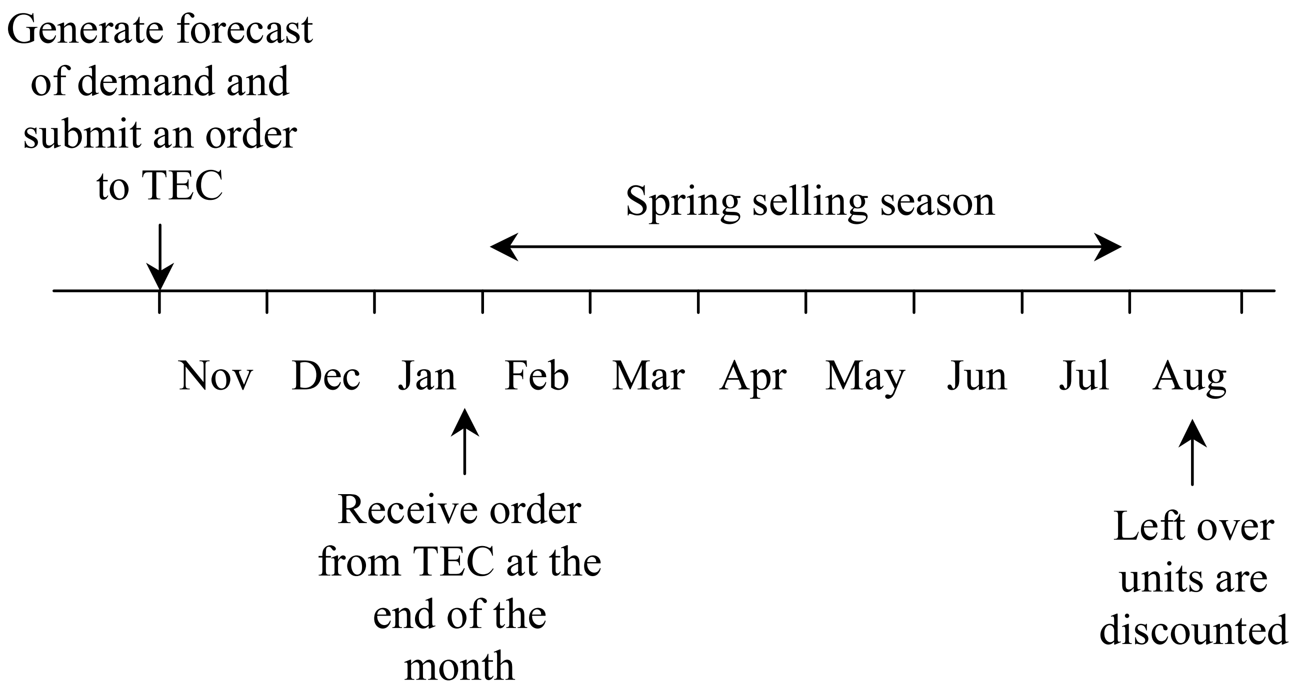
Chapter 14. Betting on Uncertain Demand: The Newsvendor Model

Hammer 3/2 timeline



- Marketing’s forecast for sales is 3200 units.

- Selling Price=$190, Purchase Price from TEC =$110
- Discount Price at the end of the season (Salvage Value) =$90

Newsvendor model implementation steps

- Generate a demand model:
  - Determine a distribution function that accurately reflects the possible demand outcomes, such as a normal distribution function.

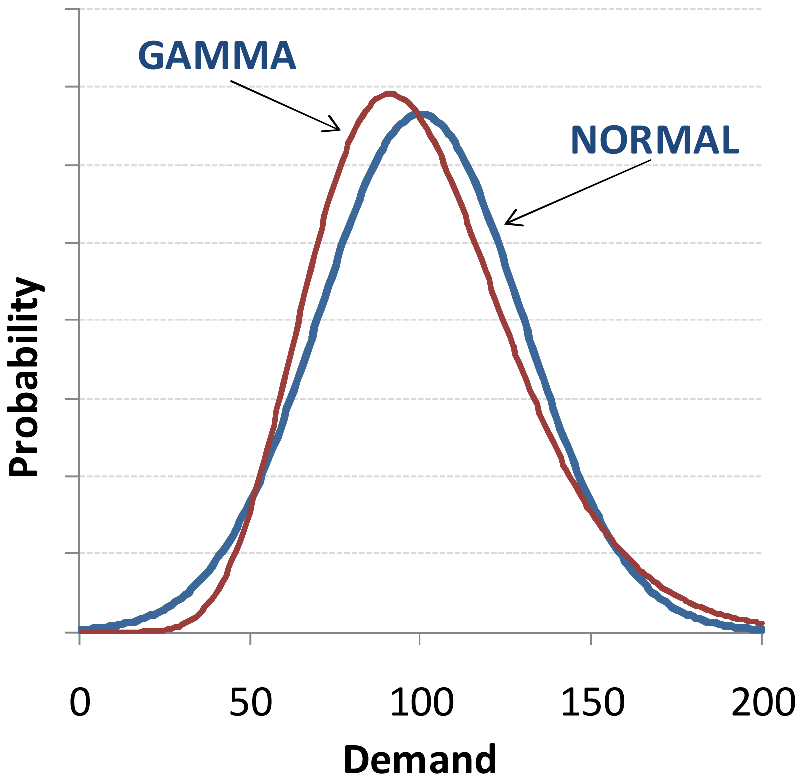
- Gather economic inputs:
  - Selling price, production/procurement cost, salvage value of inventory
  - Negative salvage value is possible!

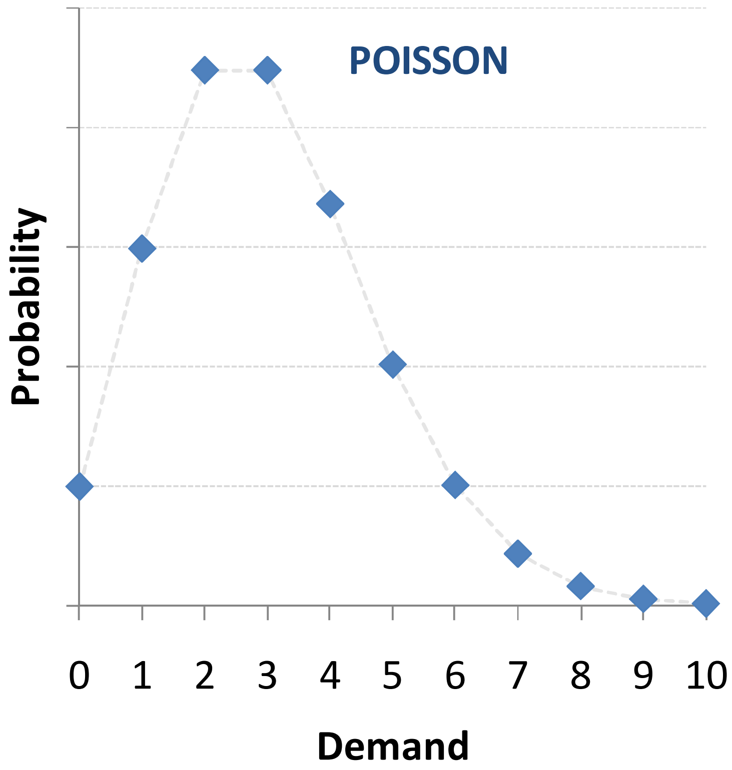
- Choose an objective:
  - e.g. maximize expected profit or satisfy an in-stock probability.

- Choose a quantity to order.

What is a demand model?

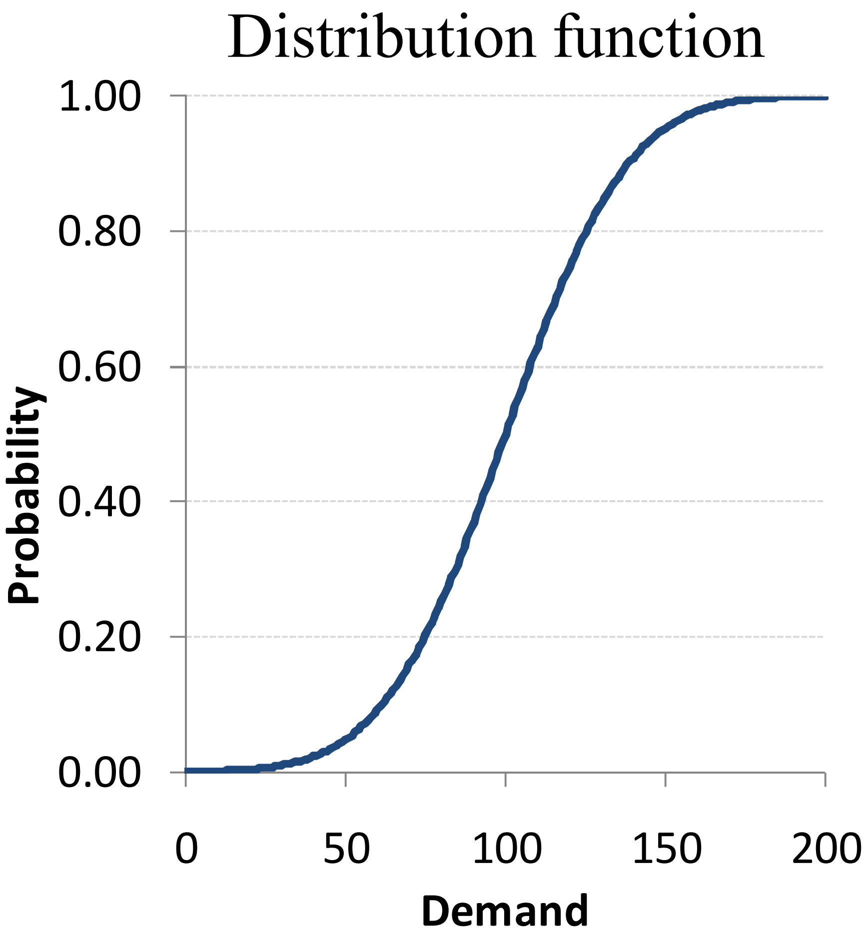
- A demand model specifies what demand outcomes are possible and the probability of these outcomes.
- Traditional distributions from statistics can be used as demand models:
  - e.g., the normal, gamma, Poisson distributions

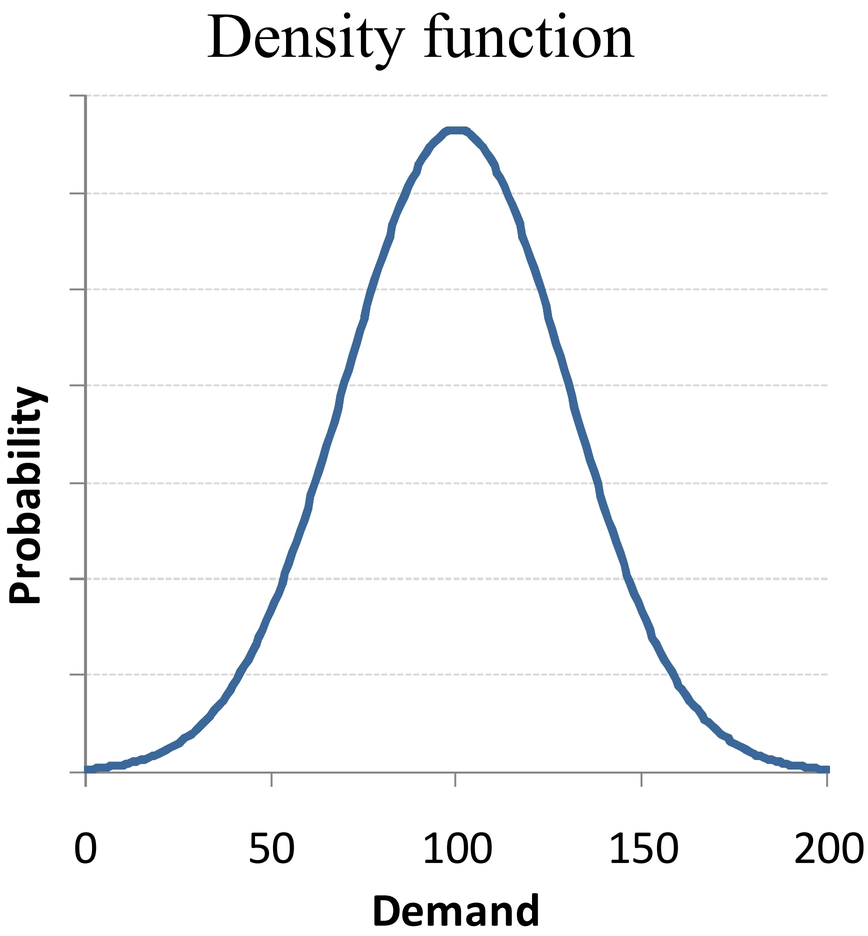




Distribution and density functions

- Two ways to describe the same demand model:





Empirical distribution function of forecast accuracy

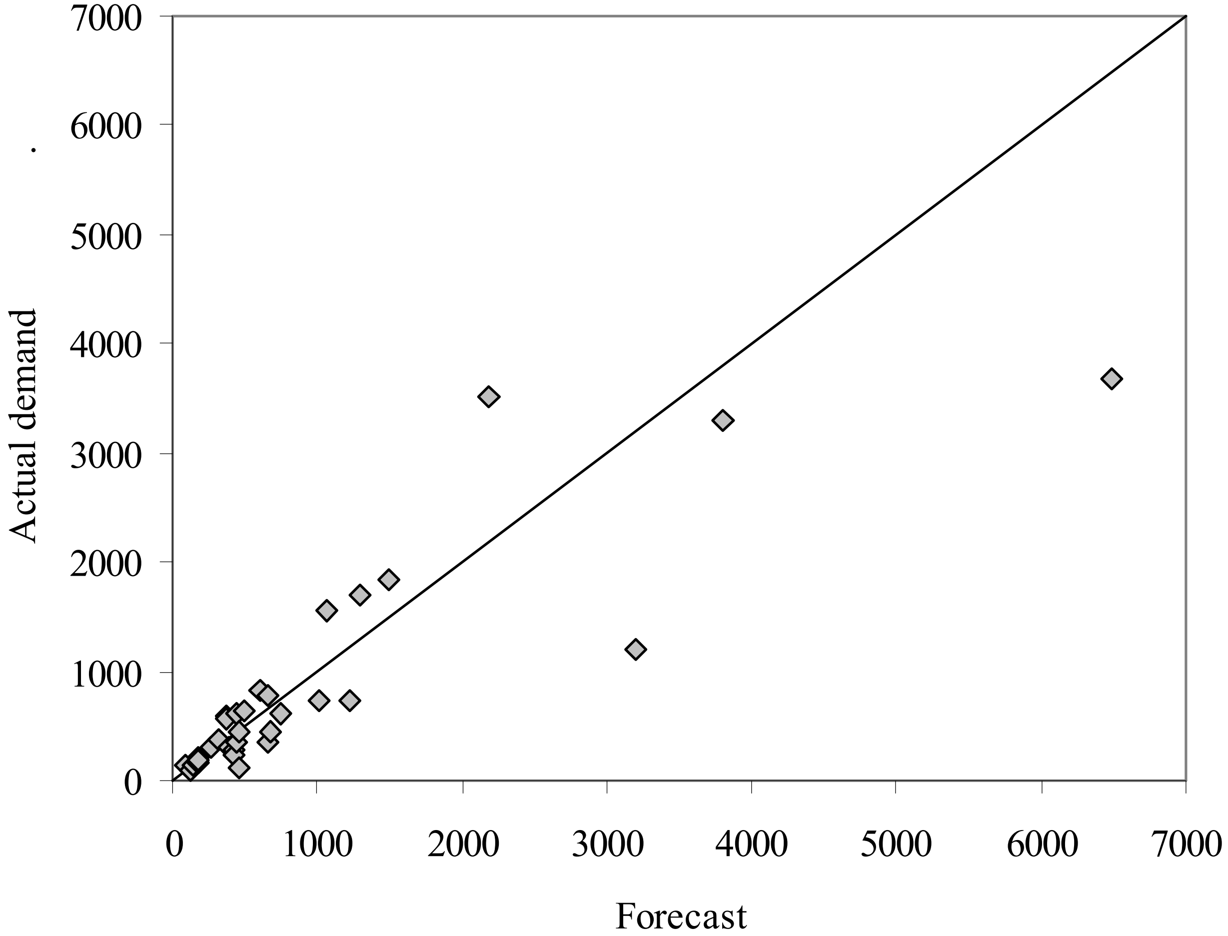
- Start by evaluating the actual to forecast ratio (the A/F ratio) from past observations.

Product description	Forecast	Actual demand	Error*	A/F Ratio**
JR ZEN FL 3/2	90	140	-50	1.56
EPIC 5/3 W/HD	120	83	37	0.69
JR ZEN 3/2	140	143	-3	1.02
WMS ZEN-ZIP 4/3	170	163	7	0.96
HEATWAVE 3/2	170	212	-42	1.25
JR EPIC 3/2	180	175	5	0.97
WMS ZEN 3/2	180	195	-15	1.08
ZEN-ZIP 5/4/3 W/HOOD	270	317	-47	1.17
WMS EPIC 5/3 W/HD	320	369	-49	1.15
EVO 3/2	380	587	-207	1.54
JR EPIC 4/3	380	571	-191	1.50
WMS EPIC 2MM FULL	390	311	79	0.80
HEATWAVE 4/3	430	274	156	0.64
ZEN 4/3	430	239	191	0.56
EVO 4/3	440	623	-183	1.42
ZEN FL 3/2	450	365	85	0.81
HEAT 4/3	460	450	10	0.98
ZEN-ZIP 2MM FULL	470	116	354	0.25
HEAT 3/2	500	635	-135	1.27
WMS EPIC 3/2	610	830	-220	1.36
WMS ELITE 3/2	650	364	286	0.56
ZEN-ZIP 3/2	660	788	-128	1.19
ZEN 2MM S/S FULL	680	453	227	0.67
EPIC 2MM S/S FULL	740	607	133	0.82
EPIC 4/3	1020	732	288	0.72
WMS EPIC 4/3	1060	1552	-492	1.46
JR HAMMER 3/2	1220	721	499	0.59
HAMMER 3/2	1300	1696	-396	1.30
HAMMER S/S FULL	1490	1832	-342	1.23
EPIC 3/2	2190	3504	-1314	1.60
ZEN 3/2	3190	1195	1995	0.37
ZEN-ZIP 4/3	3810	3289	521	0.86
WMS HAMMER 3/2 FULL	6490	3673	2817	0.57

* Error = Forecast - Actual demand

** A/F Ratio = Actual demand divided by Forecast

Historical forecast performance at O'Neill



Forecasts and actual demand for surf wet-suits from the previous season

Using historical A/F ratios to choose a normal distribution for the demand model

- Start with an initial forecast generated from hunches, guesses, etc.
  - O'Neill's initial forecast for the Hammer 3/2 = 3200 units.
- Evaluate the A/F ratios of the historical data:

$$A/F\ ratio = \frac{Actual\ demand}{Forecast}$$

- Set the mean of the normal distribution to

$$Expected\ actual\ demand = Expected\ \ A/F\ ratio \times Forecast$$

- Set the standard deviation of the normal distribution to

$$\begin{aligned} &Standard\ deviation\ of\ actual\ demand = \\ &\qquad Standard\ deviation\ of\ \ A/F\ ratios \times Forecast \end{aligned}$$

O'Neill's Hammer 3/2 normal distribution forecast

Product description	Forecast	Actual demand	Error	A/F Ratio
JR ZEN FL 3/2	90	140	-50	1.5556
EPIC 5/3 W/HD	120	83	37	0.6917
JR ZEN 3/2	140	143	-3	1.0214
WMS ZEN-ZIP 4/3	170	156	14	0.9176
• • •	• • •	• • •	• • •	• • •
ZEN 3/2	3190	1195	1995	0.3746
ZEN-ZIP 4/3	3810	3289	521	0.8633
WMS HAMMER 3/2 FULL	6490	3673	2817	0.5659
Average				0.9975
Standard deviation				0.3690

$$Expected\ actual\ demand = 0.9975 \times 3200 = 3192$$

$$Standard\ deviation\ of\ actual\ demand = 0.369 \times 3200 = 1181$$

- O'Neill can choose a normal distribution with mean 3192 and standard deviation 1181 to represent demand for the Hammer 3/2 during the Spring season.

The Newsvendor Model:

The order quantity that maximizes expected profit

Hammer 3/2 economics

- O'Neill sells each suit for p = $190
- O'Neill purchases each suit from its supplier for c = $110 per suit
- Discounted suits sell for v = $90
  - This is also called the *salvage value*.

- O'Neill has the "too much/too little problem":
  - Order too much and inventory is left over at the end of the season
  - Order too little and sales are lost.

- Recall, our demand model is a normal distribution with mean 3192 and standard deviation 1181.

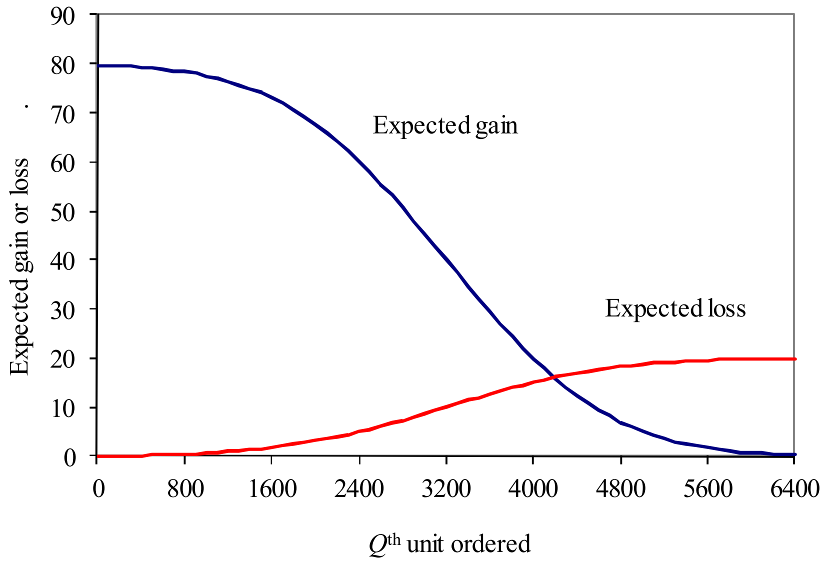
“Too much” and “too little” costs

- **C_o = overage (≡ overstock) cost**
  - The consequence of ordering one more unit than what you would have ordered had you known demand.
    - Suppose you had left over inventory (you over ordered). C_o is the increase in profit you would have enjoyed had you ordered one fewer unit.
  - For the Hammer 3/2 $C_o = Cost - Salvage\ value = c - v$ = 110 – 90 = 20

- **C_u = underage (≡ understock) cost**
  - The consequence of ordering one fewer unit than what you would have ordered had you known demand.
    - Suppose you had lost sales (you under ordered). C_u is the increase in profit you would have enjoyed had you ordered one more unit.
  - For the Hammer 3/2 $C_u = Price - Cost = p - c$ = 190 – 110 = 80

Balancing the risk and benefit of ordering a unit

- Ordering one more unit increases the chance of overage …
  - Expected loss on the Q^{th} unit = C_o x $F(Q)$
  - $F(Q)$ = Distribution function of demand = $Prob\{Demand \leq Q\}$
- … but the benefit/gain of ordering one more unit is the reduction in the chance of underage:
  - Expected gain on the Q^{th} unit = C_u x $(1-F(Q))$



- As more units are ordered, the expected benefit from ordering one unit decreases while the expected loss of ordering one more unit increases.

Maximize expected profit

- To maximize the expected profit of ordering Q units so that the expected loss on the Q^{th} unit equals the expected gain on the Q^{th} unit:

$$C_o \times F(Q) = C_u \times (1 - F(Q))$$

- Rearrange terms in the above equation $\Rightarrow$

$$F(Q) = \frac{C_u}{C_o + C_u} = \frac{p-c}{p-v}$$

- The ratio $C_u \ / \ (C_o + C_u)$ is called the ***critical ratio***.

- Hence, to maximize profit, choose Q such that the probability we satisfy all demand (i.e., demand is Q or lower) equals the critical ratio.

- For the Hammer 3/2 the critical ratio is $\dfrac{C_u}{C_o + C_u} = \dfrac{80}{20+80} = 0.80$

Hammer 3/2's expected profit maximizing order quantity

- The critical ratio is 0.80
- Find the critical ratio <u>inside</u> the *Standard Normal Distribution Function Table:*

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	$\boxed{0.7995}$	$\boxed{0.8023}$	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389

  - If the critical ratio falls between two values in the table, choose the greater *z*-statistic … this is called the ***round-up rule***.
  - Choose *z* = 0.85
- Convert the z-statistic into an order quantity :

$$
\begin{aligned}
Q &= \mu + z \times \sigma \\
&= 3192 + 0.85 \times 1181 = 4196
\end{aligned}
$$

The Newsvendor Model:

Performance Measures

Performance measures of newsvendor model

- For any order quantity we would like to evaluate the following performance measures:
  - *In-stock probability*
    - Probability all demand is satisfied
  - *Stockout probability*
    - Probability some demand is lost
  - *Expected lost sales*
    - The expected number of units by which demand will exceed the order quantity
  - *Expected sales*
    - The expected number of units sold.
  - *Expected left over inventory*
    - The expected number of units left over after demand (but before salvaging)
  - *Expected profit*

In-stock probability

- The in-stock probability is the probability that all demand is satisfied.

- All demand is satisfied if demand is the order quantity, *Q*, or smaller.
  - If Q = 3000, then to satisfy all demand, demand must be 3000 or fewer.

- The distribution function tells us the probability demand is *Q* or smaller!

- Hence, the *In-stock probability* = $F(Q) = \Phi(z)$

Stockout probability = $\left(1 - F(Q)\right)$

Evaluate the in-stock probability

- What is the in-stock probability if the order quantity is Q = 3000?

$$P(D \le Q) = P\left(\frac{D-\mu}{\sigma} \le \frac{Q-\mu}{\sigma}\right) = P\left(z \le \frac{3000-3192}{1181}\right)$$
$$= P(z \le -0.16) = 1 - 0.5636 = 0.4364$$

- *Std. Normal Distribution Function Table* (p460)

- Answer :
  - If 3000 units are ordered, then there is a 43.64% chance that all demand will be satisfied.

Choose Q subject to a minimum in-stock probability

- Suppose we wish to find the order quantity for the Hammer 3/2 that minimizes left over inventory while generating at least a 99% in-stock probability.

- Step 1:

$$P(D \le Q) = P\left(\frac{D-\mu}{\sigma} \le \frac{Q-\mu}{\sigma}\right) = P\left(z \le \frac{Q-3192}{1181}\right) \ge 0.99$$

  - Choose z = 2.33 (p461) to satisfy our in-stock probability constraint.

- Step 2:
  - Convert the z-statistic into an order quantity for the actual demand distribution.
  - Q = μ + z x σ = 3192 + 2.33 x 1181 = 5944

Other measures of service performance

- The *fill rate* is the fraction of demand that can be satisfied from stock:
  - The fill rate is also the probability a randomly chosen customer can purchase a unit (i.e., does not experience a stockout).
  - The fill rate is not the same as in-stock probability!
    - e.g. if Q=100 and demand turns out be 101?

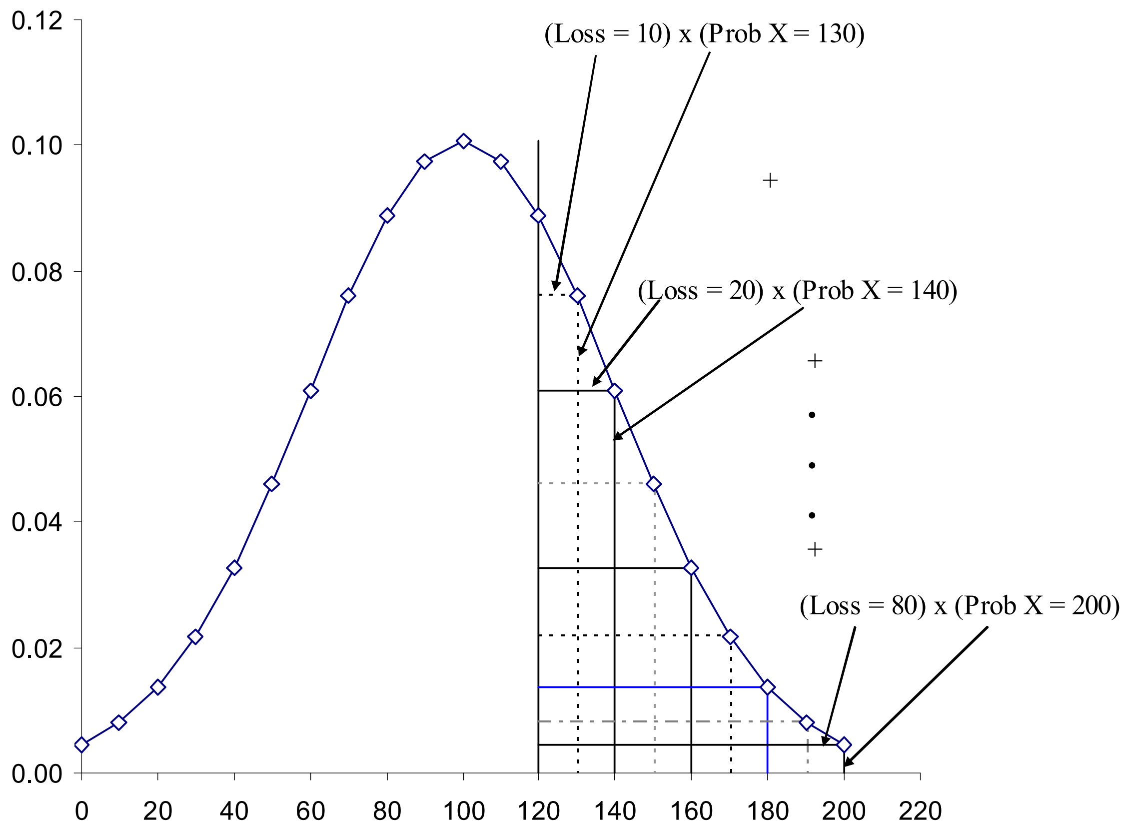
      Fill rate=99%   vs. the company did not satisfy all demand

$$FillRate = \frac{ExpectedSales}{ExpectedDemand}$$

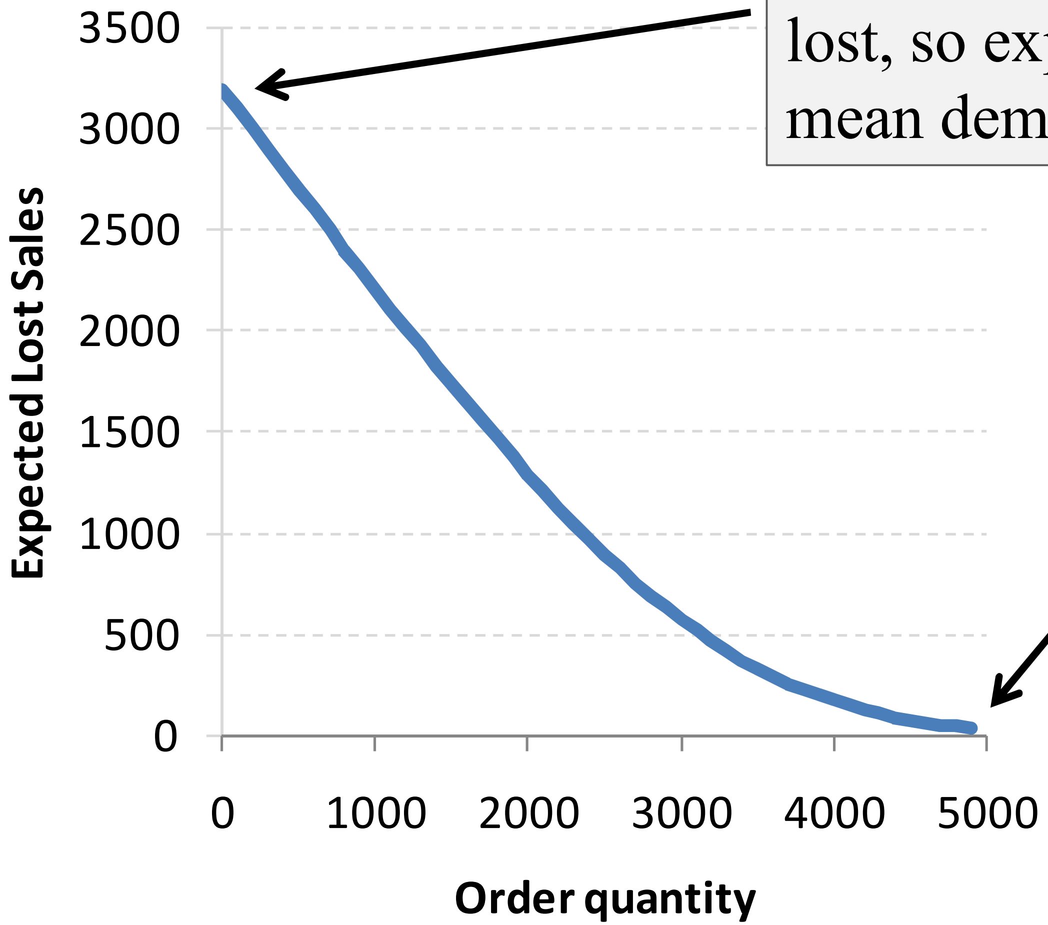
Expected lost sales: graphical explanation

- Suppose demand can be one of the following values {0,10,20,…,190,200}
- Suppose Q = 120
- Let D = actual demand
- What is the expected lost sales?
- If D ≤ Q,
  lost sales = 0
- If D = 130,
  lost sales = D – Q = 10

- Expected lost sales =
  10 x Prob{D = 130} +
  20 x Prob{D = 140} +
  … +
  80 x Prob{D = 200}



Expected lost sales of Hammer 3/2s



If 0 units are ordered, all sales are lost, so expected lost sales equals mean demand, 3192

If 5000 units are ordered, expected lost sales will be nearly zero.

Expected lost sales of Hammer 3/2s with *Q* = 3000

- Suppose O'Neill orders 3000 Hammer 3/2s.
- How many sales will be lost on average?

- To find the answer:
  - Step 1: normalize the order quantity to find its *z*-statistic.

$$z = \frac{Q-\mu}{\sigma} = \frac{3000-3192}{1181} = -0.16$$

  - Step 2: Look up in the *Standard Normal Loss Function Table* the expected lost sales for a standard normal distribution with that *z*-statistic: *L*(-0.16)=0.4840
    - or, in Excel

$$L(z) = Normdist(z,0,1,0) - z*(1 - Normsdist(z))$$

  - Step 3: Evaluate lost sales for the actual normal distribution:

$$Expected\ Lost\ Sales = \sigma \times L(z) = 1181 \times 0.4840 = 572$$

Measures that follow expected lost sales

If they order 3000 Hammer 3/2s, then …

Expected sales = μ - Expected lost sales = 3192 – 572 = 2620

Expected left over Inventory = Q - Expected sales = 3000 – 2620 = 380

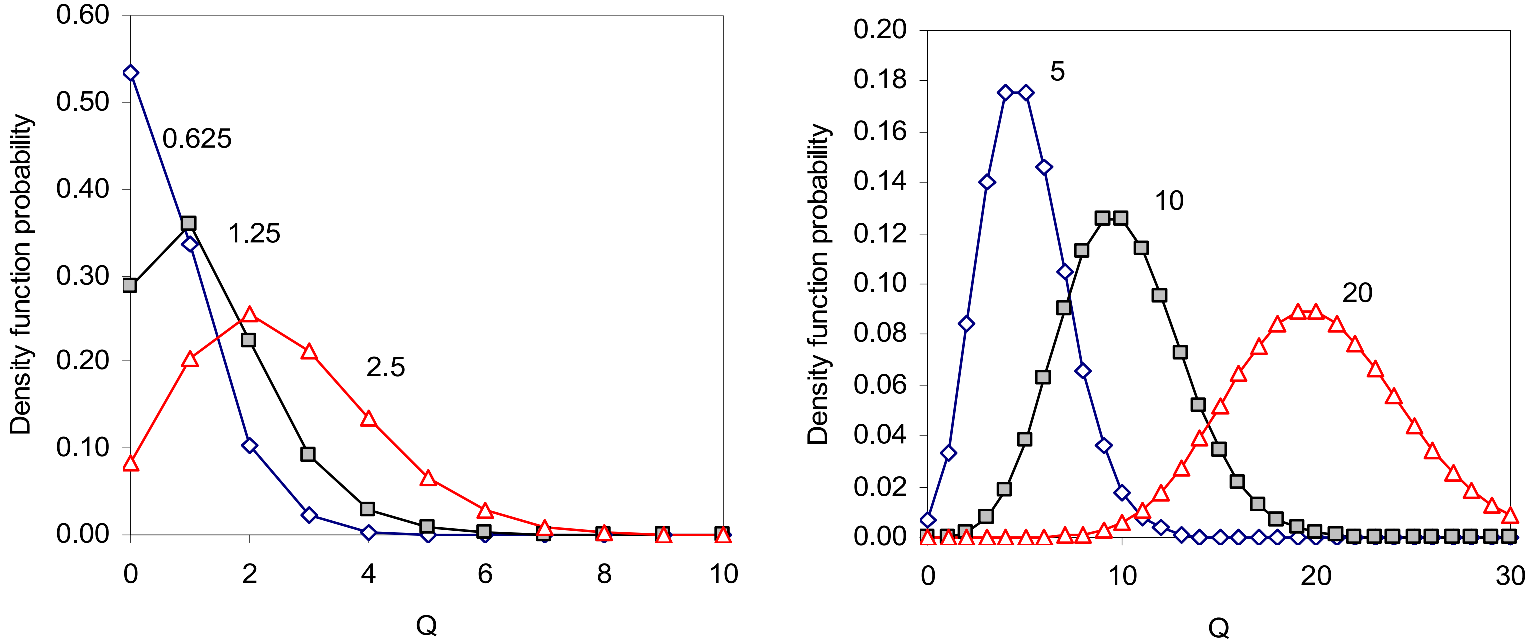
$$\begin{aligned}\text{Expected Profit} &= \left[(\text{Price - Cost}) \times \text{Expected sales}\right] \\ &\quad - \left[(\text{Cost - Salvage value}) \times \text{Expected left over inventory}\right] \\ &= \$80 \times 2620 - \$20 \times 380 \\ &= \$202{,}000\end{aligned}$$

Note: the above equations hold for any demand distribution

Newsvendor analysis with the Poisson distribution

What is the Poisson distribution and what is it good for?

Six Poisson density functions with means 0.625, 1.25, 2.5, 5,10 and 20



- Defined only by its mean (standard deviation = square root(mean))
- Does not always have a “bell” shape, especially for low demand.
- Discrete distribution function: only non-negative integers
- Good for modeling demands with low means (e.g., less than 20)
- If the inter-arrival times of customers are exponentially distributed, then the number of customers that arrive in a given interval of time has a Poisson distribution.

EcoTea Inc

- EcoTea plans to sell a gift basket of Tanzanian teas through its specialty stores during the Christmas season.
- Basket price is \$55, purchase cost is \$32 and after the holiday season left over inventory will be cleared out at \$20.
- Estimated demand at one of its stores is Poisson with mean 4.5.
- One order is made for the season.
- What is the optimal order quantity?
  - C_o=32-20=12; C_u=55-32=23; C_u / (C_u + C_o) = 23 / 35 = 0.6571
  - Use round up rule, so order 5 baskets
- If they order 6 baskets, what is their lost sales?
  - From the loss function table, they can expect to lose 0.32 in sales.

Poisson Distribution Function Table

S	…	Mean 4.25	4.5
0		0.01426	0.01111
1		0.07489	0.06110
2		0.20371	0.17358
3		0.38621	0.34230
4		0.58012	$\boxed{0.53210}$
$\boxed{5}$		0.74494	$\boxed{0.70293}$
6		0.86169	0.83105

Poisson Loss Function Table

S	…	Mean 4.25	4.5
0		4.25000	4.50000
1		3.26426	3.51111
2		2.33915	2.57221
3		1.54286	1.74579
4		0.92907	1.08808
5		0.50919	0.62019
6		0.25413	$\boxed{0.32312}$

Managerial Lessons: Forecasting Process

- It is insufficient to have just a forecast of expected demand.

- When forecasting, it is important to track actual demand.

  ⇒stockout situation: attempt a reasonable estimate of actual demand

  ⇒ actual demand includes potential sales only at the regular price.

  (e.g. sales in the previous season: 1000 units,

  600 of them at the discounted price)

- Keep track of past forecasts and forecast errors ⇒ assess std. of demand

Managerial Lessons: Order Quantity Choice Process

- The profit-maximizing order quantity generally does not equal expected demand.

- Some products may have an overage cost that is larger than the underage cost. ⇒

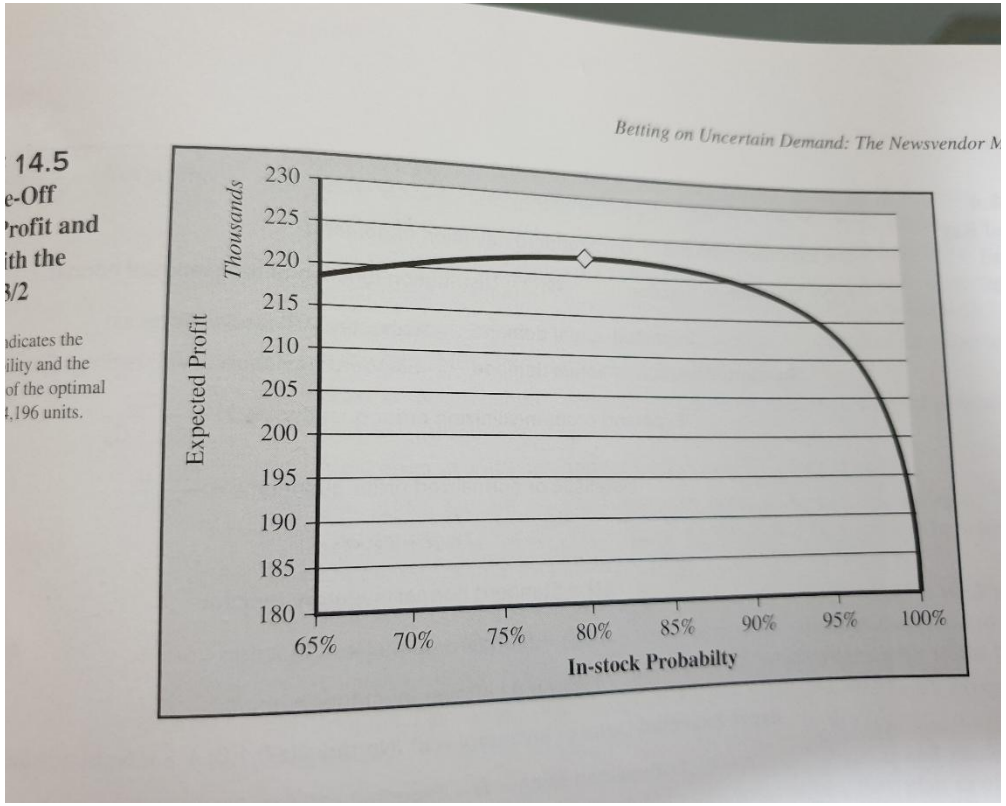
- The order quantity decision should be separated from the forecasting process. Two products with the same mean forecast may have different expected profit-maximizing order quantities ⇒ Why?

Managerial Lessons: Order Quantity Choice Process

- Choosing an order quantity to maximize expected profit is only one possible objective. ⇒ customer service?

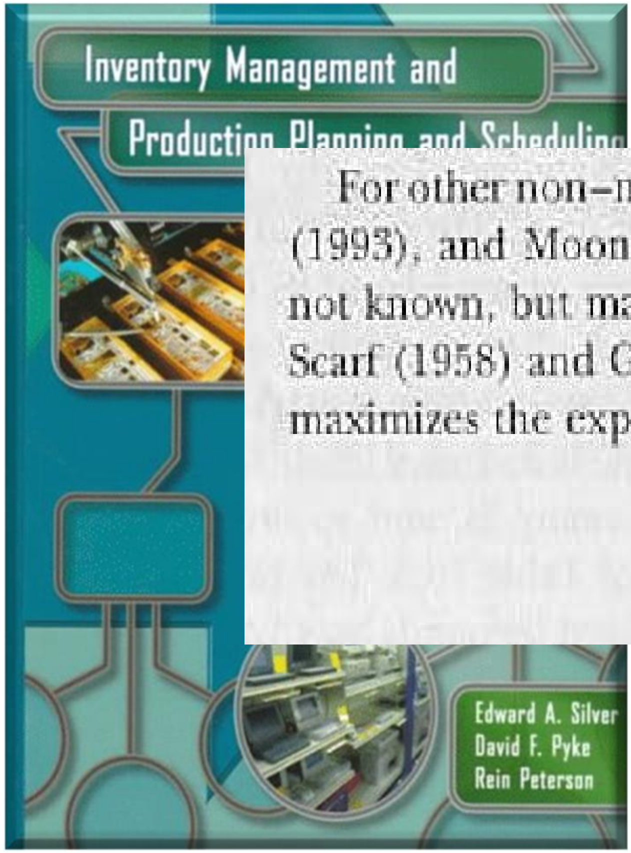
  (e.g. Hammer 3/2 ⇒ in-stock probability is only 80%)

(Q) Raising in-stock probability to 90% or 99%?



Distribution Free Newsvendor Problem (생산관리 세계적 교과서에 소개됨)

The Distribution Free Newsboy Problem : Review and Extensions. G. Gallego and I.K. Moon, Journal of the Operational Research Society (1993. 08), Vol. 44, No. 8, pp. 825-834 ☞ Cited OOO times



For other non-normal demand distributions see Scarf (1958), Gallego and Moon (1993), and Moon and Choi (1995). In particular, if the demand distribution is not known, but managers have some sense of the mean and variance of demand, Scarf (1958) and Gallego and Moon (1993) show that using the following formula maximizes the expected profit against the worst possible distribution of demand:

$$Q^* = \hat{x} + \frac{\sigma_x}{2}\left(\sqrt{\frac{p/v-1}{1-g/v}} - \sqrt{\frac{1-g/v}{p/v-1}}\right) \qquad 10.11$$

Homework: Distribution Free Newsvendor Problem (Due: 11월23일)

1. What is the main difference between the general newsvendor problem and the distribution free newsvendor problem?
2. Why has the paper been cited so many times?
   (h-index: A scholar with an index of *h* has published *h* papers each of which has been cited in other papers at least *h* times.
   It tries to measure both the productivity and impact of the published work. If one has a h-index > 30, he/she is regarded as a prominent scholar.)
3. Explain the expected value of additional information (EVAI).
   If the value is quite small, what should the decision maker do?

논문다운로드: http://scm.snu.ac.kr/publication/publ-in.asp

피인용 검색: https://www.scopus.com/authid/detail.uri?authorId=7101610478

Newsvendor model summary

- The model can be applied to settings in which …
  - There is a single order/production/replenishment opportunity.
  - Demand is uncertain.
  - There is a "too much-too little" challenge:
    - If demand exceeds the order quantity, sales are lost.
    - If demand is less than the order quantity, there is left over inventory.

- Firm must have a demand model that includes an expected demand and uncertainty in that demand.
  - With the normal distribution, uncertainty in demand is captured with the standard deviation parameter.

- At the order quantity that maximizes expected profit, the probability that demand is less than the order quantity equals the critical ratio:
  - The order quantity that maximizes expected profit balances the "too much-too little" costs.

Additional questions to consider…

- Suppose you are making an order quantity decision using the newsvendor model…
  - If your order quantity maximizes expected profit, then what is the in-stock probability?
  - If your forecast uncertainty increases, do you order more or do you order less?
  - At the end of the season, a merchandiser notices that 80% of the items he manages have left over inventory. Another merchandiser notices that only 20% of her items have left over inventory. Who makes better decisions?

Appendix D (The Round-up Rule)

- $f(d_i)$: Probability mass function of demand
- $D \in \{d_1, d_2, \cdots, d_n\}$: Discrete demand (support of the distribution)
- $C_u\,(d_{i+1} - d_i)\big(1 - F(d_i)\big)$ Note that $1 - F(d_i) = \sum_{j=i}^{n} F(d_j)$
  - The expected gain of increasing order from d_i to d_{i+1}
- $C_o\,(d_{i+1} - d_i)F(d_i)$
  - The expected loss of increasing order from d_i to d_{i+1}
- Condition for increasing order
  - $\frac{C_u}{C_o + C_u} \geq F(d_i)$ (derived from $C_u\,(d_{i+1} - d_i)\big(1 - F(d_i)\big) \geq C_o\,(d_{i+1} - d_i)F(d_i)$)
- d_{i+1} is optimal if $F(d_i) \leq \frac{C_u}{C_o + C_u} \leq F(d_{i+1})$

Appendix D (The Round-up Rule)

- Y: A value chosen by the round-up rule
- X: The largest value in the loss table satisfying $X < Y$
- Round-up rule (rule of choosing Y instead of X)
  - Discrete demand
    - Equivalent with ordering one more unit
    - Guarantees optimality
  - Continuous demand (approximated when demand is large)
    - Not equivalent with ordering one more unit
    - Does not guarantee optimality, but close to the optimal solution
    - e.g.) Textbook p. 298

      Critical ratio = 0.8

      $z = 0.85$, $\mathrm{Q} = 4{,}196$ (round-up rule)

      $z = 0.84$, $\mathrm{Q} = 4{,}184$ (selecting closer z)

      Increased by 12 units

Example

- Demand: $D \sim Poisson(1)$
- $C_u = 1, C_0 = 0.21$
- $\frac{C_u}{C_0+C_u} = 0.83$
- $F(1) = 0.74,\ F(2) = 0.92$

- Expected profit of ordering 1 unit
  - Expected lost sales: 0.36788
  - Expected sales: 0.63212 (μ – Expected lost sales)
  - Expected leftover inventory: 0.36788 (Q – Expected sales)
  - Expected profit: 0.55487 ($C_u \cdot$ Expected sales – $C_0 \cdot$ Expected leftover inventory

- Expected profit of ordering 2 units
  - Expected profit: 0.6646

- About 20% higher!