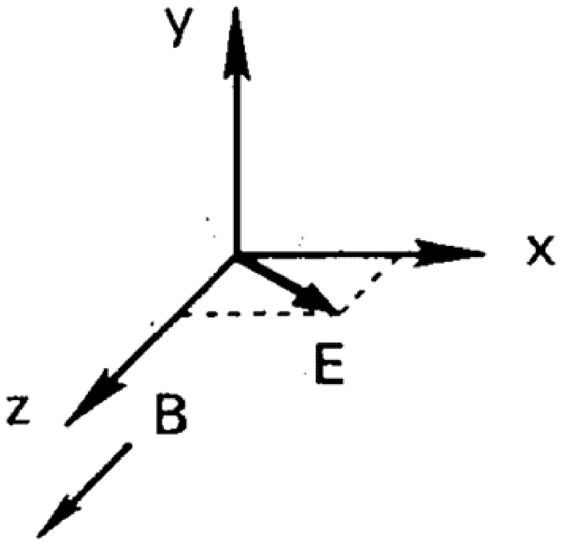


- Energy gain?

Motions in uniform E and B fields

- Equation of motion

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



- Parallel motion: **B** = B_0**z** and **E** = E_0**z**,

$$m\frac{dv_z}{dt} = qE_z$$

$$v_z = \frac{qE_z}{m}t + v_{z0}$$

→ Straightforward acceleration along B

$\boldsymbol{E}\times\boldsymbol{B}$ drift

- Transverse motion: **B** = B$_0$**z** and **E** = E$_0$**x**,

$$m\frac{dv_x}{dt} = qE_0 + qB_0 v_y$$
$$m\frac{dv_y}{dt} = -qB_0 v_x$$

- Differentiating,

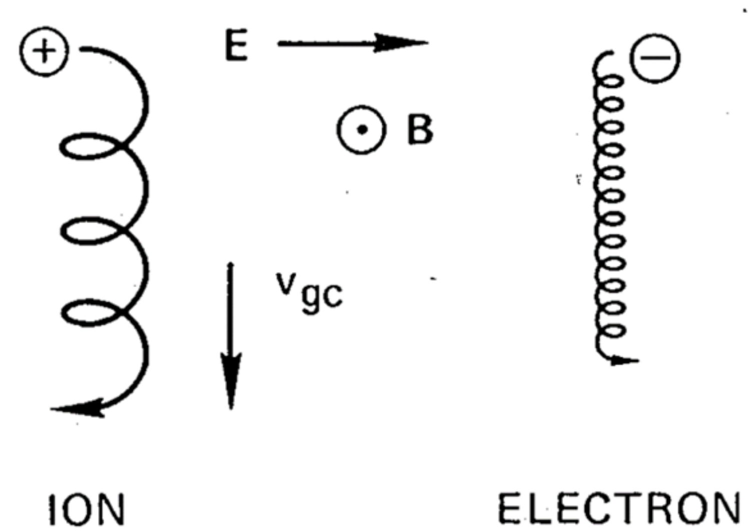
$$\frac{d^2 v_x}{dt^2} = -\omega_c^2 v_x$$
$$\frac{d^2 v_y}{dt^2} = -\omega_c^2\left(\frac{E_0}{B_0} + v_y\right)$$

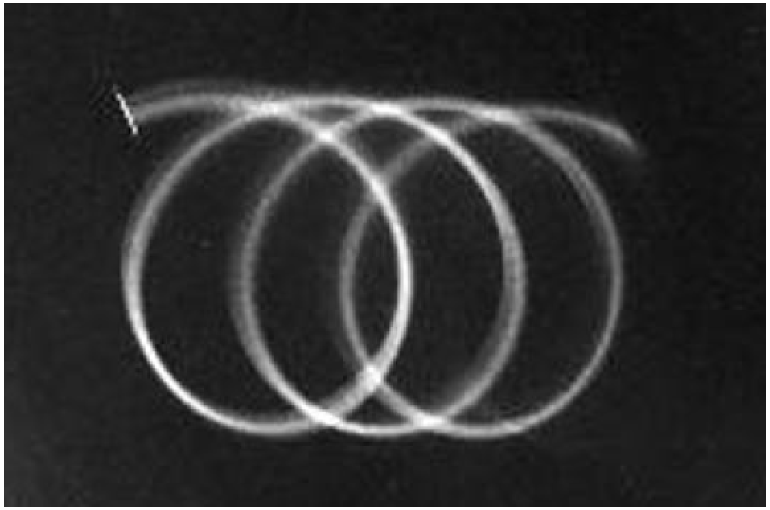
$$\boxed{\boldsymbol{v}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}}$$

- Particle velocity

$$v_x = v_\perp\cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp\sin(\omega_c t + \phi_0) - \frac{E_0}{B_0}$$

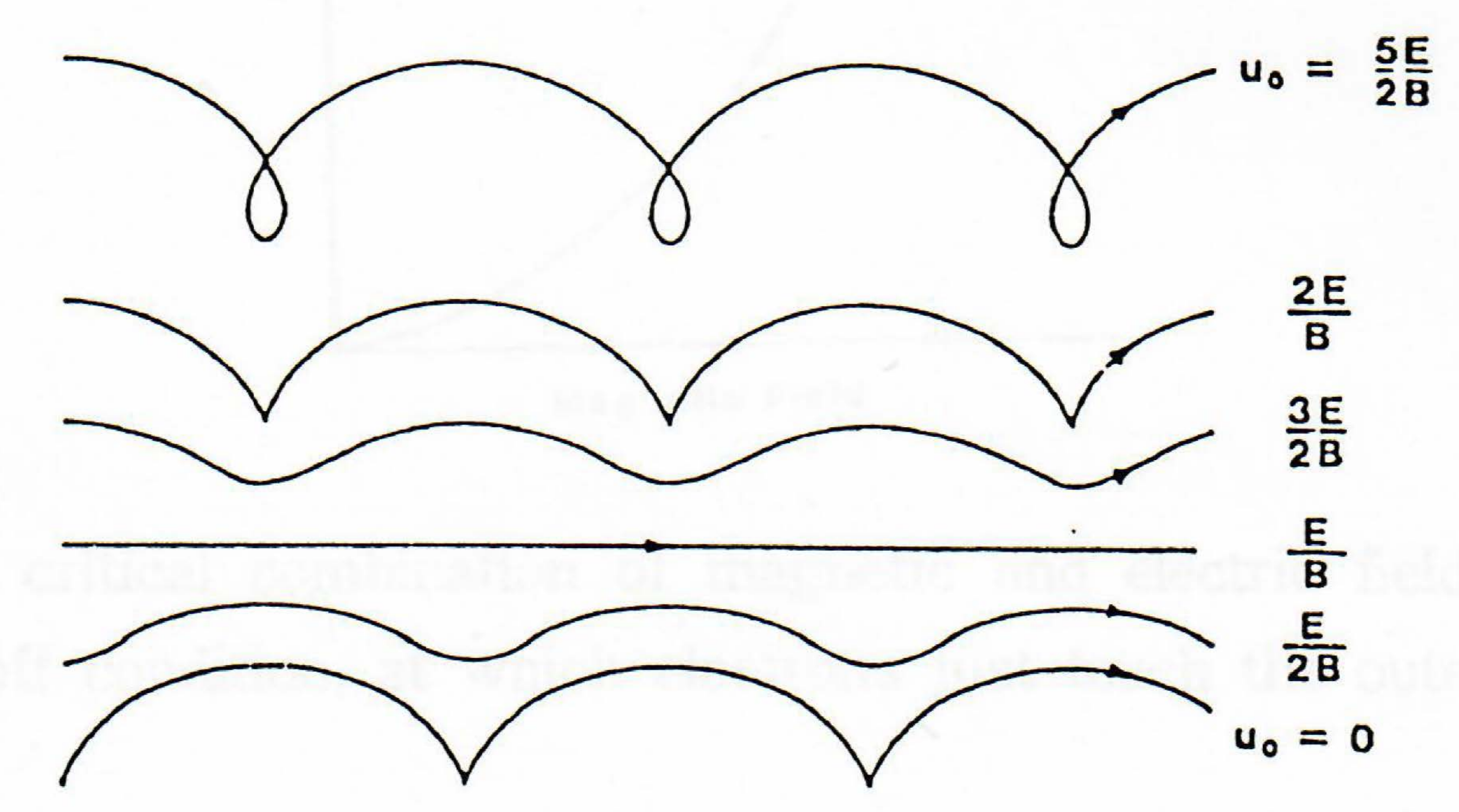
v_{gc}





$\boldsymbol{E}\times\boldsymbol{B}$ drift

- Electron trajectories for various initial velocities [H/W]



Time-varying $\boldsymbol{E}$ field: polarization drift

- Assume that $\boldsymbol{E} = E_0 e^{i\omega t}$, then $\dot{\boldsymbol{E}}_x = i\omega \boldsymbol{E}_x$

$$\frac{d^2 v_x}{dt^2} = -\omega_c^2 \left(v_x \mp \frac{i\omega}{\omega_c} \frac{\tilde{E}_x}{B} \right)$$
$$\frac{d^2 v_y}{dt^2} = -\omega_c^2 \left(v_y - \frac{\tilde{E}_x}{B} \right)$$

$$\boxed{\tilde{v}_E = \frac{\tilde{E}_x}{B}}$$

$$\boxed{\tilde{v}_p = \pm \frac{i\omega}{\omega_c} \frac{\tilde{E}_x}{B}}$$

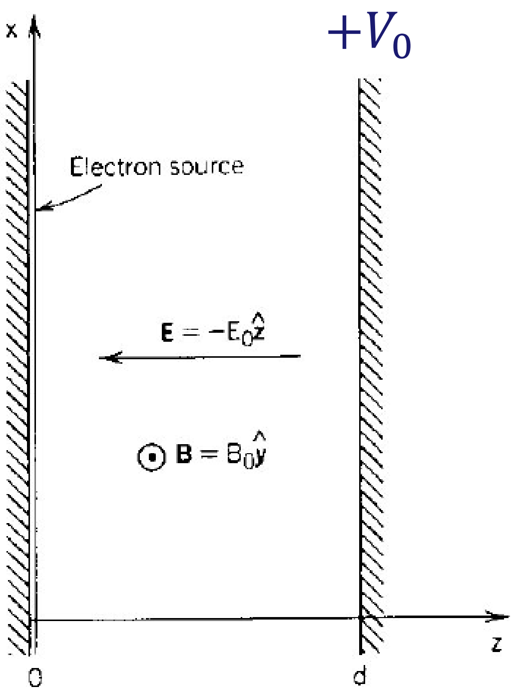
- Particle velocity for slowly-varying E field ($\omega \ll \omega_c$)

$$v_x = v_\perp e^{i\omega_c t} + \tilde{v}_p$$
$$v_y = \pm i v_\perp e^{i\omega_c t} + \tilde{v}_E$$

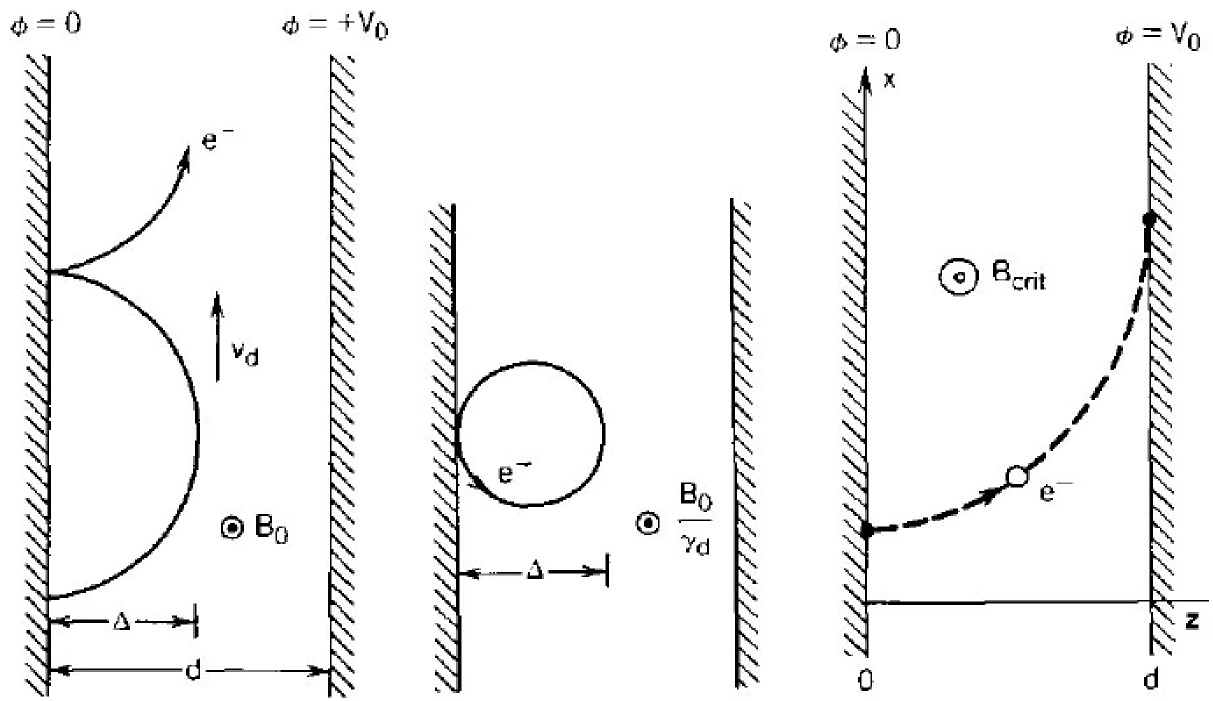
- Polarization drift

$$\boxed{\boldsymbol{v}_p = \pm \frac{1}{\omega_c B} \frac{dE}{dt}}$$

Drift motion of electrons in a planar gap



- Several microwave sources, such as magnetrons and gyrotrons, depend upon the motion of electrons in crossed electric and magnetic fields. The term crossed field means that the electric and magnetic fields are perpendicular.



- If $\Delta < d$, electrons cannot cross to the anode and the gap is magnetically insulated. This phenomenon is called magnetic insulation.

Criterion for magnetic insulation: critical B field

- The canonical momentum in the x direction P_x is conserved because all forces are uniform along x.

$$P_x = \gamma(z) m_e v_x(z) - e A_x(z)$$

$$\boldsymbol{P} = m\boldsymbol{v} + q\boldsymbol{A}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

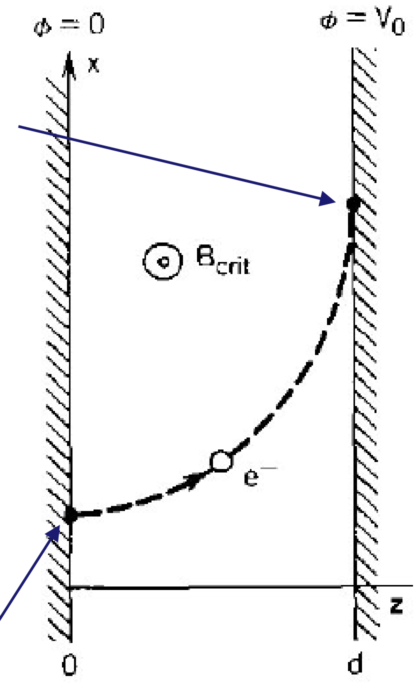
- The vector potential can be calculated as

$$B_y = \frac{\partial A_x}{\partial z} \qquad A_x(d) = \int_0^d B_y(z)dz = B_0 d$$

- An applied magnetic field equal to B_{crit} gives electron orbits that just reach the anode. Conservation of total energy implies the electrons have a relativistic γ factor of

$$\gamma(d) = 1 + \frac{eV_0}{m_e c^2} \qquad v_x(d) = c\frac{\sqrt{\gamma(d)^2 - 1}}{\gamma(d)}$$

$v_z(d) = 0$



$v_z(0) = 0 \quad v_x(0) = 0$

$A_x(0) = 0 \quad P_x(0) = 0$

- The conservation of canonical angular momentum:

$$P_x(0) = P_x(d) = \gamma(d) m_e v_x(d) - e B_{crit} d = 0$$

- We obtain the critical magnetic field:

$$B_{crit} = \frac{m_e \gamma(d) v_x(d)}{ed} = \frac{m_e c}{ed}\sqrt{\gamma(d)^2 - 1} = \frac{m_e c}{ed}\left[\frac{2eV_0}{m_e c^2} + \left(\frac{eV_0}{m_e c^2}\right)^2\right]^{1/2}$$

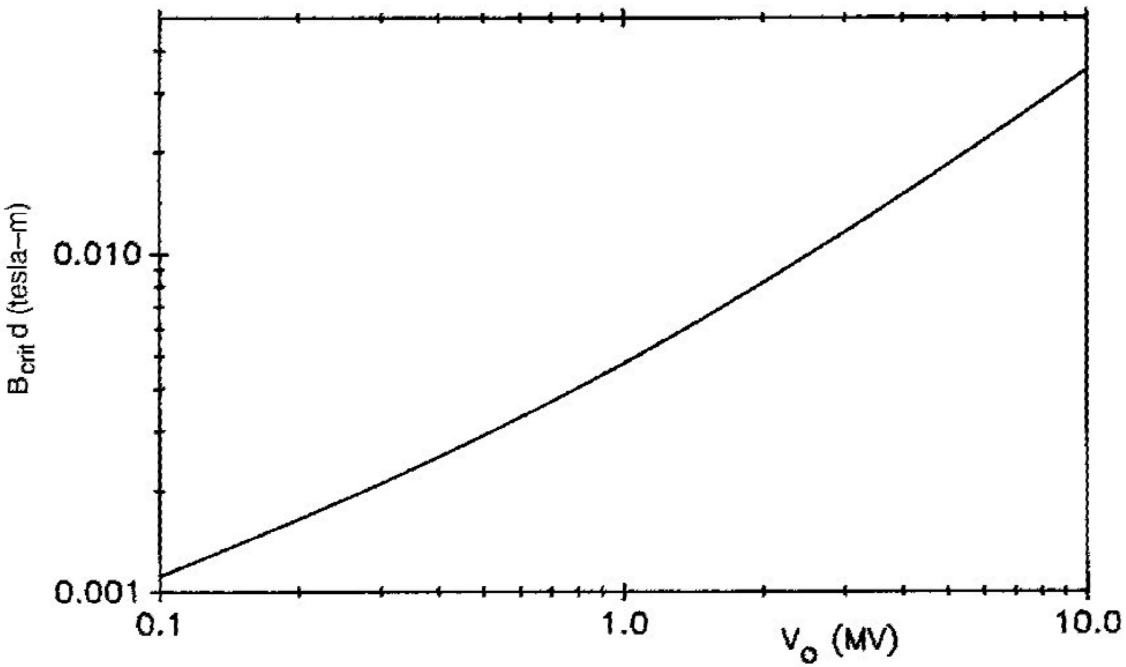
Moderate magnetic fields can insulate high-voltage gaps

- The critical magnetic field for electrons can be expressed as:

$$B_{crit} = \frac{m_e c}{ed}\left[\frac{2eV_0}{m_e c^2} + \left(\frac{eV_0}{m_e c^2}\right)^2\right]^{1/2} = B^*\underbrace{\left[1 + \frac{eV_0}{2m_e c^2}\right]^{1/2}}_{\text{Relativistic correction term}}$$

$$B^* = \frac{1}{d}\left[\frac{2m_e V_0}{e}\right]^{1/2}$$

Non-relativistic factor



In a slowly-rising gap voltage, electrons follow laminar orbits in $\boldsymbol{E} \times \boldsymbol{B}$ direction rather than cycloid drift orbits

- In a rising field, the polarization drift carries electrons away from the cathode while they move sideways at the E×B drift velocity.

- In the non-relativistic limit, the final orbit satisfies the following three equations:

$$m_e v_x(z) - (eB_0 d)(z/d) = 0$$

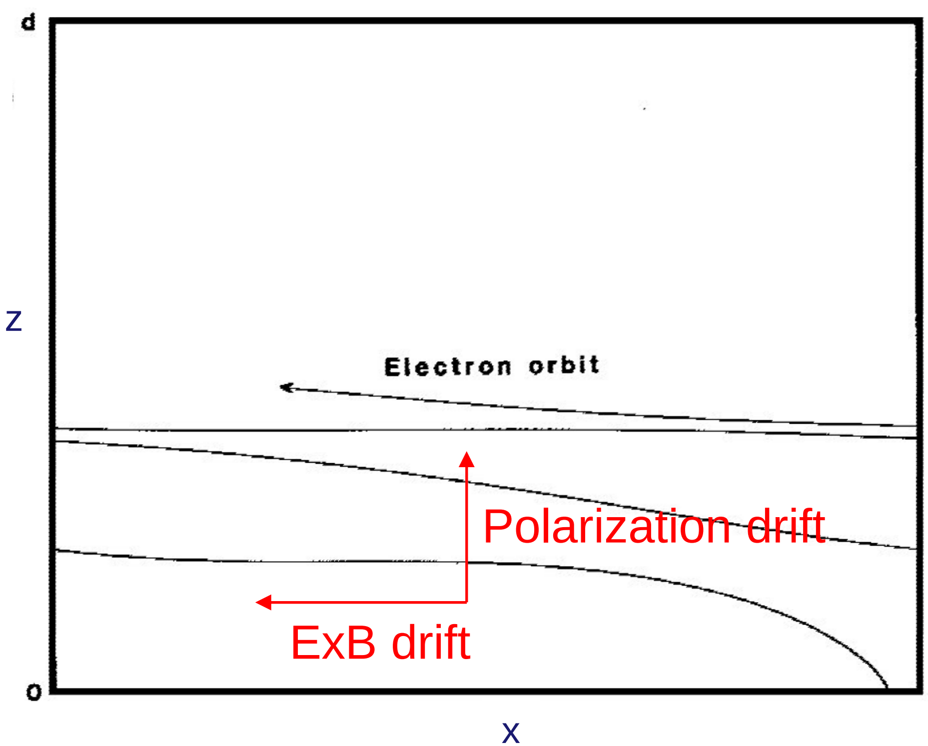
$$v_x(z) = V_0/B_0 d$$

$$v_z(z) \approx 0 \qquad B_0 = B^*$$

$$\Rightarrow \quad z = d/2$$

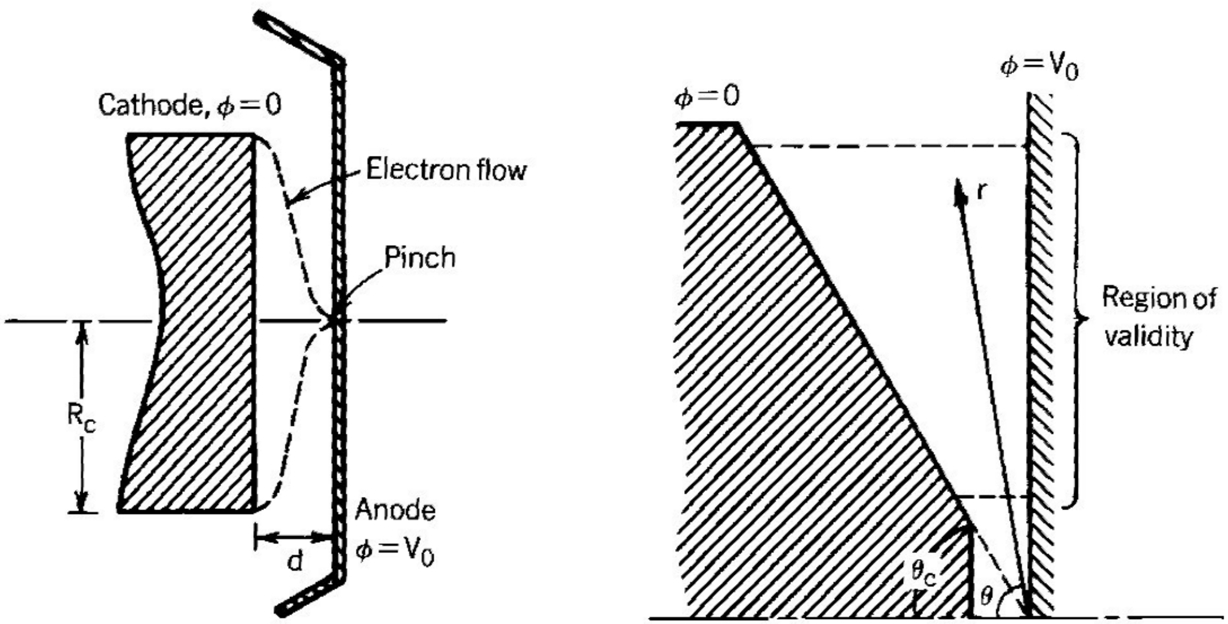
- The final position is the half of the gap distance.

Voltage rises linearly to give $B_0 = B^*$
Voltage rise time ~ $15/\omega_{ge}$



Pinched electron beam diode

- Experiments have shown that two-electrode vacuum gaps driven by a pulsed power generator can create tightly pinched electron beams. For example, with beam current in the range ~100 kA and voltage ~1 MV, almost all electrons emitted from the cathode compress to a tight focus on the axis at the anode.
- Pinching results from the strong azimuthal magnetic field generated by the high-current electron beam. A pinch occurs when the magnetic field at the edge of the cathode is strong enough to bend electron orbits so that they cannot cross directly to the anode – the magnetic field insulates the edge of the cathode.



Criterion for pinching in electron beam diode

- Space-charge-limited current in terms of the relativistic energy factor for electron arriving at the anode of V_0:

$$I_{sc} \cong \frac{4}{9}\epsilon_0\left(\frac{2e}{m_e}\right)^{1/2}\frac{(\gamma_0-1)^{3/2}(m_ec^2)^{3/2}}{d^2e^{3/2}}(\pi R_c^2)\Theta(\gamma_0) \qquad \gamma_0 = 1+\frac{eV_0}{m_ec^2}$$

Correction factor for relativistic effect $\Theta \lesssim 1$

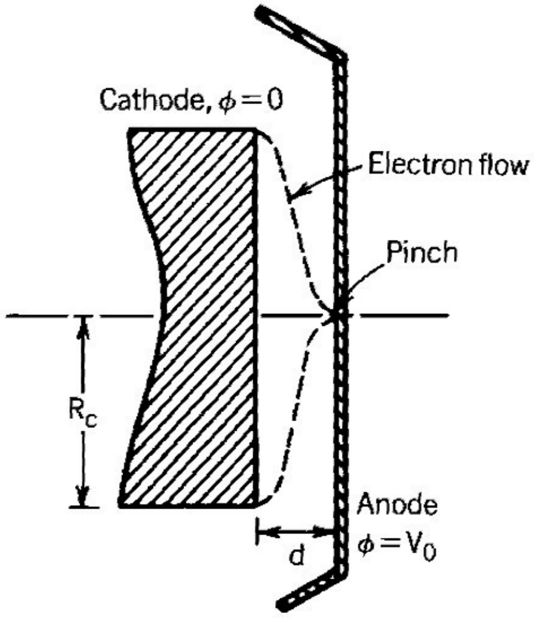
- Electrons move radially inward when the field at the cathode edge satisfies the magnetic insulation condition:

$$B(R_c) = \frac{\mu_0 I_{crit}}{2\pi R_c} = B_{crit} = \frac{m_ec}{ed}\left[\frac{2eV_0}{m_ec^2}+\left(\frac{eV_0}{m_ec^2}\right)^2\right]^{1/2}$$

$$\Rightarrow \quad I_{crit} = \left[\frac{2\pi m_ec}{e\mu_0}\right]\frac{R_c}{d}[\gamma_0^2-1]^{1/2}$$

- Criterion for pinching ($I_{sc} > I_{crit}$)

$$\frac{\sqrt{8}}{9}\left(\frac{R_c}{d}\right)\frac{(\gamma_0-1)^{3/2}\Theta(\gamma_0)}{(\gamma_0^2-1)^{1/2}} > 1 \quad \Rightarrow \quad \frac{R_c}{d} > 3.2$$



Parapotential flow model for pinched electron diode

- In the pinched-beam diode, electrons move radially inward through regions of varying $\boldsymbol{E}$ and $\boldsymbol{B}$. For laminar flow, $\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}$ must hold at all radii. The equation also implies that the electrons move perpendicular to electric field lines; therefore, the orbits lie on lines of constant electrostatic potential. This fact motivates the term parapotential - the prefix para means in the same direction. Self-consistent crossed field equilibria with laminar electron orbits are called Brillouin flow solutions.

- The expression for the total saturated parapotential current in a pinched electron beam diode is:

$$I_0 = \left[\frac{2\pi m_e c}{e\mu_0}\right]\frac{R_c}{d}\gamma_0 \ln\left[\gamma_0 + (\gamma_0 - 1)^{1/2}\right]$$

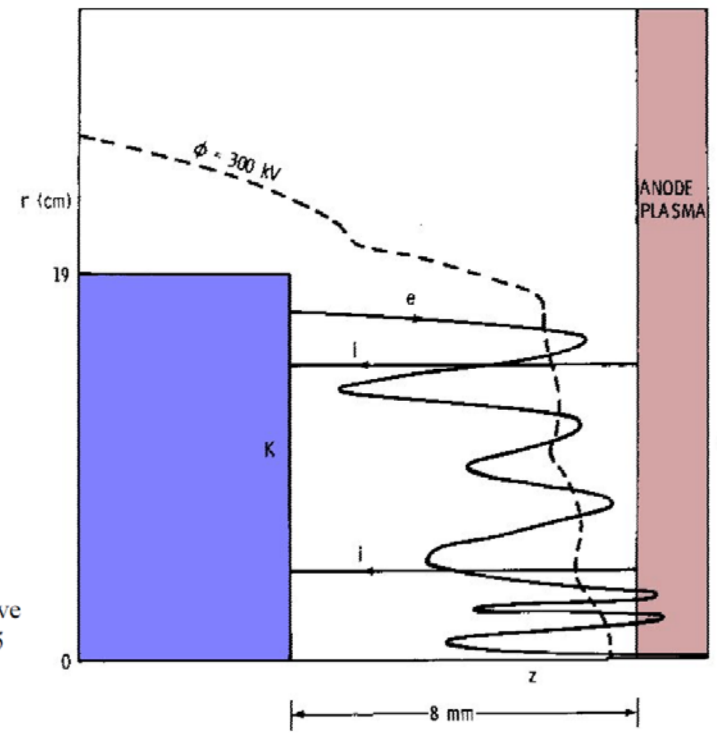


Figure 8.7. Particle-in-cell computer simulation of a pinched beam diode. Figure shows a representative electron orbit, ion orbits and a line of constant electrostatic potential; 500 kV applied voltage, $I_e = 285$ kA and $I_i = 300$ kA. (Courtesy, J. Poukey, Sandia National Laboratories).

Electron diodes with strong applied magnetic fields

- We can circumvent electron beam pinching by applying a magnetic field in the direction of electron flow. The net magnetic field forces electrons to follow spiral orbits directly across the acceleration gap.
- A rough criterion for pinch suppression is that the axial field B_0 should be comparable to or greater than the beam-generated field on the envelope B_s:

$$B_0 \geq B_s = \frac{\mu_0 I_0}{2\pi R}$$

- The electrons leave the cathode parallel to z and enter a region of magnetic field inclined at an angle θ_f:

$$\theta_f \leq \tan^{-1}\left(\frac{B_s}{B_0}\right) = \tan^{-1}\left(\frac{\mu_0 I_0}{2\pi R B_0}\right)$$

- Electrons emerging at the anode have a spread in angle between the limits $\pm 2\theta_f$.

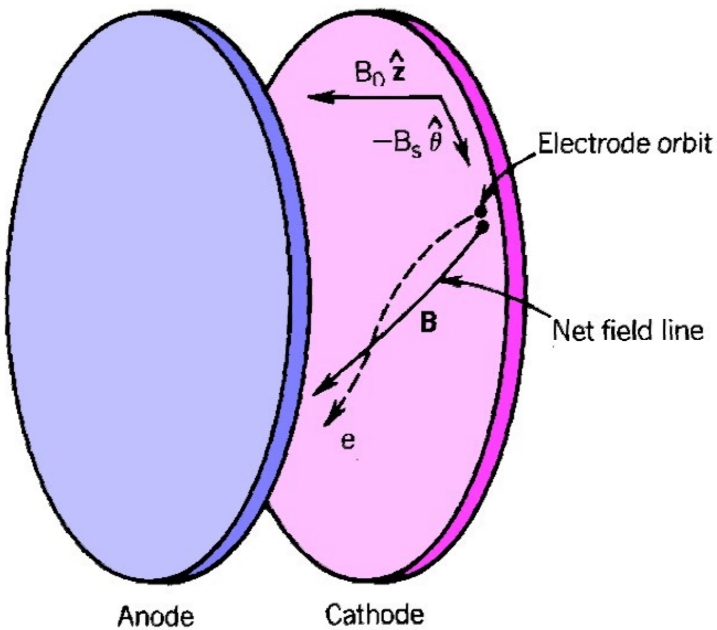
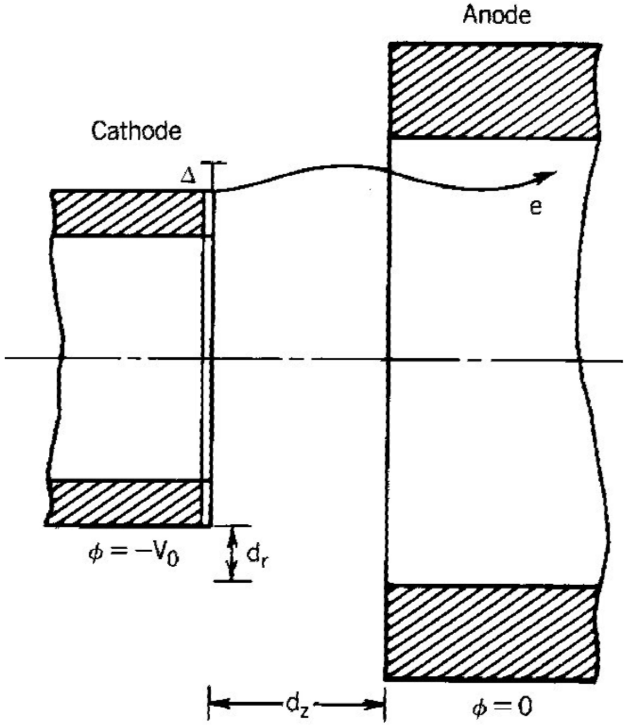


Figure 8.8. Motion of electrons near the edge of a high-current electron diode with a beam-generated toroidal field B_s and an applied solenoidal field B_o.

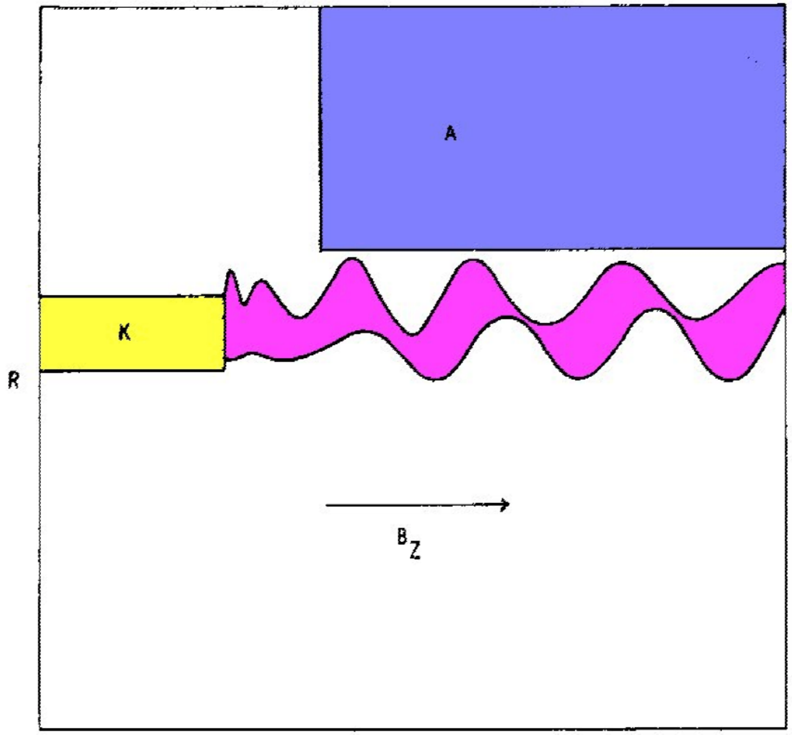
Foilless electron diodes

- With an applied axial magnetic field, it is possible to generate high-current beams that do not pass through the anode. Devices based on this principle are called foilless electron diodes.
- In this geometry, the axial component of electric field between the cathode and a ring anode accelerates the electrons while the axial magnetic field confines them so that they miss the anode and continue into a transport region.



$$E_r \sim V_0/d_r$$

$$\Delta \sim \frac{2\gamma m_e E_r}{e B_0^2}$$



Magnetic insulation of high-power transmission lines

- The magnetic fields from the flow of current to the load can inhibit a line short circuit from electron leakage. A strong magnetic field can insulate the line – breakdown electrons follow drift orbits toward the load instead of crossing the vacuum gap.
- Magnetically-insulated lines transport high-power density partly because they operate with electric fields well above the breakdown level.
- The saturated axial parapotential current between the coaxial cylinders is:

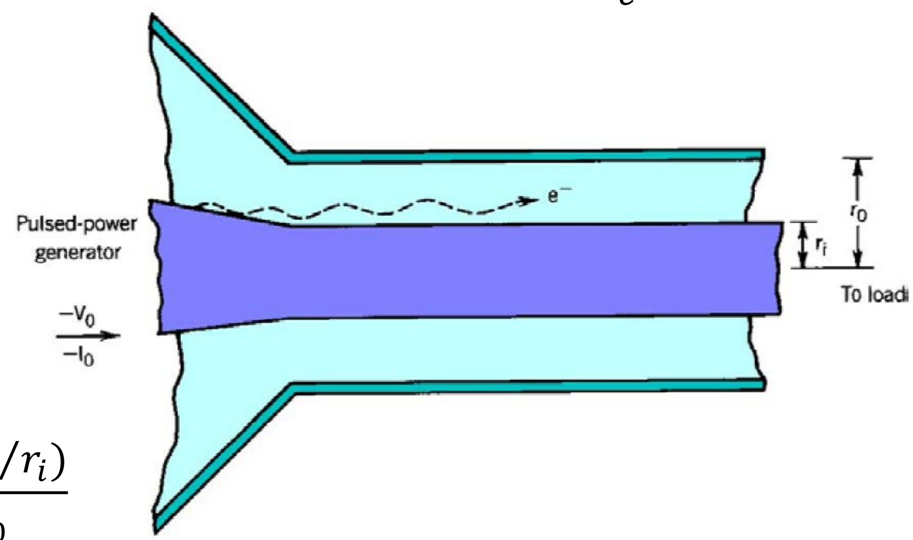
$$I_0 = \left[\frac{1}{\ln(r_o/r_i)}\right]\left[\frac{2\pi m_e c}{e\mu_0}\right]\gamma_0 \ln\left[\gamma_0 + (\gamma_0^2 - 1)^{1/2}\right] \qquad \gamma_0 = 1 + \frac{eV_0}{m_e c^2}$$

- The impedance of a transmission line:

$$Z_m = \frac{V_0}{I_0} = \left[\frac{m_e c^2}{e}\right]\frac{\gamma_0 - 1}{I_0}$$

$$Z_m = Z_0\left[\frac{\gamma_0 - 1}{\gamma_0 \ln\left[\gamma_0 + (\gamma_0^2 - 1)^{1/2}\right]}\right]$$

Impedance of a conventional line: $Z_0 = \dfrac{\mu_0 \ln(r_o/r_i)}{2\pi\epsilon_0}$

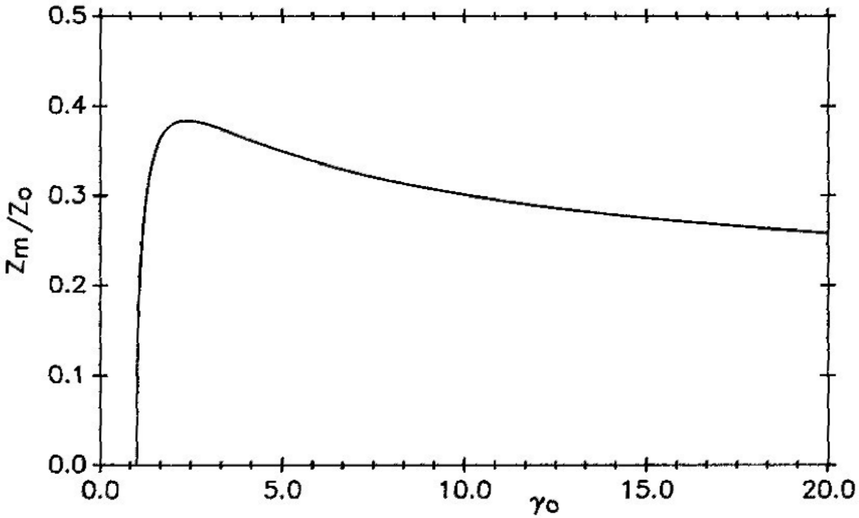


Magnetic insulation of high-power transmission lines

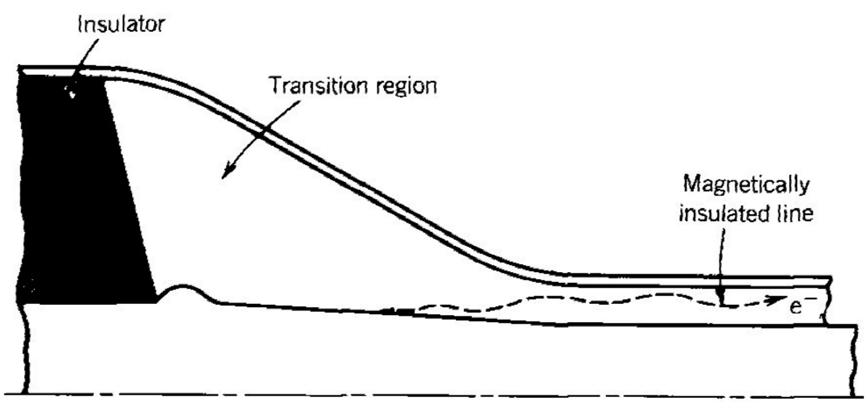
- Since Z_m is always less than Z_0, a magnetically-insulated line carries higher energy density than a conventional line. Magnetically-insulated lines transport high-power density partly due to a lower impedance than conventional one.

$$Z_m = Z_0 \left[\frac{\gamma_0 - 1}{\gamma_0 \ln[\gamma_0 + (\gamma_0^2 - 1)^{1/2}]} \right]$$

Correction factor to account for free drifting electrons

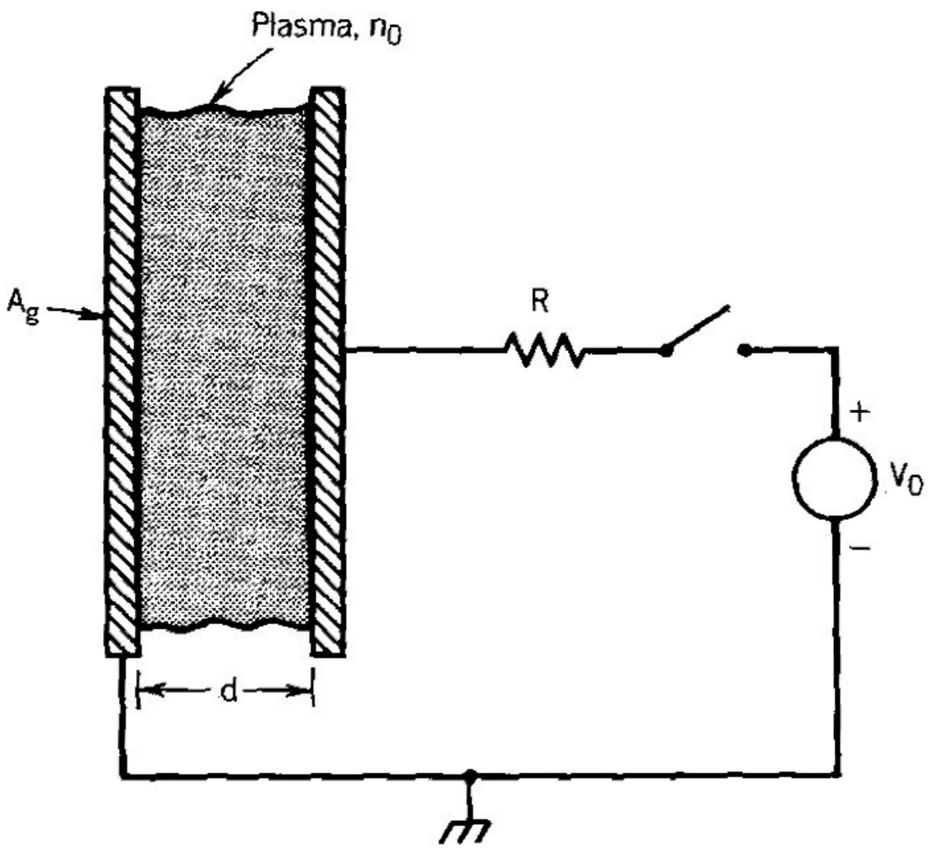


- The electric field is low near the interface and increases along the transition to the magnetically-insulated line. At some point, electron emission begins. The electrons experience a rising electric field as they drift downstream. The combination of the rising electric field and emission of more electrons pushes the drifting electrons towards the anode.

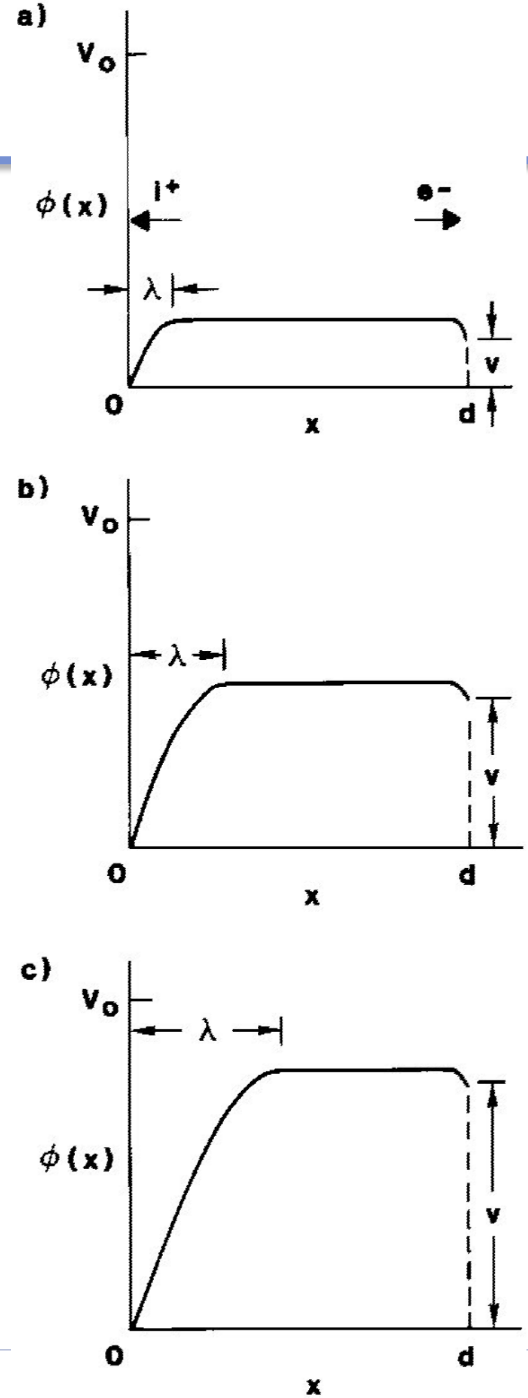


Plasma erosion: depletion of plasma by a pulsed electric field

- The term plasma erosion signifies the removal of a plasma from a gap by a pulsed electric field. In response to the field, electrons and ions flow to opposite electrodes. With no replenishment of particles, the field ultimately clears the gap.
- Ion diodes may operate at very high current densities exceeding 1 kA/cm^2. One approach to supply ions is to fill the diode gap with a high density plasma before the voltage pulse. The stored ions can support high flux for a short pulse.
- A further advantage of plasma prefill is that the ion diode exhibits a rising impedance during the voltage pulse – the consequent ramped voltage waveform may be useful for power multiplication by longitudinal ion bunching.



Plasma erosion: simple model



- Just after switching ($t = 0^+$), the source voltage appears across the series resistor because current flows easily across the gap. The initial circuit current is

$$I_0 = I(t = 0^+) = V_0/R$$

- As ions leave the region near the cathode, a vacuum region (denoted as λ) of increasing width forms. The gap voltage, $v(t)$, rises to maintain ion flow across the growing vacuum sheath. The process continues until all plasma ions leave the gap.

- Quasi-static Child law is satisfied for ion current:

$$J_i(t) \cong \frac{4}{9}\epsilon_0 \left(\frac{2e}{m_i}\right)^{1/2} \frac{v(t)^{3/2}}{\lambda(t)^2}$$

- The total current over an erosion gap of area A_g is

$$i(t) = i_i + i_e = 2A_g J_i(t) = 2A_g \frac{4}{9}\epsilon_0 \left(\frac{2e}{m_i}\right)^{1/2} \frac{v(t)^{3/2}}{\lambda(t)^2}$$

Plasma erosion: simple model

- The gap voltage is related to the circuit current by:

$$v(t) = V_0 - i(t)R$$

- The conservation of charge:

$$i(t) = \frac{\Delta Q}{\Delta t} = \frac{2en_0 A_g \Delta\lambda}{\Delta t} = 2en_0 A_g \frac{d\lambda}{dt}$$

- Normalization:

$$V = \frac{v}{V_0}, \qquad I = \frac{i}{I_0}, \qquad \Lambda = \frac{\lambda}{d}, \qquad \tau = \frac{t}{en_0 d A_g / I_0}$$

- The dimensionless equations:

$$I = \frac{V^{3/2}}{(I_0/I_f)\Lambda^2}, \qquad V = 1 - I, \qquad I = 2\frac{d\Lambda}{d\tau}$$

- The quantity I_f is the current that flows in the gap just before depletion of the plasma, which equals twice the SCL ion current:

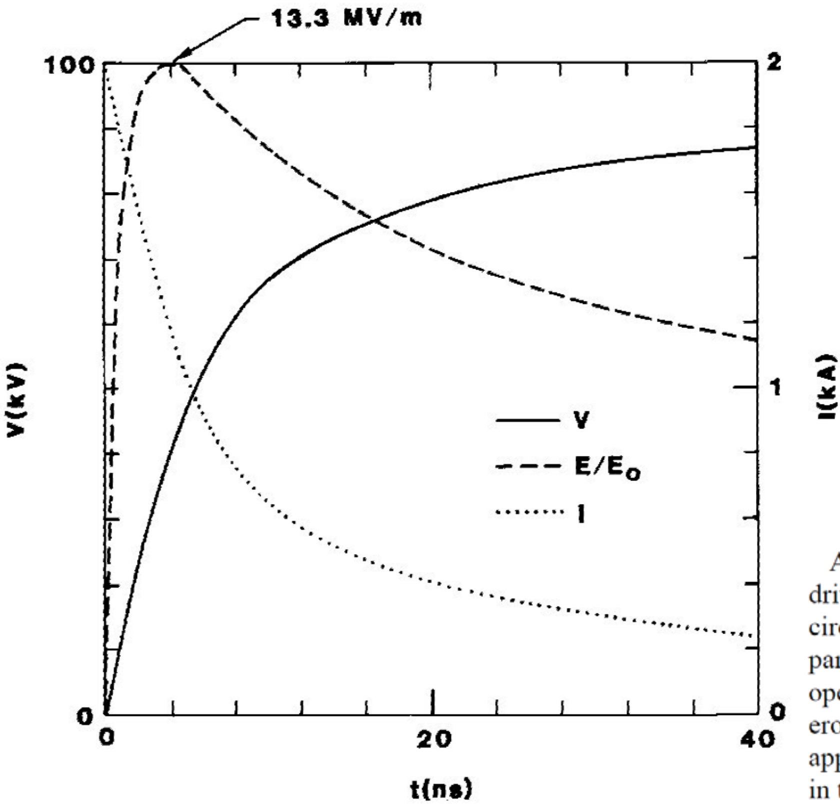
$$I_f = 2A_g \left(\frac{4\epsilon_0}{9}\right)\left(\frac{2e}{m_i}\right)^{1/2} \frac{V_0^{3/2}}{d^2}$$

Plasma erosion: simple model

- By combining normalized equations, we obtain the final form:

$$\frac{d\Lambda}{d\tau} = \frac{[1 - 2(d\Lambda/d\tau)]^{3/2}}{(I_0/I_f)\Lambda^2}$$

- Initial conditions: $V(0) \approx 0,$ $I(0) \approx 0,$ $\Lambda(0) = 0,$ $d\Lambda/d\tau = 1/2$



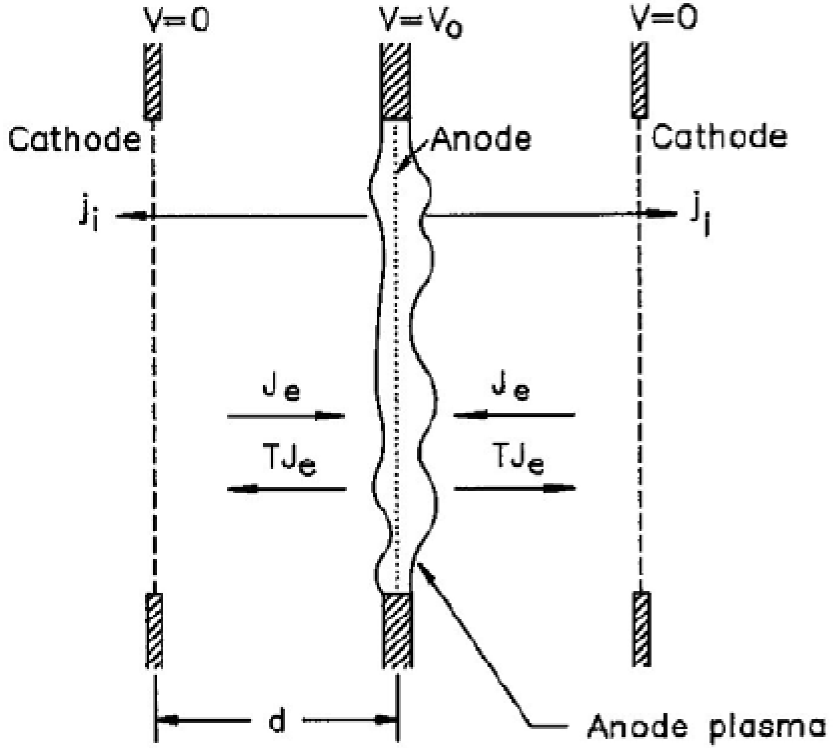
A specific example illustrates some implications of the results. Suppose a plasma gap and driving circuit have the following properties: V_o = 100 kV, I_o = 2 kA, and d = 0.05 m. The gap is circular with diameter 0.15 m – it carries an initial current density of 11 × 10^4 A/m^2. The parameters imply that cathode electric field for the example. From Figure 8.17*c* we find an opening time of only 20 ns, while the clearing time is over 1 μs. The example shows that plasma erosion can give fast switching times for isolated pulses, but it is not useful for high frequency applications. Analysis of the time dependent voltage and current shows that the energy deposited in the negative electrode over the clearing time is small, less than 50 mJ/cm^2. The electric field

Reflex triode

- The fundamental problem for the generation of high-current ion beams is energy loss to electron flow. In a one-dimensional diode, the Child law implies that the electric field to generate an ion beam is much higher than the field to generate an equal current density of electrons.
- In unlimited sources of both ions and electrons, the ion flow is smaller than the counter-streaming electron flow by a factor:

$$\frac{j_i}{j_e} \cong \sqrt{\frac{m_e}{m_i}} \sim 0.0233 \text{ for proton}$$

- The reflex triode solves some of the problems of space-charge flow at high electric field.
- Two cathodes are placed symmetrically a distance d from a central anode. The cathode surfaces are grids to allow extraction of ions. The anode is also a grid with a high geometric transparency factor T.



Reflex triode: simple model

- Assumptions:
  - The anode and cathodes supply unlimited fluxes of ions and electrons. As a result, the electric field equals zero on all electrode surfaces.
  - Particles move only in the z direction.
  - An electron that strikes the anode grid is absorbed.
  - Particle motion is nonrelativistic.
  - The system is in equilibrium – the electron and ion fluxes are uniform in triode gaps.
- The space-charge solution in a gap of the reflex triode is identical that for bipolar flow. Hence, the Langmuir condition states:

$$J_e(1+T) = j_i(m_i/m_e)^{1/2}$$

$$\alpha = \frac{j_i}{j_e}\sqrt{\frac{m_i}{m_e}} = 1$$

- The current density of electrons absorbed at the anode is:

$$j_e = J_e(1-T)$$

- The figure of merit for a triode is the ratio of the net ion current exiting the device to the electron current lost on the anode:

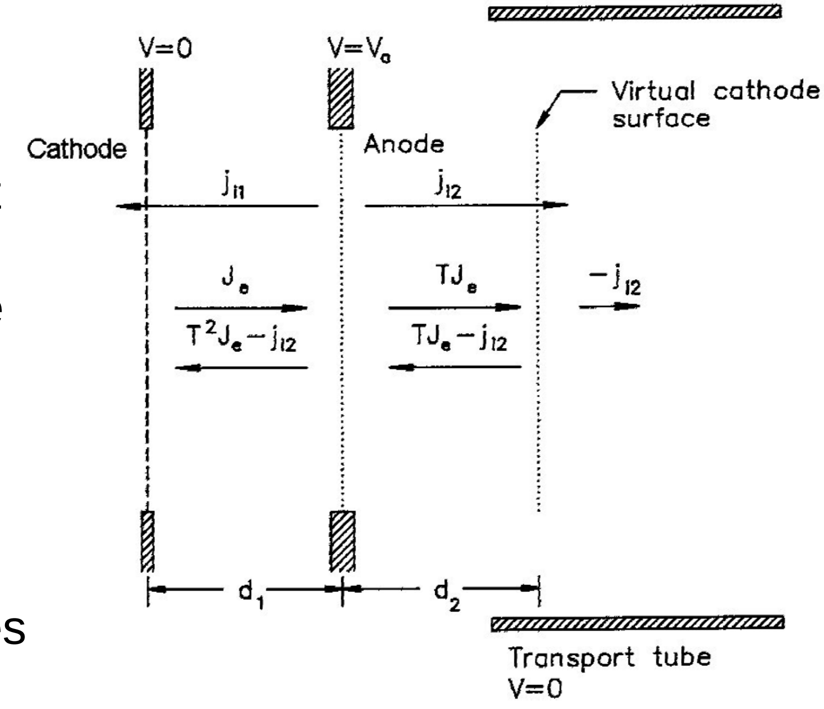
$$\frac{j_i}{j_e} = \sqrt{\frac{m_e}{m_i}}\frac{(1+T)}{(1-T)}$$

$$j_i/j_e = 0.91 \text{ for } T = 0.95$$

Reflex triode with extraction through a virtual cathode

- The ion current densities on both sides equal the bipolar flow expression with the appropriate gap spacing, d_1 or d_2. The ratio of ion current density on the virtual cathode side to that on the real cathode side is:

$$\frac{j_{i2}}{j_{i1}} = \left(\frac{d_1}{d_2}\right)^2$$



- A small fraction of the electron flux arriving at the virtual cathode continues in the forward direction to neutralize the extracted ions. The current density of lost electrons equals j_{i2}.
- The bulk of electrons arriving at the virtual cathode is reflected. The return electron current density on the right hand side equals $TJ_e - j_{i2}$. A current density $T(TJ_e - j_{i2})$ passes back through the anode mesh and returns to the real cathode. Langmuir conditions state:

(Real cathode)

$$J_e + T(TJ_e - j_{i2}) = j_{i1}(m_i/m_e)^{1/2}$$

(Virtual cathode)

$$2TJ_e - j_{i2} = j_{i2}(m_i/m_e)^{1/2}$$

Reflex triode with extraction through a virtual cathode

- We can obtain the ratios of ion current densities and gap widths on each side:

$$\frac{j_{i2}}{j_{i1}} = \frac{2T}{\left(1+\sqrt{\frac{m_e}{m_i}}\right)(1+T^2) - 2\sqrt{\frac{m_e}{m_i}}T^2}$$

$$\frac{d_2}{d_1} = \left[\frac{\left(1+\sqrt{\frac{m_e}{m_i}}\right)(1+T^2) - 2\sqrt{\frac{m_e}{m_i}}T^2}{2T}\right]^{1/2}$$

- The current density of electrons absorbed at the anode is:

$$j_e = J_e(1-T) + (TJ_e - j_{i2})(1-T) = J_e(1-T^2) - j_{i2}(1-T)$$

- The figure of merit can be defined as the ratio of the ion current through the virtual cathode to the total electron current from the real cathode to the anode:

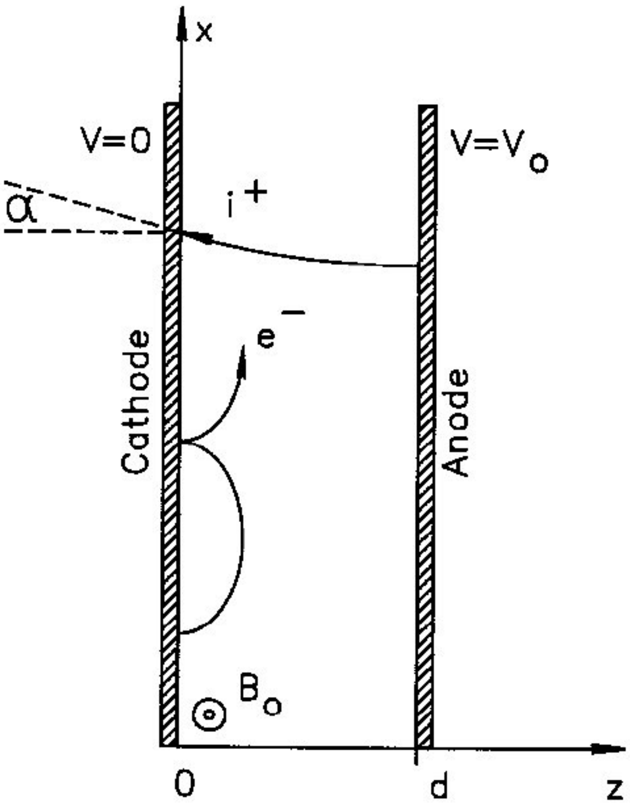
$$\frac{j_{i2}}{j_e} = \frac{2T}{(1-T)^2 + (m_i/m_e)^{1/2}(1-T^2)}$$

$j_{i2}/j_e = 0.45$ for $T = 0.95$

Magnetically-insulated ion diode

- The generation of high-intensity ion beams depends on the suppression of electron flow in regions of strong electric fields. The magnetically-insulated diode uses a magnetic field perpendicular to the accelerating electric field to stop electrons while allowing ions to pass with a small deflection.

$$B_{crit} = \frac{m_0 c}{ed}\left[\frac{2eV_0}{m_0 c^2} + \left(\frac{eV_0}{m_0 c^2}\right)^2\right]^{1/2} = B^*\left[1 + \frac{eV_0}{2m_0 c^2}\right]^{1/2}$$



- The equation for transverse ion motion is:

$$m_i \frac{dv_x}{dt} = -ev_z B_y(z) \quad \Rightarrow \quad \frac{dv_x}{dz} \cong -\frac{eB_y(z)}{m_i}$$

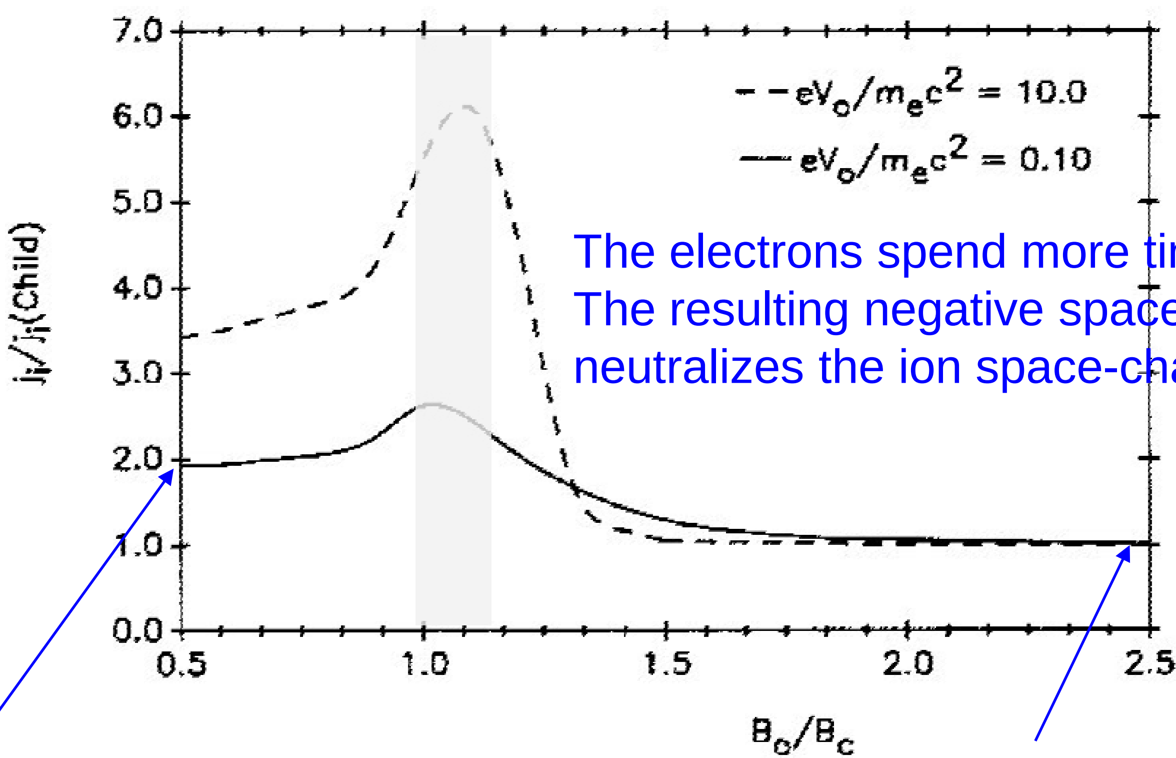
$$v_x(0) = \frac{e}{m_i}\int_0^d B_y(z)dz = \frac{eB_0 d}{m_i}$$

- The exit angle of ions emitted normal to the anode:

$$\alpha \cong \frac{v_x(0)}{v_z(0)} = \frac{\frac{eB_0 d}{m_i}}{\sqrt{\frac{2eV_0}{m_i}}} = \left(\frac{B_0}{B^*}\right)\left(\frac{m_e}{m_i}\right)^{1/2}$$

Ion flow enhancement in magnetically insulated diodes

- Numerical simulation showed that ion current density exceeds the bipolar flow limit just above the critical insulating field ($B_0/B_{crit} \geq 1$).



The electrons spend more time near the anode: The resulting negative space-charge partially neutralizes the ion space-charge

No magnetic insulation: Nonrelativistic bipolar flow (~1.86)

High magnetic field confines the electrons close to the cathode: Little effect on ion flow